Modeling and analysis of a cliff-mounted piezoelectric sea-wave energy absorption system

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Abstract. Sea waves induce significant pressures on coastal surfaces, especially on rocky vertical cliffs or breakwater structures (Peregrine 2003). In the present work, this hydrodynamic pressure is considered as the excitation acting on a piezoelectric material sheet, installed on a vertical cliff, and connected to an external electric circuit (on land). The whole hydro/piezo/electric system is modeled in the context of linear wave theory. The piezoelectric elements are assumed to be small plates, possibly of stack configuration, under a specific wiring. They are connected with an external circuit, modeled by a complex impedance, as usually happens in preliminary studies (Liang and Liao 2011). The piezoelectric elements are subjected to thickness-mode vibrations under the influence of incident harmonic water waves. Full, kinematic and dynamic, coupling is implemented along the water-solid interface, using propagation and evanescent modes (Athanassoulis and Belibassakis 1999). For most energetically interesting conditions the long-wave theory is valid, making the effect of evanescent modes negligible, and permitting us to calculate a closed-form solution for the efficiency of the energy harvesting system. It is found that the efficiency is dependent on two dimensionless hydro/piezo/electric parameters, and may become significant (as high as 30 – 50%) for appropriate combinations of parameter values, which, however, corresponds to exotically flexible piezoelectric materials. The existence or the possibility of constructing such kind of materials formulates a question to material scientists.

Keywords: renewable energy; piezoelectricity; sea wave energy

1. Introduction

Ocean waves carry huge amount of energy propagating in a thin layer near the surface of the sea and, eventually, impinging on the coastline. Being a surface phenomenon, sea waves consist one of the most intense natural energy resources. Nowadays this resource has been very well documented throughout the world ocean. See, e.g., Pontes, Athanassoulis *et al*. (1995, 1996), Cavaleri, Athanassoulis, Barstow (1999), Barstow and Mørk *et al*. (2003), Barstow *et al*. (2009), Mørk and Barstow *et al*. (2010), which describe the results of three European Commission–funded projects (WERATLAS, EUROWAVES and WORLDWAVES) studying offshore and nearshore wave conditions and wave energy resource.

In the open sea, especially in the northern oceans, the mean wave power may be more than per meter of the wave front (100 kW/m). Of course, as the waves approach the coast, shoaling

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causes breaking on the free surface and dissipation in the seabed boundary layer, resulting in lower figures for the available mean wave power per wave-front meter. Even thought, when the shoreline has the form of an (almost) vertical cliff, either rocky or manmade, with appreciable depth in front of it, waves impinge on it exerting large pressure loads. The wave climate in such sites has been extensively studied, mainly to provide information for the design of breakwaters or for the study of the erosive effects on natural coasts, as well as for assessing the available wave potential for nearshore and onshore wave energy devices. An extended list of many existing wave energy devices can be found in Wikipedia (http://en.wikipedia.org/wiki/Wave_power). In depth discussions of the physics principles and the technological aspects of the various devices are provided by the relevant papers in the technical literature; modern guides to this huge literature are the recently published books by Cruz (2008) and Khaligh and Onar (2010). Types of power take-off include: hydraulic ram, elastomeric hose pump, pump-to-shore, hydroelectric turbine, oscillating water columns in conjunction with air turbine, linear electrical generator, etc. In the present work an alternative point of view is adopted. We are going to investigate if it is possible to take off wave power directly through a piezoelectric material placed on the cliff.

Piezoelectricity, known since 1880 thanks to the experimental work by the brothers Pierre and Jacques Curie, has been intensively exploited in the recent years for designing energy harvesting devices, mainly in microscale. See, e.g., the recent review articles Sodano *et al.* (2004), Anton and Sodano (2007), Priya (2007), and the books by Priya and Inman (2009), Erturk and Inman (2011). Most of devices studied or reviewed in the literature are vibration–based energy harvesters, transducing the energy of mechanical vibrations to electric power supply of small electronic devices. Some concepts appropriate for converting energy from ambient fluid flow into useful electrical energy have appeared in the last decade or so. For example, Priya *et al.* (2005) and Myers *et al.* (2007) designed and tested a piezoelectric windmill, transducing wind energy into electricity; Taylor *et al.* (2001) and Pobering and Schwesinger (2004) studied piezoelectric flag generators, consisting of a flexible sheet placed downstream of a bluff body and excited by the von Kármán vortex sheet.

The subject of direct piezoelectric conversion of ocean wave energy is rather undeveloped. The main reason for this seems to be the very low frequency regime of sea waves (below to 0.5 Hz). Early concepts of piezoelectric wave harvesters, based on piezoelectric films or ropes made of Polyvinylidene fluoride (PVDF) (Taylor and Burns 1983, Haeusler and Stein 1985), have not been practically applied. The concept of a floating wave carpet, proposed by Koola and Ibragimov (2003) could be interesting when combined with an appropriate modeling and analysis of a flexible piezo-electric material. Murray and Rastegar (2009) proposed a two-stage piezoelectric wave energy harvester, consisting of a primary, low frequency, subsystem (e.g., a heaving buoy), which excites a secondary subsystem vibrating at its natural frequency, the latter being orders of magnitude higher than the frequency of the primary subsystem. The aforementioned piezo-electric wave energy harvesters, as well as other existing variants of them, all belong to the classes of point absorbers or attenuators.

The goal of the present paper is to investigate a terminator-type piezoelectric system that could extract electric energy from the direct impact of sea waves, impinging upon a vertical cliff. This seems to be the simplest possible configuration of a hydro/piezo/electric system, that could be deployed in large scale on the cliffs, especially those ones formed by breakwaters or floating breakwaters. Wave energy impinging upon such kind of structures induces large loads that can have only catastrophic effects. If a part of it could be transduced into electricity, two advantages would be realized: relaxing the exerted loads and gaining useful energy.

The structure of the paper is as follows: In Sec. 2 the whole system, consisting of three distinct subsystems (the hydrodynamic and the piezoelectric ones, and an external electrical circuit), is described in detail. In Sec. 3 the 3-3 mode of the piezoelectric vibration of a single piezoelement and the whole piezoelectric sheet covering the cliff, under a specific wiring, are studied. In Sec. 4 the hydrodynamic problem is formulated and a complete modal representation of the wave potential in the vicinity of the vertical cliff is given. Results from Sec. 3 and 4 are exploited in Sec. 5, where the coupling of the two subsystems is implemented through the interfacial, fluid–solid, matching conditions, taking the form of an infinite system of algebraic equations with respect to the modal coefficients. In the same section an approximate (yet accurate) closed form expression is obtained for the wave reflection coefficient, which controls the energetic coupling of the three subsystems. Finally, in Sec. 6, the ohmic resistance of the external circuit optimizing the efficiency of the hydro/piezo/electric harvester is found. The optimized efficiency is calculated analytically and investigated numerically. It is shown that efficiency may become significant (as high as 30 – 50%) for appropriate combinations of two dimensionless hydro/piezo/ electric parameters. To practically exploit this high efficiency new piezoelectric materials are needed, exhibiting much higher flexibility than the usual ones, and high values of the energy conversion factor. The possibility of manufacturing such kind of materials remains an open question.

2. System configuration

Before proceeding to the consideration of a specific, piezoelectric, wave-energy harvesting system, a description of the "virgin site" where this system could be installed in, seems to be appropriate. The virgin site would be any vertical cliff, either natural or manmade, as, e.g., a rocky cliff, a breakwater or a floating breakwater, with appreciable sea depth in front of it, so that the shoaling and dissipation effects to remain mild. Under these conditions, incoming waves induce large pressure loads on the vertical cliffs, which can be considered as rigid (non-deformable) bodies. The impinging wave energy partly dissipates (due to wave breaking and bottom friction), and partly is reflected back to the sea. The vertical cliff, being rigid and not moving, acts as a perfect barrier of the energy flow. The proposed concept of wave energy harvesting relies on the following observations: if a deformable body is interposed between the rigid vertical cliff and the incoming waves, the presence of both pressure on and deformation of the fluid-solid interface would result into an energy flow from sea waves to this body. If, in addition, the deformable body exhibits piezoelectric properties, part of the energy flowing through the fluid-solid interface would be transformed into electrical energy, which could be stored in (or consumed by) an external electric circuit, without the intervention of any other mechanical parts.

Since the present paper aims at a preliminary assessment of such an energy harvesting system, we focus on the basic physics facts, disregarding many technical details. Even though, we have to make a complete (yet simplified) modeling of three distinct subsystems: the hydrodynamic subsystem, i.e., the hydrodynamic wave field in the vicinity of the cliff, the piezoelectric subsystem, i.e., the material layer posed on the cliff and facing the action of sea waves, and an external electrical circuit, located on land.

2.1 The hydrodynamic subsystem: sea waves impinging into the cliff

Waves impinging into the cliff produce a complicated, nonlinear, slightly dissipative, impact phenomenon, resulting in the development of a fluctuating hydrodynamic pressure pattern on the fluid-solid interface. Realistic, wind-generated, sea waves are usually modeled as random waves, characterized by means of their spectrum. The angular frequencies ω may range from $0.314\,\text{rad/sec}$ to $3.14\,\text{rad/sec}$ (corresponding to periods $2\,\text{sec} < T < 20\,\text{sec}$), the actual range being strongly case and site dependent. The complete modeling of this phenomenon is an extremely difficult problem, not fully understood yet, which is out of the scope of the present paper. A general description of the phenomenon along with a survey of earlier works has been presented by Peregrine (2003). Some aspects of the nonlinear water-wave impact problem on rigid vertical surfaces have been recently studied by Molin *et al.* (2005), Jamois *et al.* (2006), Molin *et al.* (2010). Advanced methods of numerical simulation of such problems, using moving particles techniques and taking into account both nonlinearities and dissipation effects, have also been developed recently; see, e.g., Khayyer and Gotoh (2009).

For reasons explained above, we shall restrict ourselves to a reasonably convenient mathematical formulation of the hydrodynamic problem, namely the linear water-wave theory; see, e.g., Wehausen and Laitone (1960), Sec. 11. We shall also make the assumptions that the vertical cliff has an appreciable horizontal extent and the front of the incident wave is almost aligned to it, which permit us to treat the hydrodynamic problem as two-dimensional (2D). In addition, to simplify the hydrodynamic analysis, we assume that the seabed is horizontal. A vertical section of the fluid domain Ω is shown in Fig. 1. In the same figure it is also shown the Cartesian coordinate system used in the hydrodynamic analysis. The not shown y axis (perpendicular to the paper) extends along the horizontal dimension of the vertical cliff.

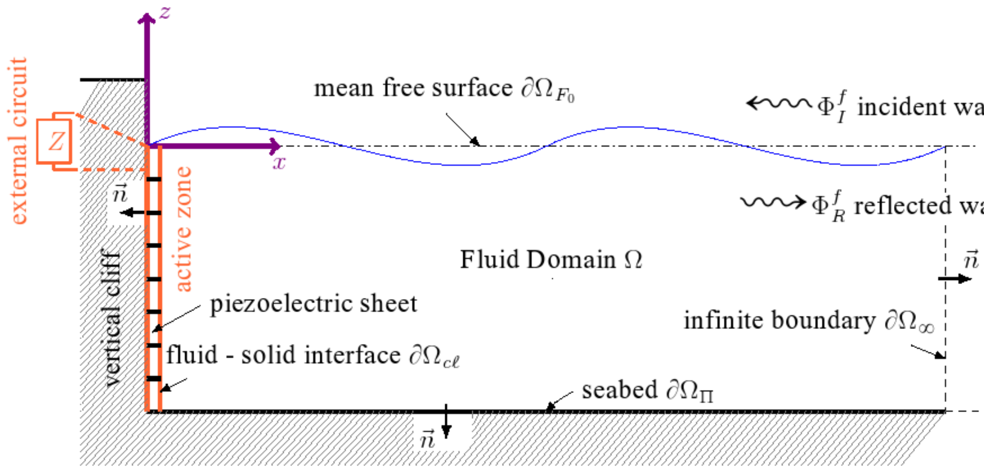

Fig. 1 Geometric configuration of the system

Under the assumption of linearity, the superposition principle is valid, which permits us to synthesize any (linear) wave pattern from the monochromatic (frequency domain) solution. Thus, focusing on the monochromatic case, we can assume that the velocity field is derived by a velocity potential $\Phi^f(x,z;t)$, which is expressed in terms of the complex phasor $\Phi^f(x,z;\omega)$ by means of the equation

$$\begin{aligned}\Phi^f(x,z;t) &= \mathrm{Re}_j\left\{\Phi^f(x,z;\omega)\exp(j\omega t)\right\} = \\ &= \mathrm{Re}_j\left\{\left(\Phi_I^f(x,z;\omega)+\Phi_R^f(x,z;\omega)+\Phi_{loc}^f(x,z;\omega)\right)\exp(j\omega t)\right\}\end{aligned} \tag{1a}$$

where

$$\Phi_I^f(x,z;\omega) = \frac{jg}{\omega}\frac{H}{2}\frac{\cosh\left[k_0\left(h_D+z\right)\right]}{\cosh(k_0 h_D)}\exp(jk_0 x) \tag{1b}$$

is the incident wave, having amplitude $H/2$

$$\Phi_R^f(x,z;\omega) = W\frac{jgH}{2\omega}\frac{\cosh\left[k_0\left(h_D+z\right)\right]}{\cosh(k_0 h_D)}\exp(-jk_0 x) \tag{1c}$$

is the reflected wave, and $\Phi^f_{\ell oc}(x,z;\omega)$ is a local wave field, vanishing exponentially far from the cliff. (The exact form of $\Phi^f_{\ell oc}(x,z;\omega)$ will be given in Sec.4). In Eqs. (1), $j=\sqrt{-1}$ is the imaginary unit, g is the acceleration due to gravity, ω is the frequency of the monochromatic incident wave, k_0 is the corresponding wave number, h_D is the sea depth in front of the vertical cliff and W is the reflection coefficient. The latter is, in general, complex valued, $W=|W|\cdot e^{j\,\mathrm{Arg}(W)}$, $|W|$ being the amplitude attenuation factor and $\mathrm{Arg}(W)$ being the phase shift with respect to the incident wave.

The hydrodynamic pressure field in the fluid, $p(x,z;\omega)$, is given by the linearized Bernoulli's law:

$$p(x,z;\omega) = -j\rho_f\,\omega\,\Phi^f(x,z;\omega) \tag{2}$$

where ρ_f is the mass density of sea water. Note that, when the nonlinear effects are taken into account, the total hydrodynamic pressure induced on the vertical cliff exhibits, in general, larger values than those obtained by means of the linear theory.

2.2 The piezoelectric subsystem: energy harvesting elements on the cliff

Piezoelectricity, initially detected in some crystalline solid materials, is a phenomenon according to which an electric field is developed in the material in response to externally applied mechanical stresses. It is a reversible process; when an external electric field is applied to the piezoelectric material, the latter exhibits deformations. Linear piezoelectricity is quantified macroscopically by means of the piezoelectric constitutive equations, connecting mechanical stress $\{\sigma_i\}_{i=1}^{i=6} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{31}, \sigma_{12}\}$ and electric displacement D_k to mechanical strain $\{e_i\}_{i=1}^{i=6} = \{e_{11}, e_{22}, e_{33}, 2e_{23}, 2e_{31}, 2e_{12}\}$ and electric intensity E_k

$$\sigma_i = c_{ik}^E e_k - \epsilon_{ki} E_k, \qquad D_i = \epsilon_{ik} e_k + \varepsilon_{ij}^S E_j \tag{3a,b}$$

where c_{ik}^E is the elastic stiffness tensor under constant electric intensity, ε_{ij}^S is the dielectric permittivity tensor under constant strain, and ϵ_{ik} is the piezoelectric stress tensor. The latter contains null elements since the piezoelectric effect disappears for certain crystallographic and limiting point symmetry groups. (Newnham 2005, Ch. 12.3).

The widely used piezoelectric materials largely consist of two classes; the piezoelectric ceramics (e.g., PZT family) and the electroactive polymers (EAP), as PVDF. Piezoelectric ceramics dominate the transducer applications, showing strong piezoelectric effect but are stiff and brittle and thus inappropriate, in the form of bulk materials, for energy harvesting applications where flexibility is needed (Brockmann 2009, Ch.4). On the other hand, traditional EAP show relatively improved flexibility but moderate piezoelectric coefficients (Bar-Cohen 2010). In between the two aforementioned classes of materials, lie the piezoelectric composites which combine high coupling factors with relatively high mechanical flexibility (Uchino 2010), but their properties cannot differ substantially from the properties of their constituent materials. Also interesting materials are those manufactured by the newer developments in EAP, such as relaxor ferroelectric copolymers and cellular polymers (Bauer and Bauer 2008). Probably the most promising materials are the dielectric electroactive polymers (DEAP), exhibiting the ability of large deformations along with high values of energy conversion ratio (Carpi *et. al.* 2008). Let it be noted, however, that the electromechanical properties of DEAP do not fit well in the classical piezoelectric modeling, followed in this work, since they show viscoelastic behavior and they are practically incompressible.

Piezoelectric materials are available either in small solid pieces or in the form of films or ropes. In this conjunction, and in order to exploit the thickness-mode oscillations, the piezo-elements considered in the present study are assumed to be small plates with transverse dimensions ℓ_1, ℓ_2, of order of magnitude of some centimeters, and thickness h, of order of magnitude of some millimeters. One of their surfaces $S = \ell_1 \times \ell_2$ is clamped on the vertical cliff and the other is free to oscillate under the influence of the wave impact. Piezoelements are installed contiguously from the sea bottom to the mean free surface and are electrically connected in series, forming a vertical array of M_1 piezoelements; see Fig. 1. The repetition of this array for an appreciable length $L_2 = M_2 \ell_2$ in the direction of the y–axis (horizontally along the cliff), in conjunction with a parallel electric connection between the vertical arrays, results in a two dimensional active zone of piezoelements, which is also called the piezoelectric sheet; Fig. 2.

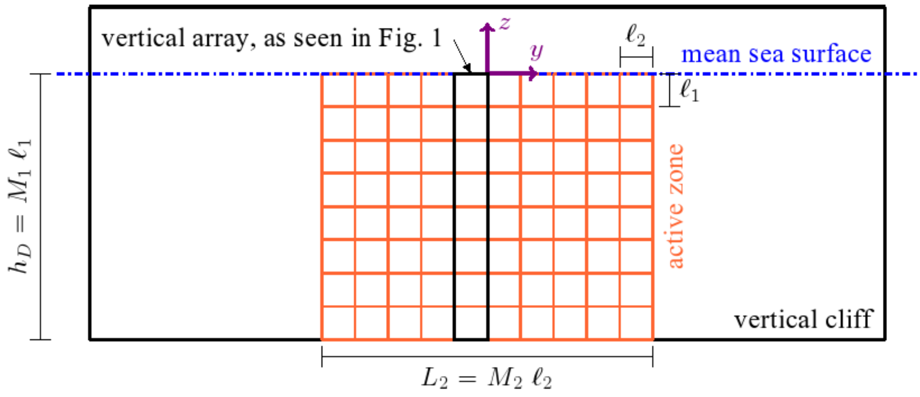

Fig. 2 The two-dimensional, cliff-mounded, piezoelectric active zone

Two basic technical issues, relevant to the formation and the installation of the piezoelectric sheet, are the insulation from the ambient sea water and the fixation on the vertical cliff. Both issues are strongly material dependent and they are out of the scope of the present work, which aims at a feasibility study of the basic concept.

2.3 The external electrical circuit

In order to take off power from the impinging waves, the output terminals of the system of piezoelements should be plugged in an external electrical circuit. A typical choice for the latter is the so-called standard energy harvesting (SEH) circuit, including a diode rectifier and a smoothing capacitor; see, e.g., Gyomar *et al.* (2005) and Shu and Lien (2006). A simpler choice, which is the usual one in most of the literature emphasizing on the mechanical part of the system, is a standard AC circuit, characterized by its impedance

$$Z(\omega) = R + j\,X(\omega) \tag{4}$$

where R models the total resistance and $X(\omega)$ models the total reactance. A thorough discussion concerning the effect of the external circuit on the energy flow in piezoelectric harvesters can be found in Liang and Liao (2011). In the context of linear theory, the angular frequency ω comes from the monochromatic wave excitation, having a very low value. By studying the considered hydro/piezo/electric system connected to the above described circuit, it is possible to find a closed form expression for the net (time average) power taken off from the waves, which reveals the main (dimensionless) parameters affecting the energy harvesting phenomenon.

3. The piezoelectric problem

3.1 The piezoelectric problem for a single piezoelement

For each piezoelement, a local, $(x_1\, x_2\, x_3)$ – Cartesian coordinate system is introduced, with x_i-axis coinciding with the corresponding principal piezoelectric axis; see Fig. 3. Each piezoelement is considered geometrically symmetric with respect to the coordinate planes $x_1 = 0$, $x_2 = 0$, $x_3 = 0$. Face $\gamma\delta$ is clamped (on the vertical cliff), while face $\alpha\beta$ is free to oscillate under the influence of incoming sea waves. [Note that in the physical position, faces $\alpha\beta$ and $\gamma\delta$ of each piezoelement are vertical; cf. Fig. 1]. Both faces $\alpha\beta$ and $\gamma\delta$ are electroded.

In this paper the thickness-mode vibration is considered, in which the resulting electric polarization vector has the same direction as the applied stress (thus $i = j = 3$). Thus, the constitutive Eqs. (3(a) and (b)) take the form

$$\sigma_3(x_3;t) = c_{33}^E e_3(x_3;t) - \epsilon_{33} E_3(x_3;t) \tag{5a}$$

$$D_3(x_3;t) = \epsilon_{33} e_3(x_3;t) + \varepsilon_3^S E_3(x_3;t) \tag{5b}$$

where $\varepsilon_3^S \equiv \varepsilon_{33}^S$. The external excitation (tensile) stress $\hat{\sigma}_3$, applied to the electroded face $\alpha\beta$, equals to $-p$, where p is the hydrodynamic pressure; the presence of minus sign is due to the fact that p is always compressive. The applied excitation $\hat{\sigma}_3$ gives rise to mechanical displacements $u_3(x_3;t)$ and voltage difference

$$\Delta V(t) = V_1(t) - V_0(t) = \Phi^{e\ell}(h/2;\, t) - \Phi^{e\ell}(-h/2;\, t) \tag{6}$$

between the two faces $\alpha\beta$ and $\gamma\delta$, where $\Phi^{e\ell}(x_3;t)$ is the electric potential field developed inside the piezoelement.

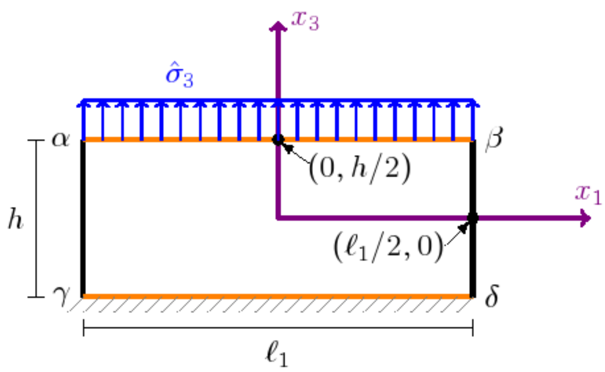

Fig. 3 Mode 3-3 vibration of a single piezoelement with $\alpha\beta$ and $\gamma\delta$ faces electroded

As the physical length (width) of each piezoelement is small in comparison with both the depth h_D of the sea in front of the vertical cliff and the wavelength of sea waves, the applied stress $\hat{\sigma}_3$ (due to sea waves) can be considered almost constant on the face $\alpha\beta$ of each piezoelement. Thus, we can consider $\hat{\sigma}_3$ equal to the mean value of the hydrodynamic pressure $-p$ over the face $\alpha\beta$, and simplify the piezoelectric problem assuming that all quantities are dependent only on x_3 coordinate. In this way, the piezoelectric phenomenon to be studied becomes essentially one dimensional (1D).

The equations governing the piezoelectric phenomenon are Newton's second law for deformable bodies, Maxwell's equations, the constitutive equations of piezoelectricity, and appropriate boundary conditions. See, e.g., Parton and Kudryavtsev (1988), Ch 1, Bardzokas and Filshtinsky (2006), Ch. 2, Meitzler *et. al.* (1987). Note that mechanical and dielectric dissipative phenomena are ignored in this study.

Since the frequency range of sea waves (exciting the piezoelements) is very low in comparison with electromagnetic waves frequencies, the equations governing the piezoelectric phenomenon are the quasi-static ones. In addition, under the assumption of monochromatic excitation, with circular frequency ω, all quantities can be represented by the corresponding phasors, i.e., $u_3\left(x_3;\,t\right) = \mathrm{Re}_j\left\{u_3\left(x_3;\,\omega\right)\exp\left(j\,\omega\,t\right)\right\}$. Then, for the present case of 1D linear problem in the frequency domain, the set of governing equations and boundary conditions takes the form

$$c_{33}^{E}\frac{\partial^2 u_3}{\partial x_3^2}\left(x_3;\omega\right) + \epsilon_{33}\frac{\partial^2 \Phi^{e\ell}}{\partial x_3^2}\left(x_3;\omega\right) = -\rho_b\,\omega^2 u_3\left(x_3;\omega\right) \tag{7}$$

$$\varepsilon_3^{S}\frac{\partial^2 \Phi^{e\ell}}{\partial x_3^2}\left(x_3;\omega\right) = \epsilon_{33}\frac{\partial^2 u_3}{\partial x_3^2}\left(x_3;\omega\right) \tag{8}$$

$$u_3\left(-h/2;\omega\right) = 0 \tag{9a}$$

$$c_{33}^{E}\frac{\partial u_3}{\partial x_3}\left(h/2;\omega\right) + \epsilon_{33}\frac{\partial \Phi^{e\ell}}{\partial x_3}\left(h/2;\omega\right) = \hat{\sigma}_3(\omega) \tag{9b}$$

$$\Phi^{e\ell}\left(-h/2;\omega\right) = V_0(\omega) \tag{10a}$$

$$\Phi^{e\ell}\left(h/2;\omega\right) = V_1\left(\omega\right) \tag{10b}$$

where ρ_b is the mass density of the piezoelement.

Eq. (7) is Newton's second law for deformable bodies containing also an electric term due to constitutive Eqs. (5(a) and (8)) is Gauss's law for the electric field containing also an elastic term due to constitutive Eq. (5(b)). Eqs. (9(a) and (b)) are mechanical boundary conditions. The problem is supplemented by the electrostatic boundary conditions Eqs. (10(a) and (b)). Eq. (10(a)) is a gauge condition which sets the level value of the potential. $V_0(\omega)$ is arbitrarily chosen, the quantity having physical meaning being the voltage difference $\Delta V(\omega)$. Eq. (10(b)) relates the unknown quantity $V_1(\omega) = \Delta V(\omega) + V_0(\omega)$ with the also unknown quantity $\Phi^{e\ell}\left(h/2;\omega\right)$. Accordingly, boundary conditions Eqs. (10(a) and (b)) do not specify boundary data; they just specify relations between unknown quantities. As a consequence, the solution of the boundary problem (7) - (10) is not unique. As we shall see in the sequel, this lack of boundary data will result in an undermined coefficient, the true value of which will be obtained later on by using information from the external electric circuit. This complicacy makes the present (direct) piezoelectric problem different from the (inverse) piezoelectric problem of free mechanical vibrations under the influence of known voltage difference (more extensively studied in the literature, see, e.g., Yang (2006b)), where the corresponding boundary conditions do not contain unknown quantities, rendering the problem uniquely solvable Ieşan (1990).

The boundary-value problem (7) - (10) is easily solved, as follows. Integrating twice Eq. (8) and using boundary conditions Eqs. (9(a)) and (10(a)), we express $\Phi^{e\ell}\left(x_3;\omega\right)$ in terms of $u_3\left(x_3;\omega\right)$ and an unknown coefficient $A(\omega)$, in the form

$$\Phi^{e\ell}\left(x_3;\omega\right) = \frac{\epsilon_{33}}{\varepsilon_3^S} u_3\left(x_3;\omega\right) + A(\omega)\left(x_3 - \frac{h}{2}\right) + V_0(\omega).$$

Substituting this expression for $\Phi^{e\ell}\left(x_3;\omega\right)$ in boundary condition Eq. (9(b)) and in Eq. (7), we formulate a boundary value problem for $u_3\left(x_3;\omega\right)$, containing also $A(\omega)$. Solving the latter, $u_3\left(x_3;\omega\right)$ is calculated in terms of $A(\omega)$. Applying the resulting solution to $x_3 = h/2$ and invoking Eq. (6), we get

$$u_3\left(x_3 = h/2;\omega\right) = \left(\hat{\sigma}_3(\omega) - \epsilon_{33} A(\omega)\right) \frac{h}{c_{33}^D} \frac{\tan(\tilde{\omega})}{\tilde{\omega}} \tag{11}$$

$$\Delta V(\omega) = V_1(\omega) - V_0(\omega) = \frac{\epsilon_{33}\hat{\sigma}_3(\omega) - \epsilon_{33}^2 A(\omega)}{\varepsilon_3^S} \frac{h}{c_{33}^D} \frac{\tan(\tilde{\omega})}{\tilde{\omega}} + A(\omega) h \tag{12}$$

where $c_{33}^D = c_{33}^E + \epsilon_{33}^2/\varepsilon_3^S$ and $\tilde{\omega} \equiv \omega h \sqrt{\rho_b / c_{33}^D}$ (13a,b)

are the elastic stiffness coefficient under constant electric displacement, and the dimensionless frequency, respectively.

Eqs. (11) and (12) can be simplified further, by observing that $\tilde{\omega}$ is a very small quantity in the context of the considered application. In fact, taking into account that for common piezo-elements the orders of magnitude of the involved quantities are: $\rho_b \sim O(10^4 kg/m^3)$, $c_{33}^D \sim O(10^{10} Pa)$, $h \le h_{\max} \sim O(10^{-2} m)$ (see, e.g., Bauer and Bauer 2008, Bloomfield 1994, Smith and Auld 1991, APC 2002), and that for sea waves $\omega \sim O(1 rad/s)$, we find that $\tilde{\omega} \sim O(10^{-7})$.

Thus, $\tan(\tilde{\omega})/\tilde{\omega} \approx 1$, and Eqs. (11) and (12) can be safely simplified to

$$u_3\left(x_3 = h/2; \omega\right) = \left[\hat{\sigma}_3(\omega) - \epsilon_{33} A(\omega)\right] \frac{h}{c_{33}^D} \tag{14}$$

$$\Delta V(\omega) = V_1(\omega) - V_0(\omega) = \frac{\left[\epsilon_{33}\hat{\sigma}_3(\omega) - \epsilon_{33}^2 A(\omega)\right]}{\varepsilon_3^S} \frac{h}{c_{33}^D} + A(\omega) h \tag{15}$$

The undetermined coefficient $A(\omega)$ will be now expressed in terms of the current $I = I(\omega)$ flowing through the piezoelement (and through the external circuit).

To bring the electric current into play, we need some (simplified) electrodynamic equation, not included in the quasi-static problem (7) - (10). Since the magnetic field is negligible, the additional electrodynamic equation can be thought in various different ways, e.g., either as the time derivative of Gauss's law (Erturk and Inman 2011, Sec. 3.1.3, Parton and Kudryavtsev 1988, Sec. 1.3), or as the conservation of electric charge, or as a degenerate form of Ampère's law which provide the definition of displacement current. In any case, for the present 1D piezoelectric problem, the additional equation for the electric current has the form

$$I/S = \dot{D}_3 \tag{16}$$

where S is the area of each of the electroded surfaces of the piezoelement. Using constitutive relation Eq.(5(b)) and Eq. (16) is written as

$$I = j\omega S\left(\epsilon_{33} e_3 + \varepsilon_3^S E_3\right) \tag{17}$$

Recalling that $e_3 = \partial u_3 / \partial x_3$ and $E_3 = -\partial \Phi^{el} / \partial x_3$, we obtain, for the case of $\tilde{\omega} << 1$

$$e_3 = \frac{\hat{\sigma}_3(\omega) - \epsilon_{33} A(\omega)}{c_{33}^D}, \qquad E_3 = -\frac{\epsilon_{33}\hat{\sigma}_3(\omega) - \epsilon_{33}^2 A(\omega)}{\varepsilon_3^S c_{33}^D} - A(\omega) \tag{18a,b}$$

Substituting Eqs. (18(a) and (b)) into Eq. (17) we get

$$I(\omega) = -j\omega\varepsilon_3^S S A(\omega) \tag{19}$$

Now, since the voltage, Eq. (15), and the current, Eq. (19), are both expressed in terms of coefficient $A(\omega)$, it is clear that coupling of the piezoelement with an external electrical circuit would provide us with a specific value of $A(\omega)$ and, thus, a complete solution of the piezoelectric problem.

For comparison purposes we consider here the two limiting cases, namely, the open-electrode piezoelement ($I(\omega)=0$), and the short-circuit piezoelement ($\Delta V(\omega) = 0$). In the first case, $A(\omega)=0$ and thus, from Eq. (15), we get $\Delta V(\omega) = \left(\epsilon_{33} / \varepsilon_3^S\right)\left(h / c_{33}^D\right)\hat{\sigma}_3(\omega)$, a result also given by APC (2002), Table 1.8. In the second case, combining $\Delta V(\omega) = 0$ with Eq. (15), we obtain $A(\omega) = -k_t^2\,\hat{\sigma}_3(\omega) \big/ \epsilon_{33}\left(1-k_t^2\right)$, where

$$k_t^2 = \frac{\epsilon_{33}^2}{\varepsilon_3^S\, c_{33}^D} \tag{20}$$

is the energy conversion (or coupling) factor. Introducing the above expression for $A(\omega)$ into Eq. (14) we obtain $u_3(\omega) = \hat{\sigma}_3(\omega)h / ((1-k_t^2)c_{33}^D) = (h / c_{33}^E)\hat{\sigma}_3(\omega)$, in accordance with Preumont (2011), Sec. 3.6.2. For a physical interpretation of k_t^2 and the derivation of equation $c_{33}^E = \left(1-k_t^2\right) c_{33}^D$, see Jaffe *et. al.* (1971), Ch. 3, Sec. C.1. Furthermore, in Jaffe *et. al.* (1971), Ch. 2, Sec. 2, it is proven that $0 < k_t^2 < 1$.

3.2 The system of piezoelements on the vertical cliff in series connection

We shall now proceed to considering the whole active zone. Various connections are possible between the electrodes of adjacent piezoelements that form each vertical array. In the present work a series connection has been selected, as depicted in Fig. 4.

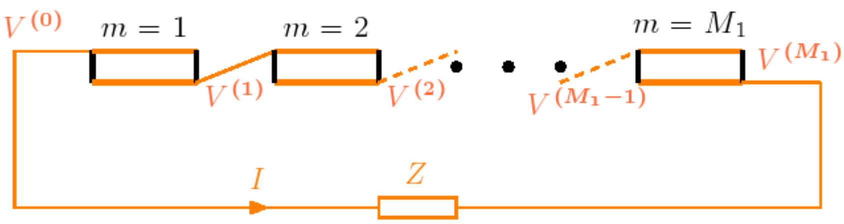

Fig. 4 Series connection of piezoelements forming one vertical array

The results obtained in previous subsection, for a single piezoelement, can be applied to each piezoelement of the group. All quantities associated with the $m-th$ piezoelement, e.g., $u_3\left(h/2,\,\omega\right)$, $\hat{\sigma}_3(\omega)$, etc., will be now distinguished by a superscript m in parenthesis, e.g.,

$u_3^{(m)}(h/2,\,\omega)$, $\hat{\sigma}_3^{(m)}(\omega)$, etc.. Considering all piezoelements being of the same material and of the same dimensions, we do not use the m superscript for material properties and element dimensions. On the basis of the series connection of adjacent piezoelements, $I^{(m)}(\omega)=I(\omega)$, $V_0^{(m)}=V^{(m-1)}$ and $V_1^{(m)}=V^{(m)}$, $m=1,2,\ldots,M_1$. Using Eq. (11), the voltages $V^{(m)}(\omega)$ at the output electrode of each piezoelement are given by (see also Fig. 4)

$$V^{(m)}(\omega)-V^{(m-1)}(\omega)=\frac{\left[\epsilon_{33}\,\hat{\sigma}_3^{(m)}(\omega)-\epsilon_{33}^2\,A^{(m)}(\omega)\,h\right]}{\varepsilon_3^S\,c_{33}^D}+A^{(m)}(\omega)\,h \tag{21}$$

where
$$\hat{\sigma}_3^{(m)}(\omega)=\frac{1}{\ell_1}\int_{\substack{m-th\\ piezoelement}}\hat{\sigma}_3(z;\omega)dz \tag{22}$$

and $\hat{\sigma}_3(z;\omega)=-\hat{p}(z;\omega),-h_D\le z\le 0$, z being the global vertical coordinate; see Fig. 1.

Setting $V^{(0)}=0$ on the first electrode of the first piezoelement, and summing up all Eqs. (21), we find the total voltage difference

$$\Delta V(\omega)=V^{(M_1)}(\omega)=\frac{\left(\epsilon_{33}\sum_{m=1}^{M_1}\hat{\sigma}_3^{(m)}(\omega)-\epsilon_{33}^2\sum_{m=1}^{M_1}A^{(m)}(\omega)\right)h}{\varepsilon_3^S\,c_{33}^D}+h\sum_{m=1}^{M_1}A^{(m)}(\omega) \tag{23}$$

Applying Ohm's law to the external circuit, represented here by an equivalent impedance $Z(\omega)$, we get the following equation for the current

$$I(\omega)=\frac{\Delta V(\omega)}{Z(\omega)}=\frac{\left(\epsilon_{33}\sum_{m=1}^{M_1}\hat{\sigma}_3^{(m)}(\omega)-\epsilon_{33}^2\sum_{m=1}^{M_1}A^{(m)}(\omega)\right)h}{\varepsilon_3^S\,c_{33}^D\,Z(\omega)}+\frac{h}{Z(\omega)}\sum_{m=1}^{M_1}A^{(m)}(\omega) \tag{24}$$

As the piezoelements are connected in series, the current $I(\omega)$ is common over the whole circuit. Thus, Eq. (19), applied to each piezoelement, takes the form

$$I(\omega)=-\,j\omega\varepsilon_3^S SA^{(m)}(\omega)\;\Rightarrow\;A^{(1)}=\ldots=A^{(m)}=\ldots=A^{(M_1)}=A \tag{25}$$

By introducing the piezoelectric constant

$$C_0=\frac{\varepsilon_3^S\,S}{h} \tag{26}$$

called the clamped capacitance (of each piezoelement), [for a physical interpretation see, e.g., Lefeuvre *et. al.* (2010), Guyomar *et. al.* (2005)], and setting

$$\mathscr{E}_t(\omega) = \frac{k_t^2}{(1-k_t^2) + j\omega\,(C_0 / M_1)\,Z(\omega)} \tag{27}$$

the system of Eqs. (24) and (25) provides the following solution for the common value of A's, $A(\omega)$

$$A(\omega) = -\frac{\mathscr{E}_t(\omega)}{\epsilon_{33}} \frac{1}{M_1} \sum_{m=1}^{M_1} \hat{\sigma}_3^{(m)}(\omega) \tag{28}$$

Substituting Eq. (28) into Eq. (24), we obtain the total voltage output

$$\Delta V(\omega) = I(\omega)\,Z(\omega) = j\omega\,C_0\,Z(\omega)\frac{\mathscr{E}_t(\omega)}{\epsilon_{33}}\,h\,\frac{1}{M_1}\sum_{m=1}^{M_1}\hat{\sigma}_3^{(m)}(\omega) \tag{29}$$

Eq. (29) provides us with the solution of the piezoelectric system connected with an external circuit of equivalent impedance $Z(\omega)$, in terms of the applied stress $\hat{\sigma}_3^{(m)}(\omega)$. However, in order to implement the coupling of this (sub)system with the water wave impinging on the cliff (given by Eqs. (42) and (43)), we need to find a relation between the piezoelements face velocities $j\omega\,u_3^{(m)}\left(x_3 = h/2;\,\omega\right)$ and the excitation stresses $\hat{\sigma}_3^{(m)}(\omega)$. To this aim, we come back to the Eq. (14), written for each piezoelement, and substitute $A(\omega)$ from Eq. (28)

$$u_3^{(m)}(x_3 = h/2;\,\omega) = \frac{h}{c_{33}^D}\hat{\sigma}_3^{(m)}(\omega) + \frac{h\,\mathscr{E}_t(\omega)}{c_{33}^D}\frac{1}{M_1}\sum_{m=1}^{M_1}\hat{\sigma}_3^{(m)}(\omega), \quad m = 1,\ldots,M \tag{30}$$

Since the vertical width ℓ_1 of each piezoelement is only a small fraction of the water-wave length, the stress variation over the face $\alpha\beta$ of each piezoelement is negligible, which implies that the mean excitation stress $\hat{\sigma}_3^{(m)}$ is approximately equal to $\hat{\sigma}_3(z;\omega) = -\hat{p}(z;\omega)$, with z restricted to vary over the face $\alpha\beta$ of the $m-th$ piezoelement. Moreover, the sum of the mean stresses applied over the totality of piezoelements, can be written, using Eq. (22), as

$$\frac{1}{M}\sum_{m=1}^{M_1}\hat{\sigma}_3^{(m)}(\omega) = \frac{1}{h_D}\int_{-h_D}^{1}\hat{\sigma}_3(z;\omega)dz = \overline{\hat{\sigma}_3(\omega)} \tag{31}$$

where $\overline{\hat{\sigma}_3(\omega)}$ is the mean excitation stress over the whole vertical cliff. Thus, Eq. (30) can be reformulated in a continuous fashion in the form

$$\hat{u}_3(z;\omega) = \frac{h}{c_{33}^D}\hat{\sigma}_3(z;\omega) + \frac{h}{c_{33}^D}\mathscr{E}_t(\omega)\overline{\hat{\sigma}_3(\omega)}, \qquad -h_D \leq z \leq 0 \tag{32}$$

where $\hat{u}_3(z;\omega) \approx u_3^{(m)}(x_3 = h/2;\omega)$, for z varying over the face $\alpha\beta$ of the $m-th$ piezo-element. Eq. (32) shows that the mechanical displacement of the interface $\partial\Omega_{c\ell}$ (the outer face of the piezoelectric sheet covering the cliff) comes from two terms, a local one and a global one. The first is of elastic nature, having a local dependence on the applied pressure and stiffness coefficient $c_{33}^D = c_{33}^E + \epsilon_{33}^2/\varepsilon_3^S$. The presence of c_{33}^D, which is greater than the standard c_{33}^E coefficient appearing in the constitutive Eq. (5(a)), models the piezoelectric stiffening phenomenon. (For a general discussion and mathematical formulation of piezoelectric stiffening see Auld (1969) and Yang (2006a), Sec. 2.2.1.) The second term is of purely piezoelectric nature, it has a global dependence on the applied pressure, and is also dependent on the external electric circuit characteristics through the factor $\mathscr{E}_t(\omega)$.

Let it be noted that, by substituting Eq. (30) into Eq. (29) for the case of the single piezoelement ($M_1 = 1$), the following relation between $\Delta V(\omega)$ and $\hat{u}_3(\omega)$ is obtained

$$\Delta V(\omega) = j\omega a Z(\omega)\,\hat{u}_3(\omega)/\left(1 + j\omega C_0 Z(\omega)\right), \qquad \text{with } a = \epsilon_{33} S/h \tag{33}$$

a result also found by Guyomar *et. al.* (2005).

3.3 Power flow relations

Our main goal in this paper is to investigate the possibility of extracting power from the incoming sea waves and deliver it to an external circuit, through the intervention of a piezoelectric sheet covering the cliff. We are now focusing on the calculation of this power flow. The net (time average) power flowing through the piezoelectric sheet covering an area $h_D \times L_2$ of the cliff (see Fig. 2) is given by the equation

$$\mathrm{P}_{c\ell}^{\text{piezo}}(\omega) = L_2 \frac{1}{T}\int_{t=0}^{t=T}\int_{z=-h_D}^{z=0} \frac{\partial \hat{u}_3(z;t)}{\partial t}\,\hat{\sigma}_3(z;t)\,dzdt \tag{34}$$

where $T = 2\pi/\omega$ is the period of the oscillating system (the same as the period of the incoming wave), and the factor L_2 accounts for the horizontal extent of the piezoelectric sheet. Using phasors, the net power flow is written in the form

$$\mathrm{P}_{c\ell}^{\text{piezo}} = \mathrm{P}_{c\ell}^{\text{piezo}}(\omega) = L_2 \frac{1}{2}\mathrm{Re}_j\left\{\int_{z=h_D}^{z=0} j\omega\,\hat{u}_3(z;\omega)\,\hat{\sigma}_3^*(z;\omega)\,dz\right\} \tag{35}$$

where the asterisk denotes the complex conjugate. Using Eq. (32), we easily see that the elastic part of the velocity ($j\,\omega\left(h/c_{33}^{D}\right)\hat{\sigma}_3(z;\,\omega)$) does not contribute to the power flow (as expected), which takes finally the form

$$\mathrm{P}_{c\ell}^{\text{piezo}}(\omega) \;=\; -\frac{1}{2}\,\omega\,\frac{h}{c_{33}^{D}}\;\mathrm{Im}\left\{\mathscr{E}_t(\omega)\right\}\,(h_D L_2)\,\left|\overline{\hat{\sigma}_3(\omega)}\right|^2 \tag{36}$$

Using Eq. (27) and decomposing the complex impedance $Z(\omega) = R + j\,X(\omega)$, it can be checked that $\mathrm{P}_{c\ell}^{\text{piezo}}(\omega) > 0 \;\Leftrightarrow\; R > 0$, the latter being always valid.

Besides, the net electric power $\mathrm{P}_Z(\omega)$ consumed by the external circuit is calculated in terms of the electric quantities $\Delta V_{\text{tot}}(\omega)$ and $I_{\text{tot}}(\omega)$. Due to the parallel electrical connection between the vertical arrays and the identical electrical quantities of each array, it holds true that $\Delta V_{\text{tot}}(\omega) = \Delta V(\omega)$ and $I_{\text{tot}} = M_2\,I = L_2\,I/\ell_2$, where M_2 is the number of vertical arrays that form the active zone. Thus, $\mathrm{P}_Z(\omega)$ is calculated as

$$\begin{aligned}\mathrm{P}_Z(\omega) &= \frac{1}{2}\mathrm{Re}_j\left\{\Delta V_{tot}(\omega) I_{tot}^{*}(\omega)\right\} = M_2\frac{1}{2}\mathrm{Re}_j\left\{\Delta V(\omega) I^{*}(\omega)\right\} = \\ &= M_2\frac{1}{2}\mathrm{Re}_j\left\{\Delta V(\omega)\frac{\Delta V^{*}(\omega)}{Z^{*}(\omega)}\right\} = M_2\frac{1}{2}\left|\frac{\Delta V^{*}(\omega)}{Z^{*}(\omega)}\right|^2 \mathrm{Re}_j\left\{Z(\omega)\right\}\end{aligned} \tag{37}$$

Using Eqs. (27) and (29), we get

$$\mathrm{P}_Z(\omega) \;=\; M_2\,\frac{1}{2}\,\omega^2\,C_0^2\,\frac{h^2}{\epsilon_{33}^2}\left|\mathscr{E}_t(\omega)\right|^2\,\left|\overline{\hat{\sigma}_3(\omega)}\right|^2 R \tag{38}$$

Calculating $\mathrm{Im}\left\{\mathscr{E}_t(\omega)\right\}$ and $\left|\mathscr{E}_t(\omega)\right|^2$, and recalling the definitions of the quantities C_0 and k_t^2 (see Eqs. (26) and (20)), we can show that $\mathrm{P}_{c\ell}^{\text{piezo}}(\omega) = \mathrm{P}_Z(\omega)$. Thus, the whole net power flowing through the piezoelectric sheet is delivered at the external circuit. This is a statement of the conservation of energy, since we have neglected the dissipation within the piezoelements.

4. The hydrodynamic problem

4.1 Mathematical formulation of the hydrodynamic boundary-value problem

The 2D liquid domain Ω extends from the seabed $\partial\Omega_{\Pi}(z=-h_D)$ up to the free surface $\partial\Omega_F\left(z=\eta(x;t)\right)$, and from the vertical cliff $\partial\Omega_{c\ell}$ up to infinity $\partial\Omega_{\infty}$; see Fig.1. Two hydrodynamic fields are involved in the problem: the velocity potential field $\Phi^f\left(x,z;\,t\right)$, and the pressure field $p(x, z; t)$. Both fields are independent from the y coordinate, in accordance with the 2D character of the problem, as discussed in subsection 2.2. In the context of linear water-wave theory, the domain of definition of velocity potential $\Phi^f(x, z; t)$ is restricted to the fixed domain $\Omega_0 = \{-h_D < z < 0,\ 0 < x < +\infty\}$, i.e., to a half strip with plane boundaries.

The complete, linearized, boundary-value problem for the total wave potential $\Phi^f(x, z; \omega)$, in the frequency domain, is formulated as follows (see, e.g., Wehausen and Laitone (1960), Sec. 11, Stoker (1957), Sec. 3.1, or Mei *et al*. (2005), Sec. 1.4)

$$\Delta\Phi^f(x, z; \omega) = 0, \quad \text{in } \Omega_0 \tag{39}$$

$$\frac{\partial\Phi^f}{\partial z}(x, z=0; \omega) - \mu_0\Phi^f(x, z=0; \omega) = 0, \qquad \mu_0 = \omega^2 / g, \qquad (\text{on} \quad \partial\Omega_{F_0}) \tag{40}$$

$$\frac{\partial\Phi^f}{\partial z}(x, z=-h_D; \omega) = 0, \qquad (\text{on} \quad \partial\Omega_{\Pi}) \tag{41}$$

$$\frac{\partial\Phi^f}{\partial x}(x=0, z; \omega) = j\omega\hat{u}_3(z; \omega), \qquad (\text{on} \quad \partial\Omega_{c\ell}) \tag{42}$$

$$p(x=0, z; \omega) = -\hat{\sigma}_3(z; \omega), \qquad (\text{on} \quad \partial\Omega_{c\ell}) \tag{43}$$

$$\Phi^f \to \Phi^f_I + \Phi^f_R, \qquad \text{when} \qquad x \to +\infty, \ (\textit{i.e.}, \text{ at } \partial\Omega_\infty) \tag{44}$$

where $\Phi^f_I = \Phi^f_I(x, z; \omega)$ is the incident wave, and $\Phi^f_R = \Phi^f_R(x, z; \omega)$ is the reflected wave, already prescribed by Eqs. (1(b) and (c)). The pressure $p(x, z; \omega)$ is given by the linearized Bernoulli's law, Eq. (2).

Conditions (42) and (43) are matching conditions, ensuring the continuity of normal velocity and normal pressure through the fluid-solid interface $\partial\Omega_{c\ell}$, respectively. These conditions match the hydrodynamic quantities Φ^f and p with the elastodynamic quantities $\hat{u}_3$ and $\hat{\sigma}_3$ and, through them, with the piezo-electric problem.

4.2 Modal representation of the wave potential

The solution of the coupled problem is greatly facilitated by means of the following modal representation of the wave potential:

Modal Representation Theorem of the Wave Potential: Every function Φ^f defined in the half strip Ω_0, satisfying the Laplace Eq. (39) therein, the free-surface boundary condition (40) on $\partial\Omega_{F_0}(z=0)$, the seabed boundary condition (41) on $\partial\Omega_{\Pi}(z=-h_D)$, and the condition $\left|\Phi^f(x,z;\omega)\right| \le M = const.,$, in Ω_0, admits of the following representation

$$\Phi^f(x,z;\omega) = \frac{j\,g\,H}{2\omega} Z_0(z)\exp\left(j\,k_0\,x\right) + W\frac{j\,g\,H}{2\omega} Z_0(z)\exp\left(-j\,k_0\,x\right) + \Phi^f_{\ell oc}(x,z;\omega) \tag{45}$$

where the first term in the right-hand side of the above equation represents the incident wave, the second term represents the reflected wave, and the third term, $\Phi^f_{\ell oc}(x,z;\omega)$, represents a local wave field, vanishing exponentially far from the cliff, which can be expanded in the form of an infinite series of evanescent modes, as follows

$$\Phi^f_{\ell oc}(x,z;\omega) = \sum\nolimits_{n=1}^{\infty} \Phi^f_n(x,z;\omega) = \sum\nolimits_{n=1}^{\infty} C_n Z_n(z)\exp(-k_n x) \tag{46}$$

$Z_0(z)$, $Z_n(z)$, $n = 1, 2, 3, \ldots$, are the vertical eigenfunctions of the water-wave problem, given by the equations

$$Z_0(z) = \frac{\cosh\left[k_0\left(h_D+z\right)\right]}{\cosh\left(k_0\,h_D\right)}, \qquad Z_n(z) = \frac{\cos\left[k_n\left(h_D+z\right)\right]}{\cos\left(k_n\,h_D\right)}, \quad n = 1, 2, \ldots . \tag{47a,b}$$

The constants k_0 and k_n, $n = 1, 2, 3, \ldots$, appearing in the above equations are the positive roots of the dispersion relation

$$\frac{\mu_0}{k_0} = \tanh\left(k_0\,h_D\right), \qquad \frac{\mu_0}{k_n} = -\tan\left(k_n\,h_D\right) \tag{48a,b}$$

where h_D is the (constant) sea depth.

The coefficients W, C_n, $n = 1, 2, 3, \ldots$, are free; they can be determined by means of the boundary (matching) conditions imposed on the vertical boundary surfaces $\partial\Omega_{c\ell}, \partial\Omega_{\infty}$. The above representation theorem traces back to Kreisel (1949). It is also discussed by Wehausen & Laitone (1960), Sec. 17 and Mei (2005), Sec. 8.4.1, and it has been extensively used in the study of various water wave problems over a locally varying bathymetry (see, e.g., Bai and Yeung (1974), Lenoir and Tounsi (1988), Athanassoulis and Belibassakis (1999), Belibassakis and Athanassoulis (2005)).

Using Eqs. (45) and (46) we easily obtain representations of the horizontal wave velocity $\hat{\Phi}^f_{,x}(z;\omega) = \partial\Phi^f(x=0,z;\omega)/\partial x$ and the pressure $\hat{p}(z;\omega) = p(x=0,z;\omega)$ on the fluid-solid interface $\partial\Omega_{c\ell}$, in terms of the unknown coefficients W, C_n, $n = 1, 2, 3, \ldots$. These representations will be exploited in the next section in order to solve the coupled problem.

4.3 Power flow relations

The net (time average) power flowing towards the cliff through a vertical section at any position $x = a$ within the liquid domain (having horizontal extent L_2, normally to the wave front) is given by the equation

$$\mathrm{P}_a^f = -L_2 \frac{1}{T} \int_{t=0}^{t=T} \int_{z=-h_D}^{z=0} p(x=a,z;t)\, \Phi_{,x}^f(x=a,z;t)\, dzdt \tag{49}$$

where $\Phi_{,x}^f(x=a,z;t) = \partial \Phi^f(x=a,z;t)/\partial x$ and $T = 2\pi/\omega$ is the period of the wave. Passing to phasors, Eq. (49) takes the form

$$\mathrm{P}_a^f(\omega) = L_2 \frac{1}{2} \mathrm{Re}_j \left\{ j\omega\, \rho_f \int_{-h_D}^{0} \Phi^f(x=a,z;\omega) \left(\Phi_{,x}^f(x=a,z;\omega) \right)^* dz \right\} \tag{50}$$

As expected from energy considerations (and it can be proved by using Green's Theorem) the above quantity is independent from the position $x = a$ of the considered section. Accordingly, the easiest way to calculate $\mathrm{P}_a^f(\omega)$ (in terms of hydrodynamic quantities) is by letting $a \to \infty$ and using Eq. (45) keeping only the first two (non-evanescent) modes. After straightforward calculations, we obtain

$$\mathrm{P}_a^f(\omega) = \frac{1}{8} \rho_f\, g\, H^2 L_2\, \omega \frac{k_0}{\mu_0} \left(1 - |W|^2 \right) \| Z_0 \|^2 \tag{51}$$

On the other hand, if we apply Eq. (49) to $x = 0$, and take into account the matching conditions (42) and (43), we readily see that $\mathrm{P}_{a=0}^f(\omega)$ is exactly the power flowing through the fluid-solid interface $\partial\Omega_{c\ell}$ towards the piezoelements, which, finally, is consumed by the external circuit; see Eq. (38). The equations

$$\mathrm{P}_\infty^f(\omega) = \mathrm{P}_a^f(\omega) = \mathrm{P}_{a=0}^f(\omega) = \mathrm{P}_{c\ell}^{\mathrm{piezo}}(\omega) = \mathrm{P}_Z(\omega) \tag{52}$$

express the conservation of energy under the idealized conditions that the dissipation during the propagation of the sea waves as well as the dissipation in the piezoelements are negligible.

5. Solution of the coupled problem

The dynamical coupling between the piezoelectric and hydrodynamic problem is realized by means of the matching conditions (42) and (43). Combining these two equations with Eq. (32), we obtain the following condition on the fluid-solid interface $\partial\Omega_{c\ell}$

$$\hat{\Phi}^{f}_{,x}(z;\omega) + j\omega \frac{h}{c^{D}_{33}} \hat{p}(z;\omega) + j\omega \frac{h}{c^{D}_{33}} \frac{\mathscr{E}_t(\omega)}{h_D} \int_{-h_D}^{0} \hat{p}(z;\omega)dz = 0 \tag{53}$$

This is a non-local (because of the last term) condition connecting the hydrodynamic fields $\Phi^{f}(x, z; \omega)$ and $p(x, z; \omega)$ at $x = 0$.

5.1 Formulation of infinite system of equations with respect to the modal coefficients

The modal expansion, given by Eqs. (45) and (46), permits us to obtain modal expansions for the functions $\Phi^{f}_{,x}(x=0,z;t) = \partial\Phi^{f}(x=0,z;t)/\partial x$ and $\hat{p}(z;\omega) = p(x=0, z; \omega)$, in terms of the expansion coefficients W, C_n, $n = 1,2,3,\ldots$. Substituting these modal expansions into Eq. (53), and performing the appropriate algebraic manipulations, we finally obtain

$$\left\{a_0 Z_0(z) + \beta_0 Z_0(z) + \gamma_0 \mathscr{T}_0\right\} W + \sum_{n=1}^{\infty} \left\{a_n Z_n(z) + \beta Z_n(z) + \gamma \mathscr{T}_n \right\} =$$
$$= a_0 Z_0(z) + \beta_0 Z_0(z) + \gamma_0 \mathscr{T}_0, \quad -h_D \le z \le 0 \tag{54}$$

where $$\alpha_0 = \frac{g k_0}{\omega} \frac{H}{2}, \quad \alpha_n = -k_n, \quad \beta_0 = j\omega\rho_f g \frac{h}{c^{D}_{33}} \frac{H}{2}, \quad \beta = \rho_f \omega^2 \frac{h}{c^{D}_{33}} \tag{55}$$

and $$\gamma_0 = j\omega \rho_f g \frac{H}{2}, \quad \frac{h}{c^{D}_{33}} \frac{\mathscr{E}_t(\omega)}{h_D}, \quad \gamma = \rho_f \omega^2 \frac{h}{c^{D}_{33}} \frac{\mathscr{E}_t(\omega)}{h_D}, \quad \mathscr{T}_n = \int_{-h_D}^{0} Z_n(z)dz \tag{56}$$

Recall now that the vertical eigenfunctions $Z_0(z), Z_n(z)$, $n = 1, 2, \ldots$, as defined by Eq. (47 (a) and (b)), constitute an orthogonal system of functions, complete in the Hilbert space $L^2(-h_D, 0)$. Accordingly, by projecting both members of Eq. (54) on each one of the basis functions $Z_0(z)$, $Z_n(z)$, $n = 1, 2, \ldots$, we obtain the following infinite system of equations with respect to the unknown coefficients W, C_n

$$\left\{\mathrm{K}^{+}_{00} + \gamma_0 \Lambda_{00}\right\} W + \sum_{n=1}^{\infty} \gamma \Lambda_{n0} C_n = \mathrm{K}^{-}_{00} - \gamma_{00} \Lambda_{00} \tag{57a}$$

$$\gamma_0 \Lambda_{0m} W + \sum_{n=1}^{\infty} \left\{\mathrm{K}_{nn} \delta_{nm} + \gamma \Lambda_{nm}\right\} C_n = -\lambda_0 \Lambda_{0m}, \quad m = 1,2.3,\ldots, \tag{57b}$$

where

$$\mathrm{K}_{00}^{+} = \left(\alpha_0 + \beta_0\right) \left\| Z_0 \right\|^2, \ \ \mathrm{K}_{00}^{-} = \left(\alpha_0 - \beta_0\right) \left\| Z_0 \right\|^2, \ \ \Lambda_{n0} = \Lambda_{0n} = \mathscr{T}_n \mathscr{T}_0 \tag{58}$$

$$\mathrm{K}_{nn} = \left(\alpha_n + \beta\right) \left\| Z_n \right\|^2, \ \ \Lambda_{nm} = \Lambda_{mn} = \mathscr{T}_n \mathscr{T}_m, \tag{59}$$

and $\left\|Z_n\right\|^2 = \int_{-h_D}^{0} Z_n^2(z)\, dz$ is the square of the norm of $Z_n(z)$ in the space $L^2\left(-h_D, 0\right)$.

Using Eq. (57(a)) we eliminate W from Eq. (57(b)), which then take the form

$$\sum_{n=1}^{\infty} \left\{ \mathrm{K}_{nn}\, \delta_{nm} + \gamma \Lambda_{nm} - \frac{\gamma \gamma_0 \Lambda_{00} \Lambda_{nm}}{\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}} \right\} C_n = \gamma_0 \Lambda_{0m} \frac{\mathrm{K}_{00}^{+} + \mathrm{K}_{00}^{-}}{\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}}, \ \ m = 1, 2, \ldots \tag{60}$$

For our purposes, the most important coefficient is the reflection coefficient W, which is expressed, in terms of C_n, by means of the equation (obtained from (57(a))

$$W = -\sum_{n=1}^{\infty} \frac{\gamma \Lambda_{n0}}{\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}} C_n + \frac{\mathrm{K}_{00}^{-} - \gamma_0 \Lambda_{00}}{\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}} \tag{61}$$

Since the coefficients $\gamma \Lambda_{n0} / \left(\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}\right)$, multiplying C_n in Eq. (61), are about four orders of magnitude smaller than the C_n – independent term $\left(\mathrm{K}_{00}^{-} - \gamma_0 \Lambda_{00}\right) / \left(\mathrm{K}_{00}^{+} + \gamma_0 \Lambda_{00}\right)$, it is expected that the effect of the C_n – dependent terms on W should be small. This has been definitely verified by means of detailed numerical calculations, shown that the effect of the C_n – dependent series on the values of W is less than 0.1%. Thus, it is safe to proceed with our analysis by keeping only the second (C_n – independent) term in Eq. (61). This approximation is compatible with the long-wave theory for water waves.

5.2 Closed-form solution for the reflection coefficient

Under the (numerically confirmed) simplification that the C_n coefficients do not practically affect the reflection coefficient W, the second term of the right-hand side of Eq. (61) provides us with a closed-form solution for W. Recalling the definitions (58) and (59) of the quantities K_{00}^{+}, Λ_{n0}, Λ_{00}, in conjunction with Eqs. (55) and (56) defining α_0, β_0, γ_0, the closed-form expression for the reflection coefficient W can be written in the form

$$W = \frac{1 - j\,\mathscr{H}\,\dfrac{h}{c_{33}^{D}}\,(1 + \mathscr{Y}\,\mathscr{E}_t(\omega))}{1 + j\,\mathscr{H}\,\dfrac{h}{c_{33}^{D}}\,(1 + \mathscr{Y}\,\mathscr{E}_t(\omega))} \tag{62}$$

where $\mathscr{H} = \rho_f\, g \tanh(k_0\, h_D)\,, \qquad \mathscr{Y} = (\mathscr{T}_0\,\mathscr{T}_0)\,/\,(h_D\,\|Z_0\|^2)$ (63a,b)

are two purely hydrodynamic, real-valued (positive), quantities. $\mathscr{H}$ is analogous to the specific weight of sea water (also affected by the sea depth), while $\mathscr{Y}$ represents the effect of the vertical structure of the hydrodynamic pressure. Note that the numerical values of these two hydrodynamic quantities satisfy the following estimates

$$\mathscr{H} \sim O(\rho_f\, g) = O(10^4\, Pa\,/\,m) \qquad \text{and} \quad 0 < \mathscr{Y} \le 1 \tag{64a,b}$$

Inequality $\mathscr{Y} \le 1$ is obtained by applying the Cauchy-Schwartz inequality to the functions $Z_0(z)$ and 1, $z \in [-h_D, 0]$.

As is seen from Eq. (62), the reflection coefficient W is dependent only on the following (dimensionless) coefficients

$$\varpi = \mathscr{H}\,\frac{h}{c_{33}^{D}} > 0 \qquad \text{and} \qquad \lambda = \mathscr{Y}\,\mathscr{E}_t(\omega) \in \mathbb{C} \tag{65a,b}$$

which realize the energetic coupling between the three subsystems (hydrodynamic, piezoelectric and external circuit). We shall call these coefficients hydro/piezo/electric compliances. From the definition of quantity $\mathscr{E}_t(\omega)$, Eq. (27), we obtain

$$\mathscr{E}_t(\omega) = \frac{k_t^2\, \Pi}{\Pi^2 + \chi^2} - j\,\frac{k_t^2\, \chi}{\Pi^2 + \chi^2} \tag{66}$$

where $\Pi \sim 1 - k_t^2 - \omega\,(C_0\,/\,M_1)\; X(\omega)$ and $\chi = \omega\,(C_0\,/\,M_1)\; R > 0$

Furthermore, since the external inductance $X(\omega)$ is expected to take small values, we can assume that $\Pi \approx 1 - k_t^2 > 0$. Taking into account Eqs. (64(b)) and (65), and keeping track of the dependence on χ, the real and the imaginary parts of λ are expressed as follows

$$\lambda_R(\chi) = \mathscr{Y}\,\mathrm{Re}_j\left\{\mathscr{E}_t\right\} = \sigma\,\frac{\Pi}{\Pi^2 + \chi^2}\,, \quad \lambda_J(\chi) = \mathscr{Y}\,\mathrm{Im}_j\left\{\mathscr{E}_t\right\} = -\sigma\,\frac{\chi}{\Pi^2 + \chi^2} \tag{68a,b}$$

where $\sigma \equiv \mathscr{Y} k_t^2$, is a partial hydro/piezo/electric compliance. Using the notation introduced above, we can write $|W|^2$ in the form

$$|W|^2(\chi) = \frac{1 + 2\varpi\lambda_J(\chi) + \varpi^2[1+\lambda_R(\chi)]^2 + \varpi^2\lambda_J^2(\chi)}{1 - 2\varpi\lambda_J(\chi) + \varpi^2[1+\lambda_R(\chi)]^2 + \varpi^2\lambda_J^2(\chi)} \equiv \frac{F(\varpi,\lambda_R(\chi),\lambda_J(\chi))}{G(\varpi,\lambda_R(\chi),\lambda_J(\chi))}. \qquad (69)$$

6. Optimization and efficiency of the hydro/piezo/electric harvester

Combining Eqs. (51), (52) with Eq. (69), we readily see that the ratio of the total power taken off the impinging waves over the incident wave power, that is, the efficiency of the hydro/piezo/electric harvester described in Sec. 2, can be expressed as

$$\mathrm{P}^f_{a=0}(\omega,\chi)/\mathrm{P}^f_I(\omega) = 1 - |W(\chi)|^2 \qquad (70)$$

where $\mathrm{P}^f_I(\omega) = \frac{1}{2}\rho_f\, g \left(\frac{H}{2}\right)^2 L_2\,\omega \frac{k_0}{\mu_0}\|Z_0\|^2$ is the incident wave power. Thus, it is clear that the coupling phenomenon between the hydrodynamic wave field, the piezoelectrically vibrating elements and the external electric circuit is solely modeled by $1 - |W(\chi)|^2$. Since in the variable $\chi = \omega\left(C_0/M_1\right) R$, the easily adjustable ohmic resistance R of the external circuit is involved, it is expedient to maximize $1 - |W(\chi)|^2$ (equivalently, the taken-off power) with regard to χ, following the common practice in piezoelectric harvesters (Guyomar *et al.* 2005, Lefeuvre *et al.* 2010). Using the first derivative test, we have to solve the equation

$$d\left[1-|W|^2(\chi)\right]/d\chi = 0 \;\Leftrightarrow\; \left\{\frac{dF}{d\chi}G - F\frac{dG}{d\chi}\right\} = 0 \qquad (71)$$

After some algebraic manipulations, we find that Eq. (71) reduces to

$$\left(\chi^2 + \Pi^2\right)^2\left(\chi^2 - \Pi^2\left(1+\mu^2\right)\right) = 0 \qquad (72)$$

where

$$\mu^2 \equiv \mu^2\left(\frac{\sigma}{\Pi}, \varpi\right) = \frac{\sigma}{\Pi}\left(\frac{\sigma}{\Pi} + 2\right)\frac{\varpi^2}{1+\varpi^2} \qquad (73)$$

That is, Eq. (71) has the double negative root $\chi^2_{1,2} = -\Pi^2$, which is of no importance for our purposes, and the positive root $\chi_3^2 = \Pi^2\left(1+\mu^2\right)$, where

$$\frac{\sigma}{\Pi} = \frac{\mathscr{Y}\, k_t^2}{1-k_t^2 - \omega\left(C_0 / M_1\right) X(\omega)} \approx \frac{\mathscr{Y}\, k_t^2}{1-k_t^2} \tag{74}$$

[The second (simplified) form of σ/Π is valid since the reactance $X(\omega)$ is expected to be much smaller than $(1-k_t^2)/\omega(C_0/M_1) \sim O(10^{12}\Omega)$.]

Thus, the value $\chi = \chi_{\text{opt}} = \omega\left(C_0/M_1\right) R_{\text{opt}} > 0$ which maximizes the taken-off power is given by the formula $\chi_{\text{opt}} = \Pi\sqrt{1+\mu^2}$, which leads to the following optimal external ohmic resistance value R_{opt}

$$R_{opt} = \left(\frac{1-k_t^2}{\omega(C_0/M_1)} - X(\omega)\right)\sqrt{1+\mu^2} \approx \frac{1-k_t^2}{\omega(C_0/M_1)}\sqrt{1+\mu^2} \tag{75}$$

Introducing χ_{opt} in Eq. (69), the following form for the electrically optimized efficiency is obtained

$$1-\left|W\right|_{opt}^2 = \mathscr{W}\left(\frac{\sigma}{\Pi},\varpi\right) =$$

$$= \frac{4\omega\dfrac{\sigma}{\Pi}\sqrt{1+\mu^2\left(\sigma/\Pi,\varpi\right)}\Big/\left(2+\mu^2\left(\sigma/\Pi,\varpi\right)\right)}{1+2\varpi\dfrac{\sigma}{\Pi}\dfrac{\sqrt{1+\mu^2\left(\sigma/\Pi,\varpi\right)}}{2+\mu^2\left(\sigma/\Pi,\varpi\right)}+\dfrac{\varpi^2}{2+\mu^2\left(\sigma/\Pi,\varpi\right)}\left[2+\mu^2\left(\sigma/\Pi,\varpi\right)+2\dfrac{\sigma}{\Pi}+(\dfrac{\sigma}{\Pi})^2\right]} \tag{76}$$

It should be stressed that the optimum value $1-\left|W\right|_{\text{opt}}^2$ is dependent only on the two dimensionless, positive-valued quantities ϖ and σ/Π, which appropriately combine the hydrodynamic, the piezoelectric and the circuit characteristics affecting the energetic coupling of the system. Furthermore, taking into account the definitions of σ and Π, and the facts that $\mathscr{Y} \in (0,1]$ and (for many interesting materials) $k_t^2 \in (0.01, 0.5)$, we easily find that σ/Π ranges (for all realistic situations) from 0 to (approximately) 1.0.

The quantity $1-\left|W\right|_{\text{opt}}^2$ as a function of the two arguments ϖ and σ/Π is shown in Fig. 5. By this figure, it is seen that, for every value of σ/Π, the efficiency of the system is maximized for values of $\varpi \sim O(10^0)$, and that the system absorbs appreciable energy in the range $O(10^{-1}) < \varpi < O(10^1)$.

The dependence of the efficiency $1-\left|W\right|_{\text{opt}}^2$ on σ/Π is monotonically increasing; the higher the value σ/Π the better the efficiency is. Since $\varpi \equiv \mathscr{H}\, h / c_{33}^D$ and $\mathscr{H} \sim O(10^4\, Pa/m)$, it is concluded that the piezoelectric material needed for an efficient harvester would be characterized by $h/c_3^D \sim O(10^{-4} m/Pa)$, having also k_t^2 as higher as possible in order that the parameter σ/Π has a relatively high value.

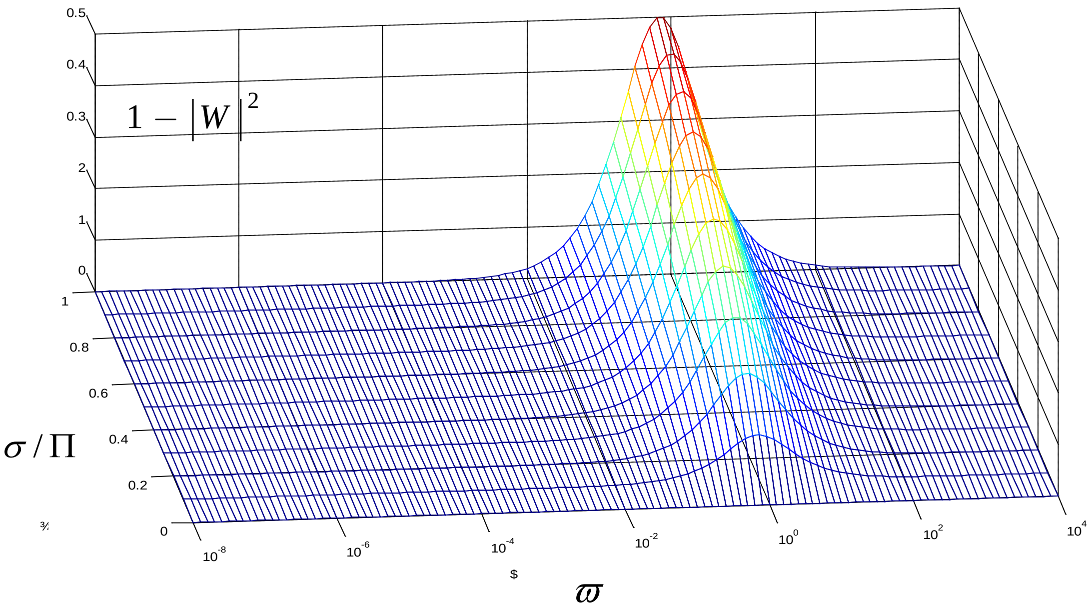

Fig. 5 The efficiency $1-|W|^2_{\text{opt}}$ of the hydro/piezo/electric harvester, as a function of the two dimensionless quantities ϖ and σ/Π

To get a first idea concerning the feasibility of the above requirements in relation with existing materials, we have compiled Table 1, showing the corresponding properties of some piezoelectric materials. From this Table we see that materials do not meet the flexibility requirement for an efficient harvester. An improvement of the flexibility coefficient h/c_{33}^D by (approximately) three orders of magnitude is necessary in order that the piezoelectric sheet absorbs enough energy from the impinging waves.

Table 1 Piezoelectric properties of some common materials, assuming h = 0.1 m

	PZT ceramics	PVDF polymers	1-3 ceramic(PZT)-polymer composites	Cellular polypropylenes	Silicone dow corning HS3 (DEAP)
h/c_{33}^D (m/Pa)	$10^{-13} - 10^{12}$ i)	(1 – 5), 10^{-11} ii), iii)	$10^{-12} - 10^{-11}$ vi)	$O(5\times10^{-8})$ ii)	$O(8\times10^{-7})$ vii)
k_t^2	0.22 – 0.40 i)	0.012 – 0.023 iv), v)	0.25 – 0.42 vi)	$O(3.6\times10^{-3})$ iv)	0.65 vii)

i) Sherman and Butler (2007), Appendix A.5, ii) Bauer and Bauer (2008), Table 6.1, iii) Bloomfield (1994), Table 1, iv) Döring *et al.* (2008), Table 2, v) Splitt (1996), Table 1, vi) Smith & Auld (1991), Figs. 3 and 4., vii) Carpi *et. al.* (2008), Ch. 4, Table 4.1

7. Conclusions

In the present work, a sea-wave energy absorption system using, as energy harvester, an active zone of thickness-oscillating piezoelements installed on a vertical cliff is studied. The considered active zone is formed by parallel-connected vertical arrays, each one consisting of piezoelements connected in series. The active zone is then connected to an external AC electric circuit modeling the consumer load. The analysis of the system performed was restricted to the linear theory for both piezoelectric and hydrodynamic subproblems, and has led to a closed form efficiency coefficient, optimized with respect to the *external resistive load*. The main conclusions drawn from the obtained solution and its numerical study can be summarized as follows:

- There are two dimensionless parameters governing the efficiency of the system, namely σ / Π and ϖ. Each of these dimensionless parameters is the product of two factors, one of piezoelectric and one of hydrodynamic nature.
- System's efficiency $\mathscr{W}$ is strongly affected by the value of parameter ϖ. In fact, $\mathscr{W}$ exhibits a resonance pattern around the value of $\varpi = 1$.
- System's efficiency $\mathscr{W}$ is mildly dependent on the parameter σ / Π, exhibiting a monotonically increasing behavior.
- The optimal resistive load takes a large value since $R_{opt} \propto 1/C \sim O(10^{12}\Omega)$. Similarly large values of optimal resistance have been obtained by Guyomar *et. al.* (2005).

Evaluating the feasibility of the studied system, we state the following:

- The elastic flexibility h / c_{33}^{D} of the common piezoelectric materials (*see* Table 1) is not large enough for parameter ϖ to reach the resonant value. Clearly, it is a question towards the material scientists if the advances in material manufacturing could lead to electroactive materials exhibiting large flexibility and appreciable coupling factor.
- Dielectric Electroactive Polymers (DEAP), could offer a solution to this problem. In cases of using such materials the modeling of the whole system should be adapted to the physics of DEAP.

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Appendix. Nomenclature

Latin Symbols

$A(\omega)$	coeff. in the piezoelectric solution defined by Eq. (28)
a	piezoelectric force factor of one piezoelement
C_0	clamped capacitance of one piezoelement
C_n	Coeffs. of evanescent sea waves
c_{ij}^E	elastic stiffness coeffs under constant electric intensity
c_{ij}^D	elastic stiffness coeffs under constant electric displacement
$D_i(x_3;\omega)$	electric displacement components
$E_i(x_3;\omega)$	electric intensity components
$e_i(x_3;\omega)$	mechanical strain components
$\mathscr{E}_t(\omega)$	generalized energy conversion factor; see Eq. (27)
g	acceleration due to gravity
$H/2$	incident wave amplitude
h	thickness of one piezoelement
h_D	sea depth in front of the vertical cliff
$\mathscr{H}$	hydrodynamic coefficient; see Eq. (63a)
$I(\omega)$	electric current
$\mathscr{T}_n$	integrals of $Z_n(z)$'s over h_D
j	the imaginary unit
k_n	eigenvalues of the water wave problem (wavenumbers)
k_t^2	piezoelectric energy conversion (or coupling) factor
L_2	length of the active zone
$\ell_1,\ \ell_2$	transverse dimensions of one piezoelement
M_1	number of piezoelements in the vertical direction
M_2	number of piezoelements in the lateral direction
$\vec{n}$	outward normal unit vector on the sea volume boundaries
$\mathrm{P}_\infty^f(\omega)$	net sea wave power flow at a liquid section away from the vertical cliff
$\mathrm{P}_a^f(\omega)$	net sea wave power flow at the liquid section $x=a$
$\mathrm{P}_{c\ell}^{\mathrm{piezo}}(\omega)$	net power flowing through the piezoelectric sheet
$\mathrm{P}_Z(\omega)$	net electric power consumed by the external circuit
$p(x,z;\omega)$	hydrodynamic pressure field in the fluid
R	total resistance of the external AC electric circuit
S	surface of one piezoelement
T	period of the oscillating system
t	time variable
$u_3(x_3;\omega)$	mechanical displacement
$(x\,y\,z)$	Cartesian axes for the hydrodynamic problem (global Cartesian axes)
$(x_1\,x_2\,x_3)$	Cartesian axes for each piezoelement (local Cartesian axes)
$X(\omega)$	total reactance of the external AC electric circuit
$V_0(\omega),\ V_1(\omega)$	voltages at the clamped and the free surface of a piezoelement
W	reflection coefficient of sea waves
$\mathscr{W}$	electrically optimized efficiency of the system; see Eq. (60)
$\mathscr{Y}$	hydrodynamic coefficient; see Eq. (63b)
$Z(\omega)$	total impedance of the external AC electric current
$Z_n(z)$	eigenfunctions of the water wave problem
$\lVert Z_n(z)\rVert$	norm of $Z_n(z)$ in the space $L^2(-h_D,0)$

Greek Symbols

$\alpha\beta,\ \gamma\delta$	surfaces of one piezoelement
$\Delta V(\omega)$	voltage difference between the surfaces of a piezoelement
$\Delta\Phi^f(x,z;\omega)$	Laplacian of the hydrodynamic potential
ϵ_{ij}	piezoelectric stress coefficients
ε_{ij}^S	dielectric permittivity coefficients under constant strain
$\lambda,\lambda_R,\lambda_J$	see Eqs. (65b) and (68a,b)
$\mu_0=\omega^2/g$	sea wave frequency parameter
μ^2	see Eq. (73)
Π	see Eq. (67a)
ϖ	see Eq. (65a)
ρ_b	mass density of piezoelectric material
ρ_f	mass density of sea water
$\sigma_i(x_3;\omega)$	mechanical stress components

$\sigma = \mathscr{Y} k_t^2$ partial hydro/piezo/electric compliance,

$\Phi^{e\ell}(x_3;\omega)$ electric potential inside each piezoelement

$\Phi^f(x, z;\omega)$ hydrodynamic velocity potential field

$\Phi_I^f(x, z;\omega)$ velocity potential of the incident wave

$\Phi_R^f(x, z;\omega)$ velocity potential of the reflected wave

$\Phi_n^f(x, z;\omega)$ velocity potential of the evanescent waves

χ see Eq. (67a)

Ω, Ω_0 sea volume; see Fig. 1

$\partial\Omega_{c\ell}$, $\partial\Omega_{F_0}$, $\partial\Omega_{\Pi}$, $\partial\Omega_{\infty}$ boundaries of the sea volume; see Fig. 1

ω frequency of the oscillating system

$\tilde{\omega}$ nondimensionalized frequency; Eq. (13b)