

## Analysis of the piled raft for three load patterns: A parametric study

H.S. Chore<sup>\*1</sup> and M.J. Siddiqui<sup>2a</sup>

<sup>1</sup>Department of Civil Engineering, Datta Meghe College of Engineering,  
Sector-3, Airoli, New Mumbai- 400708, India

<sup>2</sup>Department of Civil Engineering, Anjuman- E- Islam A.R. Kalsekar Polytechnic,  
New Panvel- 410206, India

(Received November 4, 2013, Revised January 21, 2014, Accepted January 24, 2014)

**Abstract.** The piled raft is a geotechnical construction, consisting of the three elements- piles, raft and the soil, that is applied for the foundation of a tall buildings in an increasing number. The piled rafts nowadays are preferred as the foundation to reduce the overall and differential settlements; and also, provides an economical foundation option for circumstances where the performance of the raft alone does not satisfy the design requirements. The finite element analysis of the piled raft foundation is presented in this paper. The numerical procedure is programmed into finite element based software SAFE in order to conduct the parametric study wherein soil modulus and raft thickness is varied for constant pile diameter. The problems of piled raft for three different load patterns as considered in the available literature (Sawant *et al.* 2012) are analyzed here using SAFE. The results obtained for load pattern- I using SAFE are compared with those obtained by Sawant *et al.* (2012). The fair agreement is observed in the results which demonstrate the accuracy of the procedure employed in the present investigation. Further, substantial reduction in maximum deflections and moments are found in piled raft as compared to that in raft. The reduction in deflections is observed with increase in raft thickness and soil modulus. The decrease in maximum moments with increase in soil modulus is seen in raft whereas increase in maximum moments is seen in piled raft. The raft thickness and soil modulus affects the response of the type of the foundation considered in the present investigation.

**Keywords:** piled raft; raft; pile; load pattern; soil modulus; maximum deflection; maximum moment

### 1. Introduction

The type of foundation and its design for a tall structure is based on the magnitude of the loads on it and the type of founding strata which supports it. If the founding stratum is within a reasonable depth, a shallow foundation in the form of raft is adequate. However, if the material is weak, the loads need to be transferred down to capable strata by means of deeper basements or piles. This is true, especially in case of multistoried building frames resting on weak sub soil strata where heavy structural loads acting on the frames have to be transmitted safely below to the firm

---

\* Corresponding author, Professor, E-mail: [hschore@rediffmail.com](mailto:hschore@rediffmail.com)

<sup>a</sup>Lecturer, E-mail: [junaid\\_engineer@rediffmail.com](mailto:junaid_engineer@rediffmail.com)

strata.

In the past few decades, there has been an increasing recognition that the use of pile groups in conjunction with the raft can lead to considerable economy without compromising the safety and performance of the foundation. Such a foundation makes use of both the raft and the piles, and is referred to here as a *pile-enhanced raft or a piled raft*. The piled-raft concept has also been proven to be an economical way to improve the serviceability of foundation performance by reducing settlements to acceptable levels. Although the piled-raft concept has been most notably applied to new construction involving high-rise buildings it is also potentially useful for remedial works and moderate height structures.

Methods that have been used for the analysis range from simplified calculations to numerical methods such as the boundary element method (Butterfield and Banerjee 1971, Brown and Wiesner 1975, Kuwabara 1989, Mendonca and De Paiva 2003) and the finite element method (Hooper 1973, Ottaviani 1975, Chow 1987, Liu and Novak 1991, Katzenbach and Reul 1997, Prakoso and Kulhawy 2001, Reul and Randolph 2003). In early years because of the limited availability of computer memory and processing speed, the use of numerical methods was confined to simple problems. In last three decades due to rapid development in computer technologies, numerical methods such as full three dimensional finite element methods are often used to solve the complex problems.

The foundation concept of piled rafts differs from traditional foundation design, where the loads are assumed to be carried either by the raft or by the piles, considering the safety factors in each case. Several methods of analyzing piled rafts that have developed over the years include approximation methods, finite element method, boundary element method, combined boundary element and finite element method, combined finite layer and finite element method; and variational approach.

## 2. Brief review of literature

In recent years, a variety of approaches for analyzing the piled-raft foundation system as mentioned in the preceding section have been developed over the years. All these approaches vary in the degree of sophistication of the formulations amount and the type of input parameters required, assumptions made; and in the applicability to realistic pile-soil-raft situations. Some of the significant studies are briefed approach wise in the subsequent paragraphs.

The approximation approach as presented by Chen *et al.* (1974) treated the raft as a thin plate, the piles as springs and the soil as an elastic continuum; and further, the interaction effects between the piles were ignored. Randolph (1983) presented a method to compute the interaction between a single pile and a circular raft. Clancy and Randolph (1993) employed a hybrid method in which analytical solution was combined with the finite elements. The raft was modeled by two-dimensional thin plate finite elements, the piles were modeled by one-dimensional rod finite elements and the soil response was calculated by using an analytical solution. Poulos (1994) employed a finite difference method for the raft with the consideration of the interaction effects between the piles and raft. Kitiyodom and Matsumoto (2003) developed a simplified method of numerical analysis using a hybrid model in which the flexible raft is modelled as thin plates and the piles as elastic beams and the soil is treated as springs.

The finite element method is one of the powerful tools for the analysis of the complex problems of piled raft. In order to reduce the computational efforts, the problems are sometimes simplified to

an axi-symmetric problem or a plane- strain problem. Some of the noteworthy contributions using this method include those by Hooper (1973), Madhav and Karmarkar (1982), Kakurai *et al.* (1991), Wiesner (1991), Gandhi and Maharaj (1996), Smith and Wang (1998), Franke *et al.* (2000), Katzenbach *et al.* (2000), Reul and Randolph (2003), Maharaj and Gandhi (2004), Poulos *et al.* (2006), Thoiba Singh, N. and Balweshwar Singh (2008), Noh *et al.* (2008), Cheng (2011), Xie *et al.* (2012) and Sawant *et al.* (2012). Some of the researchers analyzed the circular piled rafts while few of them, reported the performance of piled raft foundation for a multi-storeyed building. Some of the analyses were carried out in the context of non-linear behaviour of soil, few of them even used finite elements in conjunction with infinite elements. While some investigations considered sandy soil, few of them considered the cohesive sub-soil. Even, a study considered layered soil. Some studies were carried out using complete three dimensional finite element analysis; few studies were carried out in the context of simplified finite element models.

The boundary element method is a powerful tool that can be applied in engineering applications as only the boundary has to be discretized which reduces the amount of computer memory and the time to solve the problem as compared to that in finite element or finite difference method. This method provides a direct and accurate solution for the analysis. Moreover, it is fast and requires a moderate amount of computer storage space. The method has been used by many researchers (Brown and Wiesner 1975, Kuwabara 1989, Baziar *et al.* 2009, Sales *et al.* 2010) in the solution of the problem of piled raft embedded in different types of soil. Different idealizations were made for modelling different components of the foundation in question.

Integrating pros and cons some of the researchers (Hain and Lee 1978, Kakurai *et al.* 1987, Sinha 1997, Franke *et al.* 2000 and Mendonca and De Paiva 2003) even made use of combined boundary element and finite element method. Small and Booker (1984, 1986) developed an approach based on the finite layer technique in conjunction with the finite element method for analyzing the piled raft in layered soil. Maharaj and Gandhi (2004), Tan *et al.* (2005) and Chow *et al.* (2011) worked on the similar lines as that of Small and Booker (1984). The variational approach developed by Shen *et al.* (1999) makes use of the principle of minimum potential energy to simulate the response of the foundation system. Discretizations are required only at the interface between the raft and the soil. The method was extended later by many researchers (Shen and Teh 2002, Liang and Chen 2004 and Chow *et al.* 2011).

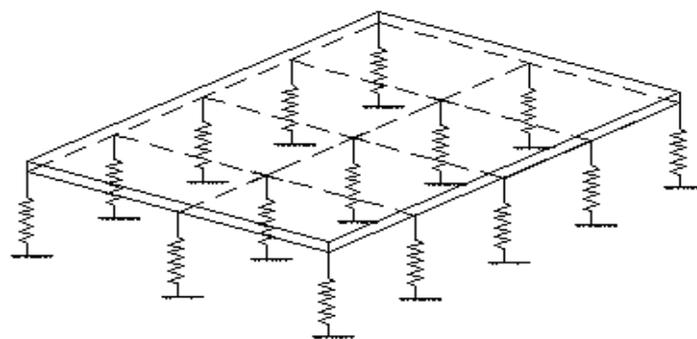


Fig. 1 Structural idealization with raft and supporting soil

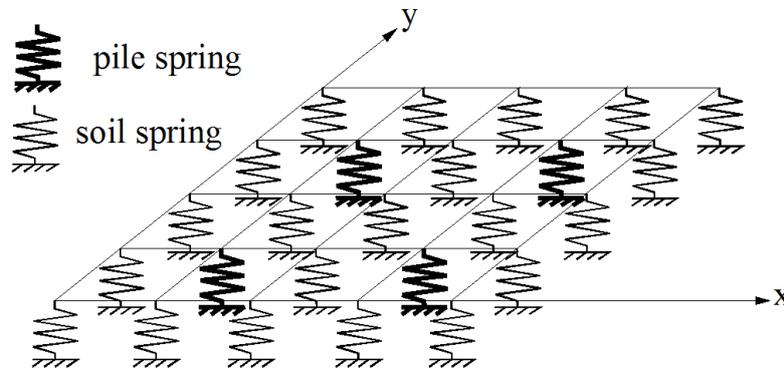


Fig. 2 Structural idealization for piled raft and supporting soil

Based on the afore-mentioned review of literature, the analysis of a piled raft is presented using the finite element based software, *SAFE*. The problems considered in a study reported by Sawant *et al.* (2012) are considered in the present investigation. The present study aims at comparing the response of the raft foundations of different thicknesses with and without pile and varying soil conditions by keeping the pile dimensions constant in respect of a particular load pattern. The effect of raft thickness, soil modulus and load pattern on the response is considered.

### 3. Idealizations made in the mathematical modeling

For the purpose of analysis, standard software *SAFE*, based on the philosophy of finite element method, is used. Initially to observe the behavior of piled raft, piles are modeled as column (spring) and raft as beam on elastic foundation. The soil is modeled as Winkler's springs along the periphery and at the pile tip. The raft takes the load from super structure and transfers it to pile as well as soil; the part load will be taken by raft and part that by pile. In this method, the piles are attached as column (point spring of equivalent stiffness). Fig. 1 indicates the structural idealization with raft and supporting sub-soil whereas Fig. 2 indicates the structural idealization for piled raft with supporting sub-soil.

The behavior of piled raft is studied through finite element based software *SAFE*. In this, the piles are uniformly distributed with uniform horizontal spacing. The distribution of bending moment and shear force in raft and piled raft also has considerable difference which results in the reduction of raft thickness and makes it economical especially when raft is subjected to large amount of forces.

### 4. Particulars of the problem

For the purpose of a parametric study undertaken in this paper, the problems as considered in the literature (Sawant *et al.* 2012) are considered. The particulars that were considered in the afore-mentioned study are as below.

- (a) Young's modulus ( $E$ ) =  $2.48 \times 10^7$  kN/m<sup>2</sup>, Poisson's ratio ( $\mu$ ): 0.3

- (b) Thickness of the raft: 0.45 m, 0.9 m, 1.5 m
- (c) Piles with length 3 m and diameter 300 mm (0.3 m) under columns.
- (d) Modulus of subgrade reaction: 40000 kN/m<sup>3</sup>, 100000 kN/m<sup>3</sup>, 200000 kN/m<sup>3</sup>, 400000 kN/m<sup>3</sup>
- (e) Column loads: Three load patterns, namely- LP-I, LP-II, LP-III
  - Load-Pattern-I: Raft size- 10 m × 10 m; Loads- 800 kN on columns placed in the corners, 1500 kN on middle columns along edges, 2500 kN on the central column as indicated in Fig. 3
  - Load-Pattern-II: Raft size- 10 m ×10 m; Loads- 1000 kN on all the columns as indicated in Fig. 4
  - Load Pattern-III: Raft Size- 14 m ×14 m; Loads: 800 kN on all the columns in the last row, 1000 kN on all the columns placed in last but one row; and thereafter, 3000 kN loads on all the columns in the front and intermediate row, as indicated in Fig. 5

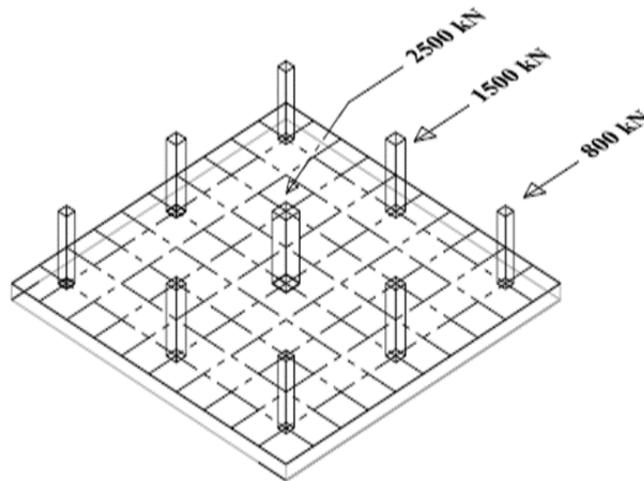


Fig. 3 Load pattern-I

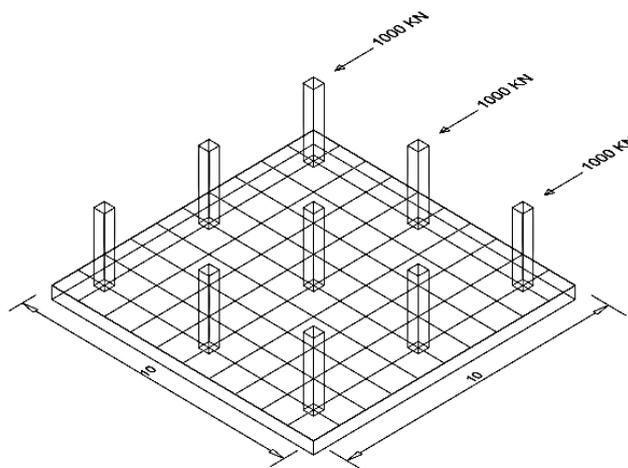


Fig. 4 Load pattern-II

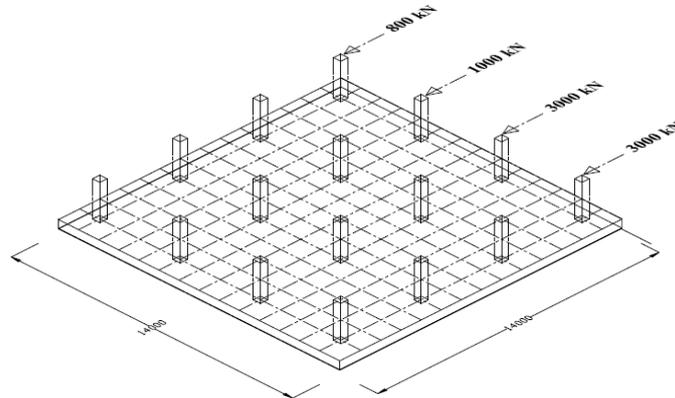


Fig. 5 Load pattern-III

It may be noted that Sawant *et al.* (2012) presented an improved solution algorithm based on the finite element method to analyse piled raft foundation. Piles were modelled as beam elements with soil springs. The finite element analysis of raft was based on the classical theory of thick plates resting on Winkler foundation that accounts for the transverse shear deformation of the plate. Four noded isoparametric rectangular elements with three degree of freedom per node were considered in the development of finite element formulation. Independent bilinear shape functions were assumed for displacement and rotational degrees of freedom.

## 5. Parametric study

For specified three load patterns, the raft thickness and the soil modulus are varied to study their effect on the response. The response is considered in terms of maximum deflection and bending moments in the raft. The analysis is carried out with the help of FEM based software SAFE. Fig. 6 shows the three dimensional (3D) extrude model for all load patterns in the standard software SAFE.

For the purpose of the validation of the numerical procedure proposed to be implemented with SAFE, the results obtained using the afore-mentioned software package in the present investigation are compared with those available in the literature (i.e., same as mentioned in the problem description load pattern-I). The results are shown in Table 1.

Table 1 Maximum deflection for different raft thickness (m) and various soil moduli (kN/m<sup>3</sup>)

Thickness (m)	Present study (SAFE)				Sawant <i>et al.</i> (2012)			
	40000	100000	200000	400000	40000	100000	200000	400000
0.45	2.26	1.59	1.22	0.95	3.25	1.6	1	0.65
0.9	1.95	1.22	0.81	0.54	2.95	1.25	0.75	0.4
1.5	1.87	1.16	0.74	0.46	2.8	1.15	0.6	0.35

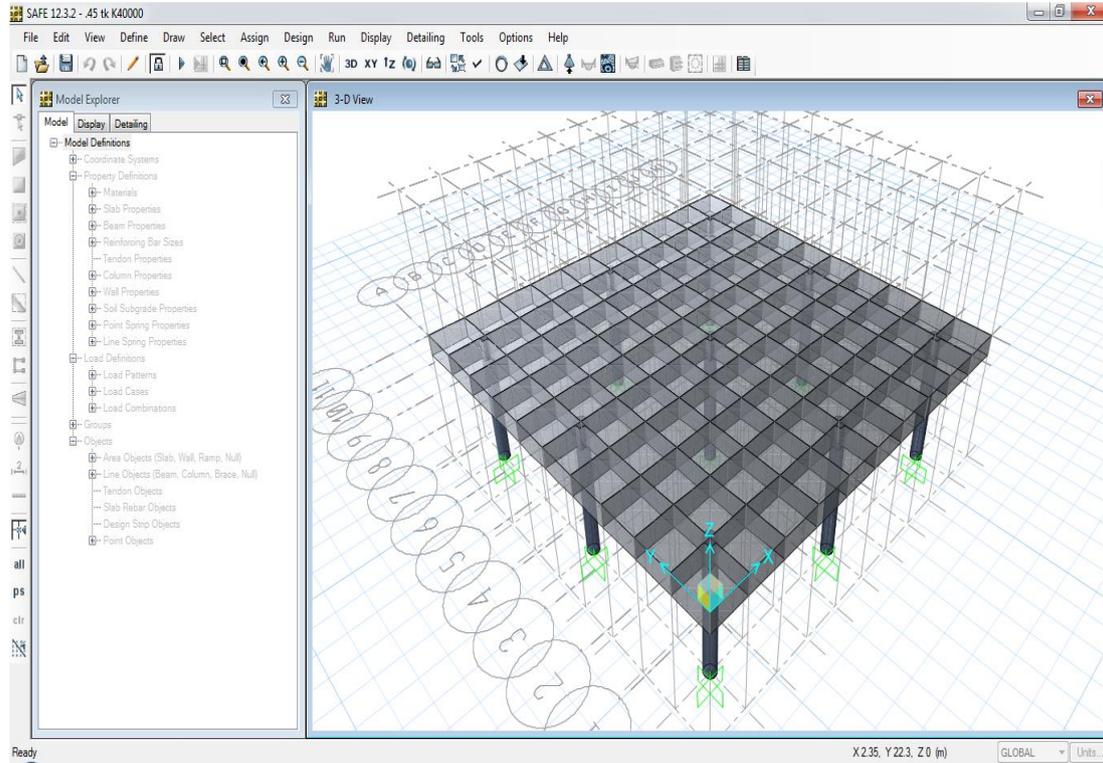


Fig. 6 Generalized F.E. model using SAFE for load pattern under consideration

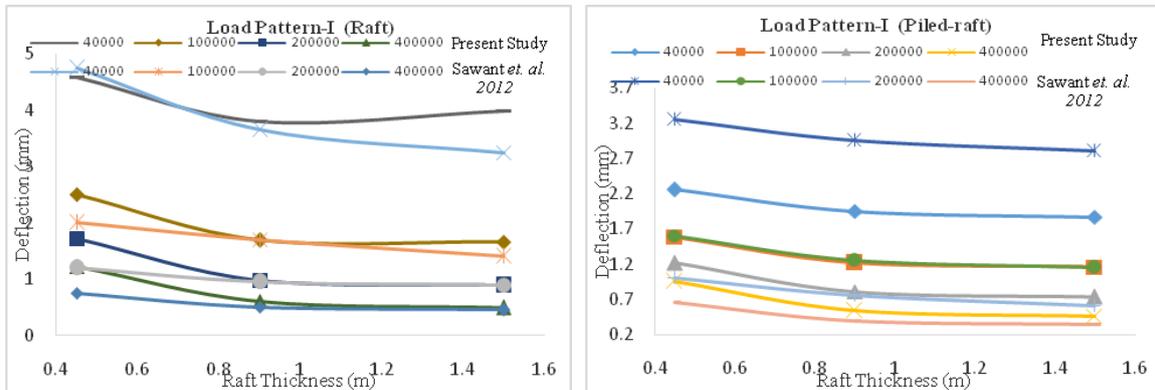


Fig. 7 Variation in maximum deflection in raft and piled raft with raft thickness for load pattern-I

The difference between the values of maximum deflection as obtained using SAFE and the one reported in the literature (Sawant *et al.* 2012) is observed to be 33% for the value of modulus of subgrade reaction to be 40000 kN/m<sup>3</sup>. Similarly, for subsequent higher values of soil modulus, the difference is observed to be 1%, 14% and 27%. It is seen from these results that the maximum deflection as observed in respect of the present study using SAFE and the one reported in literature agrees fairly. The close agreement was found in the either result (Fig. 7) which validates the

accuracy of the numerical procedure resorted to in the present study. Further, the variation in the results obtained in the present study and the one resorted to in a study reported by Sawant *et al.* (2012) could be attributed to the different axioms followed in either investigation.

### 5.1 Load pattern-I

The values of the maximum deflection in raft and piled raft as obtained for all the three load patterns are shown in Table 2. Along similar lines, the values of the maximum moments are shown in Table 3.

From Fig. 7, it is evident that the maximum deflections are found to be decreasing with increase in the raft thickness. Also the maximum deflections are decreasing with increase in the soil modulus. As thickness of the raft increases from 0.45 m to 0.9 m, deflection of the raft decreases from 20% to 100% and deflection of the piled raft decreases from 15% to 75% in the increasing order of modulus of subgrade reaction. Further, for 0.9 m to 1.5 m the deflection of the raft decreases from 4% to 22% and deflection of the piled raft decreases from 4% to 17% in the increasing order of modulus of subgrade reaction.

The percentage variation in maximum deflection of the piled raft with respect to the raft for different values of modulus of subgrade reaction is shown in Fig. 7. It is seen that with the increase in modulus of subgrade reaction, the maximum deflections in the piled raft reduces. The decrease in the deflection in respect of piled raft is observed in the range of 14%-51% when compared with that observed in respect of raft foundation. For this load pattern the piled raft is more suitable when modulus of subgrade reaction is low. In respect of higher values of modulus of subgrade reaction, the decrease in deflection is seen to be 14% as compared to that found in raft foundation.

Table 2 The maximum deflection in raft and piled raft for various load pattern

Thickness (m)	Raft Soil modulus (kN/m <sup>3</sup> )				Piled-Raft Soil modulus (kN/m <sup>3</sup> )			
	40000	100000	200000	400000	40000	100000	200000	400000
<b>Load Pattern I</b>								
0.45	4.58	2.51	1.72	1.23	2.26	1.59	1.22	0.95
0.9	3.8	1.69	0.98	0.61	1.95	1.22	0.81	0.54
1.5	3.98	1.66	0.89	0.5	1.87	1.16	0.74	0.46
<b>Load Pattern II</b>								
0.45	5.08	2.29	1.2	0.7	1.49	1.13	0.78	0.49
0.9	3.71	1.76	1	0.55	1.37	0.99	0.7	0.45
1.5	3.45	1.51	0.83	0.47	1.42	0.93	0.62	0.39
<b>Load Pattern III</b>								
0.45	14.68	6.6	3.47	2.05	4.34	3.27	2.25	1.47
0.9	11.45	5.17	2.82	1.54	4	2.82	1.95	1.25
1.5	9.89	4.39	2.4	1.31	3.97	2.64	1.76	1.09

Table 3 Values of maximum moment in raft and piled raft for various load pattern

Thickness (m)	Raft Soil modulus (kN/m <sup>3</sup> )				Piled-Raft Soil modulus (kN/m <sup>3</sup> )			
	40000	100000	200000	400000	40000	100000	200000	400000
<b>Load Pattern I</b>								
0.45	84.76	70.59	62.01	67.29	29.04	121.39	141.21	152.17
0.9	100.87	98.56	94.1	85.79	54.59	133.97	175.08	198.14
1.5	103.83	103.34	102.28	99.74	49.46	101.95	162.71	200.86
<b>Load Pattern II</b>								
0.45	70.3	49.16	35.44	23.27	48.51	78.1	95.69	107.89
0.9	141.79	107.53	83.1	62.55	86	162.12	204.03	227.18
1.5	175.6	158.09	136.9	110.13	80.29	203.21	271.17	308.42
<b>Load Pattern III</b>								
0.45	198.17	150.59	109.25	70.68	153.7	233.48	281.43	315.05
0.9	281.39	243.54	213.34	181.61	252.27	429.4	541.46	618.8
1.5	395.04	322.57	271.88	243.68	356.54	518.74	600.88	711.16

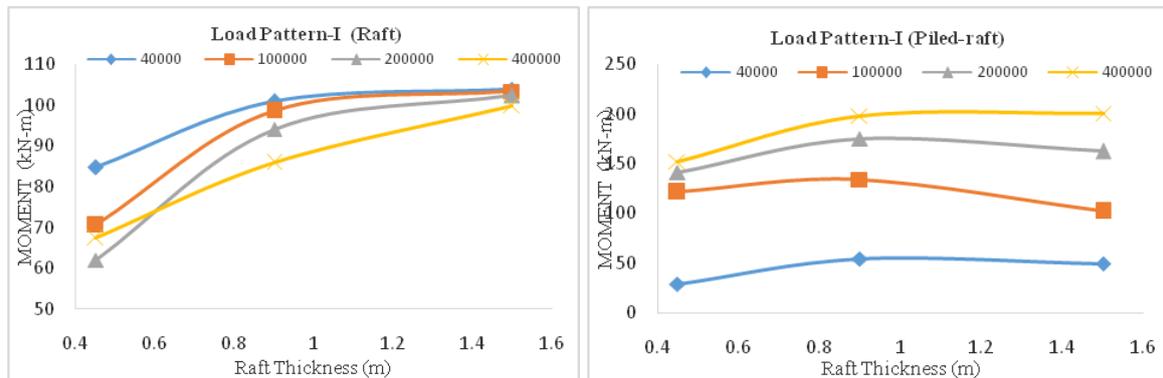


Fig. 8 Variation in maximum moment with raft thickness in raft and piled raft for load pattern-I

The percentage variation in maximum deflection of the piled raft with respect to the raft for different values of modulus of subgrade reaction is shown in Fig. 7. It is seen that with the increase in modulus of subgrade reaction, the maximum deflections in the piled raft reduces. The decrease in the deflection in respect of piled raft is observed in the range of 14%-51% when compared with that observed in respect of raft foundation. For this load pattern the piled raft is more suitable when modulus of subgrade reaction is low. In respect of higher values of modulus of subgrade reaction, the decrease in deflection is seen to be 14% as compared to that found in raft foundation.

Referring to Fig. 8, it is seen that the maximum moments are increasing with increase in raft thickness. Further, with increase in the soil modulus, the reduction in moments is observed. However, for this load pattern the reduction is marginal. Moreover, in case of piled-raft, the maximum moments are increasing with the raft thickness. With increase in the soil modulus, the

moments are found to increase; the increment of the moment is marginal.

It is found to be on lesser side by 55% as compared to that in simply raft foundation at the lower value of modulus of subgrade reaction considered in the present investigation. However, for next higher values of modulus of subgrade reaction, the maximum moments in piled raft is on higher side in the range of 36% -120%.

### 5.2 Load pattern- II

Referring to the values of maximum deflections obtained in respect of raft and pile raft for different raft thicknesses and as is evident from Fig. 9, it is seen that maximum deflections are found to be decrease with increase in the raft thickness. Also the maximum deflections are found to decrease with increase in the soil modulus. It is also observed that as the thickness of raft increases from 0.45 m to 0.9 m, the deflection of the raft decreases from 20% to 37% and that of piled raft, from 8% to 14% with increase in the values of modulus of subgrade reaction. Further, with increase in the raft thickness from 0.9 m to 1.5 m, the deflection of the raft is found to decrease in the range of 7% to 20% and that of piled raft is found to decrease in the range of 3% to 15% with increase in the values of modulus of subgrade reaction.

The values of maximum moments indicated in Fig. 10, it is seen that the maximum moments are increasing with increase in raft thickness. Further, with increase in the values of soil modulus, the reduction in moments is observed. For this particular load pattern, the maximum moments developed are on considerably lower side. This could be because of the uniform nature of loading. This effect is more pronounced for the piled raft foundation as compared to that in simple raft foundation. In case of the piled-raft foundations, the maximum moments are increasing with the raft thickness. With increase in the soil modulus, the moments are also found to increase; the increment of the moment is marginal.

It can be observed that the maximum moment in piled raft is found to be on lesser side by 41% as compared that in raft foundation for the lowest value of soil modulus, i.e., 40000 kN/m<sup>3</sup>, considered in the present investigation. With further higher values of modulus of subgrade reaction, however, the maximum moment in piled raft is found to be on higher side in the range of 46%-260% as compared to that in raft foundation.

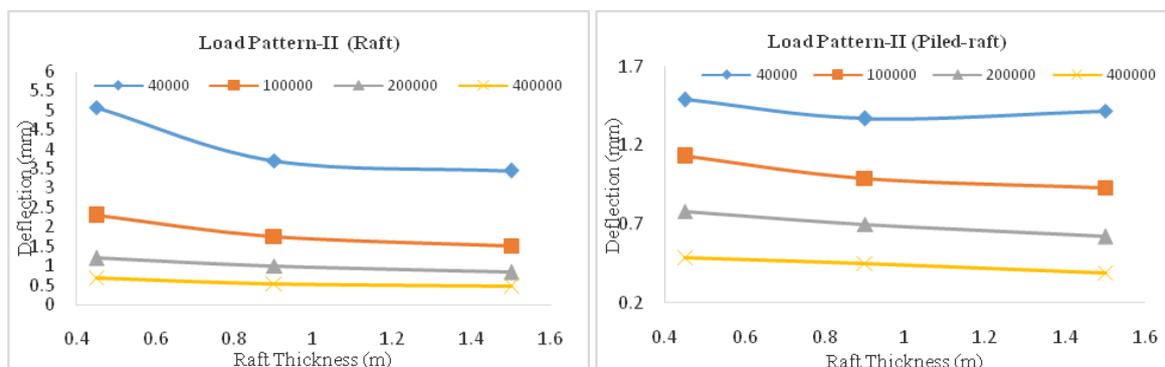


Fig. 9 Variation in maximum deflection with raft thickness in raft and piled raft for load pattern-II

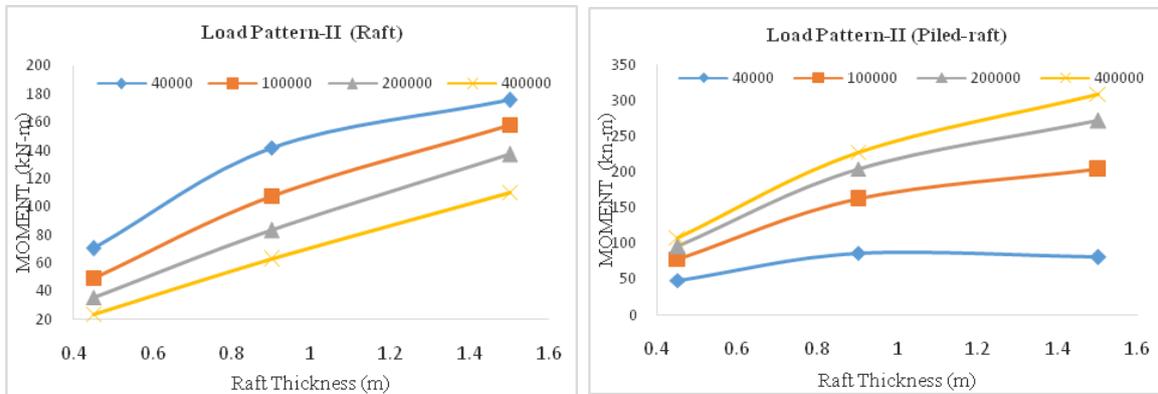


Fig. 10 Variations of maximum moment with raft thickness for raft and piled raft for load pattern-II

### 5.3 Load pattern- III

From the values of the maximum deflections as mentioned in the Load Pattern – II, it is seen that the maximum deflections are found to decrease with increase in the raft thickness. Further, it is found to decrease with increase in the soil modulus. The variation of the values of maximum deflections with raft thickness as indicated in Fig. 11.

It is also noted that as thickness of raft increases from 0.45 m to 0.9 m, deflection of the raft decreases from 23% to 33% whereas that in piled raft, from 8% to 18%, with increase in the values of modulus of subgrade reaction. For further increase in the raft thickness up to 1.5 m, the decrease in deflection of raft is observed to be in the range of 15% to 18% and that in piled raft, in the range of 1% to 15%, with increase in the values of modulus of subgrade reaction.

From the values of maximum moments as depicted in Fig. 12, it is observed that the maximum moments are increasing with increase in raft thickness. Further, with increase in the values of soil modulus, the reduction in the moments is observed. However, for this load pattern the reduction is marginal.

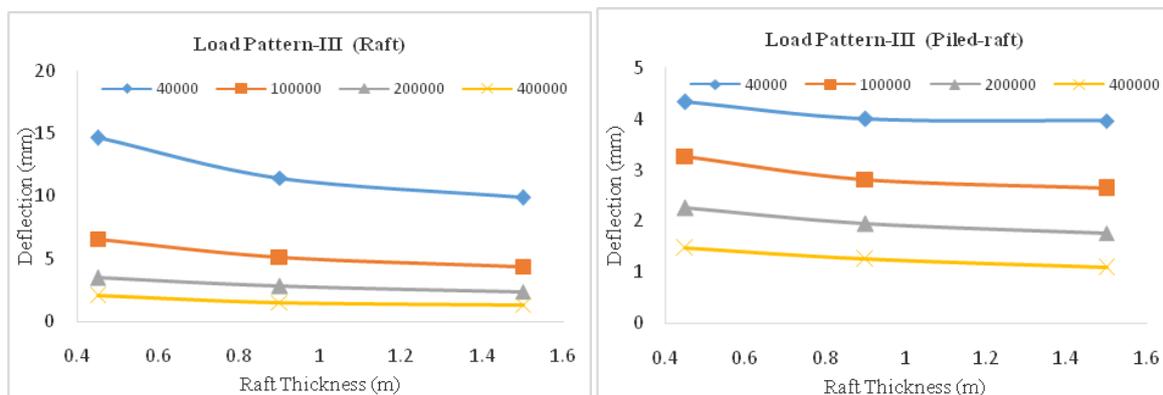


Fig. 11 Variation in maximum deflection with raft thickness in raft and piled raft for load pattern-III

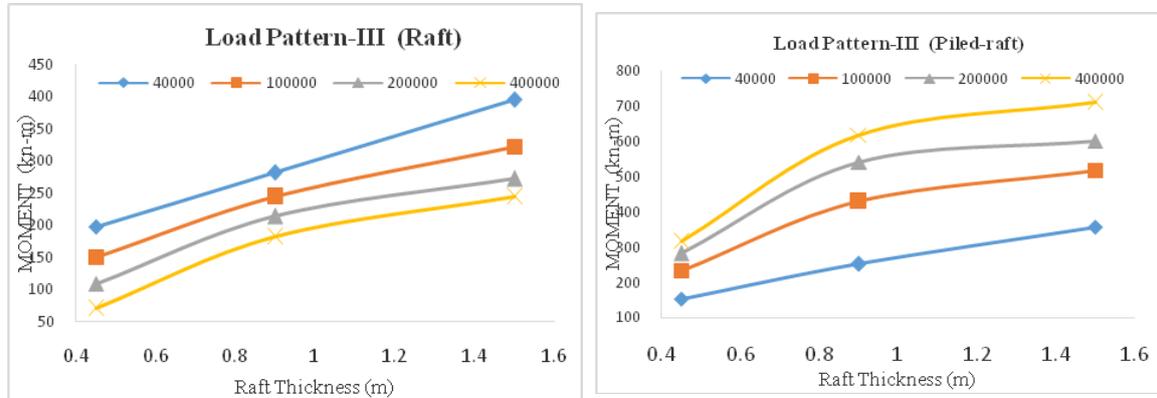


Fig. 12 Variations of maximum moment with raft thickness in raft and piled raft for load pattern-III

In case of the piled-raft, the maximum moments are increasing with the raft thickness. Further, with increase in the values of soil modulus, the moments are found to increase, the increment of moment being marginal. It is observed that the maximum moment in piled raft is found on the lesser side by 14% as compared that found in raft foundation for lowest value of soil modulus (i.e., 40000 kN/m<sup>3</sup>). On the contrary to this, for next higher values of modulus of subgrade reaction considered in the present investigation, the maximum moments in raft are on higher side in the range of 64%-260%.

In case of piled raft foundations, maximum moments are found to increase with increase in soil modulus. However, the trend as seen in case of simply raft is exactly opposite. In raft, the maximum moments are found to decrease with soil modulus. It may be noted that a raft, when considered alone, behaves in the flexible manner with increase in soil modulus; owing to which the non-dimensional flexibility factor reduces. Hence, the bending moments which are directly proportional to the flexural factor, also decrease. On the contrary to this, in case of piled raft, it (piles) introduces additional fixity causing fixing moments to increase.

## 6. Conclusions

From the parametric study presented in this paper, following significant conclusions can be drawn:

- The maximum deflections and moments reduce substantially in case of piled raft foundation as compared to simply raft foundation.
- Maximum deflections reduce with increase in raft thickness as well as increase in soil modulus.
- Maximum moments increase with increase in soil modulus in respect of piled raft whereas decrease with increase in soil modulus in respect of simply raft.
- Percentages of decrease in moments of piled raft foundation compared with raft foundation go on increasing with increase in soil modulus for load case in which all columns are subjected to same loading.

- Range of decreasing percentage of deflection in case of piled raft foundation compared to raft foundation is between 10% and 30%.

## References

- Baziar, M.H., Ghorbani, A. and Katzenbach, R. (2009), "Small-scale model test and three dimensional analysis of piled -raft foundation on medium-dense sand", *Int. J. Civil. Eng.*, **7**(3), 170-175.
- Brown, P.T. and Wiesner, T.J. (1975), "The behaviour of uniformly loaded piled strip footings", *Soils Found.*, **15**(4), 13-21.
- Butterfield, R. and Banerjee, P.K. (1971), "The problem of pile group - pile cap interaction", *Geotechnique*, **21**(2), 135-142.
- Chen, K.S., Karasudhi, P. and Lee, S.L. (1974), "Force at a point in the interior of layered elastic half-space", *Int. J. Solids Struct.*, **10**(11), 1179-1199.
- Cheng, Z. (2011), "Prediction and measurement of settlement of a piled raft foundation over thick soft ground", *Electron. J. Geotech. Eng. (EJGE)*, **16**(A), 125-136.
- Chow, Y.K. (1987), "Axial and lateral response of pile groups embedded in non-homogeneous soil", *Int. J. Numer. Anal. Meth.*, **11**(6), 621-638.
- Chow, Y.K. and Teh, C.I. (1991), "Pile-cap-pile-group interaction in non- homogeneous soil", *J. Geotech. Geoenviron.*, **117**(11), 1655-1667.
- Chow, Y.K., Poulos, H.G. and Small, J.C. (2011), "Piled raft foundations for tall buildings", *Geotech. Eng. J. (South East Asian Geotechnical Society)*, **42**(2), 78-84.
- Clancy, P. and Randolph, M.F. (1993), "An approximate analysis procedure for piled raft foundations", *Int. J. Numer. Anal. Met.*, **17**(12), 849-869.
- Franke, E., El-Mossallamy, Y. and Wittmann, P. (2000), *Design Applications of Raft Foundation*, Thomas Telford, London, UK.
- Gandhi, S.R. and Maharaj, D.K. (1996), "Analysis of piled raft foundation", *Proceedings of the 6<sup>th</sup> International Conference on Piling and Deep Foundations*, Bombay, India, January.
- Hain, S.J. and Lee, I.K. (1978), "The Analysis of flexible raft-pile systems", *Geotechnique*, **28**(1), 65-83.
- Hooper, J.A. (1973), "Observations on the behaviour of a piled-raft foundation on London Clay", *Proceedings of the Institution of Civil Engineers - Part 2*, **55**(4), 855-877.
- Kakurai, M., Yamashita, K. and Tomono, M. (1987), "Settlement behaviour of piled raft foundation on soft ground", *Proceedings of the 8<sup>th</sup> Asian Regional Conference on Soil Mechanics and Foundation Engineering*, Kyoto, Japan.
- Katzenbach, R. and Reul, O. (1997), "Design and performance of piled rafts", *Proceedings of the 14<sup>th</sup> International Conference on Soil Mechanics and Foundation Engineering*, Hamburg, Germany.
- Katzenbach, R., Arslan, U. and Moormann, C. (2000), *Design Applications of Raft Foundation*, Thomas Telford, London, UK.
- Kitiyodom, P. and Matsumoto, T. (2003), "A simplified analysis method for piled raft foundations in non-homogeneous soils", *Int. J. Numer. Anal. Met.*, **27**(2), 88-109.
- Kuwabara, F. (1989), "An elastic analysis for piled raft foundations in a homogeneous soil", *Soils Found.*, **29**(1), 82-92.
- Liu, W. and Novak, M. (1991), "Soil-pile-cap static interaction analysis by finite and infinite elements", *Can. Geotech. J.*, **28**(6), 771-783.
- Liang, F.Y. and Chen, L.Z. (2004), "A modified variation approach for the analysis of piled raft foundation", *Mech. Res. Commun.*, **31**, 593-604.
- Madhav, M.R. and Karmarkar, R.S. (1982), "Elasto-plastic settlement of rigid footings", *J. Geotech. Eng. Div. (ASCE)*, **108**(GT-3), 483-488.
- Maharaj, D.K. and Gandhi, S.R. (2004), "Non-linear finite element analysis of piled raft foundations", *Proceedings of the ICE - Geotech. Eng.*, **157**(3), 107-113.

- Mendonca, A.V. and De Paiva, J.B. (2003), "A Boundary element method for the static analysis of raft foundations on piles", *Eng. Anal. Bound. Elem.*, **24**(3), 237-247.
- Noh, E.Y., Huang, M., Surarak, C., Adamec, R. and Balasurbamian, A.S. (2008), "Finite element modeling for piled-raft in sand", *Proceedings of the 11<sup>th</sup> East Asia-Pacific Conference on Structural, Engineering and Construction (EASEC-11)*, Taipei, Taiwan.
- Ottaviani, M. (1975), "Three-dimensional finite element analysis of vertically loaded pile groups", *Geotechnique*, **25**(2), 159-174.
- Poulos, H.G. (1994), "An approximate numerical analysis of piled raft interaction", *Int. J. Numer. Anal. Met.*, **18**(2), 73-92.
- Poulos, H.G., Badelow, F., Small, J.C. and Moyes, P. (2006), "Economic foundation design for tall buildings", *Proceedings of the 10<sup>th</sup> International Conference on Piling and Deep Foundations*, Amsterdam, Netherlands.
- Prakoso, W.A. and Kulhawy, F.H. (2001), "Contribution to piled raft foundation design" *J. Geotech. Geoenviron.*, **127**(1), 17-24.
- Randolph, M.F. (1983), "Design of piled raft foundations", *Proceedings of the International Symposium on Recent Developments in Laboratory and Field Tests and Analysis of Geotechnical Problems*, Bangkok, Thailand.
- Reul, O. and Randolph, M.F. (2003), "Piled rafts in over consolidated clay: comparison of in-situ measurements and numerical analyses", *Geotechnique*, **53**(3), 301-315.
- Sales, M.M., Small, J.C. and Poulos, H.G. (2010), "Compensated piled rafts in clayey soils: behaviour, measurements, and predictions", *Can. Geotech. J.*, **47**(3), 327-345.
- Sawant, V.A., Ladhane, K. and Pawar, S. (2012), "Parametric study of piled raft for three load-patterns", *Coupled System Mech., Int. J.*, **1**(2), 115-131.
- Shen, W.Y., Chow, Y.K. and Yong, K.Y. (1999), "Variational solution for vertically loaded pile groups in an elastic half-space", *Geotechnique*, **49**(2), 199-213.
- Shen, W.Y. and Teh, C.I. (2002), "Analysis of laterally loaded pile groups using a variational approach", *Geotechnique*, **52**(3), 201-208.
- Sinha, J. (1997), "Piled raft foundations subjected to swelling and shrinking soils", Ph.D. Thesis (Unpublished), University of Sydney, Australia.
- Small, J.C. and Booker, J.R. (1984), "Finite layer analysis of layered elastic materials using flexibility approach, Part I. — Strip Loadings", *Int. J. Numer. Anal. Met.*, **20**, 1025-1037
- Small, J.C. and Booker, J.R. (1986), "Finite layer analysis of layered elastic materials using flexibility approach, Part II. — Circular and rectangular loadings", *Int. J. Numer. Anal. Met.*, **23**(5), 959-978.
- Smith, I. M. and Wang, A. (1998), "Analysis of piled rafts", *Int. J. Numer. Anal. Met.*, **22**(10), 777-790.
- Tan, Y.C., Chow, C.M. and Gue, S.S. (2005), "Piled raft with different pile length for medium rise buildings on very soft clay", *Proceedings of the 16<sup>th</sup> Int. Conf. Soil Mech. And Geot. Eng. (ICSMGE)*, Osaka, Japan, September.
- Thoiba Singh, N. and Baleshwar Singh (2008), "Interaction analysis for piled rafts in cohesive soils", *Proceedings of the 12<sup>th</sup> International Conference on International Association for Computer Methods and Advances in Geomechanics (IACMAG)*, Goa, India.
- Wiesner, T.J. (1991). "Various applications of piled raft analysis", *Proceedings of the International Conference on International Association for Computer Methods and Advances in Geomech.*, (Ed.:Beer, Booker and Carter), Balkema, Rotterdam.
- Xie X., Shou M. and Huang J. (2012), "Application study of long-short-piled raft foundation", *Appl. Mech. Mater.*, **70**, 242-245.