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Experimental investigation on vortex induced forces of oscillating cylinder at high Reynolds number

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Abstract. Hydrodynamic characteristics of a bluff cylinder oscillating along transverse direction in steady flow were experimentally investigated at Reynolds number of 2×10^5.The effects of non-dimensional frequency, oscillating amplitude and Reynolds number on drag force, lift force and phase angle are studied. Vortex shedding mechanics is applied to explain the experimental results. The results show that explicit similarities exist for hydrodynamic characteristics of an oscillating cylinder in high and low Reynolds number within subcritical regime. Consequently, it is reasonable to utilize the test data at low Reynolds number to predict vortex induced vibration of risers in real sea state when the Reynolds numbers are in the same regime.

Keywords: vortex induced vibration, forced oscillation experiment, subcritical regime, high Reynolds number

1. Introduction

With the development of offshore oil industry in deep sea, the length of the riser is increasing from a few hundred meters to several kilometers. The security of the riser is of extreme importance for oil industry as well as marine environment. Therefore, it has been an essential issue in ocean engineering to accurately predict the hydrodynamic force and response of the riser in real marine environment and optimize the design of the structure.

Fatigue damage caused by dynamic response of the riser plays pivotal role in the design and the primary reason for the damage is the vibration induced by vortex shedding behind the riser in current. Uncertainty of vortex induced vibration (VIV) leads to larger design safety factor and hence higher costs in construction. In addition, small deformation analysis is no longer applicable to flexible riser with extremely large aspect ratio, which makes the prediction of VIV even more difficult.

In recent years, lots of researches on VIV of slender marine structures have been conducted (Vandiver 2003, Larsen *et al.* 2005, Chaplin *et al.* 2005). However, to understand the mechanism of VIV and further predict it more accurately, the hydrodynamic characters of a small segment of riser, i.e., a rigid cylinder, become the key issue. Specifically, forced oscillation experiment and

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self-excited vibration experiment are involved. In forced vibration, the cylinder is driven externally with certain frequency and amplitude, rather than its wake. The primary purpose is to study the force acting on the cylinder and the energy transfer between the moving cylinder and the flow, and meanwhile obtain detailed hydrodynamic coefficients. In self-excited vibration, the moving cylinder is driven entirely by the interaction between the wake and the body, and hence the wake structure and kinematics of the cylinder are the focus of research. Although the two kinds of experiment are conducted with different purpose, there are striking similarities in the wake modes (2S, 2P) and lift force phase angle for the forced and free oscillating cylinders, and in both cases the wake undergoes a transition as lock-in occurs (Carberry *et al.* 2005). And the forced oscillation test results can be used to predict the response of an elastically mounted cylinder (Sarpkaya 2004).

The pioneering experiments of Bishop and Hassan (1964) with cylinders subjected to forced oscillations drew some valuable conclusions that: the Strouhal number remains nearly constant of 0.2 in subcritical regime and lock-in is manifested when oscillating frequency approaches the shedding frequency of the cylinder. Since then, representative scholars including Mercier (1973), Sarpkaya (1978), Staubli (1983), Gopalkrishnan (1993), Carberry (2001, 2003, 2004) and Morse（2009）have performed forced oscillation test of rigid cylinder.

In data processing, Sarpkaya decomposed the lift force into two parts, one in phase with velocity and one in phase with acceleration. Such decomposition well explains the energy transfer between the fluid and the oscillating body and hence serves as the basis for the majority of later research.

Subsequently, Gopalkrishnan carried out thorough forced oscillation experiments, obtaining the hydrodynamics coefficients which have been applied in software SHEAR7 and VIVANA. However, the Reynolds number of the experiment was still in the range of subcritical regime (10^4), which is so small that the accuracy of the prediction remains unjustified.

To gain insight on the impact of Reynolds number on hydrodynamics, Carberry conducted forced oscillation experiments with consecutive Reynolds number (2.3×10^3, 4.4×10^3, 9.1×10^3). The experiments discovered that Reynolds number has larger impacts on the hydrodynamic coefficients for the range $2.3\times10^3 < Re < 9.1\times10^3$.

Later, Morse accomplished more systematic forced oscillation experiments and applied the result to predict the response of cylinder under self-excited vibration, obtaining a good agreement with the result of self-excited oscillation experiment conducted by Govardhan and Williamson (2000). Raghavan and Bernitsas (2011) conducted self-excited vibration of VIV of rigid cylinder at high Reynolds number (8×10^3~1.5×10^5) and they found that the amplitude ratio（A/D）increased with Reynolds number within the upper branch and Reynolds number had a strong influence than the mass ratio on A/D in some cases.

In recent years, Chinese scholars focused more attention on the Computational Fluid Dynamics (CFD) evaluation method. Zong Zhi (2011) and Dong Guohai (2009) numerically simulated a cylinder undergoing vortex induced vibration with different Reynolds numbers. However, direct numerical simulation based on CFD has not been a practical way for engineering applications due to its extensive requirements on computational resources.

Despite the fact that so much work has been done, most of researches today are concerned with low Reynolds number, as shown in Table 1. This paper focuses on the similarities and differences between the effects of high and low Reynolds numbers on hydrodynamic characteristics of cylinders undergoing VIV, and additionally aims to address whether it is reasonable to utilize the hydrodynamic coefficients at low Reynolds number to predict vortex induced vibration of risers at real sea state when the Reynolds numbers are in the same regime. Such a question is attributed to the fact that in academia and industry of today, the prediction of the VIV at real sea state is normally based on the experimental data obtained at low Reynolds number. The accuracy and reliability of the prediction remains unjustifiable. As Carberry (2002) presented, such question would be left unresolved unless forced vibration experiments with high Reynolds number in subcritical regime are conducted.

Aimed at the verification, in this paper, high Reynolds number equal to 2×10^5 has been achieved in the forced vibration experiment. Such Reynolds number is in the subcritical regime and close to the critical regime. Comparison between the result of this experiment and low Reynolds number experiments results shows the influence of Reynolds number on hydrodynamics of cylinder in whole subcritical regime.

Table 1 Reynolds number in forced oscillation test

Reference	Reynolds number
Mercier (1973)	13000
Sarpkaya (1978)	5000-25000
Staubli（1983）	60000
Gopalkrishnan (1993)	10000
Carberry (2002)	2300,4400,9100
Morse(2009)	4000
Dong Guohai(2012)	100，200，10000
Wan Decheng(2009)	200，300，500
Present test	200000

2. Experimental apparatus

The experiment was conducted at Shanghai Ship and Shipping Research Institute. The towing tank is 192 m long, 10 m wide and 4.2 m deep, and the maximum speed of the towing carriage can reach 9.0 m/s. The apparatus consists of carriage, forced vibration device and the rigid cylinder model. The real experimental setup and 3-D sketches are shown in Figs. 1 and 2 respectively.

The test model is made of polypropylene, smooth and 2 m long by 0.25 m diameter. It is squeeze fitted in the forced vibration device and harmonically vibrates in transverse direction along with the track driven by the motor. Two methods have been adopted to reduce the end effects to guarantee the two dimensional flow through the cylinder model. Firstly, two dummy cylinders with the same diameter as the model are assembled at the ends of the model. And little space (<1 mm) between the dummy cylinder and the model is left to guarantee that the force transducer only measures the force on the model. Secondly, two 0.75 m diameter endplates are assembled outside of the dummy cylinder to decrease the impact of other devices on the flow field. To guarantee the two dimensional flow, we designed validation test before the forced oscillation test. In the validation test, the static cylinder was constrained in the current of 0.4 m/s (Re=8x10^4). And the horizontal force was measured by the force transducers. Then we calculated the drag coefficient of the static cylinder based on the collected data. The drag coefficient was measured 1.2, which is in good agreement with existed experimental data by Schewe (1983).

Three-component transducers are used to measure the forces on cylinder in the experiment which comprises hydrodynamic forces and inertia force and the latter should be subtracted from the total force. For the force transducers, we have calibrated the transducers 3 times in water and air to guarantee their reliabilities in the tests. Both displacement and velocity of the test model are measured using encoder of the motor.

Fig. 1 Picture of the experimental setup

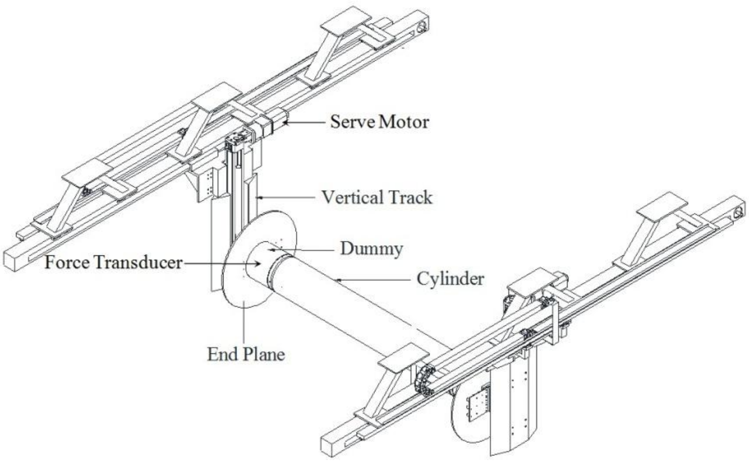


Fig. 2 Sketch definition of the experimental setup

3. Experiment procedure

The experiment was focused on the influences of oscillating amplitude and frequency on VIV characteristics of rigid cylinder with Reynolds number equal to 2×10^5. And we compared the result with that of Gopalkrishnan ($Re=1\times10^4$) to study the influences of Reynolds number. The velocity of carriage was kept at 0.8m/s to guarantee the constant Reynolds number 2×10^5. Amplitude takes 0.26D and 0.3D respectively while non-dimensional oscillating frequency $\hat{f}_0$ ($\hat{f}_0 = f_0D/U$, where f_0 is the forced vibration frequency, D is the cylinder diameter and U is the velocity of carriage) is in the range of 0.1-2.4 with interval of 0.2. The sketch of the oscillation in the experiment is shown in Fig. 3.

The acquisition time in each case is around 100s, including 6s initial phase, 4s transitional phase, 85s steady phase and 5s ending phase. Fig. 4 shows a set of time series of drag force in forced vibration (Y/D=0.30, $\hat{f}_0 = 0.220$).

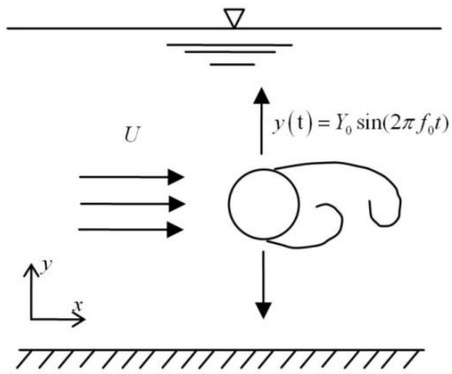


Fig. 3 Sketch definition of the oscillation

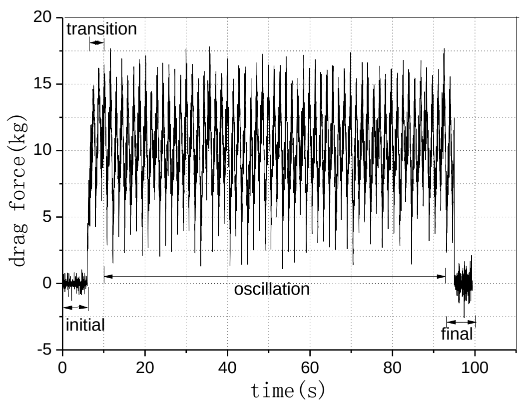


Fig. 4 Time series of drag force

4. Data processing

The original force data is low filtered to eliminate high frequency noise and the cut-off frequency is set as 2.2 f_0. And then the filtered time series is used to deduce the mean drag coefficient C_{Dm} , oscillating drag coefficient C_{Do}, life coefficient C_L and phase angle Φ_0, and hence the variation of the hydrodynamic coefficients with non-dimensional frequency.

The cylinder is forced to oscillate in the following form

$$y(t) = Y_0 \sin(2\pi f_0 t) \tag{1}$$

where Y_0 is the oscillating amplitude and f_0 is the oscillating frequency.

Both the lift force and the drag force of the oscillating cylinder include the Strouhal frequency component and oscillating frequency component.

We may model the life force by

$$L = L_0 \sin(2\pi f_0 t + \phi_0) + L_s \sin(2\pi f_s t + \phi_s) \quad (2)$$

and the drag force by

$$D = D_m + D_0 \sin(2\pi(2f_0)t + \psi_0) + D_s \sin(2\pi(2f_s)t + \psi_s) \quad (3)$$

where D_m is the mean drag, L_0 and D_0 are the amplitude of oscillating lift and drag force respectively,Φ_0 and Ψ_0 are the phase angle. Ls and D_s are the magnitudes of the oscillating Strouhal lift and drag forces respectively. Φ_s and Ψ_s are the phase angle.

Since we are only interested in the response in lock-in regime where the cylinder motion controls the shedding process and the Strouhal frequency disappears, $L_s \sin(2\pi f_s t + \phi_s)$ and $D_s \sin(2\pi(2f_s)t + \psi_s)$ in the equation can be ignored.

Non-dimension analysis is conducted in the Eqs. (2) and (3) to obtain lift force and drag force coefficient by

$$C_L = C_{L_0} \sin(2\pi f_0 t + \phi_0) \quad (4)$$

$$C_D = C_{Dm} + C_{D_0} \sin(2\pi(2f_0)t + \psi_0) \quad (5)$$

where C_{Dm} is the mean drag coefficient, C_{Lo} and C_{Do} are the amplitude of lift coefficient and drag coefficient.

According to Fourier series analysis theory, a time series can be expressed as

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi nt}{T}) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi nt}{T}) \quad (6)$$

where $a_0 = \frac{1}{T}\int_0^T x(t)dt$, $a_n = \frac{2}{T}\int_0^T x(t)\cos(\frac{2\pi nt}{T})dt$, $b_n = \frac{2}{T}\int_0^T x(t)\sin(\frac{2\pi nt}{T})dt$, and n symbolizes the number of Fourier series. As for lift coefficient, n takes 1; whereas for drag coefficient, n takes 2.

Therefore, the amplitude C_{Lo} and phase angle Φ_0 of lift coefficient can be obtained as

$$C_{L_0} = \sqrt{a_1^2 + b_1^2} \quad (7)$$

$$\phi_0 = \arctan(\frac{a_1}{b_1}) \quad (8)$$

And the amplitude of mean drag coefficient and fluctuating drag coefficient can be obtained as

$$C_{Dm} = a_0 \quad (9)$$

$$C_{Do} = \sqrt{a_2^2 + b_2^2} \quad (10)$$

The expression of lift coefficient is then expanded as

$$C_L = C_{L_0} \sin\phi_0 \cos(2\pi f_0 t) + C_{L_0} \cos\phi_0 \sin(2\pi f_0 t) \tag{11}$$

Thus, the amplitude of lift force in phase with velocity and in phase with acceleration can be expressed as

$$C_{L\_V} = C_{L_0} \sin\phi_0 \tag{12}$$

$$C_{L\_A} = C_{L_0} (-\cos\phi_0) \tag{13}$$

We can get the added mass of oscillating cylinder M_{Ao} as

$$M_{A_0} = -\frac{\frac{1}{2}\rho l D U^2 C_{L\_A}}{Y_0 (2\pi f_0)^2} \tag{14}$$

And the added mass coefficient is defined as

$$C_{M_0} = \frac{M_{A_0}}{\rho V} \tag{15}$$

where V is the displacement of the cylinder.

Combining Eqs. (14) and (15), the added mass coefficient can be further expressed

$$C_{M_0} = -\frac{C_{L\_A}}{2\pi^3 (A/D) \hat{f}_0^2} \tag{16}$$

5. Results and discussions

The experiment results are compared with those of Gopalkrishnan (1993) to study the effect of Reynolds number on the hydrodynamic coefficients. In addition, forced oscillation tests with two different amplitudes (0.26 and 0.3D) have been carried out to study the effect of oscillating amplitude on lift coefficient and phase angle. In the following, we will directly analyze the characteristics of the VIV at high Reynolds number using the hydrodynamic forces measured in the model test.

5.1 Mean and oscillating drag coefficient

As shown in Fig. 5, mean drag coefficient is about 1.2 in non-lock in region. As the non-dimensional frequency increases to lock-in region where the oscillating frequency is close to vortex shedding frequency, the mean drag coefficient will increase. We can also see number is higher.

The frequency of oscillating drag force is twice of the vortex shedding frequency, as shown in Figs. 6 and 7. The lift force is totally dominated by its first-harmonic components while the oscillating drag force by the second harmonics. We obtain the amplitude of the oscillating drag force with the frequency of $2f_0$ through the Fourier Transform, where f_0 is shedding frequency. For forced oscillation test with small amplitude, the oscillating drag coefficient is approximately 9% of mean drag coefficient, as shown in Fig. 5.

The variation of oscillating drag force with non-dimensional frequency is similar to that of mean drag force, i.e., the oscillating drag coefficient keeps nearly constant in non-lock-in region and reaches a maximum in lock-in region.

Comparing the present results with Gopalkrishnan's (1993), good agreement between the oscillating drag coefficients is found. Thus, we conclude that Reynolds number has little influence on oscillating drag coefficient in sub-critical regime.

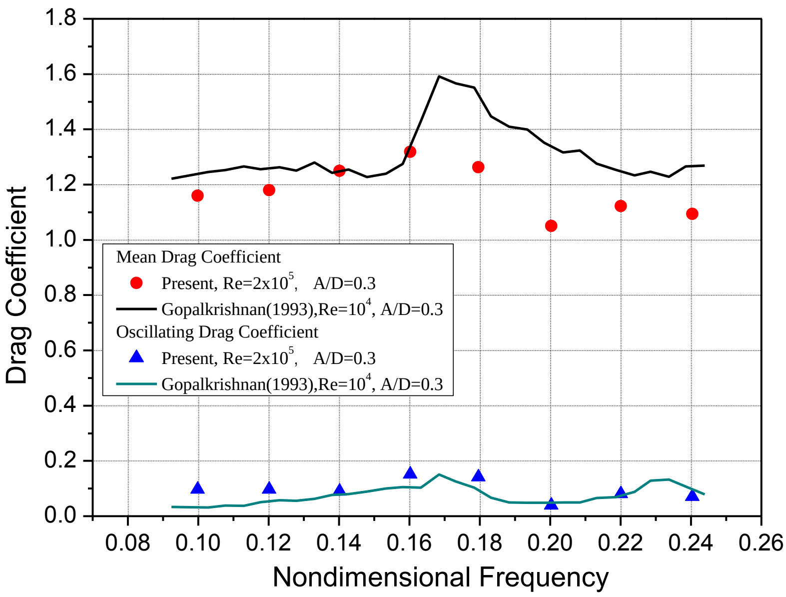


Fig. 5 Variation of mean and oscillating drag coefficient with non-dimensional frequency

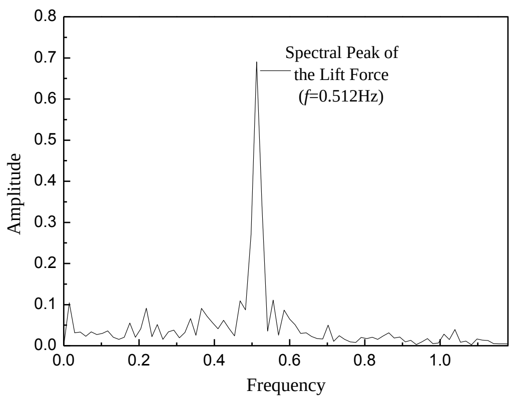


Fig. 6 Spectrum of the Lift Force

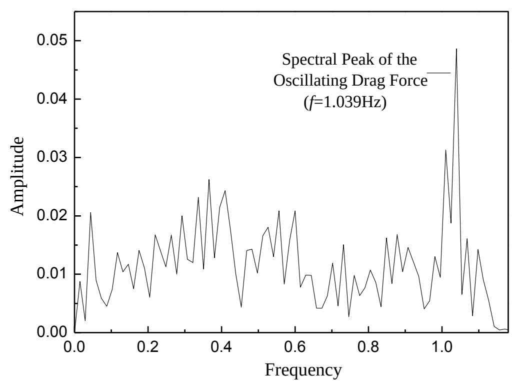


Fig. 7 Spectrum of the Oscillating Drag Force

5.2 Lift Coefficient

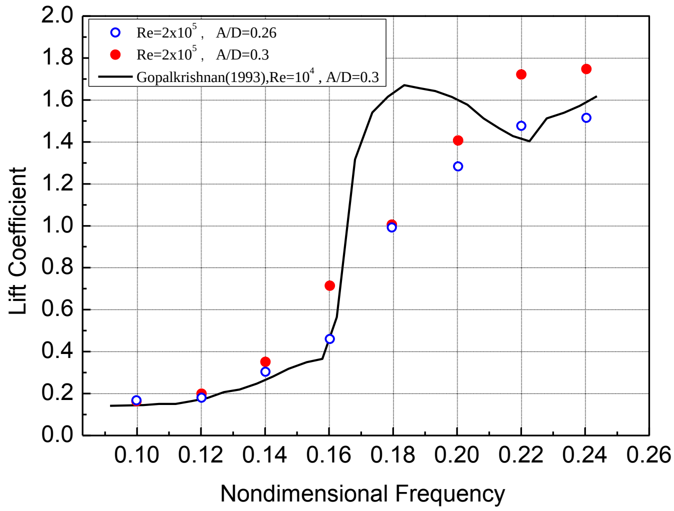


Fig. 8 Variation of lift coefficient with non-dimensional frequency

Generally speaking**,** the variation of lift force with non-dimensional frequency in this test is greatly consistent with that of low Reynolds number as shown in Fig. 8. It may be because the two cases (same A/D and f^*, but different Reynolds numbers) have similar vortex structure. In another word, the Reynolds number has little effect on the vortex structure in the sub-critical regime.

As shown in Figure 8, the changing trends of lift coefficient under high and low Reynolds number are sufficiently alike: the coefficient keeps small and varies slowly when the oscillating frequency is low. Once non-dimensional frequency reaches 0.16~0.18, lift coefficient increases rapidly owing to the lock-in phenomenon in which the vortex strength becomes concentrated. In the higher frequency area above 0.20, the lift coefficients also increase along the frequency but at a much slower rate. Yet the two curves at the same oscillating amplitude differ in details, like the rate of the change, which can be attribute to the difference of Reynolds number.

Comparing the results of different amplitudes (0.26 and 0.3D), we can see that lift coefficient is slightly larger at the higher amplitude which indicates that the vortex strength becomes intensive as the amplitude increases.

5.3 Lift force phase angle

Lift force phase angle refers to the angle between total lift force and cylinder motion, indicating the direction of the power transfer between cylinder and fluid. Values of phase angle in range $0 \le \phi_0 \le \pi$ correspond to energy transfer from the fluid to the cylinder. The variation of phase angle with non-dimensional is shown in Fig. 9.

When the oscillating frequency is low, the lift force phase angle is nearly $-\pi$, which means slight energy transfer from cylinder to fluid. The phase angle is between 0 and π for the ranges $0.12 \le \hat{f}_0 \le 0.18$ and it includes the whole exiting region where the energy transfer from fluid to cylinder i.e., the cylinder could get excited into motion by the fluid flow. As the oscillating frequency continues to increase, the phase angle keeps almost unchanged at 0, which indicates no energy transfer between the fluid and cylinder.

Of the extreme importance is the lock-in region in which the phase angle decreased from π to zero.

The variation relationship between phase angle and non-dimensional frequency matches well in both the present test and Gopalkrishnan's (1993), i.e., Reynolds number has little impact on the direction of the power transfer between cylinder and fluid.

Furthermore, we can conclude that the oscillating amplitude has little influence on the phase angle by comparing the forced oscillation test under different amplitudes.

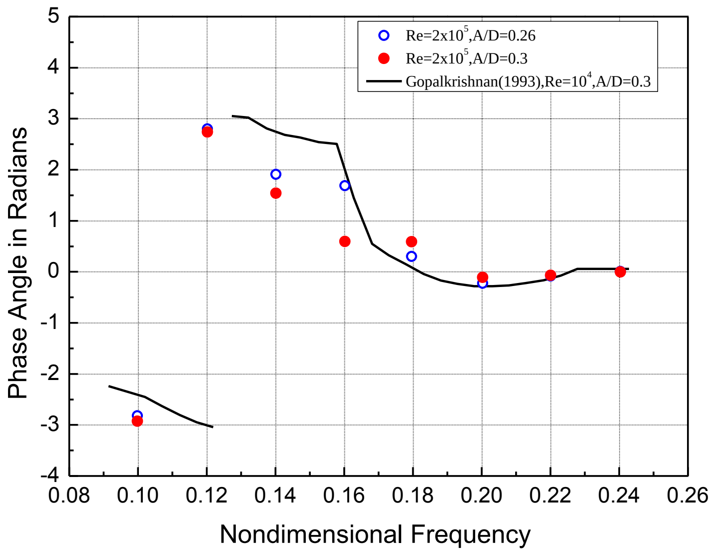


Fig. 9 Variation of lift force phase angle with non-dimensional frequency

5.4 Lift force in phase with velocity

Fig. 10 shows the variation of lift force in phase with velocity with non-dimensional frequency.

The energy transfers from fluid to cylinder when the lift force in phase with velocity C_{L_V} is positive, which is similar to phase angle. When the frequency is low, C_{L_V} increases from negative to positive as the oscillating frequency increases, i.e., the effect that the lift force have on the motions of the cylinder turns from suppressing to stimulating. In lock-in region, the lift coefficient reaches the maximum, and hence the power transfers from the fluid to the cylinder. As the oscillating frequency further increases, far from the vortex shedding frequency, the lift coefficient decreases rapidly or even to negative values.

In addition, we can see from Figs. 8, 9 and 10 that lock-in occurs at the frequency ranges $0.16 \leq \hat{f}_0 \leq 0.18$. And the Strouhal number is about 0.2 for stationary cylinder in subcritical regime (Bishop and Hassan 1964). Thus, we conclude that the Strouhal number for oscillating cylinder is smaller than stationary cylinder. Such conclusion is expectable because Strouhal number is related to the time of the interaction between the two vortexes shedding from each side of the cylinder.

And for the stationary cylinder, the distance between the two vortexes is shorter than that of oscillating cylinder, as shown in Fig. 11. So the two vortexes interact more frequently for the stationary cylinder, leading to a bigger Strouhal number.

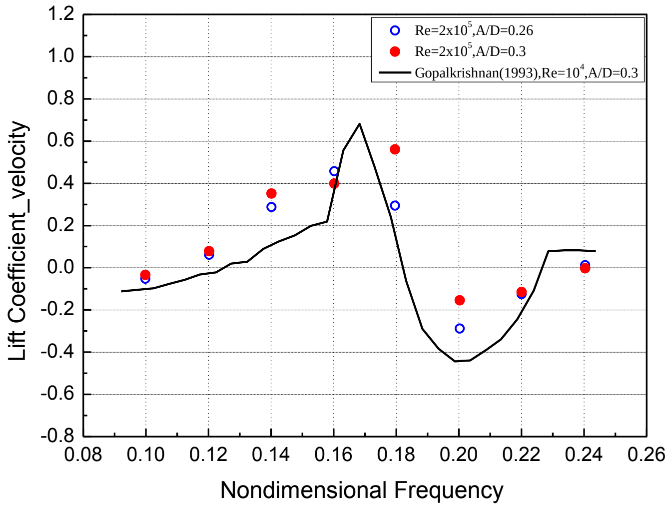
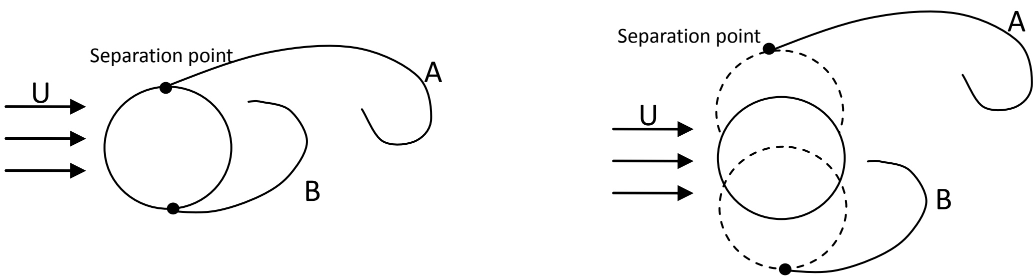


Fig.10 Variation of lift force in phase with velocity with non-dimensional frequency



(a) static cylinder (b) Oscillating cylinder

Fig. 11 Sketch showing vortex formation

Comparing the present test with Gopalkrishnan's (1993), we can find that lift force in phase with velocity is smaller while the Strouhal number is larger with lower Reynolds number. It may be caused by the move of separation point downstream as Reynolds number increases, which results in a shorter distance between the two vortexes. The Strouhal number becomes larger when the distance is shorter which has been explained above while the negative pressure area becomes small, leading to a small C_{L_V}.

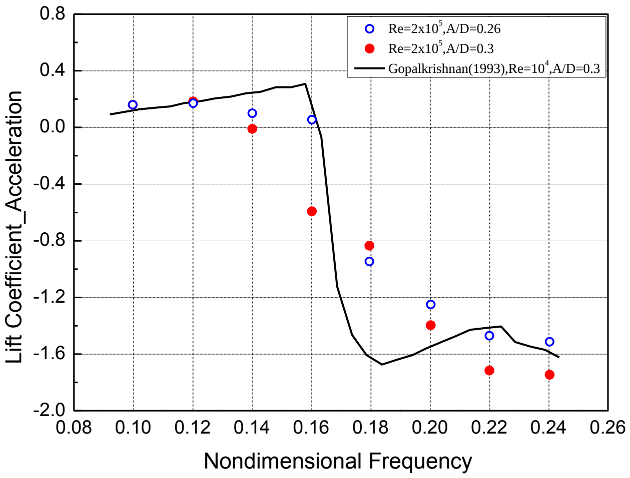


Fig. 12 Variation of lift force in phase with acceleration with non-dimensional frequency

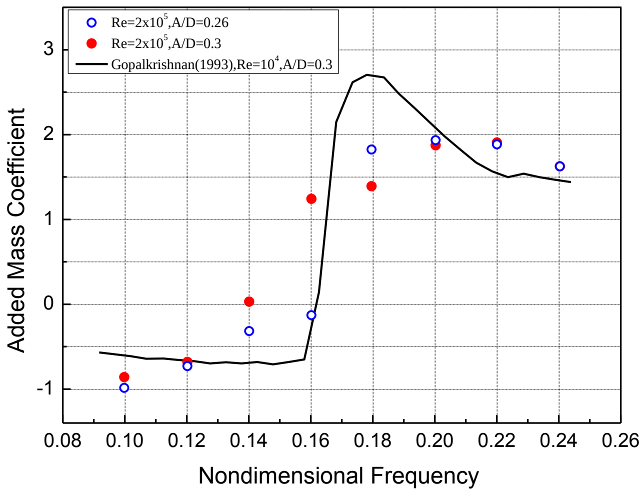


Fig. 13 Variation of added mass coefficient with non-dimensional frequency

5.5 Lift force in phase with acceleration

The variation of the lift force in phase with acceleration C_{L_A} and added mass coefficient C_{Mo} with non-dimensional frequency is shown in Figs. 12 and 13.

According to potential theory, the added mass coefficient for a cylinder is a constant value.

However, in VIV problem, the added mass coefficient is related with oscillating frequency and amplitude, probably owing to that the lift force in phase with acceleration consists of ‘potential force’ component and ‘vortex force’ component. The former is proportional to the potential added-mass coefficient which equals to 1 for a cylinder. But the latter, the vortex force component, is related to vortex shedding and, hence, to the Reynolds number, oscillating amplitude and frequency.

At low values of non-dimensional frequency $\hat{f}_0$, added mass coefficient is less than 1. As $\hat{f}_0$ increases, added mass coefficient varies slightly until $\hat{f}_0$ reaches 0.16. At this value, there is an abrupt increase in C_{Mo} and it reaches the maximum. When $\hat{f}_0$ increases further, added mass coefficient keeps almost constant.

We conclude from Figs. 12 and 13 that Reynolds number and oscillating amplitude have little effect on the lift force in phase with acceleration.

6. Conclusions

Hydrodynamic characteristics of a bluff cylinder oscillating in transverse direction in steady current were experimentally investigated at Reynolds number of 2×10^5. The effects of non-dimensional frequency, oscillating amplitude and Reynolds number on drag force, lift force and phase angle are studied. The following remarks and conclusions can be drawn.

- Within subcritical regime, compared with low Reynolds number, high Reynolds number can result in low mean drag coefficient while has little impact on lift coefficient and phase angle. Generally to say, lift coefficient and phase angle are the key parameters in the prediction of VIV response of rises. Thus it is reasonable to utilize the vortex induced forces of the oscillating cylinder under low Reynolds number into the prediction of the vortex induced vibration of risers in real sea state when the Reynolds numbers are in the same subcritical regime.
- Similar to cases of low Reynolds number in subcritical regime, there is an explicit similarity in the variation with the oscillating frequency for the drag coefficient, lift coefficient and phase angle at high Reynolds number. As the non-dimensional frequency reaches 0.16-0.18, the coefficients change dramatically.
- Comparing the forced oscillation test under different amplitudes, we can say that the Strouhal number decreases while the lift coefficient increases as the oscillating amplitude increases. However, the phase angle almost remains constant.

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