Structural Monitoring and Maintenance, *Vol. 5, No. 3 (2018) 379-390*
DOI: https:// doi.org/10.12989/smm.2018.5.3.379

Prediction of unmeasured mode shapes and structural damage detection using least squares support vector machine

Seyed Sina Kourehli*

Department of Civil Engineering, Ahar Branch, Islamic Azad University, Ahar, Iran

(Received June 28, 2018, Revised July 27, 2018, Accepted August 1, 2018)

Abstract. In this paper, a novel and effective damage diagnosis algorithm is proposed to detect and estimate damage using two stages least squares support vector machine (LS-SVM) and limited number of attached sensors on structures. In the first stage, LS-SVM1 is used to predict the unmeasured mode shapes data based on limited measured modal data and in the second stage, LS-SVM2 is used to predicting the damage location and severity using the complete modal data from the first-stage LS-SVM1. The presented methods are applied to a three story irregular frame and cantilever plate. To investigate the noise effects and modeling errors, two uncertainty levels have been considered. Moreover, the performance of the proposed methods has been verified through using experimental modal data of a mass-stiffness system. The obtained damage identification results show the suitable performance of the proposed damage identification method for structures in spite of different uncertainty levels.

Keywords: unmeasured mode shapes; two stages method; LS-SVM; sparse sensors; uncertainty levels

1. Introduction

Damage detection and estimation in engineering structures during their service life has received increasing attention in the last few decades. One of the many nondestructive evaluation methods is based on the change of vibration parameters with a change in the structural properties (Carden and Fanning 2004, Fan and Qiao 2011).

As the number of sensors used to measure modal data is normally limited and usually are less than the number of DOFs in the finite element model, either the model reduction method should be used to match with incomplete measured mode shapes or the measured mode shapes must be expanded to the dimension of the analytical mode shapes (Kourehli *et al.* 2012). Therefore, it is essential to develop algorithms for damage diagnosis using modal data obtained by a limited number of sensors, which means using an incomplete set of modal data (Hosseinzadeh *et al*. 2014). Some researchers used the mode shape expansion methods for structural damage detection (Chen and Bicanic 2000, Au *et al*. 2003) in which others used the model reduction methods (Kourehli *et al*. 2013, Li *et al*. 2008, Kourehli 2015, Rasouli *et al*. 2014). Also, Goh *et al.* (2013) presented an approach combines a two-stage ANN model and statistical method to detect damage based on the limited number of sensors with consideration of uncertainties.

*Corresponding author, Assistant Professor, E-mail: s-kourehli@iau-ahar.ac.ir

http://www.techno-press.org/?journal=smm&subpage=7 ISSN: 2288-6605 (Print), 2288-6613 (Online)

The least squares support vector machine (LS-SVM) is an advanced version of the standard SVM, which was first introduced by Suykens *et al.* (1999). LS-SVM has been widely used in different fields of engineering as robust and promising method for classification and function estimation. Tang *et al*. (2006) proposes an online sequential weighted Least Squares Support Vector Machine (LS-SVM) technique to identify the structural parameters and their changes when vibration data involve damage events. The proposed method is capable of tracking abrupt or slow time changes of the system parameters from which the damage event and the severity of the structural damage can be detected and evaluated. Also, Xie (2010) developed improved LS-SVM combined with Hilbert transform to extract the characteristics of monitoring signals and detect damage locations for the composite laminated plate. In other work, Shyamala *et al.* (2018) presented numerical and experimental investigation for damage detection in FRP composite plates using SVM. In this study, a rectangular fiber reinforced plastic composite plate investigated both numerically and experimentally to observe the efficiency of the SVM algorithm for damage detection. Also, Xie (2010) study Fuzzy Least Square Support Vector Machine (FLS-SVM) combining Fuzzy Logic with LS-SVM, and a real-coded Quantum Genetic Algorithm (QGA) is applied to optimize parameters of FLS-SVM. Then, the method of FLS-SVM integrated QGA is used to detect damages for fiber smart structures. The proposed method of FLS-SVM integrated QGA is effective and efficient for structural damage detection. In other work, Cao *et al*. (2016) proposed a damage identification method for Da-Sheng-Guan (DSG) high-speed railway truss arch bridge using fuzzy clustering analysis.

In this paper, a new two stage method is introduced to detect and estimate damage in structures using LS-SVM. In this method LS-SVM is used to predict the unmeasured mode shapes data and predicting the damage location and severity. The presented method for damage identification has been applied to two numerical examples, namely a three-story plane frame and cantilever plate. In addition, the experimental data from the vibration test of a mass-stiffness system are used in the present approach. The highlights of the proposed method are:

- High performance of least squares support vector machine in prediction of unmeasured mode shapes.
- High accuracy of LS-SVM in detecting damage in structural elements.
- The proposed method is robust in spite of different noise levels and perturbations of stiffness and mass at different structural elements.

2. Proposed method

The proposed multi-stage damage detection method is divided into LS-SVM1 and LS-SVM2 as shown in Figs. 1 and 2. LS-SVM1 is used to relate the measured mode shapes and the mode shapes values at unmeasured points. Thus, the input parameters for LS-SVM1 are the measured nodal points in the mode shape for the first two modes of mode shapes and, while the outputs are the unmeasured nodal points in the mode shape for the first two modes of mode shapes as shown in Fig. 1.

LS-SVM 2 receives information from LS-SVM1 to determine the damage location and severity. The estimated mode shape values at unmeasured points from LS-SVM1 are combined with the measured mode shapes and frequencies to form a set of input variables for LS-SVM2. Then LS-SVM2 is used to predict the damage values of each element in the structure.

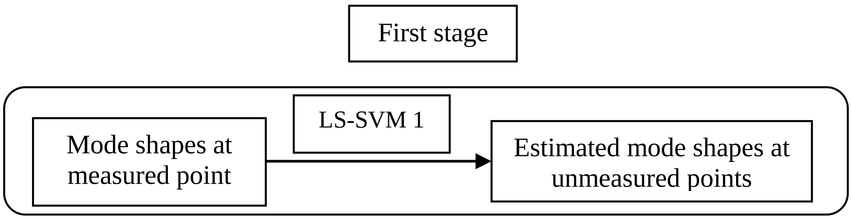

Fig. 1 Flowchart of the first stage LS-SVM1

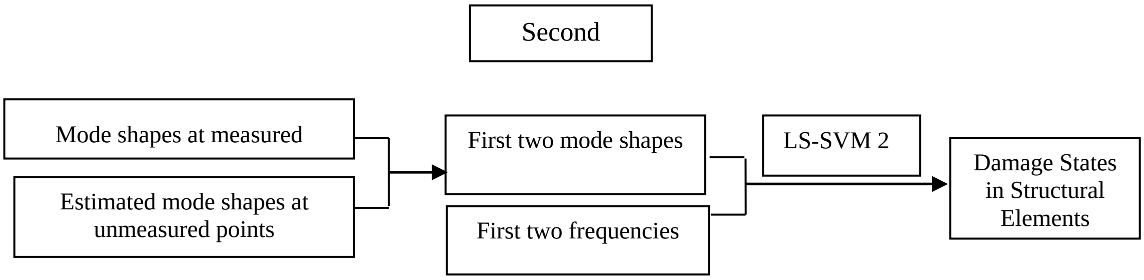

Fig. 2 Flowchart of the second stage LS-SVM2

Table 1 Obtained hyper-parameters for different examples

	Obtained hyper-parameters	Three story irregular frame	Cantilever Plate	Experimental study
First stage	Gamma(γ)	1000	127×10^6	17.617
	$\sin 2\sigma^2$	11	17.451	222.291
Second stage	Gamma(γ)	920×10^{12}	2.146×10^{17}	10.197
	$\sin 2\sigma^2$	322×10^2	853×10^2	28.221

Several factors such as erosions, cracks, and holes in a structure can reduce structure stiffness and they may be considered as damages. Stiffness reduction can be simulated by reducing the modulus of elasticity.

$$E_e^d = (1 - d_e) E_e \quad , 0 \leq d_e \leq 1 \tag{1}$$

Where E_e and E_e^d are respectively the modulus of elasticity of the e^{th} undamaged and the damaged elements. d_e is a parameter between *0* and *1*, which can respectively express the undamaged and fractured elements.
In this paper, the LS-SVM method has been used to structural damage detection and estimation based on incomplete measured mode shapes and frequencies. Also, the LS-SVMlab1.8 code package was added into the Matlab toolbox (2013), which used the interface provided by LSSVM to realize the regression function. There are two parameters that need to be chosen in the LS-SVM model, which are the bandwidth of the Gaussian RBF kernel "σ" and the regularization parameter "γ". Hyper-parameters (σ^2,γ) have a great influence on the performance of the resulting LS-SVM (Kourehli 2016). Also, the obtained hyper-parameters for different examples are summarized in Table 1.

3. Numerical examples

In this section, the efficiency and effectiveness of the proposed methods is evaluated through some numerically simulated damage identification tests. A three story irregular frame and cantilever plate are chosen with two different scenarios of damage for each of them for the purpose.

3.1 Three story irregular frame

A three-story plane steel frame as illustrated in Fig. 3 with finite-element model consists of eleven elements (seven columns and four beams) and seven free nodes are considered. The numerical studies are carried out within the MATLAB (2016) environment, which is used for the solution of finite element problems. For the considered steel frame, the material properties of the steel include Young's modulus of E=200 GPa, mass density of ρ=7800 kg/m^3. The mass per unit length, moment of inertia, and cross-sectional area of the columns are: m=127 kg/m, I=3.082 $\times 10^{-4}$ m^4 and A=1.61 $\times 10^{-2}$ m^2, respectively; for the beams are: m=134 kg/m, I=3.666 $\times 10^{-4}$ m^4 and A=1.71 $\times 10^{-2}$ m^2. Also, the damage severity in each element is given by the reduction factor listed in Table 2.
In this case, only the first 11 DOFs selected as measured DOFs in the process of damage detection and quantification.

Table 2 Damage scenarios for three story irregular frame

Scenario 1		Scenario 2	
Element 5	0.2	Element 1	0.2
Element 8	0.2	Element 4	0.2
		Element 7	0.2
		Element 11	0.2

Table 3 Statistics for training and testing for two story plane frame

		First stage		Second stage	
	Samples	MSE	R	MSE	R
Training	1750	1.405×10^{-7}	0.9991	3.243×10^{-7}	0.9991
Testing	194	1.231×10^{-6}	0.9961	1.382×10^{-6}	0.9992

Table 4 The obtained results in the first stage for three story irregular frame

Modes	Scenario 1			Scenario 2		
	Measured	Unmeasured	Estimated	Measured	Unmeasured	Estimated
FIRST MODE	0.26242	0.00930	0.00930	0.26015	0.011431	0.011429
	0.72087	0.02800	0.02801	0.69932	0.037539	0.037513
	0.99999	0.00353	0.00353	0.99983	0.003652	0.003655
	0.26062	0.04379	0.04374	0.25899	0.043662	0.04369
	0.72070	0.00762	0.00762	0.69877	0.006868	0.006871
	1.00000	0.06264	0.06262	1.00000	0.067163	0.067164
	0.25819	0.00858	0.00858	0.25658	0.008008	0.008011
	0.00507	0.02784	0.02783	0.00649	0.027997	0.027990
	0.07561	0.00155	0.00155	0.06152	0.001546	0.001541
	0.00834	0.03553	0.03548	0.01052	0.035251	0.035399
	0.06203			0.05806		
SECOND MODE	0.96070	0.03008	0.03008	0.93900	0.03231	0.032308
	0.38754	0.15876	0.15881	0.41041	0.19138	0.191272
	1.00000	0.01532	0.01532	0.99969	0.01376	0.013768
	0.96445	0.04400	0.04404	0.93941	0.03271	0.032707
	0.38660	0.03194	0.03195	0.41067	0.02585	0.025858
	0.999871	0.19485	0.19480	1	0.17616	0.176194
	0.964145	0.037731	0.037744	0.938585	0.032316	0.032321
	0.011244	0.162411	0.162317	0.012411	0.161904	0.161868
	0.083632	0.003413	0.003414	0.07225	0.003196	0.003185
	0.024336	0.159004	0.158865	0.027268	0.157765	0.158243
	0.206335			0.195959		

To generate the training patterns, a number of structures with different modal properties, using damage severity equal to 0, 20% for all elements were considered. The total number of combinations of the assigned damage severities is 2048. After training, the mean-square error (MSE) and Correlation Coefficient (R) of the measured and predicted damage severity values are determined. According to Table 3, the low value of MSE and high value of R achieved using the proposed method.

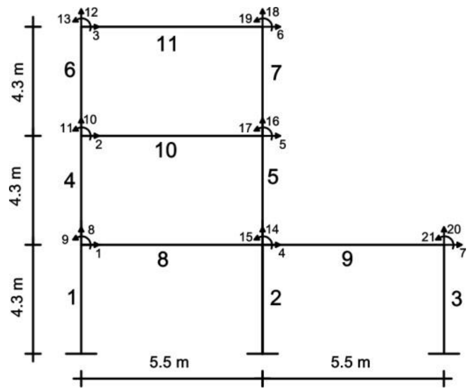

Fig. 3 Three-story plane frame with the finite element model

Table 5 Two uncertainty levels considered for three-story plane steel frame

Uncertainty Level 1 (UL1)		
Noise level in frequencies (%)	Perturbations (%) of stiffness at elements no. 3, 7	Perturbations (%) of mass at elements no. 6, 9
2	2	2
Uncertainty Level 2 (UL2)		
Noise level in frequencies (%)	Perturbations (%) of stiffness at elements no. 1, 4, 10	Perturbations (%) of mass at elements no. 2, 5, 11
1	1	1

Also, Table 4 shows the obtained results in the first stage to predict unmeasured mode shapes from measured mode shapes. It can be seen that the proposed algorithm has a good performance in predicting unmeasured mode shapes.

To investigate the noise effects and modeling errors, two uncertainty levels have been considered.

Table 5 shows the noise level and perturbations of stiffness and mass at different elements for three-story plane steel frame.

Using the complete modal data from the first-stage, damage location and severity is predicted in the second stage. Fig. 4 shows the identified damaged elements using the complete modal data from the first-stage, which may be noisy or noise free. It can be seen that the damage severity and locations can be obtained, for two different scenarios considered.

3.2 Cantilever plate

A cantilever plate as illustrated in Fig. 5 with finite element model consists of 9 elements and 16 nodes are considered. The thickness of considered plate is t=0.1 m and the material properties include Young's modulus of E=20 GPa, mass density of ρ=2400 kg/m^3 and Poisson's ratio of μ=0.2. Also, the damage severity in each element is given by the reduction factor listed in Table 6.

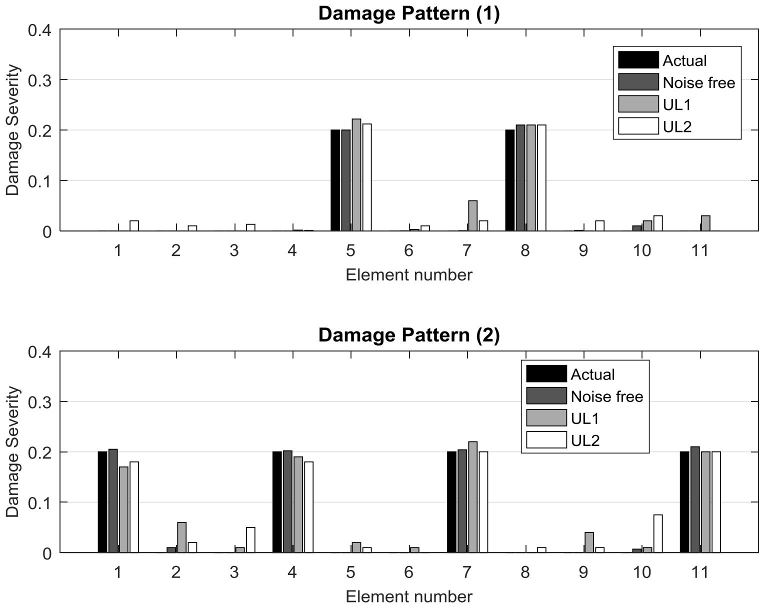

Fig. 4 Results of second stage for the three-story plane steel frame with two uncertainty level

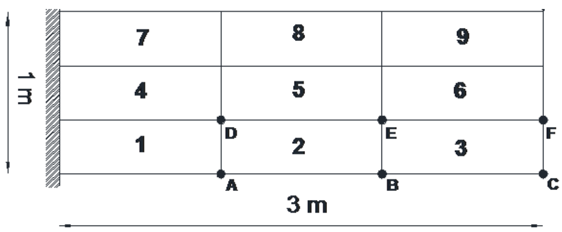

Fig. 5 The cantilever plate with finite element model

The master DOFs consist of eighteen DOFs related to nodes shown by A, B, C, D, E and F in Fig. 5. To generate the training patterns, a number of structures with different dynamic responses, using damage severity equal to 0, 20, 40% and 0, 20% for elements numbered 2,4,6,8 and elements numbered 1,3,5,7,9 ; respectively were considered. The total number of combinations of the assigned damage severities is 2592. The results were used for training patterns.

Table 6 shows the LS-SVM performance in training and testing stages. According to Table 7, the low value of MSE and high value of R achieved using the proposed method.

Table 8 shows the obtained results in the first stage to predict unmeasured mode shapes data based on limited measured modal data. The obtained results show high accuracy of LS-SVM1 in predicting unmeasured mode shapes.

Table 9 shows the noise level and perturbations of stiffness and mass at different elements for cantilever plate.

Table 6 Damage scenarios for cantilever plate

Scenario 1		Scenario 2	
Element 1	0.2	Element 4	0.2
Element 2	0.2	Element 5	0.1
		Element 8	0.3
		Element 9	0.3

Table 7 Statistics for training and testing for cantilever plate

		First stage (Predicted unmeasured mode shapes)		Second stage (Predicted damage)	
	Samples	MSE	R	MSE	R
Training	2333	2.325×10^{-11}	0.9995	7.303×10^{-9}	0.9993
Testing	259	2.200×10^{-6}	0.9999	1.476×10^{-7}	0.9991

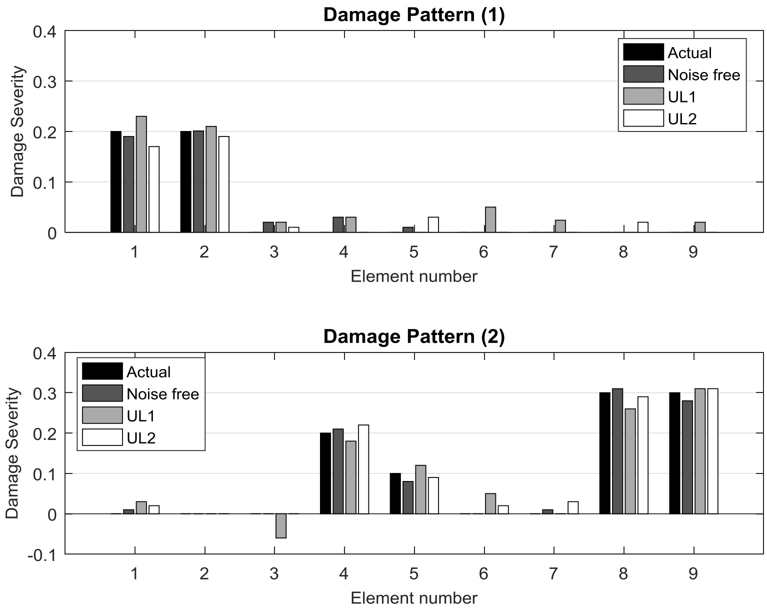

Fig. 6 Results of second stage for the cantilever plate with two uncertainty level

Fig. 6 shows the identified damaged elements in the second stage. It can be seen that the damage severity and locations can be obtained, for two different patterns in spite of modeling errors and noisy data.

Table 8 The obtained results in the first stage for cantilever plate

	Scenario 1			Scenario 2		
	Measured	Unmeasured	Estimated	Measured	Unmeasured	Estimated
FIRST MODE	0.1587147	0.1625828	0.1625828	0.1562249	0.1594925	0.1594896
	0.0216016	0.0074607	0.0074607	0.0209635	0.0079472	0.0079751
	0.2964876	0.2958869	0.2958869	0.2925419	0.289717	0.2897538
	0.5410871	0.5402413	0.5402413	0.5358789	0.5375195	0.5375228
	0.0032924	0.0068526	0.0068526	0.007599	0.0038257	0.0038061
	0.4402829	0.4358805	0.4358806	0.4399617	0.4402305	0.4401827
	1	0.9972381	0.9972381	0.9982976	0.9998297	0.9998371
	0.0030135	0.0054521	0.0054521	0.0032748	0.0012433	0.0008458
	0.4685491	0.4669782	0.4669782	0.4732841	0.4730261	0.4730449
	0.1629444	0.157568	0.1575679	0.1601597	0.154185	0.1541671
	0.005406	0.0240511	0.0240512	0.0039338	0.0254015	0.0254518
	0.2966195	0.2944407	0.2944407	0.2934713	0.2842862	0.2841719
	0.5415561	0.5371275	0.5371275	0.5377021	0.5353225	0.5353442
	0.00109	0.0112638	0.0112638	0.0027708	0.0087145	0.0086401
	0.4371621	0.4371668	0.4371668	0.4390705	0.4428903	0.4429495
	0.99883	0.9952433	0.9952433	0.9992338	1	1.0000115
	0.0040559	0.0064787	0.0064787	0.002314	8.85E-05	0.0005025
	0.4669947	0.4682374	0.4682374	0.4723474	0.47527	0.4752832
SECOND MODE	0.452307	0.309814	0.309814	0.3058353	0.3561674	0.3561122
	0.2391599	0.1855148	0.1855154	0.0539261	0.0967617	0.096112
	0.4626645	0.3084385	0.3084386	0.3116405	0.3833814	0.3831923
	0.4932449	0.2274891	0.2274891	0.228464	0.3291348	0.3291793
	0.4561087	0.3475082	0.3475075	0.0913759	0.2046147	0.2034814
	0.4639169	0.5196231	0.5196232	0.5247805	0.4969716	0.4964054
	0.2923311	0.593475	0.593475	0.6147933	0.4952082	0.4951901
	0.4639429	0.4288504	0.4288505	0.1641252	0.2023413	0.20125
	0.9407582	0.9703894	0.9703894	1	0.9884993	0.9887563
	0.3765148	0.2525579	0.252558	0.3273329	0.3905398	0.3904776
	0.214281	0.1595515	0.1595511	0.0754331	0.1092886	0.1082815
	0.3856776	0.2494465	0.2494467	0.3342125	0.4350142	0.4352223
	0.3517346	0.1208383	0.1208382	0.2695345	0.4065778	0.4066238
	0.3977215	0.2900583	0.2900562	0.1526402	0.2657235	0.2677834
	0.4902768	0.5584064	0.5584066	0.5115562	0.4760299	0.4772386
	0.4460284	0.7328366	0.7328365	0.5585638	0.4241343	0.4240726
	0.454008	0.4125072	0.4125087	0.1775026	0.2178977	0.2176173
	0.9522097	1	1	0.9880886	0.9952051	0.9953998

Table 9 Two uncertainty levels considered for cantilever plate

Uncertainty Level 1 (UL1)		
Noise level in frequencies (%)	Perturbations (%) of stiffness at elements no. 2, 9	Perturbations (%) of mass at elements no. 3, 8
2	2	2
Uncertainty Level 2 (UL2)		
Noise level in frequencies (%)	Perturbations (%) of stiffness at elements no. 1, 3, 7	Perturbations (%) of mass at elements no. 4, 5, 6
1	1	1

4. Experimental validation study

In the previous section, the proposed method was demonstrated through some numerical examples. However, it is useful to examine the experimental performance of the proposed method, using measured data from an experimental study. Therefore, in this section, the performance of the proposed damage detection method is verified thorough the first two experimental mode shapes measured from an 8-DOF (degrees of freedom) spring–mass system tested by Duffey *et al*. (2001). This system was designed by Los Alamos National Laboratory to study the effectiveness of various vibration based damage identification techniques. The system is formed by eight translating masses connected by springs, as it is shown in Fig. 7. For the undamaged case, the stiffness of all springs is 56.7 kN/m. For the damaged case, the stiffness of the fifth spring (between masses 5, 6) has reduced by 14%. The weight of the first mass is 559.3 grams and the weight of Masses 2 to 8 is 419.4 grams.

Based on the measured data, two first frequencies and mode shapes which are measured only in the last 5 DOFs, were utilized for damage detection and quantification.

To generate the training patterns, a number of structures with different modal properties, using damage severity equal to 0, 15, 30% for all elements were considered. The total number of combinations of the assigned damage severities is 2187. Table 10 shows the value of MSE and R achieved using the proposed method.

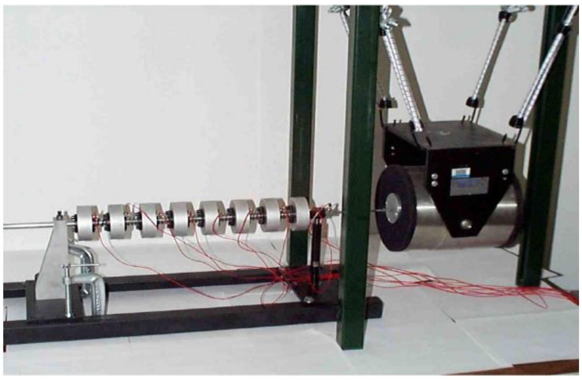

Fig. 7 Experimental 8 DOFs system with excitation shaker attached (Duffey *et al*. 2001)

Table 10 Statistics for training and testing for two story plane frame

		First stage		Second stage	
	Samples	MSE	R	MSE	R
Training	1969	3.218E -3	0.6193	2.705E -3	0.8862
Testing	218	0.04	0.6073	3.254E -3	0.8527

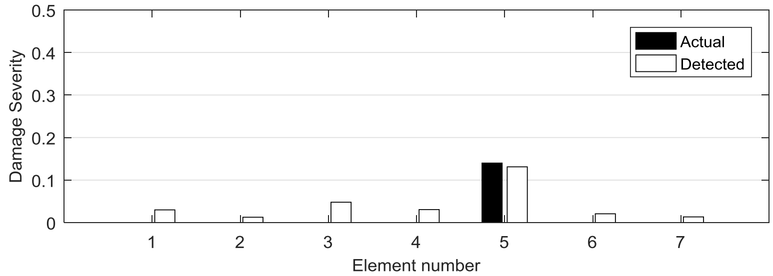

Fig. 8 The obtained result for the experimental 8 DOFs system

Fig. 8 shows the capability of the proposed method for detection and quantification of the damage in the experimental 8 DOFs system. The obtained results indicated that the proposed method can be characterized as a robust and viable method for damage detection and quantification of actual structures.

5. Conclusions

In this paper, a two stage damage detection and estimation method was introduced. For this purpose, an algorithm using incomplete modal data and LS-SVM was presented. Least squares support vector machine (LS-SVM) is used to predict the unmeasured mode shapes data and detect damage location and severity using the complete modal data from the first-stage. In numerical examples, the proposed method is applied to a three-story irregular frame and cantilever plate. Also, an experimental validation using a mass-stiffness system has been done. The results show that the presented method is sensitive to the location and severity of structural damage.

References

Au, F.T.K., Cheng, Y.S., Tham, L.G. and Bai, Z.Z. (2003), “Structural damage detection based on a microgenetic algorithm using incomplete and noisy modal test data”, *J. Sound Vib.*, **259**(5), 1081-1094.

Available at http://www.esat.kuleuven.be/sista/lssvmlab/

Cao, B., Ding, Y., Zhao, H. and Song, Y. (2016), “Damage identification for high-speed railway truss arch bridge using fuzzy clustering analysis”, *Struct. Monit. Maint.*, **3**(4), 315-333.

Carden, E.P. and Fanning, P. (2004), “Vibration based condition monitoring: a review”, *Struct. Health Monit..*, **3**, 355-377.

Chen, H.P. and Bicanic, N. (2000), “Assessment of damage in continuum structures based on incomplete modal information”, *Comput. Struct*., **74**(5), 559-570.

Duffey, T.A., Doebling, S.W., Farrar, C.R., Baker, W.E. and Rhee, W.H. (2001), “Vibration-based damage identification in structures exhibiting axial and torsional response”, *J. Vib. Acoust*., **123(**1), 84-92.

Fan, W. and Qiao, P. (2011), “Vibration based damage identification methods: a review and comparative study”, *Struct. Health Monit*., **10**, 83-111.

Goh, L.D., Bakhary, N., Rahman, A.A. and Ahmad, B.H. (2013), “Application of neural network for prediction of unmeasured mode shape in damage detection”, *Adv. Struct. Eng*., **16**, 99-113.

Hosseinzadeh, A.Z., Bagheri, A., Ghodrati Amiri, G. and Koo, K.Y. (2014), “A flexibility-based method via the iterated improved reduction system and the cuckoo optimization algorithm for damage quantification with limited sensors”, *Smart Mater. Struct*., **23**(4), 045019.

Kourehli, S.S. (2016), “Structural damage diagnosis using incomplete static responses and LS-SVM”, *Inverse Problems in science and engineering*, DOI: 10.1080/17415977.2016.1169277.

Kourehli, S.S. (2016), “LS-SVM regression for structural damage diagnosis using the iterated improved reduction system”, *Int. J. Str. Stab. Dyn*., **16**(6), 1550018. DOI: 10.1142/S0219455415500182.

Kourehli, S.S. (2015), “Damage assessment in structures using incomplete modal data and artificial neural network”, *Int. J. Str. Stab. Dyn*., **15**(6), 1450087. DOI: 10.1142/S0219455414500874.

Kourehli, S.S., Ghodrati Amiri, G., Ghafory-Ashtiany, M. and Bagheri, A. (2012), “Structural damage detection based on incomplete modal data using pattern search algorithm”, *J. Vib. Control*, **19**, 821-833.

Kourehli, S.S., Bagheri, A., Ghodrati Amiri, G. and Ghafory-Ashtiany, M. (2013), “Structural damage detection using incomplete modal data and incomplete static response”, *J. Civil Eng.-KSCE*, **17**(1), 216-223.

Li, H., Wang, J. and Hu, S.L.J. (2008), “Using incomplete modal data for damage detection in offshore jacket structures”, *Ocean Eng*., **35**(17-18), 1793-1799.

MATLAB (2016), Matlab User Manual, Mathwork Inc. Lowell, MA, U.S.A.

Rasouli, A., Ghodrati Amiri, G., Kheyroddin, A., Ghafory-Ashtiany, M. and Kourehli, S.S. (2014), “A new method for damage prognosis based on incomplete modal data via an evolutionary algorithm”, *Eur. J. Environ. Civil Eng*., **18**(3), 253-270.

Shyamala, P., Mondal, S. and Chakraborty, S. (2018), “Numerical and experimental investigation for damage detection in FRP composite plates using support vector machine algorithm”, *Struct. Monit. Maint*., **5**(2), 243-260.

Suykens, J.A.K. and Vandewalle, J. (1999), “Least squares support vector machine classifiers”, *Neural Process. Lett*., **9**(3), 293-300.

Tang, H.S., Xue, S.T., Chen, R. and Sato, T. (2006), “Online weighted LS-SVM for hysteretic structural system identification”, *Eng. Struct*., **28**(12), 1728-1735.

Xie, J. (2010), “Improved least square support vector machine for structural damage detection”, *Proceedings of the 2nd International Conference on Computer Engineering and Technology*, 6:V6-237-V6-240.

Xie, J.H. (2010), “Structural damage detection based on fuzzy LS-SVM integrated quantum genetic algorithm”, *Appl. Mech. Mater.*, **20-23**, 1365-1371.

TY