Coupled Systems Mechanics, *Vol. 2, No. 4 (2013) 375-387*
DOI: http://dx.doi.org/10.12989/csm.2013.2.4.375

Use of infinite elements in simulating liquefaction phenomenon using coupled approach

Sunita Kumari[a] and V.A. Sawant[*]

Department of Civil Engineering, Indian Institute of Technology Roorkee, Roorkee, Uttarakhand 247667, India

(Received February 15, 2014, Revised March 9, 2014, Accepted March 10, 2014)

Abstract. Soils consist of an assemblage of particles with different sizes and shapes which form a skeleton whose voids are filled with water and air. Hence, soil behaviour must be analyzed by incorporating the effects of the transient flow of the pore-fluid through the voids, and therefore requires a two-phase continuum formulation for saturated porous media. The present paper presents briefly the Biot's basic theory of dynamics of saturated porous media with *u–P* formulation to determine the responses of pore fluid and soil skeleton during cyclic loading. Kelvin elements are attached to transmitting boundary. The Pastor–Zienkiewicz–Chan model has been used to describe the inelastic behavior of soils under isotropic cyclic loadings. Newmark-Beta method is employed to discretize the time domain. The response of fluid-saturated porous media which are subjected to time dependent loads has been simulated numerically to predict the liquefaction potential of a semi-infinite saturated sandy layer using finite-infinite elements. A settlement of 17.1 cm is observed at top surface. It is also noticed that liquefaction occurs at shallow depth. The mathematical advantage of the coupled finite element analysis is that the excess pore pressure and displacement can be evaluated simultaneously without using any empirical relationship.

Keywords: 2-D isoparametric continuum element; kelvin element; transmitting boundary; 2-D infinite elements; liquefaction; pastor–zienkiewicz–chan model

1. Introduction

Liquefaction occurs frequently in saturated loose granular materials under earthquake and other dynamic loadings. During past earthquakes, the liquefaction of saturated loose sands has been the cause of severe damage to various buildings and other structures. The catastrophic nature of this type of failure attracted the attention of researchers, and considerable work has been reported to evaluate liquefaction susceptibility (Seed and Lee 1966, Seed and Idriss 1971, Seed 1979, Seed *et al*. 1985, Kramer 1996). A quantitative prediction of the phenomena leading to permanent deformation or unacceptably high build-up of pore pressures is therefore essential to guarantee the safe behaviour of structures.

[*] Associate Professor and Corresponding Author, E-mail : sawntfce@iitr.ac.in
[a] Research Scholar, E-mail : sunita.nitpatna@gmail.com

http://www.techno-press.org/?journal=csm&subpage=7 ISSN: 2234-2184 (Print), 2234-2192 (Online)

In a saturated soil, the steady-state pore-fluid pressures depend only on the hydraulic conditions and are independent of the soil skeleton response to external loads during free drainage conditions which show a single-phase continuum description of soil behaviour. Similarly, a single-phase description of soil behaviour is also adequate when no drainage (no flow) conditions prevail. However, in intermediate cases in which some flow can take place, there is an interaction between the skeleton strains and the pore-fluid flow. The solution of these problems requires that soil behaviour be analyzed by incorporating the effects of the transient flow of the pore-fluid through the voids, which requires a two-phase continuum formulation for saturated porous media named 'mixture theory'.

There are several different approaches to model the behaviour of a two-phase medium which is generally classified as uncoupled and coupled analyses. Numerous researchers (Finn *et al*. 1977, Nemat-Nasser and Shokooh 1979, Liyanapathirana and Poulos 2002) had perfomed the uncoupled analysis of liquefaction in which the response of saturated soil is modeled without considering the effect of soil–water interaction, and then the pore water pressure is included separately by means of a pore pressure generation model. The major deficiency in the uncoupled approach is that it is unable to take into account the progressive stiffness degradation caused by the increase in pore pressures in the soil. Only coupled approach can model the gradual loss of soil strength due to build-up of pore water pressures.

In the coupled analysis a formulation is used where all unknowns are computed simultaneously at each time step. This is a more realistic representation of the physical phenomena than that provided by uncoupled formulation. Biot (1955, 1956) and Biot and Willis (1957) was the first researcher who developed mixture theory for an elastic porous medium. Applications of Biot's theory to finite element analysis of saturated porous media have been presented by Simon *et al*. (1986) for wide range of existing finite element formulations. Prevost (1985, 1989) focused on the integration of the discretized field equations based on the mixture theory and presented a general analytical procedure for encompassing nonlinear constitutive models. Oka *et al*. (1994) discussed the FEM-FDM coupled liquefaction analysis of a porous soil using elasto-plastic model. Elgamal (2003) developed a computational model for analysis of cyclic mobility scenarios which was based on general fully coupled (solid–fluid) finite element formulations. Mesgouez *et al*. (2005) presented the applications of Biot's theory in transient wave propagation in saturated porous media. Taiebat (2007) worked on numerical analyses of liquefiable sand using critical state two-surface plasticity model and densification model for bounded soil domain.

The objective of the present paper is to determine the responses of pore fluid and soil skeleton during cyclic loading using Biot's basic theory of dynamics of saturated porous media with *u–P* formulation in unbounded soil domain. Kelvin elements are attached to transmitting boundary. The purpose of Kelvin elements is to absorb the wave energy and prevent back propagation of wave into the domain. The Pastor–Zienkiewicz–Chan model has been used to describe the inelastic behaviour of soils under isotropic cyclic loadings, including liquefaction and cyclic mobility. Newmark-Beta method is employed to discretize the time domain. Effect of some of the key parameters is examined.

2. Finite element implementation

Continuum based formulations for modeling liquefaction problems have been presented for over two decades. In a landmark paper, Zienkiewicz and Shiomi (1984) presented three possible coupled formulations for modeling of soil skeleton – pore fluid problems. The most general and simplest one is *u-p* formulation which captures the movements of the soil skeleton and the change of the pore pressure. This formulation neglects the differential accelerations of the pore fluid. It does account for acceleration of pore fluid together with soil skeleton, but not the differential one if exist and also neglects the compressibility of the fluid. Using the finite element method for spatial discretization, the *u–P* formulation is as follows

$$[M]\{\ddot{q}\}_e + [K]\{q\}_e - [Q]\{p\}_e = \{f_u\} \tag{1}$$

$$[G]\{\ddot{q}\}_e + [Q]^T\{\dot{q}\}_e + [S]\{\dot{p}\} + [H]\{p\}_e = \{f_P\} \tag{2}$$

Numerical solutions of Eqs. (1) and (2) can be achieved by integrating the equations in the time domain which can extrapolated to the next time instance (t_{n+1}), using known previous initial conditions by employing generalized Newmark method (Katona and Zienkiewicz 1985). In terms of incremental displacements Δq_e and pore pressure Δp_e as primary unknowns, the final set of equations is obtained as follows

$$\left(C_1[M]+[K]\right)\{\Delta q\}_i - [Q]\{\Delta p\}_i = \Delta F_u + [M]\left(C_2\dot{q}_{i-1} + C_3\ddot{q}_{i-1}\right) \tag{3}$$

$$\begin{gathered}\left(C_1[G]+C_4[Q]^T\right)\{\Delta q_i\} - \left(C_2[S]+[H]\right)\{\Delta p_i\} = \Delta F_p + [G]\left(C_2\,\dot{q}_{i-1} + C_3\,\ddot{q}_{i-1}\right) + [Q]^T\left(C_5\dot{q}_{i-1} + C_6\ddot{q}_{i-1}\right) \\ C_1 = 1\big/\left(\beta\,\Delta t^2\right);\, C_2 = 1\big/\left(\beta\,\Delta t\right);\, C_3 = 0.5/\beta;\, C_4 = \alpha\big/\left(\beta\,\Delta t\right);\, C_5 = \alpha/\beta;\, C_6 = 0.5/\beta - 1\end{gathered} \tag{4}$$

In which, α and β the parameters of the generalized Newmark method and Δt is the time step. The vectors, $\dot{q}_t, \ddot{q}_t$ and $\dot{p}_t$ can be evaluated explicitly from the information available at time t_n.

Viscous damping is also incorporated into the dynamic equation of the solid phase in the form of $[C_m]\{\dot{u}\}$, where

$$[C_m] = \alpha_1[M] + \alpha_2[K] \tag{5}$$

is called the Rayleigh damping (Ladhane and Sawant 2012). The coefficients α_1 and α_2 can be obtained by selecting a damping ratio ξ_n and a certain frequency ω_n such that

$$\xi_r = \frac{\alpha_1}{2\omega_i} + \frac{\alpha_2\omega_i}{2} \tag{6}$$

Nonlinear elasticity and theory of generalized plasticity are used to determine the relationships between incremental stresses and strains. The incremental stress–strain relationship is expressed as

$$d\sigma = D^{ep} : d\varepsilon \tag{7}$$

where $d\sigma$ and D_{ep} = incremental stress tensor and elastoplastic stiffness tensor, respectively.

The elastoplastic stiffness tensor is given as (Zienkiewicz and Mroz 1984).

$$D^{ep} = D^e - \frac{D^e : n_{gL/U} : n^T : D^e}{H_{L/U} + n^T : D^e : n_{gL/U}} \tag{8}$$

where D_e, n, $n_{gL/U}$, and $H_{L/U}$ are elastic stiffness tensor, loading direction vector, flow direction vector under loading or unloading conditions, and loading or unloading plastic modulus, respectively.

2.1 Pastor–zienkiewicz–chan model

Soil behaviour under cyclic loading is complex. Hence, the constitutive model used in a numerical code should be able to capture important features of soil behaviour under cyclic loading such as permanent deformation, dilatancy, and hysteresis loops to obtain reliable solutions of displacements and pore water pressure. For this study the constitutive model described by Pastor *et al*. (1990) was used for the sand. The P-Z Mark III model is a generalized plasticity-bounding surface-non associative type model. The model is described by means of yield surfaces and potential surfaces which are described by these equations respectively

$$f=\left\{q-M_f p\left(1+\frac{1}{\alpha_f}\right)\left[1-\left(\frac{p}{p_c}\right)^{\alpha_f}\right]\right\}$$
$$g=\left\{q-M_g p\left(1+\frac{1}{\alpha_g}\right)\left[1-\left(\frac{p}{p_g}\right)^{\alpha_g}\right]\right\} \tag{9}$$

In which, p is mean confining stress; q is deviatoric shear stress; M_g is slope of the critical state line; α_f and α_g are constant; p_c and p_g are size parameter.

The dilatancy of the sand in the P-Z Mark III model is approximated using the linear function of the stress ratio $\eta = q/p$ as suggested by Nova and Wood (1982) as

$$d_g=\frac{d\varepsilon_v^p}{d\varepsilon_s^p}=\left(1+\alpha_g\right)\left(M_g-\eta\right) \tag{10}$$

and $d\varepsilon_v^p$ and $d\varepsilon_s^p$ are incremental plastic volumetric and deviatoric strains, respectively.

M_g is related to the angle of angle of friction ϕ' by the Mohr-Coulomb relations

$$M_g = 6S\sin\phi' \big/ \left(3S - \sin\phi'\right) \tag{11}$$

Value of S is ±1 based on compression or extension.

The plastic flow direction under loading n_{gL} is given in the triaxial space as

$$\eta_{gL}=\frac{1}{\sqrt{1+d_g}}\begin{Bmatrix} d_g \\ 1 \end{Bmatrix} \tag{12}$$

The non-associate flow rule is followed so that the loading direction is expressed as

$$\eta=\frac{1}{\sqrt{1+d_f}}\begin{Bmatrix} d_f \\ 1 \end{Bmatrix} \tag{13}$$

$$d_f = (1+\alpha_f)(M_f - \eta) \tag{14}$$

M_f maintains a constant ratio with M_g. Pastor *et al*. (1990) assumed this ratio to be dependent on relative density (D_r) suggesting relation for M_f as

$$M_f = M_g \times D_r \tag{15}$$

In the P-Z Mark III model, the plastic modulus for loading (H_L) is obtained as

$$\begin{gathered} H_L = H_0 P(1-\eta/\eta_f)^4\,(1-\eta/M_g + \beta_0\beta_1 e^{-\beta_0\xi}) \\ \eta_f = (1+1/\alpha)M_f \ \ and\ \ \xi = \int d\xi = \int\left|d\varepsilon_q^p\right| \end{gathered} \tag{16}$$

where H_0, β_0 and β_1 are model parameters; and $d\xi$ is plastic deviatoric strain increment.

The undrained triaxial tests predict rapid pore pressure build up on unloading. This highlights the necessity to predict plastic strains on unloading in a constitutive model. The P-Z Mark III model uses the following expression for the plastic flow direction and the unloading plastic modulus H_u)

$$\eta_{gU} = \frac{1}{\sqrt{1+d_g}}\begin{Bmatrix} -\left|d_g\right| \\ 1 \end{Bmatrix} \tag{17}$$

$$\begin{gathered} H_u = H_{u0}\left(\frac{M_g}{\eta_u}\right)^{\gamma u}\ for\left|\frac{M_g}{\eta_u}\right| > 1 \\ H_u = H_{u0}\ for\left|\frac{M_g}{\eta_u}\right| < 1 \end{gathered} \tag{18}$$

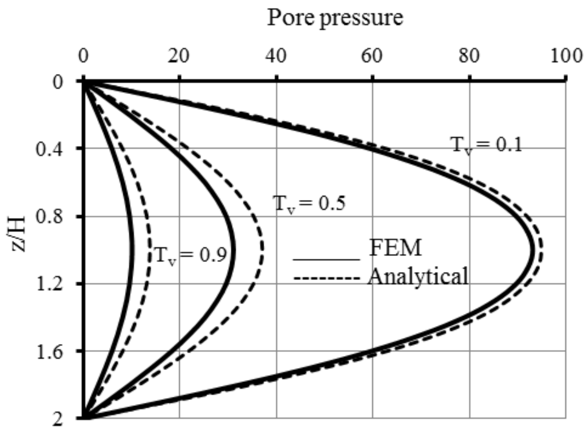

Fig. 1 Variation in pore pressure along depth

η_u is called the unloading stress ratio given by

$$\eta_u = (q / p)_u \tag{19}$$

H_{uo} and γ_u are specified material constants.

3. Validation

The correctness and accuracy of the proposed FEM based solution algorithm are validated by comparing the numerical results obtained with the analytical solutions in the literature. For this purpose, the standard problem of consolidation process was considered. Initial applied pressure on the top of clay layer is 100 kPa. Variation of pore pressure along the depth for Time factors T_v 0.1, 0.5 and 0.9 are compared in Fig. 1 with analytical solution. A good agreement is observed between FEM result and analytical solution.

4. Numerical simulation

The liquefaction phenomenon had been numerically modeled considering two-dimensional plane-strain conditions. The saturated loose sand layer of thickness 10 m, underlain by 4m depth of gravel had been considered for numerical simulation. The unbounded soil domain is discretized into 224 elements of uniform finite-infinite element mesh as shown in Fig. 2. The 8–4 Node mixed

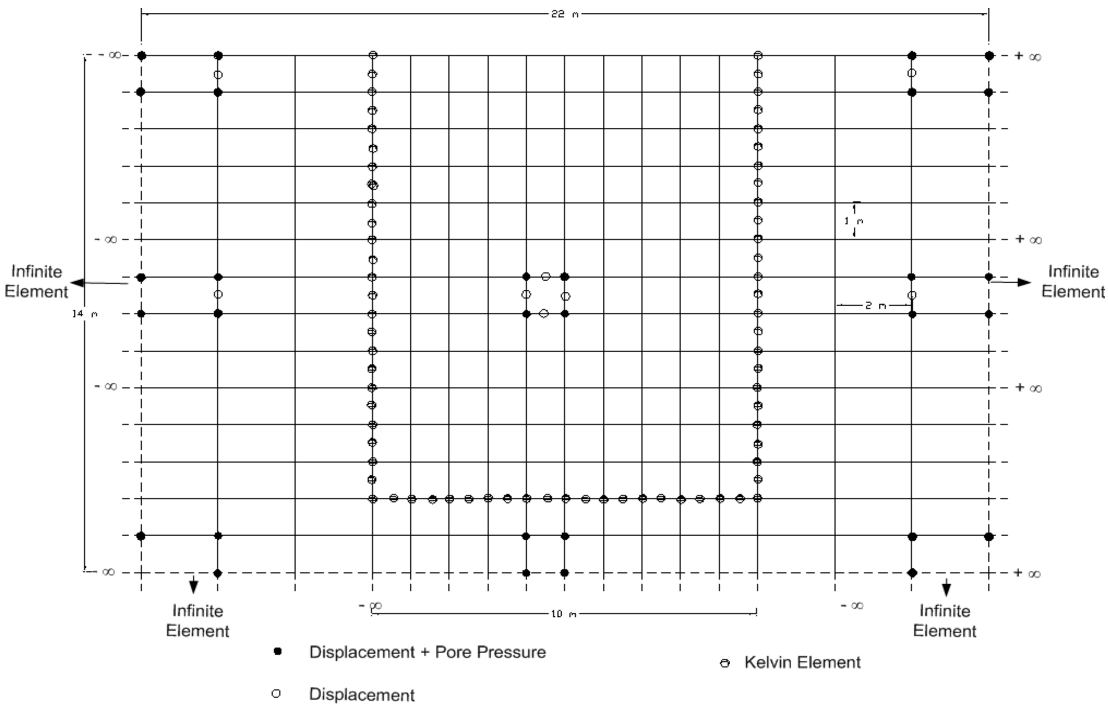

Fig. 2 Unbounded soil domain under consideration

Table 1 Material properties and model parameters (Sadeghian and Manouchehr 2012)

Parameters	Description	Value
γ	Unit weight of soil	18 kN/m^3
μ	Poisson's ratio	0.31
k	Co-efficient of permeability	6.5×10^{-6} m/s
K	Bulk Modulus	20000 KPa
M_g	Slope of the critical state line (CSL)	1.15
M_f	Yield surface parameter	1.03
D_R	Relative density	0.4
β_0	Shear hardening parameter (proposed by modellers)	4.2
β_1	Shear hardening parameter (proposed by modellers)	0.2
α_f and α_g	Dilatancy parameters (proposed by modellers)	0.45
H_0	Found by matching the shape of q–p' plot	600
H_{u0}	Unloading plastic modulus	4000
γ_u	Unloading plastic deformation parameter	2
ϕ^o	Friction angle	30^o

element having 8 displacement nodes and 4 pore pressure nodes satisfying the Babuska-Brezzi stability condition were used for finite element. As a result, displacements are continuous biquadratic and pore pressures are continuous bilinear in the element.

The soil domain was extended infinitesimally in the longitudinal direction and vertical direction. Hence, finite elements could be extended by attaching 5–4 Node mixed infinite elements, having 5 displacement nodes and 4 pore pressure nodes, at the horizontal and vertical boundary, whereas corner element should be extended by attaching 3- noded infinite elements (as shown in Fig. 2) as to appropriately model the infinite boundary conditions (Patil *et al*. 2013a). Kelvin elements are attached to transmitting boundary. The purpose of Kelvin elements is to absorb the wave energy and prevent back propagation of wave into the domain (Patil *et al*. 2013b).

Material properties of the purposed model were reported in Table 1. Dissipation of pore pressure was allowed to occur only through the top surface of the sand layer; while the lateral boundaries and the base were considered to infinite extent. The variation of displacement and pore pressure with time at a particular depth had been calculated using finite element code written in FORTRAN-90. The variation of both parameters with time was considered for comparing the response. Analyses were performed in two steps: (1) Static analysis and (2) Dynamic analysis.

4.1 Static analysis

A static analysis was performed to apply the gravitational forces due to self weight of the soil before cyclic excitation. The resulted hydrostatic pressures of fluid and the stress state along a soil column were used as initial conditions for the subsequent dynamic analysis. The coupled equations considered for static analysis were as follows

$$[K]\{q\}_e - [Q]\{p\}_e = \{f_u\} \tag{20}$$

$$[Q]^T\{\dot{q}\}_e + [H]\{p\}_e = \{f_p\} \tag{21}$$

4.2 Dynamic analysis

When equilibrium condition was achieved for initial stress condition, a nonlinear analysis was performed for the cyclic load with the supplied horizontal and vertical cyclic acceleration $a = a_0 \sin \omega t$. The dynamic analyses were performed using a Generalized Newmark (Katona and Zienkiewicz 1985) scheme with nonlinear iterations using initial linear elastic tangential global matrix. The numerical integration parameters of the generalized Newmark's method were selected as $\alpha = 0.60$ and $\beta = 0.3025$ for the dynamic analysis. The material parameters used are described in table.1.

The time step used is usually depends on time of cyclic loading and frequency of the input. Void ratio, permeability and other geometric properties were kept constant during the analysis. Rayleigh damping of 5% is applied at the dominant frequency in the earthquake-like motion input to enhance the energy dissipation characteristic of the constitutive model. The numerical simulation has been performed for 24 cycles of the loading. The amplitude and frequency of the cyclic loading were $a_0 = 0.22$ g and 1 Hz respectively.

5. Numerical results and discussion

The liquefaction behavior of saturated sand has been numerically simulated using the fully coupled formulation. Fig. 3 displays the computed horizontal and vertical displacement at different depth. The maximum values of horizontal and vertical settlement, 2.4 cm and 17 cm are predicted at the top of soil layer respectively. It has been observed that most of the settlements occur during the shaking period. Generally, the horizontal settlement is less than vertical settlement at different depths but it is higher at bottom because of proper drainage and infinite boundary. A relatively less value of settlement observed at 2 m depth, whereas displacements at 12 m depth are negligible in the gravel layer indicating marginal effect of seismic loading due to higher permeability.

The stress paths depicted in Fig. 4 show the typical mechanism of cyclic decrease in effective stress due to pore pressure build-up, captured using the Pastor–Zienkiewicz–Chan model. It is observed that maximum stress ratio q/p is 0.87 at the depth of 0.5 m, which decreases with depth mainly due to effect of overburden pressure.

Fig. 5 displays the computed excess pore pressure at different depth. The computed pore pressure time histories indicate that soil at the depth of 2 m and 4 m is liquefied because excess pore pressure (EPP) is higher than initial vertical stress. A state close to liquefaction was captured at 6 m and 8 m depth because EPP is nearly equal to the initial vertical stress. At bottom of soil domain, rise in excess pore water pressure is less due to gravel layer of high permeability, hence no liquefaction occurs. It also seems that dissipation of EPP is fast at shallow depth after completion of cyclic loading. The reason behind this behavior may due to the change of the coefficient of permeability at the end of shaking takes place in such a way that it increases at shallow depths and decreases at increased depths. In addition, at shallow depth the drainage path is shorter for dissipation of excess pore pressure. These observations indicate that the coefficient of permeability is not a stationary parameter during shaking as well as during drainage processes. Therefore using a constant value for permeability in a numerical analysis possesses an inherent pitfall by which the drainage cannot be simulated in a desirable manner which requires further investigation.

Figs. 6 and 7 shows the computed horizontal and vertical accelerations time histories at

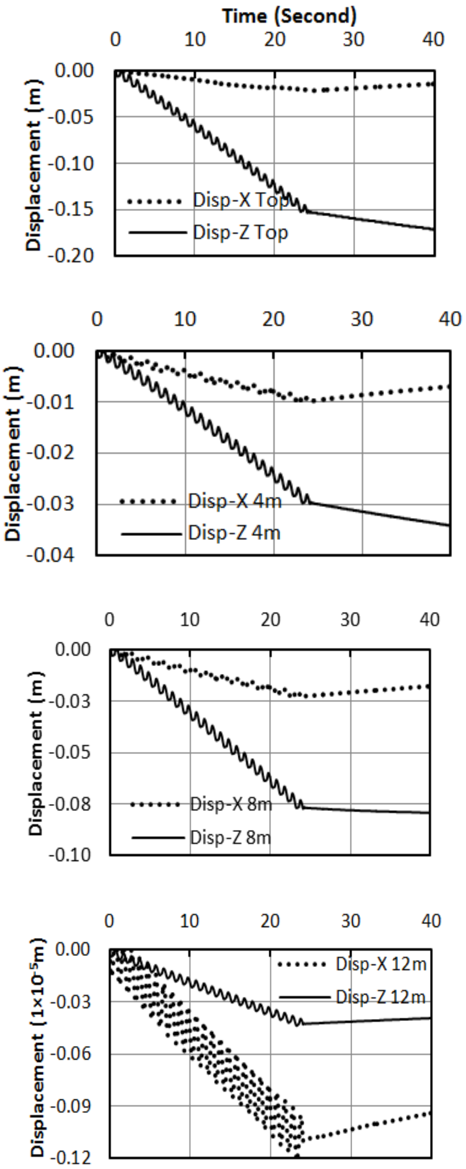

Fig. 3 Computed lateral and vertical displacement time histories at different depth

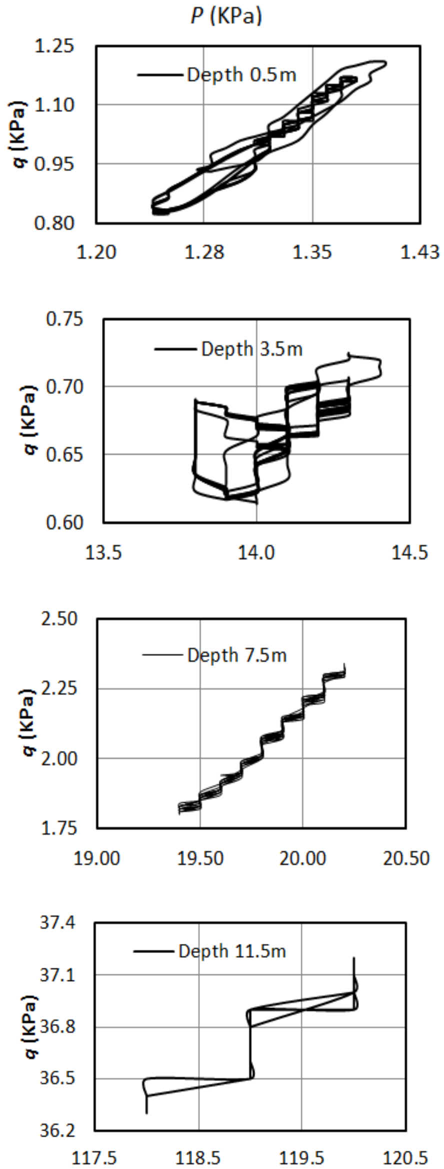

Fig. 4 Computed effective stress path at different depths

different depths. It has been observed that the peak value of these parameters are found to be about 0.3 m/s^2 and 0.75 m/s^2 at top surface, resulting higher settlement. A relatively less value of accelerations are seen at bottom, corresponding to lesser excess pore water pressure. A negligible acceleration is reported in both directions after the end of 24 cycle of loading. Results indicate amplification of earthquake input motion from base to the top surface showing maximum amplification at top level.

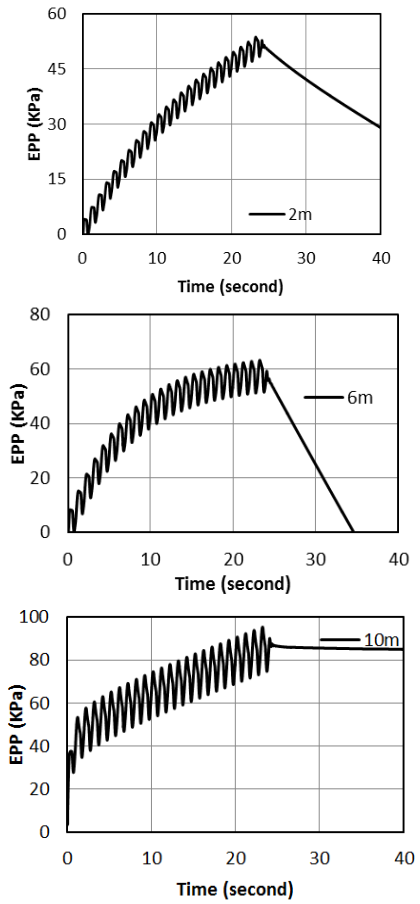

Fig. 5 Computed excess pore pressure time histories at different depth

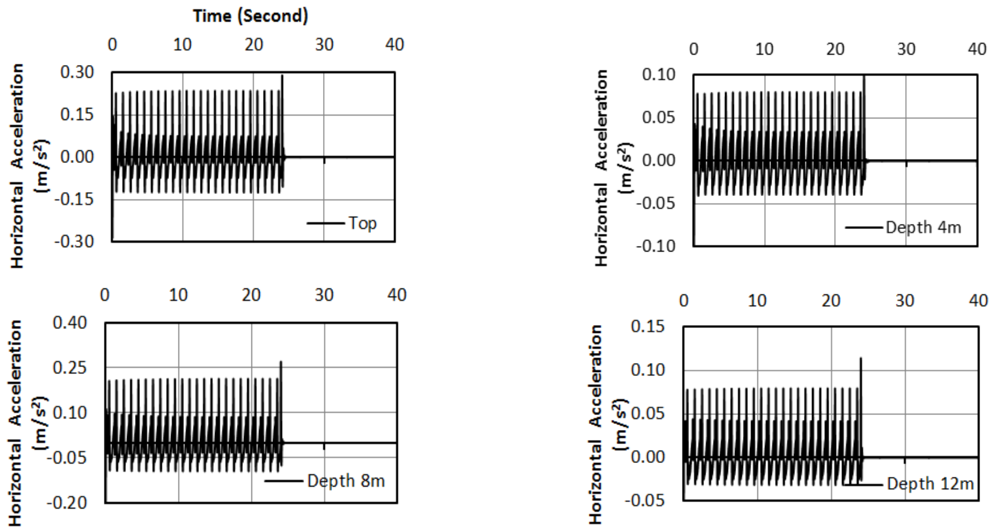

Fig. 6 Computed Horizontal acceleration time histories at different depth

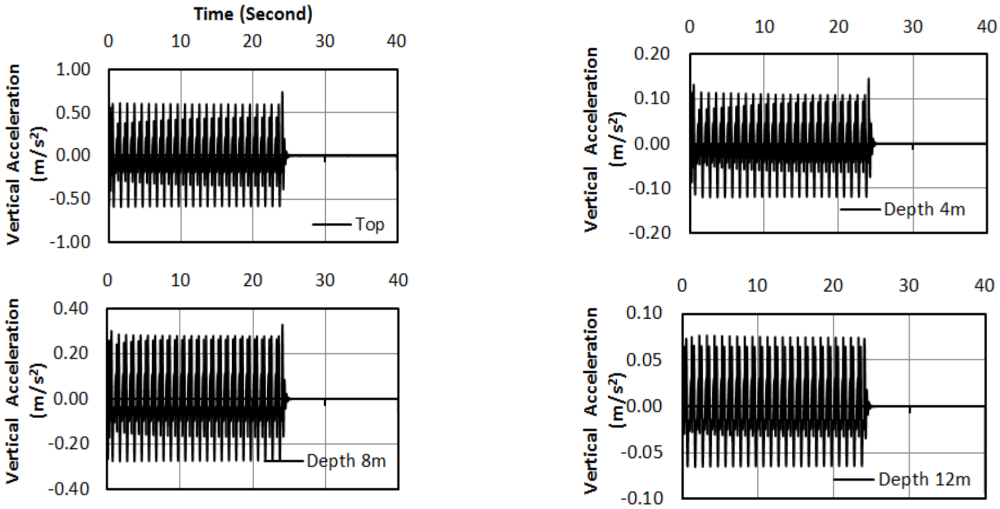

Fig. 7 Computed vertical acceleration time histories at different depth

6. Conclusions

In the present investigation, a computer program based on Coupled dynamic field equations of extended Biot's theory with *u–P* formulation is developed to predict the liquefaction potential of a saturated sandy layer. The method predicts the phenomenological features of dynamic response of saturated sand layers that commonly occur as pore water pressure rises in the sand during cyclic loading. It allows the distribution of pore-water pressure and the effects that drainage and internal flow have on the time of liquefaction to be determined quantitatively. A vertical settlement of 17.1 cm and horizontal displacement of 2.2 cm are observed at top surface. It is noticed that liquefaction occurs at shallow depth. It is observed that maximum stress ratio q/p is 0.87 at the depth of 0.5 m, which decreases with depth mainly due to effect of overburden pressure. This results in development of higher excess pore pressure at shallow depth.

Liquefaction usually causes a significant increase of the coefficient of permeability, but rapid changes in the pattern of excess pore pressure in the soil column during shaking indicates that permeability is not a constant parameter in the liquefaction process and it may either increase or decrease at different depths. Hence, assuming a constant value of the coefficient of permeability may be conservative approach. Finding a realistic assumption for variation of the coefficient of permeability during liquefaction requires further investigation.

The mathematical advantage of the coupled finite element analysis is that the excess pore pressure and displacement can be evaluated simultaneously without using any empirical relationship. The developed numerical formulation can be easily adapted to provide confidence for practicing engineers to use fully coupled procedures for predicting the dynamic performance of geotechnical site.

References

Biot, M.A. and Willis, D.G. (1957), "The elastic coefficients of a theory of consolidation", *J. Appl. Mech. T.*

ASME, **29,** 594-601.

Biot, M.A. (1955), "Theory of elasticity and consolidation for a porous anisotropic solid", *J. Appl. Phys.*, **26**(2), 182-185.

Biot, M.A. (1956), "Theory of propagation of elastic waves in a fluid-saturated porous solid", *J. Acou. Soc. Am.*, **28**(2), 168-178.

Elgamal, A., Yang, Z., Parra, E. and Ragheb, A. (2003), "Modeling of cyclic mobility in saturated cohesionless soils", *Int. J. Plasticity*, **19**(6), 883-905.

Finn, W.D.L., Lee, K.W. and Martin, G.R. (1977), "An effective stress model for liquefaction", *J. Geotech. Eng. Div. ASCE*, **103**(GT6), 513-533.

Katona, M.G. and Zienkiewicz, O.C. (1985), "A unified set of single step algorithms Part 3: the Beta-*m* method, a generalization of the Newmark scheme", *Int. J. Numer. Meth. Eng.*, **21**(7), 1345-1359.

Kramer, S.L. and Seed, B.H. (1988), "Initiation of static liquefaction under static loading conditions", *J. Geotech. Eng. Div. ASCE*, **114**(4), 412-430.

Ladhane, K.B. and Sawant, V.A. (2012), "Dynamic response of 2 piles in series and parallel arrangement", *J. Eng.*, **16**(4), 63-72.

Liyanapathirana, D.S. and Poulos, H.G. (2002), "Numerical simulation of soil liquefaction due to earthquake loading", *Soil Dyn. Earthq. Eng.*, **22**(7), 511-523.

Mesgouez, A., Lefeuve-Mesgouez, G. and Chambarel, A. (2005), "Transient mechanical wave propagation in semi-infinite porous media using a finite element approach", *Soil Dyn. Earthq. Eng.*, **25**(6), 421-430.

Nemat-Nasser, S. and Shokooh, A. (1979), "A unified approach to densification and liquefaction of cohesionless sand in cyclic shearing", *Can. Geotech. J.*, **16**(4), 659-678.

Nova, R. and Wood, D.M. (1982), *A Constitutive Model for Soil Under Monotonic and Cyclic Loading, Soil Mechanics-Transient and Cyclic Loads* (Edited by G.N. Pande and O.C. Zienkiewicz), John Wiley & Sons Ltd., New York, USA.

Oka, F., Yashima, A., Shibata, T., Kato, M. and Uzuoka, R. (1994), "FEM-FDM coupled liquefaction analysis of a porous soil using an elasto-plastic model", *Appl. Sci.*, **52**(3), 209-245.

Pastor, M., Zienkiewicz, O.C. and Chan, A.H.C. (1990), "Generalized plasticity and the modeling of soil behavior", *Int. J. Numer. Analyt. Meth. Geomech.*, **14**(3), 151-190.

Patil, V.A., Sawant, V.A. and Deb, K. (2013b), "3-D finite element dynamic analysis of rigid pavement using infinite elements", *Int. J. Geomech. ASCE*, **13**(5), 533-544.

Patil, V.A., Sawant, V.A. and Deb, K. (2013a), "2-D finite element analysis of rigid pavement considering dynamic vehicle pavement interaction effects", *Appl. Math. Model.*, **37**(3), 1282-1294.

Prevost, J.H. (1985), "A simple plasticity theory for frictional cohesionless soils", *Soil. Dyn. Earthq. Eng.*, **4**(1), 9-17.

Prevost, J.H. (1989), *DYNA1D, A Computer Program for Nonlinear Seismic Site Response Analysis: Technical Documentation, Technical Report NCEER-89–0025, National Center for Earthquake Engineering Research*, State University of New York at Buffalo, NY, USA.

Sadeghian, S. and Manouchehr, L.N. (2012), "Using state parameter to improve numerical prediction of a generalized plasticity constitutive model", *J. Comp. Geosci.*, **51**, 255-268.

Seed, H.B. and Idriss, I.M. (1971), "Simplified procedure for evaluating soil liquefaction potential", *J. Soil Mech. Found. Div. ASCE*, **92**(SM6), 1249-1273.

Seed, H.B. and Lee, K.L. (1966), "Liquefaction of saturated sands during cyclic loading", *J. Geotech. Eng. ASCE*, **92**(SM6), 105-134.

Seed, H.B., Tokimatsu, K., Harder, L.F. and Chung, R. (1985), "Influence of SPT procedures in soil liquefaction resistance evaluations", *J. Geotech. Eng., ASCE*, **111**(12), 1425-1445.

Seed, H.B. (1979), "Soil liquefaction and cyclic mobility evaluation for level ground during earthquakes", *J. Geotech. Eng. Div. ASCE*, **105**(GT2), 201-255.

Simon, B.R., Wu. J.S.S., Zienkiewicz, O.C. and Paul, D.K. (1986), "Evaluation of u–w and u–π finite element methods for the dynamic response of saturated porous media using one-dimensional models", SIAM. *J. Numer. Anal.*, **10**(5), 461-482.

Taiebat, M., Shahir, H. and Pak, A. (2007), "Study of pore pressure variation during liquefaction using two

constitutive models for sand", *Soil*. *Dyn*. *Earthq*. *Eng*., **27**(1), 60-72.
Zienkiewicz, O.C. and Mroz, Z. (1984), *Generalized Plasticity Formulation and Applications to Geomechanics*, *Mech*. *Eng*. *Mater*., (Edited by C.S. Desai and R.H. Gallagher), Wiley, New York, USA.
Zienkiewicz, O.C. and Shiomi, T. (1984), "Dynamic behaviour of saturated porous media; The generalized biot formulation and its numerical solution", *Numer*. *Meth*., **8**(1), 71-96.

CC