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# Weight optimization of coupling with bolted rim using metaheuristics algorithms

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**Abstract.** The effectiveness of coupling with a bolted rim is assessed in this research using a newly designed optimization algorithm. The current study, which is provided here, evaluates 10 contemporary metaheuristic approaches for enhancing the coupling with bolted rim design problem. The algorithms used are particle swarm optimization (PSO), crow search algorithm (CSA), enhanced honeybee mating optimization (EHBMO), Harmony search algorithm (HSA), Krill heard algorithm (KHA), Pattern search algorithm (PSA), Charged system search algorithm (CSSA), Salp swarm algorithm (SSA), Big bang big crunch optimization (B-BBBCO), Gradient based Algorithm (GBA). The contribution of the paper is to optimize the coupling with bolted rim problem by comparing these 10 algorithms and to find which algorithm gives the best optimized result. These algorithm's performance is evaluated statistically and subjectively.

Keywords: coupling with bolted rim; meta-heuristic; non-traditional optimization

# 1. Introduction

The coupling with bolted rim problem is proposed by Giraud-Moreau and Lafon. Couplings are often utilised in mechanical transmission design. Because of the vast and very diversified fields of action of the equipment and installations they outfit, flexible couplings with non- metallic materials are extremely important (compressors, pumps, generators, pulleys, cranes, conveyors, mixers, piston motors, general industrial applications, as well as in the metallurgical industry, the mining industry, the paper industry, and the pulp industry). In applications, couplings are constructed in a wide range of constructive solutions, making a unified and widely accepted classification difficult. Coupling has been employed in a wide range of applications around the world, including light towers, air pumps, welding sets, and other gear with high driving inertia. Two cast iron hubs, a super-elastic rubber element, and locking hardware comprise the connection. Torque is delivered from one part of the coupling to the other by bolts in a rigid coupling, and shafts must be aligned in this configuration. Let us consider the elementary mechanism represented in Fig. 1, made up of some elementary connections, and intended to transmit by adhesion a torque between two shafts coaxial.

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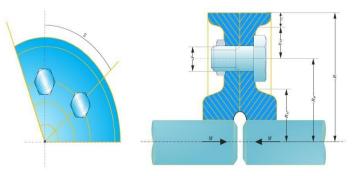


Fig. 1 Coupling bolted rim (Ali Riza et al. 2019)

## 2. Objective

The objective of the paper is to minimize the weight of the coupling by using 10 non- traditional optimization techniques and validating the results using simulation in ANSYS.

#### 3. Literature survey

Ali Rıza Yıldız implements the butterfly optimization approach which is used to solve the problem of coupling with a bolted rim (BOA). Finally, the BOA is used to a shape optimization problem. Suspension arm of the vehicle. It comes from the Kriging metamodeling system. This method is for obtaining objective and constraint equations functions in the optimization of shapes (Betul *et al.* 2020). He also introduced the new adaptive mixed differential (NAMDE) algorithm for the coupling with bolted rim problem. To update the values of mutation and crossover factors, the system employs a self-adaptive process. The objective function for the coupling with a bolted rim problem increased by 10%. Hammoudi Abderazek offers the Adaptive Mixed Differential Evolution (AMDE), a unique evolutionary approach for optimising the coupling with bolted rim problem with mixed variables (Hammoudi *et al.* 2019). Laurence Giraud-Moreau compared two evolutionary algorithms – the genetic algorithm and the evolution method-in terms of coupling with bolted rim problems (Laurance *et al.* 2002). Pascal Lafon has formulated the coupling with plates problem to minimize its radius, number of bolts and torque (Lafon *et al.* 1994). Ali Riza Yildiz is also compared six mechanical models by using ten meta-heuristic methods to optimize these six designs (Ali Riza *et al.* 2019).

In the specialist literature, engineering design optimization problems are commonly used to demonstrate the efficiency of new restricted optimization techniques. To overcome these nonlinear engineering issues, many researchers adopted unconventional solutions. These nonlinear engineering problems typically involve a heterogeneous set of design variables (discrete or continuous), nonlinear objective functions, and nonlinear constraints, some of which may be active at the global optimum. Constraints are particularly significant in engineering design issues since they are generally placed on the problem statement. It can be tough to fulfil at times, making the search laborious and inefficient.

The optimization is carried out with various solvers for the two extreme values of the parameters. Because they are stochastic, the outcomes may differ from trial to trial. As a result, the solver uses the parameter's final value after running the problem for 20 attempts (Emad *et al.* 2005). To overcome the local optima trap shortcoming and improve the solution quality of a recently introduced arithmetic optimization algorithm (AOA), the Nelder–Mead local search methodology has been incorporated into the basic AOA framework (Betul *et al.* 2023). a novel generalized normal distribution algorithm that is integrated with elite oppositional-based learning (HGNDO-EOBL) is studied and employed to optimize the design of the eight benchmark engineering functions (Pranav *et al.* 2023). A novel metaheuristic called Chaotic Marine Predators Algorithm (CMPA) is proposed and investigated for the optimization of engineering problems (Sumit *et al.* 2023). A novel hybrid metaheuristic optimization algorithm named chaotic Runge Kutta optimization (CRUN). He 10 diverse chaotic maps are being incorporated with the base Runge Kutta optimization (RUN) algorithm to improve their performance (Betul *et al.* 2022).

### 4. Mathematical modelling

The coupling with bolted rim is mathematically formulated as follows. The objective function includes three terms with weighting coefficients. d is discrete, N is an integer,  $R_B$  and M are continuous variables. The problem is subjected to eleven inequality constraints

#### 4.1 Initial expression

The study we are doing here assumes that the shaft-plate connection has already been chosen and sized. We will also admit that the shape of the plates is imposed by manufacturing conditions.

The quantity to be minimized is clearly identifiable and is written, with the notations of the Fig. 1:

$$R = R_M + b_1 + b_2 + c$$

 $R_M$ : Larger radius of the shaft - hub connections.

C: Thickness of the outer protective rim, depending on the process for obtaining trays.

The functional relationships that can be written about this mechanism are as follows:

The functional equation defining the minimum torque transmissible by this coupling gives:

$$M = Q_m \cdot f_m \cdot N \cdot R_B$$

 $Q_m$ : Minimum preload in the bolt.

 $f_m$ : Minimum coefficient of friction between the plates.

By using the calculation models of bolted joints resulting from one can write the following relationships:

Maximum Von Mises equivalent stress in the bolt

$$\sigma_B = \sqrt{(\frac{Q_M^2}{\vec{S}_e}) + 3(\frac{16C^2}{nd_e^3})}$$

 $Q_N$ : Maximum preload in the bolt.

 $S_e$ : Equivalent resistant cross section of the screw.

 $d_e$ : Diameter of the resistant section.

$$C = (16.0p)Q_M$$

*p*: The pitch of the screw.

 $d_2$ : Average diameter on the side of the screw threads.  $f_1$ : Coefficient of friction screw - nut. Preload installed in the bolt during tightening:  $Q_M = \alpha Q_m$ With  $\alpha$  Uncertainty coefficient, linked to the tightening tool used The functional equations resulting from geometric conditions are as follows: Resistance section of  $S_e$  the bolt:  $S_e = \frac{\pi d_e^2}{4}$ Interval between bolts:  $s = \frac{2\pi R_B}{N}$ On the shelves:  $R_B = R_M + b_1$ On the diameters:  $d_e = \phi_1(d)$  $d_2 = \phi_2(d)$ On the step:  $p = \phi_3(d)$ Geometric dimensions of the tightening tool:  $b_m = \phi_4(d) \ s_m = \phi_5(d)$ Finally, the following limiting functional conditions should be added: On the static resistance of the bolt:  $\sigma_B = 0.9R_e$ Re Yield strength of the quality class of the screws considered. On the torque transmitted:  $M \ge M_T$ On the number of bolts:  $N \ge N_m$ On the diameter of the bolts:  $d \ge d_m$ On the overall dimensions of the tightening tool:  $b_2 \ge b_m$  $b_1 \ge b_m$  $s \ge s_m$ The above relationships involve the following 26 parameters: Functional parameters:  $M, M_T, Q_m, Q_M, C, \alpha, \sigma_B$ 

Geometric parameters:  $d_m$ , d,  $d_e$ ,  $d_2$ , p,  $S_e$ , s,  $b_1$ ,  $b_2$ ,  $b_m$ ,  $s_m$ , N,  $N_m$ ,  $R_3$ ,  $R_M$ , c Material parameters of bolts and plates:  $R_e$ ,  $f_m$ ,  $f_1$ 

The relations  $\varphi_i$  relating the various geometrical parameters concerning the bolt and the

tightening are not explained here. They express the fact that the parameters like the step and the mean and flank diameter of threads depend on the diameter of the bolts d. These are values normalized accessible only by the value of d. In subsequent calculations the values of these parameters will be extracted from an array from the value of d. In the expression of problem, the functions  $\varphi_i$  will be kept to-simplify the writing of the conditions functional.

We will assume that the values of the following 9 parameters:

 $M_T$ ,  $f_m$ ,  $f_1$ ,  $\alpha$ ,  $R_e$ ,  $N_m$ ,  $R_M$ , c,  $d_m$ 

have been fixed by a global functional analysis, they will be considered as data from the problem.

Eleven constraints and an objective function are in this optimization problem. N bolts of diameter d are set at radius  $R_B$  to convey twist via adhesion. The goal is to discover the coupling with the smallest radius, the fewest bolts, and the least twist. This is a multi-criteria objective function with weighting coefficients of  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$ . In the restricted linkage between the shaft and the coupler is postulated in this investigation selected. One discrete variable (d), one integer variable (N), two continuous variables ( $R_B$ , M), 11 inequality constraints, and 5 discrete bolt parameters ( $\phi_i(d)$ , i=1, ..., 5). make up the specified optimization problem.

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#### 4.2 Final expression: reduction of the number of variables

We will express this problem according to the following 4 variables

 $d, N, R_B, M$ 

By combining the functional equations with the condition functional limit, we obtain the relation

$$\frac{\alpha.M}{N.R_B} < K(d) \text{ With: } K(d) = \frac{0.9f_m R_e \pi(\phi(d))^2}{4\sqrt{1+3(0.16\phi_3(d)^{J_1}/\phi_1(d))^2}}$$

Likewise, the combinations of the functional equations with the respective inequalities and allow to write:

$$\frac{2\pi R_B}{N} \ge \phi_5(d)$$
$$R_B \ge R_M + \phi_4(d)$$

We try to minimize the expression:

$$R = R_M + b_1 + b_2 + c = R_B + b_2 + c$$
$$b_2 \ge b_m.$$

So, we will have systematically *R* minimal for  $b_2=b_m=\phi_4(d)$ . We bring each other back then to the following problem, comprising 4 description variables and 6 constrained functions inequality: Minimize the expression:  $R \ge R_B + \phi_4(d) + c$ 

Under constrained functions:  $M \ge M_T$ 

$$\frac{\alpha.M}{N.R_B} \le K(d)$$

$$\frac{2\pi R_B}{N} \ge \phi_5(d)$$

$$R_B \ge R_M + \phi_4(d)$$

$$N \ge N_m$$

$$d \ge d_m$$

To present all the optimal design issues dealt with in this book, we adopt the following script which has the advantage of clearly identifying the chosen variables

Minimize the objective function:  $F(d, N, R_B, M) = R = R_B + \phi_4(d) + c$ 

Under constrained functions:  $c_1(d, N, R_B, M) = M_R - M \le 0$ 

$$c_{2}(d, N, R_{B}, M) = \frac{\alpha . M}{N . R_{B}} - K(d) \le 0$$

$$c_{3}(d, N, R_{B}, M) = \phi_{5}(d) - \frac{2\pi R_{B}}{N} \le 0$$

$$c_{4}(d, N, R_{B}, M) = R_{M} + \phi_{4}(d) - R_{B} \le 0$$

$$c_{5}(d, N, R_{B}, M) = N_{m} - N \le 0$$

$$c_{6}(d, N, R_{B}, M) = d_{m} - d \le 0$$

Variable vector:  $x = \{d, N, R_B, M\}^T$ 

From this example very representative of the type of optimization problems that we will have to deal with in optimal design, we can draw the following conclusions:

We note that several types of variables intervene in this formulation:

• Variables that may vary continuously between the limits defined by the field of solutions, such as moment M and radius  $R_B$ . We will name this type of variables: continuous variables.

• Variables constrained to take integer values such as the number of bolts N or again discrete

variables such as the diameter of the bolts resulting from the standardization.

The optimal design problem is therefore very often an optimization problem in mixed variables. Let us also underline the particularity of certain discrete variables such as here the diameter of the bolts. Indeed, some parameters are directly dependent on this type of variables, they act in a way as discrete "secondary" variables whose values are only accessible through tables. This situation is encountered for many of standardized mechanical construction elements (bearings for example) or standard (the joints). This type of problem is generally very constrained, in other words, the number of constrained functions is very often greater than the number of variables. However, the analytical expressions of the different functional relations often allow us to eliminate by substitution of the functional equations so that we very rarely have constrained functions equalities. Note finally that these are generally nonlinear problems, comprising frequently monotonous functions (Lafon *et al.* 1994).

Minimize: 
$$f(x) = \beta_1 \left(\frac{N}{N_M}\right) + \beta_2 \left(\frac{R_B + \phi_4(d) + c}{R_M}\right) + \beta_2 \left(\frac{M}{M_R}\right)$$
  
Subject to the constraints:  $g_1(x) = \frac{\alpha M}{NR_B K(d)} - 1 \le 0$   
 $g_2(x) = 1 - \frac{2\pi R_B}{\phi_5(d)N} \le 0$   
 $g_3(x) = 1 - \frac{R_B}{\phi_4(d)} + R_M$   
 $g_4(x) = N - N_{max} \le 0$   
 $g_5(x) = R - R_{max} \le 0$   
 $g_4(x) = N_m - N \le 0$   
 $g_7(x) = R_M - R_B \le 0$   
 $g_8(x) = M - M_{max} \le 0$   
 $g_9(x) = M_R - M \le 0$   
 $g_{10}(x) = d - 24 \le 0$   
 $g_{11}(x) = 6 - d \le 0$   
 $K(d) = \frac{0.9 f_m R_e \pi(\phi(d))^2}{4\sqrt{1 + 3(0.16\phi_3(d)f_1/\phi_1(d))^2}}$   
 $M_T$ =40 Nm,  $M_{max}$ =1000 Nm,  $f_m$ =0.15,  $f_1$ =0.15,  $\alpha$ =1.5,  $R_e$ =627 MPa,  $N_m$ =8,  
 $N_{max}$ =100,  $R_M$ =50 mm,  $R_{max}$ =1000 mm, C=5 mm,  $\beta_1 = \beta_2 = \beta_3 = 1$ ,  $6 \le d \le 248 \le N \le 100$ ,  
 $50 \le R_B \le 1000, 40 \le M \le 1000$ .

# Simplified:

Minimize:	$(N)$ $(B_{p}+\phi_{1}(d)+5)$ $(M)$	
	$\left(\frac{N}{8}\right) + \left(\frac{R_B + \phi_4(d) + 5}{50}\right) + \left(\frac{M}{40}\right)$	
Subject to the constraints:	1.5 <i>M</i>	
	$g_1(x):\frac{1.5M}{NR_BK(d)} - 1 \le 0$ 2(3.14)R_B	
	$g_2(x): 1 - \frac{2(3.14)R_B}{\phi_5(d)N} \le 0$	

$g_3(x): 1 - \frac{R_B}{\phi_4(d)N} + 50$
$g_4(x): N \le 100$
$g_5(x): R \le 1000$
$g_6(x): 8 \le N$
$g_7(x)$ : 50 $\leq R_B$
$g_8(x): M \le 1000$
$g_9(x)$ : $40 \le M$
$g_{10}(x): d \le 24$
$g_{11}(x): 6 \le d$
$(265.7853)(\phi(d))^2$
$K(d) = \frac{(1+1)(1+1)(1+1)}{4\sqrt{1+3(0.16\phi_3(d)0.15/\phi_1(d))^2}}$
$8 \le N \le 100, 50 \le R_B \le 1000, 40 \le M \le 1000.$

## 5. Optimization algorithm

The following are some of the most common problems with classic gradient methods and traditional direct approaches:

- It converges to an optimal solution based on the original solution chosen.
- Most algorithms are prone to limiting themselves to a sun-optimal answer.
- A problem solved by one algorithm may not be efficient when applied to another.

• Algorithms are inefficient for solving problems with non-linear objectives, discrete variables, and a large number of restrictions.

• On a parallel computer, algorithms cannot be employed efficiently.

In general, standard techniques such as steepest descent, dynamic programming, and linear programming make it difficult to address large-scale issues with nonlinear objectives functions. Traditional algorithms cannot address non-differentiable problems because they require gradient information. Some optimization problems have a large number of local optima. As a result of this issue, there is a need to build more powerful optimization approaches, and research has discovered our non-traditional optimization (Emad *et al.* 2005).

Comparing to the traditional with non-traditional methods,

• Non-traditional methods will give global results, but in traditional method we can get only local results,

• Non-traditional methods can be used to solve any methods and, for traditional we can use only specific methods.

Time consumption will be low in the non-traditional method, it takes more time in traditional method.

The following non-traditional optimization algorithms are used.

- 1. Particle swarm optimization (PSO),
- 2. Crow search algorithm (CSA),
- 3. Enhanced honeybee mating optimization (EHBMO),
- 4. Harmony search algorithm (HSA),
- 5. Krill heard algorithm (KHA),
- 6. Pattern search algorithm (PSA),

- 7. Charged system search algorithm (CSSA),
- 8. Salp swarm algorithm (SSA),
- 9. Big bang big crunch optimization (B-BBBCO),
- 10. Gradient based Algorithm (GBA).

#### 5.1 Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) is initiated by Kennedy and Eberhart in 1995. Particle Swarm Optimization is a swarm-intelligence technique, where swarm intelligence refers to any algorithm or problem-solving tool inspired by the mutual behaviour of social insect colonies and other animal societies. Particle swarm optimization got inspired by flocking of birds and fish schooling. Example of Swarm,

- A swarm of bees surrounds their hives
- An ant colony with each and every one of its delegates as ants
- A flock of birds is a swarm of birds
- An immune system is a swarm of cells
- A crowd is a swarm of people

Self-organization and exertion disunity are two fundamental aspects of this swarm intelligence. The situation and velocity of the bird or particle are associated in particle swarm optimization (James *et al.* 2010).

# 5.2 Crow Search Algorithm (CSA)

Askar Zadeh introduced CSA, a new population-based algorithm that mimics the behaviour of a crow hiding food. Crows are gifted birds that can recognise people's faces and warn their families when they are in danger. Crows are more cunning when it comes to concealing their food and remembering where it is hidden. The goal is to keep researchers interested in optimization and swarm intelligence algorithms. When all food locations are compared, the place with the most food heaped or stored is considered the global optimal solution, and the amount of food is the objective function. When the crow's gifted behaviour is applied to a variety of optimization issues, the results are astonishing (Alireza *et al.* 2016).

## 5.3 Harmony Search Algorithm (HSA)

Harmony search algorithm is evolved by Geem in 2001. HS is developed depend on the symphonic presentation process. Typically, the numerous classifications of decision variables with extreme equality and inequality which carries the non-linear and non-convex objective function in engineering optimization problem. Subsequently, executing traditional methods in optimization problems which faces numerous dilemmas. To vanquish the dilemma of the complex optimization problems, Meta-heuristic optimization algorithm will be the systematic replacement. In music spontaneity procedure, preconceived group of musicians strive to tune the pitch on their instruments to attain a pleasant harmony (Mubina *et al.* 2021a).

#### 5.4 Enhanced honeybee-mating optimization (EHBMO)

Enhanced honeybee-mating optimization (EHBMO) is new method on honeybee-mating

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optimization (HBMO). This HBMO is swarm-based procedure that is inclined by some process on honeybee-mating. Comparing to other meta-heuristic optimization algorithms, EHBMO is highly competitive. EHBMO is the algorithm which is introduced newly. EHBMO gave so many ideas for the researchers, and they've learnt so many techniques on HBMO. An implementation of Enhanced Honeybee Mating optimization algorithm (EHBMO) in order to reflection in plant growth is planned for solving the problem in the power system which has fault estimation. Here, Simulating, and original power systems are not wisely examined that can be pondered by different situations. And the other application of EHBMO is a recent HBMOA for non-smooth economic send off. A basic concept of the HBMO conveys about the work of the honeybee, and they are social insects. They build their own hives, and they toil in the extremely organized pecking order. There are three forms in the honeybee community: the queen, drones, and workers (Mubina *et al.* 2021b).

#### 5.5 Charged System Search Algorithm (CSSA)

The Charged System Search (CSS) employs a number of charged particles that interact with one another based only on their fitness values and separation lengths as determined by the Coulomb regulating law. To illustrate the similarities and differences between the CSS set of rules and some well-known meta-heuristics, a comparison is made. The CSS set of rules has been used to optimise a few benchmarks frame examples. CSS's results are compared to those of existing meta-heuristic algorithms, indicating the new set of rules' robustness (Ali Kaveh *et al.* 2019).

## 5.6 Big-Bang Big Crunch Optimization (B-BBCO)

Genetic algorithms (GA) and simulated annealing (SA) procedures are examples of innovations derived from nature. Traditional evolution algorithms are all human-based search methods with random variables and options. The study's principal implication is that it proposes a technique of writing a novel based on one of the universe's development hypotheses, namely the Big Bang and the Big Crunch Theory. Power distribution causes disruption and randomness during the Big Bang phase, while particles that are randomly distributed are drawn to order during the Big Crunch phase. Encouraged by this hypothesis, the Big Bang-Big Crunch (BB-BC) approach was developed, which creates random points in the Big Bang category and then narrows them down to a single point of representation through a heavy or low-income institution in the Big Bang fall phase. The authors of this study show that the performance of the new approach (BB-BC) suggests a high level of enhanced genetic search algorithm, which outperforms the traditional genetic algorithm (GA) for many benchmark tasks (Pavel *et al.*).

#### 5.7 Krill Herd Algorithm (KHA)

Krill Herd (KH) is a current and recent meta heuristic optimization algorithm, that has been lately preferred by Gandomi and Alavi in 2012. Krill Herd algorithm is based on the replication of the herding character of krill individuals. The short distances of every individual krill from food and from highest solidity of the herd are examined as the objective function for the krill movement. To solve the engineering optimization problem, this krill herd (KH) is instigated. KH can be applied to some of the design problems to find a feasible solution. When compared to other optimization algorithm, the presentation of KH represents the state-of-the-art in the part. The outcome is a better global optimization solution (Amir *et al.* 2016).

Algorithm	Parameter Settings
Aigontinii	Taranicici Settings
PSO	$w_{\min}=0.9, w_{\max}=0.4, c_1=2, c_2=2$
CSA	$c_1 = c_2 = c_3 = 2, \ \omega = 0.5, \ AP = 0.2, \ fl = 2, \ V_{max} = [2]^D$
EHBMO	No. of drones=40, No. of broods=10, No. of selected genes in crossover=8
HSA	HMS=50, HMCR=0.5 fixed, PAR=0.5
KHA	$N^{\max} = 0.01, V_f = 0.02, D^{\max} = 0.005$
PSA	Only the common parameters (Fes and NP)
CSSA	rand-Random value between [0,1], $c=0.1$ , $\varepsilon=0.001$
SSA	Only the common parameters (Fes and NP)
B-BBCO	Npop=100, $k_{ls}$ =30, $\alpha$ =0.8, $N_s$ =5
GBA	Only the common parameters (Fes and NP)

Table1 Specific parameter settings of used algorithms

 $w_{\min}$ ,  $w_{\max}$  are respectively the min and max inertia weight 0.4,  $c_1$  and  $c_2$  are acceleration factors. HMS-Harmony Memory Size, PAR-Pitch Adjustment rate, HMCR-Harmony Memory Consideration rate, Npop-Population size, K<sub>1s</sub>- no. of non-improvement iteration,  $\alpha$ -Reduction rate,  $N_s$ -no. of neighbours created in each generation,  $N^{\max}$ -Maximum induced speed,  $V_f$ -The foraging speed,  $D^{\max}$ -The maximum diffusion speed,  $c_1$ ,  $c_2$ ,  $c_3$ -acceleration,  $\omega$ - inertia weight, *fl*-length of the crow's flight, AP-perceptual probability of crow,  $V_{\max}$ upper limit of the particle update velocity

## 5.8 Pattern Search Algorithm (PSA)

Pattern search algorithm (PSA) is to compose a group of local search algorithm assisted by solid convergence proofs. They run by the principle of assessing the objective function repetitively in a stencil-based way. Pattern search algorithm is utilized to find the pattern or substring from another bigger string. To solve the engineering optimization problem, this Pattern search (PSA) is applied. PSA can be applied to some of the design problems to find a feasible solution. When compared to other optimization algorithms, the presentation of PSA represents the state-of-the-art in the area. This results in a better global optimization solution (Miloš *et al.* 2014).

# 5.9 Salp Swarm Algorithm (SSA)

Salp Swarm Algorithm (SSA) is the novel optimization algorithm which is used to solve the optimization problems with single and multi-objectives. The major inspiration of salp swarm algorithm is the behaviour of the salp's swarming on their navigation and foraging in the oceans. To solve the engineering optimization problem, this Salp Swarm Algorithm (SSA) is instigated. SSA can applied to some of the design problems to find feasible solution. When compared to other optimization algorithm, the presentation of SSA represents the state-of-the- art in the area. This results in better global optimization solution (Laith *et al.* 2019).

## 5.10 Gradient-Based Algorithm (GBA)

Gradient-Based Algorithm (GBA) is a current and recent meta heuristic optimization algorithm, which needs gradient or delicate details in order to function evaluations and to decide enough search directions for a more fit design through the optimization replications. In optimization problems,

Table 2 FEs number and the NP size for the algorithms

Problem	NP	t <sub>max</sub>	FEs
Coupling with bolted rim	20	250	5000

Trail no.         PSO         EHDMO         HSA         CSA         CSSA         BBCO         GBA         KHA         PSA           1         6.0000         6.0012         6.1253         6.1122         6.3265         6.4565         6.5237         6.4565         6.2563	SSA 6.1230 6.1254
	6.1254
2 6.0000 6.0023 6.1232 6.1122 6.9865 6.4565 6.5632 6.5645 6.2563	
3 6.0000 6.0052 6.1254 6.1122 6.8956 6.4565 6.5412 6.5645 6.2563	6.1256
4 6.0000 6.0045 6.1252 6.1122 6.6585 6.4565 6.5698 6.6545 6.2563	6.1254
5 6.0000 6.0012 6.1200 6.1122 6.5897 6.4565 6.5237 6.4565 6.2563	6.1254
6 6.0000 6.0025 6.1230 6.1122 6.5485 6.4565 6.5413 6.5465 6.2563	6.1236
7 6.0000 6.0013 6.1230 6.1122 6.5699 6.4565 6.5633 6.5654 6.2563	6.1235
8 6.0000 6.0021 6.2301 6.1122 6.6666 6.4565 6.5215 6.6545 6.2563	6.1254
9 6.0000 6.0022 6.2013 6.1122 6.5854 6.4565 6.5422 6.5465 6.2563	6.1237
10 6.0000 6.0011 6.3201 6.1122 6.5455 6.4565 6.5370 6.6566 6.2563	6.1525
11 6.0000 6.0052 6.2323 6.1122 6.5556 6.4565 6.5237 6.6666 6.2563	6.1254
12 6.0000 6.0045 6.3261 6.1122 6.6547 6.4565 6.5700 6.5645 6.2563	6.1452
13 6.0000 6.0045 6.3210 6.1122 6.6655 6.4565 6.5637 6.5555 6.2563	6.1255
14 6.0000 6.0014 6.3201 6.1122 6.5667 6.4565 6.5413 6.4544 6.2563	6.1254
15 6.0000 6.0015 6.3223 6.1122 6.6546 6.4565 6.5699 6.5545 6.2563	6.1254
16 6.0000 6.0078 6.3220 6.1122 6.5859 6.4565 6.5237 6.5565 6.2563	6.1290
17 6.0000 6.0085 6.3211 6.1122 6.9999 6.4565 6.5423 6.6655 6.2563	6.1857
18 6.0000 6.0015 6.3250 6.1122 6.5688 6.4565 6.5245 6.5544 6.2563	6.1479
19 6.0000 6.0015 6.3210 6.1122 6.5666 6.4565 6.5698 6.5645 6.2563	6.1570
20 6.0000 6.0015 6.3201 6.1122 6.6590 6.4565 6.5488 6.6544 6.2563	6.1523
Average 6.0000 6.0031 6.2374 6.1122 6.6425 6.4565 6.5452 6.5728 6.2563	6.1346

Table 3 Diameter of the bolts (d)

intent and restraint functions are frequently known as performance estimate. To solve the engineering optimization problem, this Gradient-Based Algorithm (GBA) is instigated. GBA can be applied to some of the design problems to find a feasible solution. When we compared to other optimization algorithm, the presentation of GBA represents the state-of-the-art in the area. This results the better global optimization solution (Jinseog *et al.* 2008).

# 6. Methodology

The performance of the non-traditional algorithm will vary for every run, and it is assured that the solution is global optimum. So, for every problem twenty trail runs were performed in all algorithms and the average value of the solution was obtained from all the trails (Hammoudi *et al.* 2018). The Specific parameters are set for different algorithms as in Table 1 and the FEs number and NP size are taken as in Table 2.

## DIAMETER OF THE BOLTS (d)

To determine the diameter of a bolt, measure the distance between the outer threads on one side

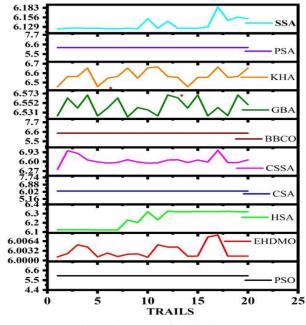


Fig. 2 Diameter of the bolts (d)

Table 4 Number of bolts (N)

Trial no	PSO	EHDMO	HSA	CSA	CSSA	BBCO	GBA	KHA	PSA	SSA
1	8.0000	8.0011	8.0123	8.2564	8.7896	8.5645	8.5632	8.5645	8.5469	8.0023
2	8.0000	8.0012	8.0123	8.2564	8.7898	8.5645	8.5412	8.6545	8.5469	8.0056
3	8.0000	8.0015	8.3210	8.2564	8.8660	8.5645	8.5699	8.6666	8.5469	8.0054
4	8.0000	8.0021	8.3210	8.2564	8.8990	8.5645	8.5413	8.5466	8.5469	8.0059
5	8.0000	8.0025	8.0213	8.2564	8.8788	8.5645	8.5699	8.5555	8.5469	8.0023
6	8.0000	8.0021	8.3212	8.2564	8.6589	8.5645	8.5215	8.5645	8.5469	8.0054
7	8.0000	8.0011	8.3212	8.2564	8.6589	8.5645	8.5633	8.4455	8.5469	8.0087
8	8.0000	8.0000	8.3232	8.2564	8.6589	8.5645	8.5623	8.5565	8.5469	8.0056
9	8.0000	8.0025	8.1212	8.2564	8.6599	8.5645	8.5326	8.4445	8.5469	8.0024
10	8.0000	8.0036	8.0202	8.2564	8.9999	8.5645	8.5632	8.6545	8.5469	8.0012
11	8.0000	8.0025	8.0120	8.2564	8.7777	8.5645	8.5624	8.5655	8.5469	8.0045
12	8.0000	8.0025	8.0321	8.2564	8.6869	8.5645	8.5690	8.5554	8.5469	8.0078
13	8.0000	8.0045	8.3201	8.2564	8.5657	8.5645	8.5478	8.5556	8.5469	8.0056
14	8.0000	8.0071	8.3201	8.2564	8.5649	8.5645	8.5698	8.5556	8.5469	8.0098
15	8.0000	8.0015	8.3201	8.2564	8.5699	8.5645	8.5412	8.5555	8.5469	8.0025
16	8.0000	8.0015	8.3250	8.2564	8.5470	8.5645	8.5632	8.5554	8.5469	8.0069
17	8.0000	8.0061	8.0236	8.2564	8.5699	8.5645	8.5478	8.5666	8.5469	8.0091
18	8.0000	8.0052	8.0215	8.2564	8.5699	8.5645	8.5698	8.5555	8.5469	8.0073
19	8.0000	8.0000	8.2365	8.2564	8.5699	8.5645	8.5478	8.6545	8.5469	8.0055
20	8.0000	8.0045	8.3210	8.2564	8.5699	8.5645	8.5622	8.6565	8.5469	8.0056
Average	8.0000	8.0027	8.1863	8.2564	8.6925	8.5645	8.5555	8.5715	8.5469	8.0055

and the outside threads on the other. This is known as the main diameter, and it is usually the correct size of the bolt. All ten Optimization methods are used and the value of d in tabulated as in Table 3.

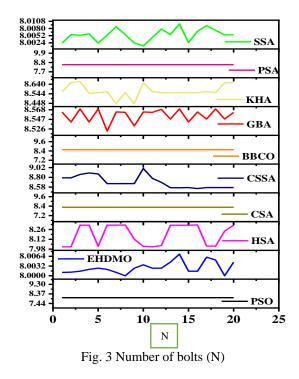


Table 5	Radius	of the	bolts	(RB)
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Trial no	PSO	EHDMO	HAS	CSA	CSSA	BBCO	GBA	KHA	PSA	SSA
1	56.5432	59.0000	58.9658	58.2356	59.6589	58.6596	58.9857	58.6596	57.8659	57.2360
2	56.5432	59.2365	58.9865	58.2356	59.6986	58.6596	58.9745	58.3265	57.8659	57.2370
3	56.5432	59.6584	58.6598	58.2356	59.6585	58.6596	58.9652	58.6598	57.8659	57.5237
4	56.5432	59.3265	58.6549	58.2356	59.6855	58.6596	58.9633	58.2366	57.8659	57.2145
5	56.5432	59.3266	58.9685	58.2356	59.6859	58.6596	58.9654	58.6598	57.8659	57.3652
6	56.5432	59.6590	58.9875	58.2356	59.6854	58.6596	58.6599	58.9865	57.8659	57.3265
7	56.5432	59.6548	58.9325	58.2356	59.4875	58.6596	58.9652	58.9689	57.8659	57.2146
8	56.5432	59.6895	58.9658	58.2356	59.8888	58.6596	58.9875	58.9865	57.8659	57.6986
9	56.5432	59.6854	58.6599	58.2356	59.6686	58.6596	58.9633	58.3652	57.8659	57.2365
10	56.5432	59.6854	58.6985	58.2356	59.6546	58.6596	58.9633	58.2563	57.8659	57.2365
11	56.5432	59.3254	58.9857	58.2356	59.6856	58.6596	58.9658	58.6523	57.8659	57.3265
12	56.5432	59.1254	58.9654	58.2356	59.6546	58.6596	58.9633	58.6985	57.8659	57.2365
13	56.5432	59.2341	58.9658	58.2356	59.6857	58.6596	58.9633	58.3652	57.8659	57.2666
14	56.5432	59.7854	58.9654	58.2356	59.6856	58.6596	58.9632	58.2365	57.8659	57.8962
15	56.5432	59.3652	58.9685	58.2356	59.6855	58.6596	58.9633	58.5698	57.8659	57.2650
16	56.5432	59.5699	58.9685	58.2356	59.6556	58.6596	58.9521	58.5632	57.8659	57.3699
17	56.5432	59.6588	58.9855	58.2356	59.6668	58.6596	58.9875	58.8965	57.8659	57.6895
18	56.5432	59.6589	58.8570	58.2356	59.6589	58.6596	58.9235	58.6235	57.8659	57.8695
19	56.5432	59.6542	58.9658	58.2356	59.6668	58.6596	58.9563	58.9568	57.8659	57.6325
20	56.5432	59.5000	58.9865	58.2356	59.6559	58.6596	58.9536	58.3562	57.8659	57.2365
Average	56.5432	59.4900	58.9047	58.2356	59.6736	58.6596	58.9492	58.6012	57.8659	57.4039

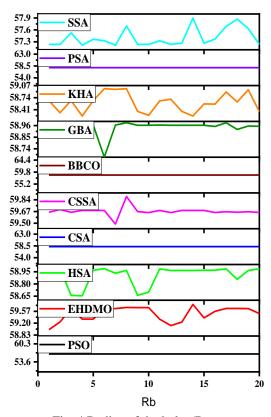


Fig. 4 Radius of the bolts (RB)

## NUMBER OF BOLTS (N)

There are certain number of bolts should be used for the process of the coupling with the bolted rim. Number of bolts also should be minimized to optimize the coupling with bolted rim. The values are tabulated in Table 4.

## RADIUS OF THE BOLT (RB)

In bolted joints, the effective radius is a critical element in determining frictional torque beneath the bolt. Because the effective radius is difficult to measure accurately in practise, the mean radius is utilized, which is the mean value of the inner and outer radii of the contact region under the bolt head and the radius of the bolts are tabulated in Table 5.

#### TORQUE OF THE BOLTS (M)

Torque is a twisting and turning force, as opposed to tension, which formed by a straight pull. Torque, on the other hand, is used to generate tension. The optimized values of the torque of the bolts are tabulated in Table 6. The bolts thread angle transforms the force into tension in the bolt shank. The amount of stress created in the bolt is essential.

Table 6 Torque of the bolts (M)

Trial no	PSO	EHDMO	HAS	CSA	CSSA	BBCO	GBA	KHA	PSA	SSA
1	40.0000	40.2563	40.5214	40.9856	40.3265	40.3265	40.5896	40.3265	40.2355	40.0002
2	40.0000	40.2564	40.3256	40.9856	40.5699	40.3265	40.5412	40.7856	40.2355	40.0006
3	40.0000	40.2566	40.1111	40.9856	40.5869	40.3265	40.5699	40.9854	40.2355	40.0023
4	40.0000	40.2514	40.2234	40.9856	40.5900	40.3265	40.5424	40.4532	40.2355	40.0052
5	40.0000	40.1232	40.2563	40.9856	40.5699	40.3265	40.5242	40.6521	40.2355	40.0033
6	40.0000	40.1251	40.2563	40.9856	40.8788	40.3265	40.5362	40.9874	40.2355	40.0021
7	40.0000	40.1201	40.2653	40.9856	40.8888	40.3265	40.5986	40.6541	40.2355	40.0056
8	40.0000	40.1250	40.3256	40.9856	40.9857	40.3265	40.5784	40.6321	40.2355	40.0058
9	40.0000	40.1236	40.2365	40.9856	40.9999	40.3265	40.5862	40.9654	40.2355	40.0057
10	40.0000	40.2512	40.2365	40.9856	40.5656	40.3265	40.5320	40.8523	40.2355	40.0024
11	40.0000	40.2512	40.2365	40.9856	40.5855	40.3265	40.5793	40.8521	40.2355	40.0089
12	40.0000	40.2365	40.2365	40.9856	40.5698	40.3265	40.5862	40.7412	40.2355	40.0088
13	40.0000	40.2514	40.2365	40.9856	40.5699	40.3265	40.5486	40.8523	40.2355	40.0052
14	40.0000	40.2514	40.2365	40.9856	40.5855	40.3265	40.5268	40.9632	40.2355	40.0066
15	40.0000	40.2365	40.2365	40.9856	40.5698	40.3265	40.5963	40.8521	40.2355	40.0058
16	40.0000	40.2365	40.2154	40.9856	40.5875	40.3265	40.5852	40.1250	40.2355	40.0065
17	40.0000	40.2540	40.2514	40.9856	40.5698	40.3265	40.5741	40.3610	40.2355	40.0070
18	40.0000	40.2540	40.2580	40.9856	40.5680	40.3265	40.5123	40.2365	40.2355	40.0026
19	40.0000	40.2150	40.2541	40.9856	40.5698	40.3265	40.5456	40.3266	40.2355	40.0057
20	40.0000	40.2514	40.2563	40.9856	40.5698	40.3265	40.5789	40.5632	40.2355	40.0066
Average	40.0000	40.2163	40.2588	40.9856	40.6354	40.3265	40.5616	40.6584	40.2355	40.0048

#### 7. Result and discussion

#### 7.1 Consistency

The consistency table gives the parameters that remain constant for all the trails. All the solvers give the value of PSO, CSA, BBCO and PSA for all the runs, which in turn indicates that the requirements are in the acceptable range.

*d*-PSO (6.0000), CSA (6.1122), BBCO (6.4565), PSA (6.2563) N-PSO (8.0000), CSA (8.2564), BBCO (8.5645), PSA (8.5469) *RB*-PSO (56.5432), CSA (58.2356), BBCO (58.6596), PSA (57.8659) M-PSO (40.0000), CSA (40.9856), BBCO (40.3265), PSA (40.2355) So, we see that the solvers PSO, CSA, BBCO, PSA remains constant throughout their runs.

# 7.2 Simplicity of Algorithm

Of all the algorithm, we have taken PSO is the simplest followed by EHBMO, SSA, HSA, BBCO.

#### 7.3 Minimum values of variables

Table 7, Table 8 and Fig. 2 present the best optimal solution and the statistical simulation results obtained by the algorithms for the coupling with a bolted rim problem. From Table 7, it can be seen

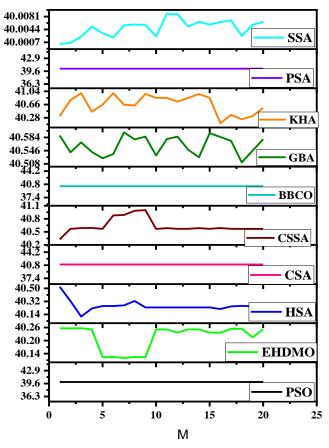


Fig. 5 Torque of the bolts (M)

Table 7 Comparison of the best optimum solution for the coupling with bolted rim problem

	PSO	EHBMO	HSA	CSA	CSSA	BBCO	GBA	KHA	PSA	SSA
d (mm)	6.0000	6.0031	6.2374	6.1122	6.6425	6.4565	6.5452	6.5728	6.2563	6.1346
N	8	8.0027	8.186348	8.2564	8.6926	8.5645	8.5555	8.5715	8.5469	8.0055
Rb (mm)	56.5432	59.4900	58.9047	58.2356	59.6737	58.6596	58.9493	58.6012	57.8659	57.40388
<i>M</i> (Nm)	40	40.2163	40.2588	40.9856	40.6354	40.3265	40.5616	40.6584	40.2355	40.00484
$f_{\min}$	3.27	3.4854	3.4529	3.5236	3.46653	3.48569	3.4824	3.48604	3.312566	3.314609

d-diameter of the bolts (mm), N-number of bolts, M-adhesion of torque (Nm), RB-radius (mm)

that all used approaches are able to find the global feasible solution. However, the PSO algorithm is the most robust in solving this problem with standard deviation values of 1.8225E-15, followed by EHBMO, SSA, HAS, CSA, CSSA, BBCO, PSA, GBA, KHA.

- *d*-PSO (6) is better than EHBMO (6.0031) and SSA (6.1346)
- M-PSO (40) is better than EHBMO (40.2163) and SSA (40.00484)
- N-PSO (8) is better that EHBMO (8.0027) and SSA (8.0055)
- RB-PSO (56.5432) is better than EBHMO (59.4900) and SSA (57.40388)

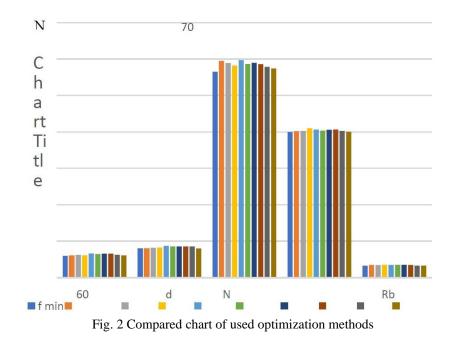


Table 8 Statistical result of the used algorithms for the coupling with a bolted rim

Algorithm	Best	Mean	Worst	SD	FEs
PSO	3.27	3.27	3.27	1.8225E-15	5000
EHDMO	3.45896	3.4854138	3.48999	0.006785632	5000
HAS	3.45213	3.4528825	3.45622	0.001446476	5000
CSA	3.5236	3.5236	3.5236	9.11252E-16	5000
CSSA	3.4569	3.46653275	3.48965	0.01092365	5000
BBCO	3.48569	3.48569	3.48569	9.11252E-16	5000
GBA	3.05698	3.482424	3.507846	0.100156805	5000
KHA	3.481254	3.48603996	3.489875	0.001730581	5000
PSA	3.312566	3.312566	3.312566	1.8225E-15	5000
SSA	3.48	3.31460911	3.317896	0.001966737	5000

# 8. Conclusions

The following are some of the most common problems with classic gradient methods and traditional direct approaches:

• It converges to an optimal solution based on the original solution chosen.

Most algorithms are prone to limiting themselves to a sun-optimal answer.

• A problem solved by one algorithm may not be efficient when applied to another.

• Algorithms are inefficient for solving problems with non-linear objectives, discrete variables, and a large number of restrictions.

• On a parallel computer, algorithms cannot be employed efficiently.

In general, standard techniques such as steepest descent, dynamic programming, and linear programming make it difficult to address large-scale issues with nonlinear objectives functions.

Traditional algorithms cannot address non-differentiable problems because they require gradient information. Some optimization problems have a large number of local optima. As a result of this issue, there is a need to build more powerful optimization approaches, and research has discovered our non-traditional optimization [6].

In this paper, we compared 10 meta-heuristic algorithms to solve the coupling with bolted rim problem. The algorithms used are particle swarm optimization (PSO), crow search algorithm (CSA), enhanced honeybee mating optimization (EHBMO), Harmony search algorithm (HSA), Krill heard algorithm (KHA), Pattern search algorithm (PSA), Charged system search algorithm (CSSA), Salp swarm algorithm (SSA), Big bang big crunch optimization (B-BBBCO), Gradient based Algorithm (GBA). These algorithm's performance is evaluated statistically and subjectively.

By comparing these methods, we've proved that PSO is the best optimization method comparing with other nine methods which we discussed in the result analysis. To minimize the number of bolts(N), diameter of the bolts(d), adhesion of torque(M) and the radius ( $R_B$ ),

Particle Swarm Optimization (PSO) got the minimum value comparing with Enhanced Honey-Bee Mating (EHBMO) and Salp Swarm Optimization (SSA). Therefore, for coupling with bolted rim problem Particle Swarm Optimization (PSO) is the best method. These results will be validated using simulation by ANSYS.

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