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Transversely isotropic thick plate with two temperature & GN type-III in frequency domain

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Abstract. This investigation is focused on the variations in transversely isotropic thick circular plate due to time harmonic thermomechanical sources. The homogeneous thick circular plate in presence and absence of energy dissipation and two temperatures has been considered. Hankel transform is used for solving field equations. The analytical expressions of conductive temperature, displacement components, and stress components are computed in the transformed domain. The effects of frequency at different values are represented graphically. Some specific cases are also figured out from the current research.

Keywords: frequency; hankel transformation; thermoelastic; thick circular plate; time harmonic sources; transversely isotropic

1. Introduction

Classical theory (CT) of elasticity is concerned with the systematic study of the stress and strain distribution that develops in an elastic body due to the application of forces or change in temperature. Thermoelasticity in presence of two temperatures represents an overview of both theories i.e., theory of heat conduction and theory of elasticity in solids. Temperature changes cause thermal effects on materials like thermal stress, strain, and deformation. Thermal dependency is the primary contrast of thermoelasticity concerning to the classical theory of thermomechanics. It may also be mentioned that modern laminated media which are being used more and more in engineering and other applications, behave anisotropically locally (thermally and elastically). Thus, there is imperative need to consider the anisotropic media particularly transversely isotropic. However, due to a greater number of elastic and thermal coefficients involved, there are not so many solutions available as there are for isotropic media.

Chen *et al.* (1968a, 1968b, 1969) formulated a two-temperature thermoelasticity of deformable bodies for the conduction of heat depending on two types of temperatures. Green and Naghdi (1991, 1992, 1993) dealt with the linear and the nonlinear theories of thermoelastic body in presence and absence of energy dissipation. Three novel thermoelastic theories were proposed by them, based on entropy equality. Their theories are known as thermoelasticity type I theory, the thermoelasticity type II theory (i.e., thermoelasticity without energy dissipation) and the

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thermoelasticity type III theory (i.e., thermoelasticity with energy dissipation). On linearization, type I becomes the classical heat equation whereas on linearization type-II as well as type-III theories gives finite speed of thermal wave propagation.

Keivani *et al.* (2014) discussed the forced vibration problem of an Euler-Bernoulli beam with a semi-infinite field by considering it a BVP in the frequency domain. Tripathi *et al.* (2015) presented the effect of axisymmetric heat supply for diffusion in an infinite as well as finitely thick thermoelastic copper plate with relaxation time. Delfim *et al.* (2015) presented a coupled FEM-BEM strategy for elastodynamic problems having infinite-domain models and complex heterogeneous media by using frequency domain analyses and an iterative FEM-BEM coupling technique. In addition, Kumar *et al.* (2016) had explored of variations due to thermomechanical sources (concentrated and distributed) using Laplace and Fourier transform technique in a transversely isotropic thermoelastic homogeneous medium in presence of rotation and two temperatures. Alzahrani (2016) investigated 2D generalized magneto-thermoelastic of a fiber-reinforced anisotropic material problem under GN theory- III type. Tripathi *et al.* (2016) presented the thermoelastic diffusion interactions in a thick circular copper material plate. Kumar and Sharma (2017) investigated transmission and reflection of plane waves at an elastic and piezo thermoelastic solid half space with fractional order derivative. Vinyas *et al.* (2017) discovered a multiphysics behaviour of magneto-electro-elastic (MEE) cantilever beam using thermo-mechanical loading. Moreover, Kumar *et al.* (2017) investigated the homogeneous isotropic thermoelastic thick circular plate with dual phase lags and two temperatures. Akbaş (2017) study the nonlinear static deflections of functionally graded (FG) porous under thermal effect using total lagrangian FEM within 2D continuum model in the Newton-Raphson iteration method.

Kumar *et al.* (2017) investigated the Rayleigh waves in a homogeneous transversely isotropic magneto-thermoelastic in the presence of two temperature, Hall current and rotation. Kant and Mukhopadhyay (2017) studied the thermoelastic effect of axisymmetric temperature distribution applied on infinitely extended thick plate using GN-I, GN-II, dual phase-lag and GL models and under memory-dependent generalized thermoelasticity. Kumar *et al.* (2017) presented the effect of plane harmonic waves in a thermoelastic medium in presence of two-temperature thermoelasticity and two relaxation parameters. Navarro *et al.* (2018) proposed a fully coupled thermodynamic oriented transient finite element formulation for magnetic, electric, mechanic and thermal field's interactions. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1997), Marin (2008), Marin (2016), Marin and Baleanu (2016), Ezzat *et al.* (2016), Ezzat *et al.* (2015), Ezzat and El-Bary (2016, 2017), Ezzat *et al.* (2017).

A lot of research had been carried out by the various researches in different fields of thermoelasticity to remove the limitations of the classical coupled theory (CCT) of thermoelasticity i.e., poor description of thermoelastic behaviour at low temperature and infinite speed of propagation of thermoelasticity deformation. Inspite of these, not much work has been carried out in study of the deformation due to time harmonic thermomechanical sources. The deformation at some point of the medium is beneficial to dissect the deformed field near mining shocks, seismic and volcanic sources; thermal power plants, high-energy particle accelerators, and many emerging technologies. In this paper, we have attempted to study the deformation in transversely isotropic thick circular plate due to mechanical and thermal sources by considering the disturbances harmonically time-dependent. The expressions of displacement components, conductive temperature and stresses components due to time harmonic thermomechanical sources over the circular region are calculated in transformed domain by using the Hankel transform. Numerical inversion technique is used to find the resulting quantities in the physical domain and effects of frequency at different values have been represented graphically.

2. Basic equations

The field equations and basic relations for an anisotropic thermoelastic medium in Green-Naghdi type-III theory in absence of heat source and body forces following Chandrasekharaiah (1998), Youssef (2011) and Green and Naghdi (1992) are

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T, \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} = \rho\, \ddot{u}_i, \tag{2}$$

$$K_{ij}\varphi_{,ij} + K_{ij}^{*}\dot{\varphi}_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T}, \tag{3}$$

Where

$$T = \varphi - a_{ij}\varphi_{,ij}, \tag{4}$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij}, \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3. \tag{6}$$

$\beta_{ij} = \beta_i\delta_{ij},\ \ K_{ij} = K_i\delta_{ij},\ K_{ij}^{*} = K_i^{*}\delta_{ij}$, i is not summed,

3. Formulation of the problem

We take a transversely isotropic thick circular plate with thickness 2b covering the area D given by $0 \le r \le \infty$,$-b \le z \le b$ and an axisymmetric heat source is used on its axial and radial direction. We take a cylindrical polar co-ordinate system (r,θ,z) with symmetry about Z-axis. The initial temperature in the transversely isotropic thick circular plate is assumed by a constant temperature T_0 and heat flux $g_0F(r,z)$ prescribed on the lower and upper surfaces. For the axisymmetric plane, the field component$(v = 0)$, and $(u, w, and\ \varphi)$ are independent of θ and our research become 2D problem with $\vec{u} = (u,0,w)$. In addition, the equations for transversely isotropic thermoelastic solid without energy dissipation and with two temperature, using the proper transformation on Eqs. (1)-(3) following Slaughter (2002) are as under

$$C_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r}u\right) + C_{13}\left(\frac{\partial^2 w}{\partial r\partial z}\right) + C_{44}\frac{\partial^2 u}{\partial z^2} + C_{44}\left(\frac{\partial^2 w}{\partial r\partial z}\right) - \beta_1\frac{\partial}{\partial r}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$(C_{11} + C_{44})\left(\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r}\frac{\partial u}{\partial z}\right) + C_{44}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - a_1\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right) - a_3\frac{\partial^2\varphi}{\partial z^2}\right\} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{8}$$

$$\left(K_1+K_1^*\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)+\left(K_3+K_3^*\frac{\partial}{\partial t}\right)\frac{\partial^2\varphi}{\partial z^2}= T_0\frac{\partial^2}{\partial t^2}\left(\beta_1\frac{\partial u}{\partial r}+\beta_3\frac{\partial w}{\partial z}\right)+\rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2}\right\}. \tag{9}$$

Essential equations for transversely isotropic medium are

$$t_{rr}= c_{11}e_{rr}+ c_{12}e_{\theta\theta}+ c_{13}e_{zz}-\beta_1 T, \tag{10}$$

$$t_{zr}= 2c_{44}e_{rz}, \tag{11}$$

$$t_{zz}= c_{13}e_{rr}+ c_{13}e_{\theta\theta}+ c_{33}e_{zz}-\beta_3 T, \tag{12}$$

$$t_{\theta\theta}= c_{12}e_{rr}+ c_{11}e_{\theta\theta}+ c_{13}e_{zz}-\beta_3 T, \tag{13}$$

where

$$e_{rz}= \frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right), e_{rr}=\frac{\partial u}{\partial r}, e_{\theta\theta}= \frac{u}{r}, e_{zz}=\frac{\partial w}{\partial z},$$

$$T= \varphi-a_1\left(\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)-a_3\frac{\partial^2\varphi}{\partial z^2},$$

$$\beta_{ij}= \beta_i\delta_{ij},\ K_{ij}= K_i\delta_{ij}\ ,$$

$$\beta_1= (c_{11}+ c_{12})\alpha_1+c_{13}\alpha_{3,}$$

$$\beta_3= 2c_{13}\alpha_1+c_{33}\alpha_3.$$

To simplify the solution, mention below dimensionless quantities are used

$$r'=\frac{r}{L},\quad z'=\frac{z}{L},\quad t'=\frac{c_1}{L}t,\quad u'=\frac{\rho c_1^2}{L\beta_1 T_0}u,\quad w'=\frac{\rho c_1^2}{L\beta_1 T_0}w,\quad T'=\frac{T}{T_0},$$
$$t'_{zr}=\frac{t_{zr}}{\beta_1 T_0},\quad t'_{zz}=\frac{t_{zz}}{\beta_1 T_0},\quad t'_{rr}=\frac{t_{rr}}{\beta_1 T_0},\quad \varphi'=\frac{\varphi}{T_0},\quad a'_1=\frac{a_1}{L^2},\quad a'_3=\frac{a_3}{L^2}\ . \tag{14}$$

Assume the time harmonic behaviour as

$$(u,w,\varphi)(r,z,t)=(u,w,\varphi)(r,z)e^{i\omega t}. \tag{15}$$

and Hankel transforms defined by

$$\tilde{f}(\xi,z,s)=\int_0^\infty f(r,z,s)rJ_n(r\xi)dr \tag{16}$$

Using dimensionless quantities defined by (14) in Eqs. (7)-(13) and then stifling the primes and utilizing (15) and (16) on the resulting quantities, we obtain

$$(-\xi^2+\omega^2+\delta_2 D^2)\tilde{u}+(1-\xi)\delta_1 D\widetilde{w}+\left(-(1-\xi)(1-a_3D^2)+a_1\xi^3)\right)\tilde{\varphi}=0 \tag{17}$$

$$(1-\xi)\delta_1 D\tilde{u}+\ (\delta_3 D^2-\xi^2\delta_2+\omega^2)\widetilde{w}-\frac{\beta_3}{\beta_1}D(1+\xi^2 a_1-a_3D^2)\tilde{\varphi}=0, \tag{18}$$

$$\begin{aligned}&\delta_6\omega^2(1-\xi)\tilde{u}+\frac{\beta_3}{\beta_1}\delta_6\omega^2 D\widetilde{w}\\&+\left(\delta_7\omega^2(1+\xi^2 a_1-a_3D^2)+\xi^2(K_1+\delta_4\omega)-D^2(K_3+\delta_5\omega)\right)\tilde{\varphi}=0,\end{aligned} \tag{19}$$

where

$$\delta_1=\frac{c_{13}+c_{44}}{c_{11}},\qquad \delta_2=\frac{c_{44}}{c_{11}},\qquad \delta_3=\frac{c_{33}}{c_{11}},\qquad \delta_4=\frac{K_1^* C_1 i}{L},$$

$$\delta_5=\frac{K_3^* C_1 i}{L},\ \delta_6=-\frac{T_0\beta_1^2}{\rho},$$

$$\delta_7=-\rho C_E C_1^2,\ i=\sqrt{-1}\ .$$

$$\widetilde{t_{zz}}=\sum A_i(\xi,\omega)\eta_i cosh(q_i z)+\sum \mu_i A_i(\xi,\omega)\sinh(q_i z) \tag{20}$$

$$\widetilde{t_{rz}}=\sum A_i(\xi,\omega)d_i cosh(q_i z)+\xi\sum A_i(\xi,\omega)q_i sinh(q_i z), \tag{21}$$

$$\widetilde{t_{rr}}=\sum A_i(\xi,\omega)R_i cosh(q_i z)+\sum S_i A_i(\xi,\omega)\sinh(q_i z), \tag{22}$$

$$\widetilde{t_{\theta\theta}}=\sum A_i(\xi,\omega)M_i cosh(q_i z)+\sum N_i A_i(\xi,\omega)\sinh(q_i z), \tag{23}$$

where

$$\eta_i=\ \delta_9\xi-\frac{\beta_3}{\beta_1}(1+a_1\xi^2)\mathrm{l_i}-\frac{\beta_3}{\beta_1}\mathrm{a_3 l_i q_i^2},$$

$$\mu_i=(\delta_9+\delta_3 d_i)q_i,$$

$$M_i=\left(1+\frac{\xi}{2}\right)-\frac{\beta_3}{\beta_1}(1+a_1\xi^2)\mathrm{l_i}+\frac{\beta_3}{\beta_1}\mathrm{a_3 l_i q_i^2},$$

$$N_i=(\delta_8+\delta_9 d_i)q_i,$$

$$R_i=\delta_8\left(1+\frac{\xi}{2}\right)-l_i(1+a_1\xi^2)+\mathrm{a_3}l_i\mathrm{q_i^2},$$

$$S_i=q_i(1+\delta_3 d_i),\qquad i=1,2,3.$$

The non-trivial solution of (17)-(19) yields

$$(AD^6+BD^4+CD^2+E)(\tilde{u},\widetilde{w},\tilde{\varphi})=0. \tag{24}$$

where

$$A = \delta_2\delta_3\zeta_8 - \zeta_{11}\delta_2\zeta_7,$$

$$B = \delta_2\zeta_5\zeta_8 + \delta_3\zeta_1\zeta_8 + \delta_2\delta_3\zeta_9 - \delta_2\zeta_7\zeta_{10} - \zeta_7\zeta_1\zeta_{11} - \zeta_2^2\zeta_8 + \zeta_2\zeta_6\zeta_{11} + \zeta_4\zeta_7\zeta_2 - \delta_3\zeta_6\zeta_4,$$

$$C = \zeta_1\zeta_5\zeta_8 + \delta_2\zeta_9\zeta_5 + \delta_3\zeta_1\zeta_9 - \zeta_7\zeta_1\zeta_{10} - \zeta_2^2\zeta_9 + \zeta_2\zeta_6\zeta_{10} + \zeta_3\zeta_2\zeta_7 - \zeta_3\zeta_6\delta_3 - \zeta_4\zeta_6\zeta_5,$$

$$E = \zeta_5\zeta_1\zeta_9 - \zeta_6\zeta_5\zeta_3.$$

$$\zeta_1 = -\xi^2 + \omega^2,$$

$$\zeta_2 = \delta_1(1 - \xi),$$

$$\zeta_3 = a_1\xi^2 - (1 - \xi),$$

$$\zeta_4 = a_3(1 - \xi),$$

$$\zeta_5 = \omega^2 - \xi^2\delta_2,$$

$$\zeta_6 = \delta_6\omega^2(1 - \xi),$$

$$\zeta_7 = \delta_6\omega^2\frac{\beta_3}{\beta_1},$$

$$\zeta_8 = -(K_3 + \delta_5\omega) - \delta_7\omega^2 a_3,$$

$$\zeta_9 = \delta_7\omega^2(1 + a_1\xi^2 + \xi^2(K_1 + \delta_4\omega),$$

$$\zeta_{10} = -(1 + a_1\xi^2)\frac{\beta_3}{\beta_1},$$

$$\zeta_{11} = a_3\frac{\beta_3}{\beta_1}$$

The results of the Eq. (24) can be written in the form

$$\tilde{u} = \sum A_i(\xi, \omega)cosh(q_i z), \tag{25}$$

$$\tilde{w} = \sum d_i A_i(\xi, \omega)cosh(q_i z), \tag{26}$$

$$\tilde{\varphi} = \sum l_i A_i(\xi, \omega)cosh(q_i z), \tag{27}$$

where $A_{i,}$ $i = 1, 2, 3$ being undetermined constants.
and $\pm q_i(i = 1,2,3)$ are the roots of the Eq. (24) and d_i and l_i are given by

$$d_i = \frac{\delta_2\zeta_8 q_i^4 + (\zeta_8\zeta_1 - \zeta_4\zeta_6 + \delta_2\zeta_9)q_i^2 + \zeta_1\zeta_9 - \zeta_6\zeta_3}{(\delta_3\zeta_8 - \zeta_7\zeta_{11})q_i^4 + (\delta_3\zeta_9 + \zeta_5\zeta_8 - \zeta_7\zeta_{10})q_i^2 + \zeta_5\zeta_9},$$

$$l_i = \frac{\delta_2\delta_3 q_i^4 + (\delta_2\zeta_5 + \zeta_1\delta_3 - \zeta_2^2)q_i^2 + \zeta_1\zeta_5}{(\delta_3\zeta_8 - \zeta_7\zeta_{11})q_i^4 + (\delta_3\zeta_9 + \zeta_5\zeta_8 - \zeta_7\zeta_{10})q_i^2 + \zeta_5\zeta_9}.$$

4. Boundary conditions

We contemplate a cubiform thermal source and normal force of unit magnitude with dispersing of tangential stress at the stress free surface at z = ±b. scientifically, these can be written as

$$\frac{\partial\varphi}{\partial z} = \pm g_o F(r,z), \tag{28}$$

$$t_{zz}(r,z,t) = f(r,t), \tag{29}$$

$$t_{rz}(r,z,t) = 0. \tag{30}$$

By putting the values $\tilde{\varphi}$, $\widetilde{t_{zz}}$, $\widetilde{t_{rz}}$ from (20)-(20) and (27) in boundary conditions (28)-(30) and applying Hankel transform on the resulting equations yields

$$\sum A_i\,(\xi,\omega) l_i q_i \vartheta_i = \pm g_o \tilde{F}(\xi,z), \tag{31}$$

$$\sum A_i(\xi,\omega)\eta_i\theta_i + \sum \mu_i\, A_i(\xi,\omega)\vartheta_i = \tilde{f}(\xi,\omega), \tag{32}$$

$$\sum A_i(\xi,\omega)(\delta_2 q_i\vartheta_i + (1-\xi)l_i\theta_i) = 0. \tag{33}$$

solving (31)-(33) for A_i, and putting in (25)-(27) and (20)-(23) we get the different components of displacement, conductive temperature and stress components as

$$\tilde{u} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1\theta_1 + \chi_2\theta_2 - \chi_3\theta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\theta_1 - \chi_5\,\theta_2 + \chi_6\theta_3\}, \tag{34}$$

$$\tilde{w} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 d_1\theta_1 + \chi_2 d_2\theta_2 - \chi_3 d_3\theta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4\, d_1\,\theta_1 - \chi_5\, d_2\theta_2 + \chi_6 d_3\theta_3\}, \tag{35}$$

$$\tilde{\varphi} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1 l_1\theta_1 + \chi_2 l_2\theta_2 - \chi_3 l_3\theta_3\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4 l_1\theta_1 - \chi_5\, l_2\theta_2 + \chi_6 l_3\theta_3\}, \tag{36}$$

$$\begin{aligned}\widetilde{t_{zz}} = {} & \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1(\eta_1\theta_1 + \mu_1\vartheta_1) + \chi_2(\eta_2\theta_2 + \mu_2\vartheta_2) - \chi_3(\eta_3\theta_3 + \mu_3\vartheta_3)\} \\ & + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(\eta_1\theta_1 + \mu_1\vartheta_1) - \chi_5\,(\eta_2\theta_2 + \mu_2\vartheta_2) + \chi_6(\eta_3\theta_3 + \mu_3\vartheta_3)\},\end{aligned} \tag{37}$$

$$\widetilde{t_{zr}} = \frac{\widetilde{f(\xi,\omega)}}{\Delta}\{-\chi_1(l_1(1-\xi)\theta_1 + \delta_2 q_1\vartheta_1) + \chi_2(l_2(1-\xi)\theta_2 + \delta_2 q_2\vartheta_2) - \chi_3(l_3\,(1-\xi)\,\theta_3 + \delta_2 q_3\vartheta_3)\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(l_1(1-\xi)\theta_1 + \delta_2 q_1\vartheta_1) - \chi_5\,(l_2(1-\xi)\theta_2 + \delta_2 q_2\vartheta_2) + \chi_6 l_3\,(1-\xi)\,\theta_3 + \delta_2 q_3\vartheta_3)\}, \tag{38}$$

$$\widetilde{t_{rr}} = \frac{\tilde{f}(\xi,\omega)}{\Delta}\{-\chi_1(R_1\theta_1 + S_1\vartheta_1) + \chi_2(R_2\theta_2 + S_2\vartheta_2) - \chi_3(R_3\theta_3 + S_3\vartheta_3)\} + \frac{g_o\tilde{F}(\xi,z)}{\Delta}\{\chi_4(R_1\theta_1 + S_1\vartheta_1) - \chi_5\,(R_2\theta_2 + S_2\vartheta_2) + \chi_6((R_3\theta_3 + S_3\vartheta_3)\}, \tag{39}$$

where

$$\Delta = \mathrm{G}_1\chi_4 - \mathrm{G}_2\chi_5 + \mathrm{G}_3\chi_6,$$

$$\Delta_1 = -\tilde{f}(\xi,s)\chi_1 + g_o\tilde{F}(\xi,z)\chi_4,$$

$$\Delta_2 = \tilde{f}(\xi,s)\chi_2 - g_o\tilde{F}(\xi,z)\chi_5,$$

$$\Delta_3 = -\tilde{f}(\xi,s)\chi_3 + g_o\tilde{F}(\xi,z)\chi_6,$$

$$\chi_1 = [\mathrm{G}_2\mathrm{G}_9 - \mathrm{G}_8\mathrm{G}_3],$$

$$\chi_2 = [\mathrm{G}_1\mathrm{G}_9 - \mathrm{G}_7\mathrm{G}_3],$$

$$\chi_3 = [\mathrm{G}_1\mathrm{G}_8 - \mathrm{G}_2\mathrm{G}_7],$$

$$\chi_4 = [\mathrm{G}_5\mathrm{G}_9 - \mathrm{G}_8\mathrm{G}_6],$$

$$\chi_5 = [\mathrm{G}_4\mathrm{G}_9 - \mathrm{G}_6\mathrm{G}_7],$$

$$\chi_6 = [\mathrm{G}_4\mathrm{G}_8 - \mathrm{G}_5\mathrm{G}_7],$$

$$\mathrm{G}_\mathrm{i} = l_i q_i \vartheta_i,$$

$$\mathrm{G}_{\mathrm{i}+3} = \eta_i\theta_i + \mu_i\vartheta_i,$$

$$\mathrm{G}_{\mathrm{i}+6} = \delta_2 q_i\vartheta_i + (1-\xi)\,l_i\,\theta_i,$$

$$\cosh(q_i z) = \theta_i, \quad \sinh(q_i z) = \vartheta_i, i = 1,2,3.$$

5. Applications

As an application of the problem, we take the source function $F(r, z)$ which decays exponentially as moving away from the centre of the thick circular plate in the radial direction and symmetrically increases along the axial directions is specified by

$$F(\mathrm{r},\mathrm{z}) = \mathrm{z}^2\mathrm{e}^{-\delta \mathrm{r}},\ \delta > 0, \tag{40}$$

$$f(r,t) = H(\alpha - r)e^{i\omega t} \tag{41}$$

Where $H(\alpha - r)$ is the Heaviside function.
Applying Hankel Transform, on Eqs. (28) and (29), gives

$$\tilde{F}(\xi,z) = \frac{z^2\delta}{(\xi^2+\delta^2)^{\frac{3}{2}}} \tag{42}$$

$$\tilde{f}(\xi,\omega) = \frac{\alpha J_1\,(\xi\alpha)}{\xi}e^{i\omega t} \tag{43}$$

The expressions of components of displacement, stress components, can be obtained from Eqs. (34)-(39), by substituting the value of $\tilde{F}$ (ξ,z) and $\tilde{f}$ (ξ,ω) from (42) and (43).

6. Inversion of the transforms

For obtaining the solution in physical domain, invert the Hankel transforms in Eqs. (34)-(39) using

$$f(r,z,\omega) = \int_0^\infty \xi\tilde{f}(\xi,z,\omega)J_n(\xi r)d\xi. \tag{44}$$

and integrate the Eq. (44) as described in Press *et al.* (1986).

7. Numerical results and discussion

To demonstrate our theoretical results and effect of frequency and two temperature, the physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal and Singh (1980) is given as

$$c_{11} = 3.07 \times 10^{11} Nm^{-2},$$

$$c_{12} = 1.650 \times 10^{11} Nm^{-2},$$

$$c_{13} = 1.027 \times 10^{10} Nm^{-2},$$

$$c_{33} = 3.581 \times 10^{11} Nm^{-2}$$

$$c_{44} = 1.510 \times 10^{11} Nm^{-2},$$

$$C_E = 4.27 \times 10^2 jKg^{-1}deg^{-1},$$

$$\beta_1 = 7.04 \times 10^6 Nm^{-2}deg^{-1}, \rho = 8.836 \times 10^3 Kgm^{-3}$$

$$\beta_3 = 6.90 \times 10^6 Nm^{-2}deg^{-1},$$

$$K_1 = 0.690 \times 10^2 Wm^{-1}Kdeg^{-1},\ K_3 = 0.690 \times 10^2 Wm^{-1}K^{-1},$$

$$K_1^* = 0.02 \times 10^2 NSec^{-2}deg^{-1},$$

$$K_3^* = 0.04 \times 10^2 NSec^{-2}deg^{-1}.$$

A comparison of the dimensionless form of the field variables normal force stress t_{zz}, tangential stress t_{zr}, radial stress t_{rr}, and conductive temperature $\varphi$for a transversely isotropic plate with two temperature and frequency is demonstrated graphically as:

i. The black line with square symbol relates to frequency i.e., $\omega = 0.25$ for a_1 =0.02, $a_3 =$ 0.04,

ii. The blue line with circle symbol relates to frequency i.e., $\omega = 0.50$ for a_1 =0.02, $a_3 =$ 0.04,

iii. The red line with triangle symbol relates to frequency i.e., $\omega = 0.75$ for a_1 =0.02, $a_3 =$ 0.04,

iv. The green line with star symbol relates frequency i.e., $\omega = 1.00$ for a_1 =0.02, $a_3 = 0.04$,

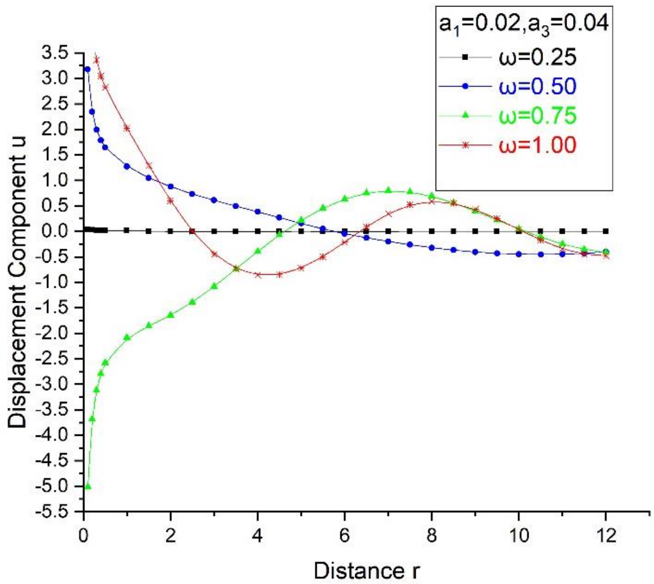

Fig. 1 Variations of displacement component u with distance r

Fig. 1 demonstrates the deviations of the displacement component u. The displacement component u, with two temperature a_1 =0.02, $a_3 = 0.04$ follow oscillatory pattern for ω=1.00, although for ω=.25, deviations are very small. In the starting range of distance r, there is a sharp increase in the value of displacement component for the curves when the ω=0.75, whereas there is a sharp decrease in the value of displacement component for ω=0.5. It is clear that two temperature with ω have major effect on the displacement component in all the cases. Behaviour of displacement component u, is oscillatory with variance in the magnitude corresponding to the four different frequencies.

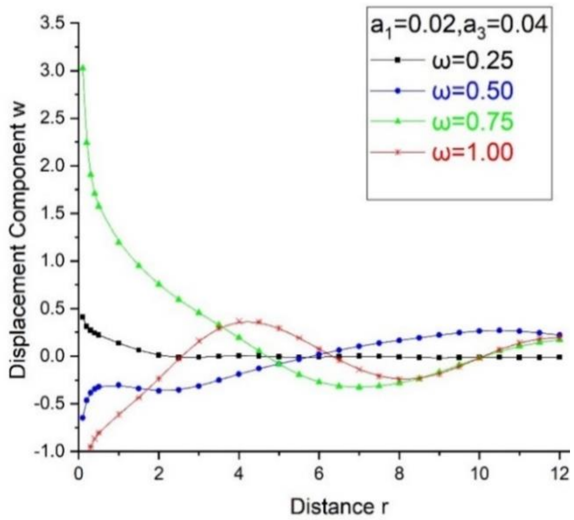

Fig. 2 Variations of displacement component w with distance r

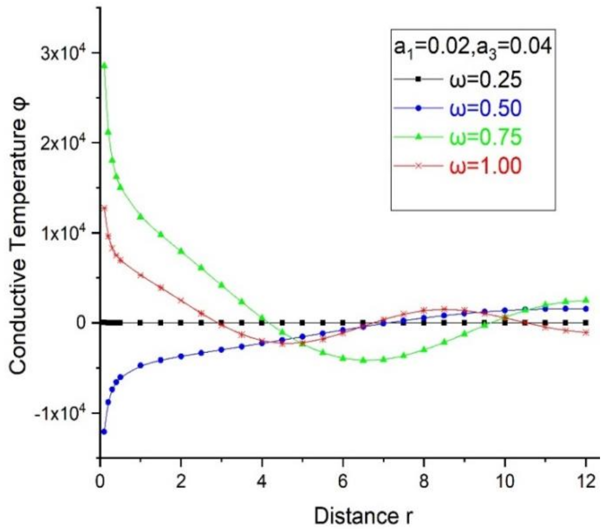

Fig. 3 Depicts the behaviour of conductive temperature φ

Fig. 2 depicts the displacement component w with distance r. In the initial range $0 \leq r \leq 4$of distance r, there is an increase in the value of displacement component when ω=0.50 and ω=1.00

and there is a sharp decrease in the value of displacement component for ω=0.25 and ω=0.75 but again away from the loading surface, it follows opposite oscillatory patterns near the zero value.

Fig. 3 demonstrations the deviations of conductive temperature φ with r. There is a sharp increase in the value of displacement component when ω=0.50 and there is a sharp decrease in the value of conductive temperature φ for ω=0.75 and ω=1.00 in the initial range of distance r, but again away from the loading surface, it shows opposite oscillatory behaviour near the zero value. The angular frequency major effect in the range $0 \leq r \leq 4$ for all the cases and curves show reverse oscillation in the remaining range.

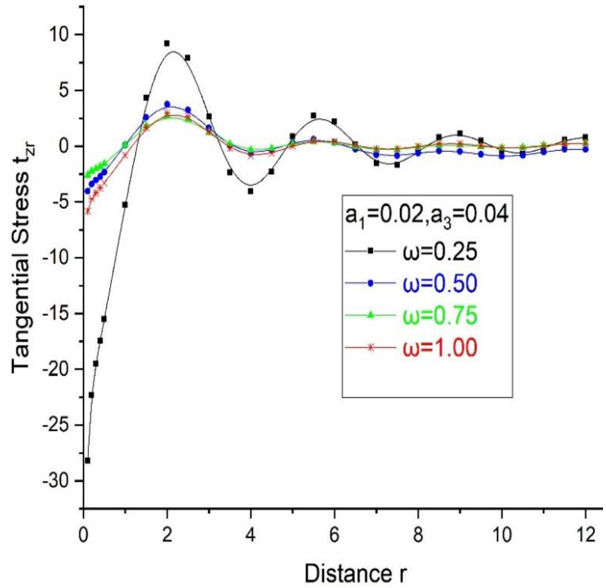

Fig. 4 Variations of tangential stress t_{zr} with r

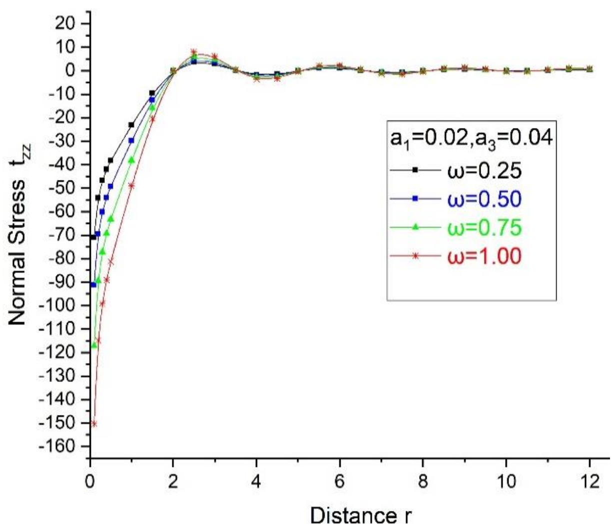

Fig. 5 Variations of normal stress t_{zz} with distance r

Fig. 4 illustrates the deviations of tangential stress t_{zr} with r. In $0 \leq r \leq 5$ range of r, the value of t_{zr} follow oscillatory pattern for all the curves when the two temperatures are a_1 =0.02, $a_3 =$ 0.04 for *ω*=1.00, *ω*=0.75, *ω*=0.50 and *ω*=0.25.

Fig. 5 shows the deviations of normal stress t_{zz} with r. There is a sharp increase in the value of normal stress t_{zz} for a_1 =0.02, $a_3 =$ 0.04 for *ω*=1.00, *ω*=0.75, *ω*=0.50 and *ω*=0.25.for 0 < r < 2 in the range of r, and then the variations are very small.

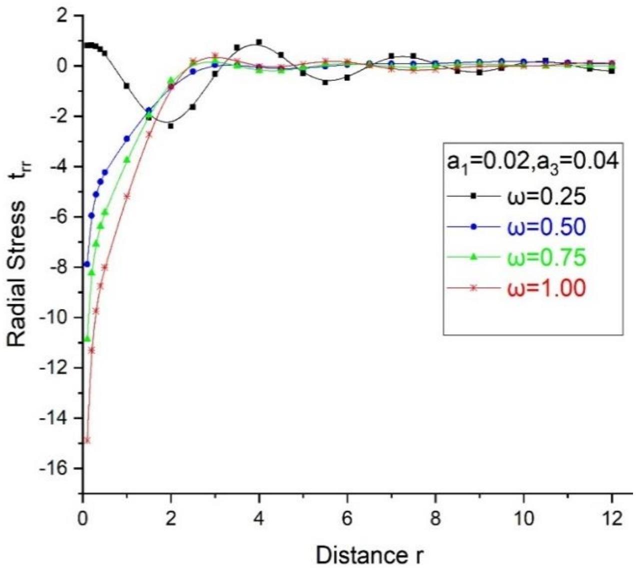

Fig. 6 Variations of radial stress t_{rr} with distance r

Fig. 6 displays the deviations of radial stress t_{rr} with r. There is a sharp increase in the value of radial stress t_{rr} with in the initial range of distance r, when the two temperatures are a_1 =0.02, $a_3 =$ 0.04 for *ω*=1.00, *ω*=0.75, *ω*=0.50 then the variations are very small. For *ω*=0.25 first it shows decrease in the value of radial stress t_{rr} with distance r and then it follows an oscillatory pattern.

8. Conclusions

From the analysis of the graphs, it is clear there is a significant influence of transversely isotropy on the deformation of various displacement components, conductive temperature and various stress components of thick circular plate while relating the influence of frequency *ω* with two temperatures. The effect of time harmonic sources frequency in transversely isotropic thick circular plate with two temperatures plays a significant role in the analysis of the deformed medium. As distance r, varied from the point of use of the time harmonic source, variations of displacement components, conductive temperature and various stress components undergoes sudden changes, causing an inconsistent patterns of curves and shows an oscillatory pattern. The shape of curves shows the impact of frequency *ω* on the body and fulfils the purpose of the study.

The outcomes of this research are extremely helpful in the 2-Dproblemwith dynamic response of time harmonic sources in transversely isotropic thermoelastic solid with two temperature, which beneficial to dissect the deformation field such as geothermal engineering; advanced aircraft structure design, thermal power plants, composite engineering, geology, high-energy particle accelerators, and many emerging technologies.

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CC

Nomenclature

δ_{ij}	Kronecker delta,
C_{ijkl}	Elastic parameters,
β_{ij}	Thermal elastic coupling tensor,
T	Absolute temperature,

T_0	Reference temperature,
φ	conductive temperature,
t_{ij}	Stress tensors,
e_{ij}	Strain tensors,
u_i	Components of displacement,
ρ	Medium density,
C_E	Specific heat,
a_{ij}	Two temperature parameters,
α_{ij}	Linear thermal expansion coefficient,
K_{ij}	Materialistic constant,
K_{ij}^*	Thermal conductivity,
ω	Frequency