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Time harmonic interactions due to inclined load in an orthotropic thermoelastic rotating media with fractional order heat transfer and two-temperature

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Abstract. The objective of this paper is to study the effect of frequency in a two-dimensional orthotropic thermoelastic rotating solid with fractional order heat transfer in generalized thermoelasticity with two-temperature due to inclined load. As an application the bounding surface is subjected to uniformly and linearly distributed loads (mechanical and thermal source). The problem is solved with the help of Fourier transform. Assuming the disturbances to be harmonically time dependent, the expressions for displacement components, stress components, conductive temperature and temperature change are derived in frequency domain. Numerical inversion technique has been used to determine the results in physical domain. The results are depicted graphically to show the effect of frequency on various components. Some particular cases are also discussed in the present research.

Keywords: Fourier transform; fractional order; frequency domain; inclined load; orthotropic; rotation; two-temperature; uniformly and linearly distributed loads

1. Introduction

Thermoelasticity is related to the deformation produced in the elastic solid body due to thermal and mechanical changes occur in the body. The elastic material deforms by the action of external load and a certain amount of heat exchanged with the surroundings. From a long ago, a lot of attention has been given to thermoelastic theories with heat equation which gives finite speed of heat signals. Nowadays, fractional thermoelasticity is one of the important and productive areas of research in solid mechanics and continuum dynamics. The solutions of the problems which contain differential equations of non-integer order are obtained by taking into account the fractional order theory. Many researchers are working on this theory. The definition of fractional order (α) was given by Caputo (1967), which divides the materials into three categories based on their conductivity i.e., weak, normal and strong. Miller and Ross (1993) gave some alternative definition of fractional order. Moreover, thermoelasticity with two-temperature is also one of the interesting areas of research. The two-temperature theory of thermoelasticity for deformable

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bodies was formulated by Chen and Gurtin (1968), Chen *et al*. (1968, 1969), which depends upon two different types of temperatures thermodynamical temperature (T) and the conductive temperature and (ϕ).The difference between these two temperatures is proportional to the heat supply. The two temperatures are equal in the absence of heat supply for time independent problems. For time dependent problems, the two temperatures are different regardless of the presence of heat supply. Youssef (2006) formulated a new theory of generalized thermoelasticity by considering two-temperature generalized thermoelasticity theory for a homogeneous isotropic body without energy dissipation.

Sharma *et al*. (2008) studied the dynamical behavior of generalized thermoelastic diffusion with two relaxation times in frequency domain. Ailawalia and Narah (2009) studied the effect of rotation in generalized thermoelastic solid under the influence of gravity with an overlying infinite thermoelastic fluid. Abbas *et al*. (2009) studied the effect of thermal dispersion on free convection in a fluid saturated porous medium. Ailawalia *et al*. (2010) studied the effect of rotation in a generalized thermoelastic medium with two-temperature under hydrostatic initial stress and gravity. Abo-Dahab and Abbas (2011) studied the thermal shock problem of generalized magneto-thermoelasticity for an infinitely long annular cylinder with variable thermal conductivity. Abbas (2011) studied the two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation. Zakaria (2012) studied the effects of hall current and rotation on magneto-micropolar generalized thermoelasticity due to ramp-type heating. Abd-Alla *et al*. (2012) investigated the propagation of Rayleigh waves in a rotating orthotropic material elastic half-space under initial stress and gravity. Marin and Stan (2013) studied the weak solutions in elasticity of dipolar bodies with stretch. Naggar *et al*. (2013) studied the effect of initial stress, magnetic field, voids and rotation on plane waves in generalized thermoelasticity. Abbas and Kumar (2014) studied the deformation due to thermal source in micropolar generalized thermoelastic half-space by finite element method. Abd-Alla *et al*. (2012) studied the plane waves in a transversely isotropic magneto-thermoelastic medium under the effect of a constant angular velocity.

Marin *et al*. (2015) studied the double porosity structure for micropolar bodies. Mahmoud *et al*. (2015) studied the effect of initial stress and rotation on free vibrations in transversely isotropic human dry bone. Sharma *et al*. (2015) studied the disturbance due to inclined load in transversely isotropic thermoelastic medium with two temperatures and without energy dissipation. Marin *et al*. (2015) formulated the extension of the domain of influence theorem for generalized thermoelasticity of anisotropic material with voids. Ezzat and Bary (2016) studied the magneto-thermoelectric viscoelastic materials with memory dependent derivatives. Ezzat and Bary (2017a) studied the fractional magneto-thermoelastic materials using GN-theory with phase-lags. Ezzat *et al*. (2017b) observed the effect of temperature and fractional order parameter on the copper like material by using GN-theory with phase-lags. Kumar *et al*. (2017) studied the Rayleigh waves in anisotropic magneto thermoelastic medium. Abbas (2018) studied the fractional order theory in thermoelastic half-space under thermal loading and free vibrations of nano-scale beam under two-temperature Green-Naghdi model. Abo-Dahab *et al*. (2018) studied the reflection of plane waves in thermoelastic microstructured materials under the influence of gravitation. Othman *et al*. (2019) analyzed the plane waves through magneto-thermoelastic microstretch rotating medium with temperature dependent elastic properties.

Kaur and Lata (2019) studied the effect of inclined load in transversely isotropic magneto-thermoelastic medium due to time harmonic sources. Lata (2019) investigated the time harmonic interactions in fractional thermoelastic diffusive thick circular plate. Lata and Zakhmi (2019, 2020) studied the various thermoelastic problems in orthotropic medium. Biswas and Abo-Dahab

(2020) studied the three dimensional thermal shock problem in orthotropic magneto-thermoelastic medium. Saeed *et al*. (2020) studied the GL-model on thermo-elastic interactions in a poroelastic material using finite element method. Akbas (2020) studied the dynamic analysis of a laminated beam under harmonic load. Alzahrani and Abbas (2020) studied the Photo-thermal interactions in a semiconducting media with a spherical cavity under hyperbolic two-temperature model. Biswas (2020) studied the propagation of Rayleigh surface waves in a homogeneous orthotropic medium based on Eringen's nonlocal thermoelasticity. Marin *et al*. (2020) established a domain of influence theorem for the mixed initial-boundary value problem in the context of the Moore-Gibson-Thompson theory of thermoelasticity for dipolar bodies. Zenkour (2020) studied the magneto-thermal shock for a fiber-reinforced anisotropic half-space with a refined multi-dual-phase-lag model. Lata and Himanshi (2021a, 2021b, 2021c) studied the various thermoelastic problems in an orthotropic medium. Abbas *et al*. (2021) studied the photo-thermal interactions in a semiconductor medium with a cylindrical cavity with two-temperature. Khamis *et al*. (2021) studied the effect of modified ohm's and Fourier's laws on magneto thermo-viscoelastic waves with Green-Naghdi theory in a homogeneous isotropic hollow cylinder. Singh and Aarti (2021) studied the Reflection of plane waves from the boundary of a thermo-magneto-electroelastic solid half space.

In the view of above work, we observed that time harmonic interactions with fractional order heat transfer in an orthotropic media with combined effect of rotation and two-temperature due to inclined load has not been considered yet. So in the present article, we have investigated the interactions due to inclined load in an orthotropic thermoelastic rotating media by using fractional order heat conduction equation in generalized thermoelasticity with and without energy dissipation and two-temperature in frequency domain. The results are presented graphically to show the effect of frequency corresponding to different values of angle of inclination on all the components.

2. Basic equations

Following Lata and Zakhmi (2020), the equation of motion for an orthotropic thermoelastic medium rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega\vec{n}$ where $\vec{n}$ is a unit vector representing the direction of axis of rotation is given as

$$\sigma_{ij,j} = \rho\,[\,\ddot{u}_i + (\mathbf{\Omega}\times(\mathbf{\Omega}\times\vec{u}))_i + (\,2\Omega\times\dot{\vec{u}})_i] \tag{1}$$

The additional terms $\mathbf{\Omega}\times(\mathbf{\Omega}\times\vec{u})$ and $2\,\mathbf{\Omega}\times\dot{\vec{u}}$ on the right side of above Eq. (1) are centripetal acceleration and Coriolis acceleration respectively.

Following Lata and Zakhmi (2020), heat equation in anisotropic medium with fractional order heat transfer and with three-phase-lags is given by

$$K_{ij}\,\left(1+\frac{\tau_t^{\alpha}}{\alpha!}\frac{\partial}{\partial t^{\alpha}}\right)\dot{\phi}_{,ji} + K_{ij}^{*}\left(1+\frac{\tau_v^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\phi_{,ji} = \left(1+\frac{\tau_q^{\alpha}}{\alpha!}+\frac{\tau_q^{2\alpha!}}{2\alpha!}\right)\,[\rho\, C_E\ddot{T} + \beta_{ij}\,T_0\,\ddot{e}_{ij}\,], \tag{2}$$

where $\beta_{ij} = c_{ijkl}\ \alpha_{ij}$, $\beta_{ij} = \beta_i\delta_{ij}$, $K_{ij} = K_i\,\delta_{ij}$, $K_{ij}^{*} = K_i^{*}\delta_{ij}$; i is not summed $(i,j = 1,2,3)$ and δ_{ij} is Kronecker delta.

Also the strain displacement relations are

$$e_{ij} = \frac{1}{2}(\,u_{i,j} + u_{j,i}),\ \ i,j = 1,2,3. \tag{3}$$

Following Youssef (2006), the two temperature relation is given by

$$T = \phi - a_{ij}\phi_{,ij} \quad \text{where } i,j = 1,2,3 \tag{4}$$

Here, dot (.) represents the partial derivative w.r.t time and (,) denote the partial derivative w.r.t spatial coordinate, $c_{ijkl}(=c_{klij}=c_{jikl}=c_{ijlk})$ is the tensor of elastic constant, ρ is the density, T_0 is the reference temperature such that $|\frac{T}{T_0}| \ll 1$, u_i are the components of displacement vector $\vec{u}$, C_E is the specific heat at constant strain, , $\sigma_{ij} = (\sigma_{ji})$ and are the components of stress tensor. T is the absolute temperature, ϕ is the conductive temperature, α_{ij} is the coefficient of linear thermal expansion, β_{ij} is the tensor of thermal moduli, $\mathbf{\Omega}$ is the angular velocity of the solid and K_{ij} , K_{ij}^* are the thermal conductivity and material characteristic constants respectively. Also τ_t , τ_q and τ_v are the phase-lag of the temperature gradient, phase-lag of the heat flux and the phase-lag of the thermal displacement respectively.

3. Formulation of the problem

We consider a perfectly conducting homogeneous orthotropic thermoelastic medium rotating with an angular velocity $\mathbf{\Omega} = \Omega\vec{n}$ initially at uniform temperature T_0 with and without energy dissipation in generalized thermoelasticity. We take a rectangular coordinate axis (x, y, z) having origin on the surface $z = 0$ with z-axis pointing vertically downwards into the medium is introduced. The surface of the half space is subjected to a time harmonic source. For 2D problem in xz-plane, the components of displacement vector and conductive temperature have the form

$$u = u(x,z,t),\ v = 0\ ,\ w = w\,(x,z,t) \text{ and } \phi = \phi(x,z,t) \tag{5}$$

We also assume that

$$\mathbf{\Omega} = (\,0, \Omega, 0), \tag{6}$$

Following Kumar and Chawla (2014), the constitutive relations for an orthotropic solid can be written as

$$\sigma_{xx} = C_{11}\, e_{xx} + C_{13}\, e_{zz} - \beta_1 \mathrm{T}, \tag{7}$$

$$\sigma_{zz} = C_{13}\, e_{xx} + C_{33}\, e_{zz} - \beta_3 \mathrm{T}, \tag{8}$$

$$\sigma_{xz} = 2\, C_{55}\, e_{xz}\,, \tag{9}$$

Where

$$e_{xx} = \frac{\partial u}{\partial x}\,,\ e_{zz} = \frac{\partial w}{\partial z},\ e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\ ,\ T = \phi - \left(a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2}\right), \tag{10}$$

Eqs. (1) and (2) with the help of Eqs. (5)-(10) takes the form

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{55}\frac{\partial^2 u}{\partial z^2} + (C_{13}+C_{55})\frac{\partial^2 w}{\partial x \partial z} - \beta_1 \frac{\partial}{\partial x}\left\{\phi - \left(a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2}\right)\right\} = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}\right), \tag{11}$$

$$(C_{13}+C_{55})\frac{\partial^2 u}{\partial x \partial z} + C_{55}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z}\left\{\phi - \left(a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2}\right)\right\} = \rho\left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t}\right), \tag{12}$$

$$K_1\left(1+\frac{\tau_t^{\alpha}}{\alpha!}\frac{\partial}{\partial t^{\alpha}}\right)\dot{\phi}_{,11}+K_3\left(1+\frac{\tau_t^{\alpha}}{\alpha!}\frac{\partial}{\partial t^{\alpha}}\right)\dot{\phi}_{33}+K_1^{*}\left(1+\frac{\tau_v^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\phi_{,11}+K_3^{*}\left(1+\frac{\tau_v^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\phi_{,33}=$$
$$\left[1+\frac{\tau_q^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}+\frac{\tau_q^{2\alpha!}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right]\left[\rho C_E\frac{\partial^2}{\partial t^2}\left\{\phi-\left(a_1\frac{\partial^2\phi}{\partial x^2}+a_3\frac{\partial^2\phi}{\partial z^2}\right)\right\}+T_0\{\beta_1\ddot{u}_{1,1}+\beta_3\ddot{u}_{3,3}\}\right]. \quad (13)$$

In the above equations we use the contracting subscript notations

$(11\to 1,\ 22\to 2,\ 33\to 3, 23\to 4,\ 13\to 5,\ 12\to 6)$ to relate C_{ijkl} to C_{mn}

where $i,j,k,l=1,2,3$ and $m,n=1,2,3,4,5,6$

We assume that the medium is initially at rest then the initial and regularity conditions are given by

$u_1(x_1,x_3,0)=0=\dot{u}_1(x_1,x_3,0)$
$u_3(x_1,x_3,0)=0=\dot{u}_3(x_1,x_3,0)$
$\phi(x_1,x_3,0)=0=\dot{\phi}(x_1,x_3,0)$ For $x_3\geq 0, -\infty<x_1<\infty$;
$u_1(x_1,x_3,t)=u_3(x_1,x_3,t)=\phi(x_1,x_3,t)=0$ For $t>0$ when $x_3\to\infty$

We define the following dimensionless quantities to facilitate the solution

$$x'=\frac{x}{L},\ z'=\frac{z}{L},\ u'=\frac{\rho c_1^2}{LT_0\beta_1}u,\ w'=\frac{\rho c_1^2}{LT_0\beta_1}w,\ t'=\frac{C_1}{L}t,\ \sigma'_{33}=\frac{\sigma_{33}}{T_0\beta_1},\ \sigma'_{31}=\frac{\sigma_{31}}{T_0\beta_1},\ \sigma'_{11}=\frac{\sigma_{11}}{T_0\beta_1},\ \phi'=\frac{\phi}{T_0},$$
$$a_1'=\frac{a_1}{L},\ a_3'=\frac{a_3}{L},\ T'=\frac{T}{T_0},\ \Omega'=\frac{L}{C_1}\Omega. \quad (14)$$

where $c_1^2=\frac{c_{11}}{\rho}$

Assuming the harmonic behaviour as

$$(u,w,\phi)(x,z,t)=(u,w,\phi)(x,z)\,e^{i\omega t}, \quad (15)$$

where ω is angular frequency

The Fourier transform is defined by

$$\hat{f}(\xi,z,\omega)=\int_{-\infty}^{\infty}\overline{f}\,(x,z,\omega)e^{i\xi x}\,dx, \quad (16)$$

Using the dimensionless quantities defined by Eq. (14) on the Eqs. (11)-(13) and suppressing the primes for convenience and applying the harmonic behaviour and Fourier Transform defined in Eqs. (15)-(16) on Eqs. (19)-(21), we obtain a system of three homogeneous equations in $(\hat{u},\hat{w},\hat{\phi})$

$$[\delta_1 D^2+p_1]\hat{u}+[p_{12}D-p_2]\hat{w}-i\xi[p_7-p_8D^2]\hat{\phi}=0, \quad (17)$$

$$[p_{12}D+p_2]\hat{u}+[\delta_3 D^2+p_3]\hat{w}-\varepsilon[p_8D^3+p_7D]\hat{\phi}=0, \quad (18)$$

$$[p_4T_3]\hat{u}+[p_5T_3D]\hat{w}+[p_9T_1+p_{10}T_1D^2+\varepsilon_4T_2D^2+T_2p_{11}+T_3\omega^2(p_7-p_8D^2]\hat{\phi}=0, \quad (19)$$

where $D=\frac{d}{dz}$,
$p_1=-\xi^2+\omega^2+\Omega^2,\ p_2=2i\omega\Omega,$
$p_3=-\delta_1\xi^2+\omega^2+\Omega^2,\ p_4=i\xi\,\varepsilon_5\,\omega^2,\ p_5=\varepsilon\varepsilon_5\omega^2,\ p_6=\omega^2,\ p_7=(1+\frac{a_1}{L}\xi^2),$
$p_8=\frac{a_3}{L},\ p_9=-i\omega\xi^2\varepsilon_1,\ p_{10}=i\varepsilon_2\omega,\ p_{11}=-\varepsilon_3\xi^2,\ p_{12}=i\xi\delta_2,$
$\delta_1=\frac{c_{55}}{c_{11}},\ \delta_2=\frac{c_{13}+c_{15}}{c_{11}},\ \delta_3=\frac{c_{33}}{c_{11}},\ \epsilon_1=\frac{K_1}{\rho\,LC_1C_E},\ \epsilon_2=\frac{K_3}{\rho\,LC_1C_E},\ \epsilon_3=\frac{K_1^{*}}{\rho c_1^2C_E},\ \epsilon_4=\frac{K_3^{*}}{\rho c_1^2C_E},$
$\epsilon_5=\frac{\beta_1^2T_0}{\rho^2c_1^2C_E},\ L=1,\ T_1=1+\frac{\tau_t^{\alpha}}{\alpha!}(i\omega)^{\alpha},\ T_2=1+\frac{\tau_v^{\alpha}}{\alpha!}(i\omega)^{\alpha},\ T_3=1+\frac{\tau_q^{\alpha}}{\alpha!}(i\omega)^{\alpha}+\frac{\tau_q^{2\alpha}}{2\alpha!}(i\omega)^{2\alpha}.$

These resulting equations have non-trivial solution if the determinant of the coefficient $(\hat{u},\hat{w},\hat{\phi})$

vanishes, which yield to the following characteristic equation.

$$(\mathrm{P}D^6 + QD^4 + RD^2 + S)\left(\hat{u}, \hat{w}, \hat{\phi}\right) = 0, \tag{20}$$

Where

$D = \frac{d}{dz}$,

$P = T_1(\delta_3\delta_1 p_{10}) + T_2(\delta_3\delta_1\varepsilon_4) + T_3(-\delta_3\delta_1 p_8\omega^2 + \varepsilon\delta_1 p_8 p_5)$,

$Q = T_1(\delta_3 p_1 p_{10} + \delta_3\delta_1 p_9 + \delta_1 p_{10} p_3 - p_{10}p_{12}^2) + T_2(\delta_3 p_1\varepsilon_4 + \delta_3\delta_1 p_{11} + \delta_1 p_3\varepsilon_4 - \varepsilon_4 p_{12}^2) + T_3(-\omega^2 p_1 p_8\delta_3 + \varepsilon_1 p_1 p_5 p_8 + \delta_3\delta_1 p_7\omega^2 - \omega^2 p_3 p_8\delta_1 + \varepsilon p_5\delta_1(1+\xi^2) + p_8\omega^2 p_{12}^2 - \varepsilon p_8 p_4 p_{12} + i\xi p_8 p_5 p_{12} - i\xi p_8 p_4\delta_3)$,

$R = T_1(\delta_3 p_1 p_9 + p_1 p_3 p_{10} + p_3 p_9\delta_1 + {p_2}^2 p_{10} - {p_{12}}^2 p_9) + T_2(\delta_3 p_1 p_{11} + \varepsilon_4 p_1 p_3 + p_3 p_{11}\delta_1 + {p_2}^2\varepsilon_4 - {p_{12}}^2 p_{11}) + T_3(\delta_3 p_1 p_7\omega^2 - p_1 p_3 p_8\omega^2 + \varepsilon p_1 p_5(1+\xi^2) + \omega^2(\delta_1 p_3 p_7 - {p_2}^2 p_8 - {p_{12}}^2 p_7) - \varepsilon p_4 p_{12}(1+\xi^2) - i\xi p_5 p_{12} p_7 + i\xi p_4\delta_3 p_7 - i\xi p_4 p_3 p_8)$,

$S = T_1(p_1 p_3 p_9 + {p_2}^2 p_9) + T_2(p_1 p_3 p_{11} + {p_2}^2 p_{11}) + T_3(\omega^2 p_1 p_3 p_7 + \omega^2 p_7 {p_2}^2 + i\xi p_4 p_3 p_7)$.

The roots of the Eq. (20) are $\pm\lambda_i$ (i=1,2,3); the solution of the equation satisfying the radiation conditions can be written as

$$\tilde{u} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z}, \tag{21}$$

$$\widetilde{w} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z}, \tag{22}$$

$$\tilde{\phi} = l_1 A_1 e^{-\lambda_1 z} + l_2 A_2 e^{-\lambda_2 z} + l_3 A_3 e^{-\lambda_3 z}, \tag{23}$$

Where d_i and l_i are given by

$$d_i = \frac{\lambda_i^4 A' + \lambda_i^2 B' + C'}{\lambda_i^4 A + \lambda_i^2 B + C};\ i = 1,2,3$$

$$l_i = \frac{\lambda_i^4 P^* + \lambda_i^2 Q^* + R^*}{\lambda_i^4 A + \lambda_i^2 B + C};\ i = 1,2,3$$

Where

$A = \{T_1(\delta_3 p_{10}) + T_2(\delta_3\epsilon_4) + T_3(-\omega^2\delta_3 p_8 + \varepsilon p_5 p_8)\}$,

$B = \{T_1(\delta_3 p_9 + p_3 p_{10}) + T_2(\delta_3 p_{11} + \epsilon_4 p_3) + T_3(\omega^2\delta_3 p_7 - \omega^2 p_3 p_8 + \varepsilon p_5\xi^2(p_7 - 1)\}$,

$C = \{T_1(p_3 p_9) + T_2(p_3 p_{11}) + T_3(\omega^2 p_3 p_7)\}$,

$A' = \{T_1(\delta_1 p_{10}) + T_2(\delta_1\epsilon_4) + T_3(-\omega^2\delta_1 p_8)\}$,

$B' = \{T_1(p_1 p_{10} + \delta_1 p_9) + T_2(p_1\epsilon_4 + \delta_1 p_{11}) + T_3(-\omega^2 p_1 p_8 + \omega^2 p_7\delta_1 - i\xi p_4 p_8)\}$,

$C' = \{T_1(p_1 p_9) + T_2(p_{11}p_1) + T_3(\omega^2 p_1 p_7 + i\xi p_4 p_7)\}$,

$P^* = \{\delta_1\delta_3\}$,

$Q^* = \{p_3\delta_1 + p_1\delta_3 - p_{12}^2\}$,

$R^* = \{p_1 p_3 + p_2^2\}$.

4. Boundary conditions

Following Lata and Kaur (2019), the boundary conditions are taken as

$$1)\ \sigma_{zz} = -F_1\psi_1(x)\,e^{i\omega t}, \tag{24}$$

$$2)\ \sigma_{xz} = -F_2\psi_2(x)\,e^{i\omega t}, \tag{25}$$

$$3)\ \frac{\partial \phi}{\partial z} = 0, \text{ at } z = 0, \tag{26}$$

Where F_1 and F_2 are the magnitudes of forces applied on the boundary of the surface, $\psi_1(x)$ and $\psi_2(x)$describes the vertical and horizontal load distribution functions along x-axis. By applying Fourier transform defined by (16) on the boundary conditions (24)-(26) and with the help of Eqs. (7)-(10), (14) and (21)-(23), we obtain the displacement components, stress components, conductive temperature and temperature change as follows

$$\tilde{u} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\Delta_1^* e^{-\lambda_1 z} + \Delta_2^* e^{-\lambda_2 z} + \Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{27}$$

$$\tilde{w} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\mathrm{d}_1\Delta_1 e^{-\lambda_1 z} + \mathrm{d}_2 \Delta_2 e^{-\lambda_2 z} + \mathrm{d}_3\Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\mathrm{d}_1\Delta_1^* e^{-\lambda_1 z} + \mathrm{d}_2\Delta_2^* e^{-\lambda_2 z} + \mathrm{d}_3\Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{28}$$

$$\hat{\phi} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\mathrm{l}_1\Delta_1 e^{-\lambda_1 z} + \mathrm{l}_2 \Delta_2 e^{-\lambda_2 z} + \mathrm{l}_3\Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\mathrm{l}_1\Delta_1^* e^{-\lambda_1 z} + \mathrm{l}_2\Delta_2^* e^{-\lambda_2 z} + \mathrm{l}_3\Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{29}$$

$$\widehat{\sigma_{33}} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\Delta_{11}\Delta_1 e^{-\lambda_1 z} + \Delta_{12}\Delta_2 e^{-\lambda_2 z} + \Delta_{13}\Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\Delta_{11}\Delta_1^* e^{-\lambda_1 z} + \Delta_{12}\Delta_2^* e^{-\lambda_2 z} + \Delta_{13}\Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{30}$$

$$\widehat{\sigma_{13}} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\Delta_{21}\Delta_1 e^{-\lambda_1 z} + \Delta_{22}\Delta_2 e^{-\lambda_2 z} + \Delta_{23}\Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\Delta_{21}\Delta_1^* e^{-\lambda_1 z} + \Delta_{22}\Delta_2^* e^{-\lambda_2 z} + \Delta_{23}\Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{31}$$

$$\widehat{\sigma_{11}} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\Delta_{41}\Delta_1 e^{-\lambda_1 z} + \Delta_{42}\Delta_2 e^{-\lambda_2 z} + \Delta_{43}\Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\Delta_{41}\Delta_1^* e^{-\lambda_1 z} + \Delta_{42}\Delta_2^* e^{-\lambda_2 z} + \Delta_{43}\Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{32}$$

$$\hat{T} = -\frac{F_1 \tilde{\psi}_1(\xi)}{\Delta}(\Delta_{51}\Delta_1 e^{-\lambda_1 z} + \Delta_{52}\Delta_2 e^{-\lambda_2 z} + \Delta_{53}\Delta_3 e^{-\lambda_3 z}) e^{i\omega t} - \frac{F_2 \tilde{\psi}_2(\xi)}{\Delta}(\Delta_{51}\Delta_1^* e^{-\lambda_1 z} + \Delta_{52}\Delta_2^* e^{-\lambda_2 z} + \Delta_{53}\Delta_3^* e^{-\lambda_3 z}) e^{i\omega t}, \tag{33}$$

where

$\Delta = \Delta_{11}(\Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}) - \Delta_{12}(\Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}) + \Delta_{13}(\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31})$,
$\Delta_1 = (\Delta_{33}\Delta_{22} - \Delta_{32}\Delta_{23})$, $\Delta_2 = -(\Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31})$, $\Delta_3 = (\Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31})$,
$\Delta_1^* = (\Delta_{12}\Delta_{23} - \Delta_{13}\Delta_{22})$, $\Delta_2^* = (\Delta_{13}\Delta_{21} - \Delta_{11}\Delta_{23})$, $\Delta_3^* = (\Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21})$,

$$\Delta_{1j} = \frac{C_{13}\, i\xi}{\rho c_1^2} - \frac{C_{33} d_j \lambda_j}{\rho c_1^2} - \varepsilon l_j\left(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}\lambda_j^2\right), \qquad j = 1,2,3$$

$$\Delta_{2j} = \frac{C_{55}}{\rho c_1^2}(-\lambda_j + i\xi d_j),\ \Delta_{3j} = \frac{-T_0}{L}(l_j\lambda_j), \qquad j = 1,2,3$$

$$\Delta_{4j} = i\xi - \frac{C_{13} d_j \lambda_j}{\rho c_1^2} - l_j\left(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}\lambda_j^2\right), \qquad j = 1,2,3$$

$$\Delta_{5j} = l_j\left(1 + \frac{a_1}{L}\xi^2 - \frac{a_3}{L}\lambda_j^2\right), \qquad j = 1,2,3$$

4.1 Linearly distributed force

The solution due to linearly distributed force is obtained by setting

$$\{\psi_1(x),\ \psi_2(x)\} = \begin{Bmatrix} 1\ if\ |x| \le m \\ 0\ if\ |x| > m \end{Bmatrix}, \tag{34}$$

Using (34) in Eqs. (24) and (25). The Fourier transforms of $\psi_1(x)$and $\psi_2(x)$with respect to the pair (x, ξ) in case of linearly distributed load of non-dimensional width 2 m applied at origin of co-ordinate system $x = z = 0$ is given by

$$\{\widehat{\psi_1}(\xi),\ \widehat{\psi_2}(\xi)\} = [2\,(1 - cos(\xi m)/\,\xi^2 m]\ ,\xi \ne 0. \tag{35}$$

Using (35) in (27)-(33), we get the components of tangential stress, normal stress, tangential displacement, normal displacement, conductive temperature and temperature change.

4.2 Uniformly distributed force

The solution due to uniformly distributed force is obtained by setting

$$\{\psi_1(x),\ \psi_2(x)\} = \begin{Bmatrix} 1\ if\ |x| \le m \\ 0\ if\ |x| > m \end{Bmatrix} \tag{36}$$

Using (36) in Eqs. (26) and (27), the Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x, ξ) in case of uniformly distributed load of non-dimensional width 2 m applied at origin of co-ordinate system $x = z = 0$ is given by

$$\left\{\widehat{\psi_1}(\xi),\ \widehat{\psi_2}(\xi)\right\} = \left[2\,sin\frac{\xi m}{\xi}\right]\ ,\xi \ne 0. \tag{37}$$

Using (37) in (27)-(33), we obtain the stress components, displacement components, temperature change and conductive temperature.

5. Applications

Suppose an inclined load F_0 per unit length with an inclination θ is acting along the *y*-axis.

$$F_1 = F_0\cos\theta\ ,\quad F_2 = F_0\sin\theta \tag{38}$$

Using (38) in (27)-(33) and with the aid of Eqs. (34)-(37), we obtain the expressions for stress components, displacement components, temperature change and conductive temperature for an orthotropic thermoelastic solid due to uniformly and linearly distributed loads.

6. Particular cases

1. If $C_{11} = C_{33,}\ 2C_{55} = C_{11} - C_{33}$, then our problem reduced for the case transversely

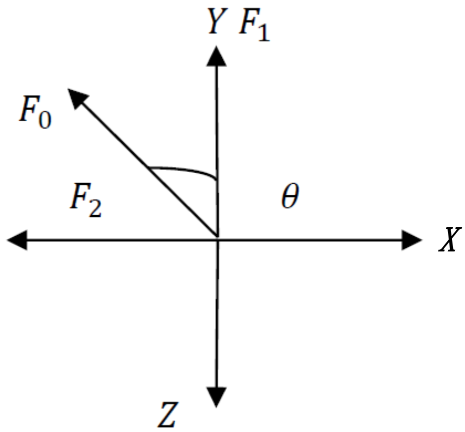

Fig. 1 Inclined load on an orthotropic thermoelastic solid

isotropic thermoelastic solid of type GN-III with combined effect of rotation, inclination and two-temperature in generalized thermoelasticity with fractional order heat transfer in frequency domain.

2. If $C_{11} = C_{33} = \lambda + 2\mu, C_{13} = \lambda, C_{55} = \mu,\ \beta_1 = \beta_3 = \beta,\ K_1^* = K_3^* = K^*,$ we get the expressions for isotropic thermoelastic solid with combined effect of inclination, rotation and two-temperature in generalized thermoelasticity with and without energy dissipation with fractional order heat transfer in frequency domain.

3. If we put $\Omega = 0,$ then we get the results for an orthotropic thermoelastic solid without rotation including the effect of two-temperature and inclination in frequency domain.

4. If we take $\tau_t = \tau_v = \tau_q = 0$ in Eqs. (27)-(33), then our problem reduces for an orthotropic thermoelastic solid of type GN-III

5. If we put $a_1 = a_3 = 0$ in Eqs. (27)-(33), then we get the expressions for orthotropic thermoelastic solid in generalized thermoelasticity with combined effect of inclination and rotation without two-temperature with fractional order heat transfer in frequency domain.

6. If we put $\theta = 0$ in the Eqs. (27)-(33), we obtain the expressions for fractional orthotropic thermoelastic rotating solid in generalized thermoelasticity with combined two-temperature without an inclination effect in frequency domain.

7. If we put $K_1 = K_3 = 0$ in the Eqs. (27)-(33), we get the expressions for an orthotropic thermoelastic solid of type GN-II with fractional order and combined effect of, inclination, rotation and two-temperature in frequency domain.

8. If we put $K_1^* = K_3^* = 0$ in the Eqs. (27)-(33), we obtain the results for an orthotropic thermoelastic solid of type GN-I including the effect of, inclination, rotation and two-temperature with fractional order heat transfer in frequency domain.

7. Inversion of transformation

To obtain the solution of the problem in physical domain, we must invert the transformations in Eqs. (27)-(33). Here the displacement components, tangential and normal stresses and temperature change are functions of z and the parameters of Fourier transforms ξ and hence are of the form f

(ξ, z). To obtain the function $f(x,z)$ in the physical domain, we first invert the Fourier Transform used by Sharma *et al*. (2008).

$$\mathrm{f}\,(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\xi x_1}\,\hat{f}\,(\xi,\mathrm{z})\,\mathrm{d}\xi=\frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x)\,f_e-\,isin(\xi x)f_0|\,\mathrm{d}\xi, \tag{39}$$

Where f_0 and f_e are respectively the odd and even parts of $\hat{f}(\xi, \mathrm{z})$. The method for evaluating this integral is described in Press *et al*. (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion:

Following Lata and Zakhmi (2020), for numerical computations we take the following values of the relevant parameter for an orthotropic thermoelastic material.

Quantity	Value	Unit
c_{11}	18.78×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{13}	8.0×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{33}	10.2×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{55}	10.06×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_E	4.27×10^{2}	$\mathrm{JKg^{-1}K^{-1}}$
β_1	1.96×10^{-5}	$\mathrm{Nm^{2}K^{-1}}$
β_3	1.4×10^{-5}	$\mathrm{Nm^{2}K^{-1}}$
T_0	293	K
K_1	0.12	$\mathrm{Wm^{-1}K^{-1}}$
K_3	0.33	$\mathrm{Wm^{-1}K^{-1}}$
K_1^*	1.313×10^{2}	$\mathrm{Ws^{-1}}$
K_3^*	1.54×10^{2}	$\mathrm{Ws^{-1}}$
ρ	8.836×10^{3}	$\mathrm{Kgm^{-3}}$
τ_v	1.5×10^{-8}	S
τ_q	2.0×10^{-7}	S
τ_t	1.5×10^{-7}	S

By using the above parametric values, the graphical representation of all the components with distance x has been made for an orthotropic thermoelastic body to show the effect of frequency due to inclined load with fix value of fractional order parameter $\alpha = 0.8$, $\Omega = 0.6$, $\mathrm{L} = 1$ and two-temperature parameter a_1 and a_3 are taken as 0.05, 0.07 respectively.

(1) The red solid line with centre symbol triangle (Δ) holds for $\omega = 0.5$ and angle of inclination $\theta = 60°$ for an orthotropic thermoelastic solid.

(2) The green solid line with centre symbol circle ($\square$) holds for $\omega = 0.75$ and angle of inclination $\theta = 60°$ for an orthotropic thermoelastic solid.

(3) The blue solid line with centre symbol circle ($\lozenge$) holds for $\omega = 0.5$ and angle of inclination

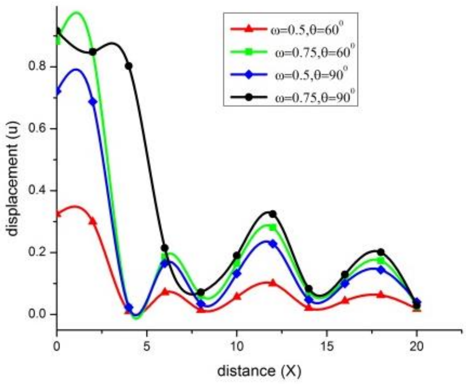

Fig. 2 Variation of displacement u with distance x (uniformly distributed force)

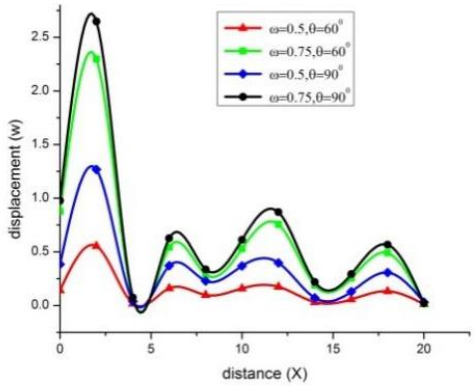

Fig. 3 Variation of displacement w with distance x (uniformly distributed force)

$\theta = 90°$ for an orthotropic magneto-thermoelastic solid.

(4) The black solid line with centre symbol star (o) holds for $\omega = 0.75$ and angle of inclination $\theta = 90°$ for an orthotropic magneto-thermoelastic solid.

8.1 Uniformly distributed force

Figs. 2 and 3 shows the effect of frequency with different angles of inclination ($\theta = 60°, 90°$) on the transverse and normal displacements with distance x corresponding to two values of frequency $\omega = 0.5$, $\omega = 0.75$ respectively. We noticed that in the initial range $0 < x < 0.25$ the pair of curves for $\omega = 0.5, 0.75$ corresponding to both values of angle of inclination ($\theta = 60°, 90°$) increases slightly then decreases and follow an oscillatory pattern with different amplitudes. While for the case $\omega = 0.75$ and $\theta = 90°$ its value falls near the boundary then rises and shows an oscillatory behaviour in the rest of the range. Fig. 2 depicts the behavior of normal displacement with distance. We see that in all the four cases trends are similar (i.e., oscillatory) for $\omega = 0.5, 0.75$ corresponding to both values of angle of inclination ($\theta = 60°, 90°$) with little difference in their magnitude of oscillations. It can be observed that the oscillatory curves attain the highest peak in the range $0 < x < 0.25$ and we also noticed that for $\omega = 0.75$ amplitudes are high as compared to $\omega = 0.5$ corresponding to both values of angle of inclination ($\theta = 60°, 90°$) respectively. Figs. 4, 5 and 6 describes the behaviour of stress components σ_{33}, σ_{13} and $\sigma_{11}$with distance x corresponding to different angles of inclination $\theta = 60°, 90°$ for $\omega = 0.5, 0.75$ respectively. We see that in the starting range near the boundary surface the value of normal stress σ_{33} falls for all the four cases then rises and attains a highest peak at $x = 7.5$, however oscillates. The trends are same (i.e., oscillatory) for the tangential stress σ_{13} corresponding to two different values of frequency and angle of inclination. Fig. 6 interprets the nature of normal stress component σ_{11} with distance. The behaviour is quite different from other two stress components. It can be observed that near the boundary its value decreases sharply for all the four cases and the curves are little oscillatory with small amplitudes. The value of conductive temperature (ϕ) and temperature change T has been shown in Figs. 7 and 8. The value of both the temperatures decreases sharply near the boundary surface for $\omega = 0.5$, $\omega = 0.75$ and angle of inclination $\theta = 60°, 90°$ respectively. After that their value increases and follows a smooth oscillatory pattern.

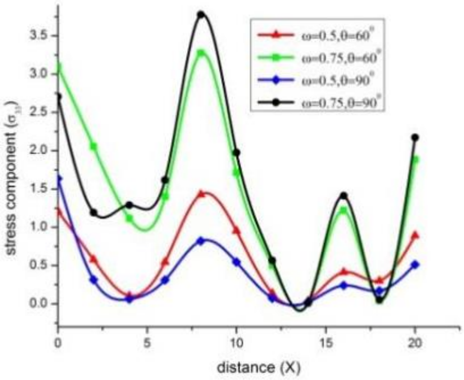

Fig. 4 Variation of stress component σ_{33} with distance x (uniformly distributed force)

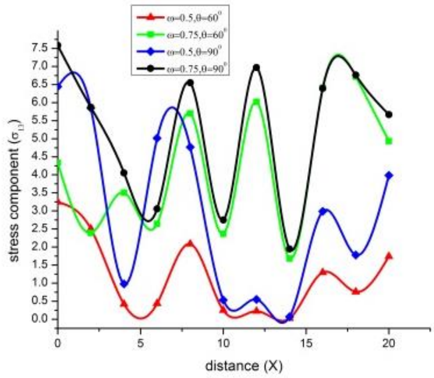

Fig. 5 Variation of stress component σ_{13} with distance x (uniformly distributed force)

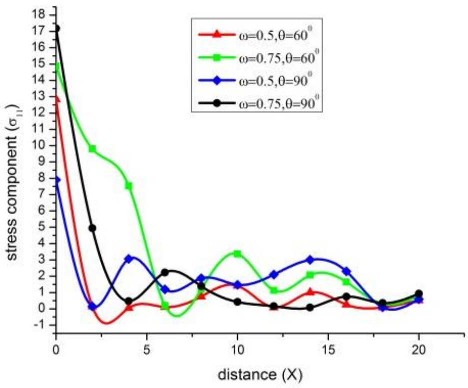

Fig. 6 Variation of stress component σ_{11} with distance x (uniformly distributed force)

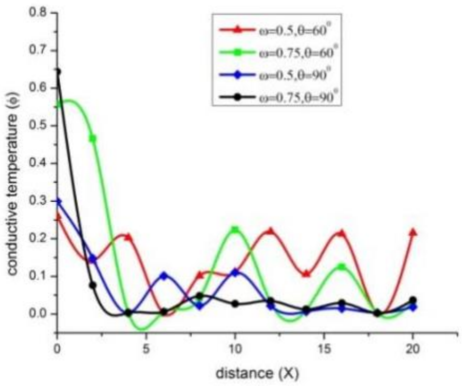

Fig. 7 Variation of conductive temperature ϕ with distance x (uniformly distributed force)

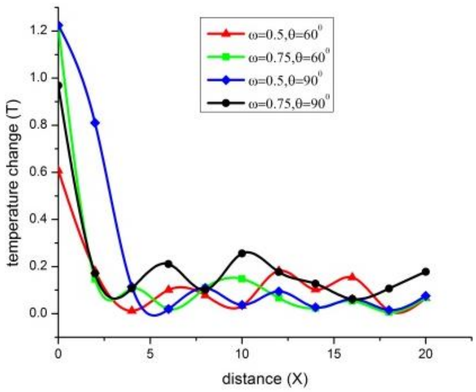

Fig. 8 Variation of temperature change T with distance x (uniformly distributed force)

The difference between the two is that the magnitude of oscillations is high in the case of conductive temperature as compared to other temperature.

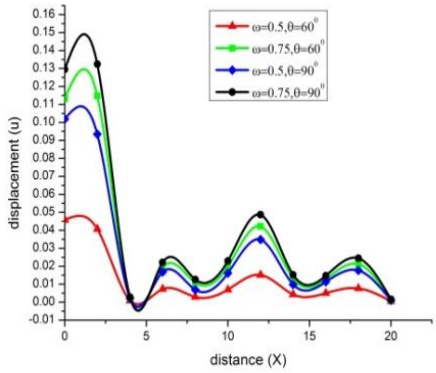

Fig. 9 Variation of displacement u with distance x (linearly distributed force)

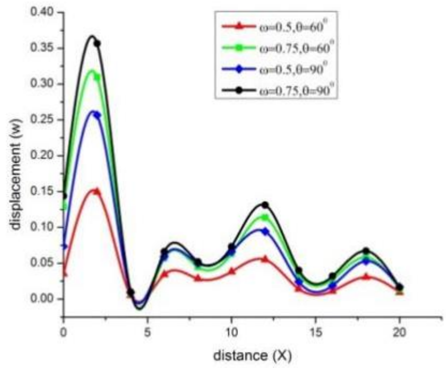

Fig. 10 Variation of displacement w with distance x (linearly distributed force)

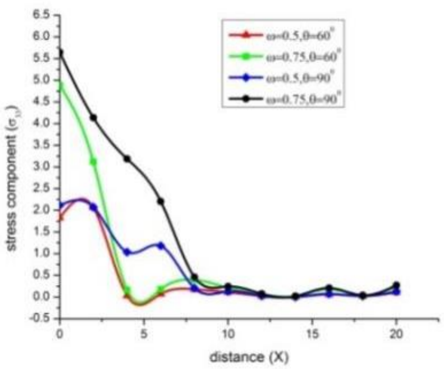

Fig 11 Variation of stress σ_{33} with distance x (uniformly distributed force)

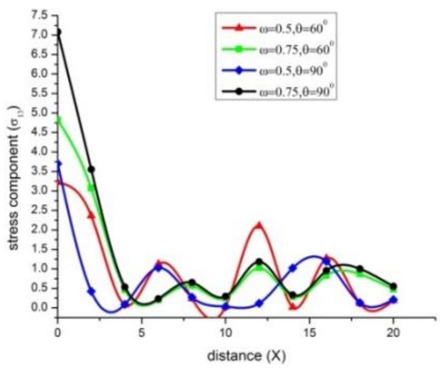

Fig 12 Variation of stress σ_{31} with distance x (linearly distributed force)

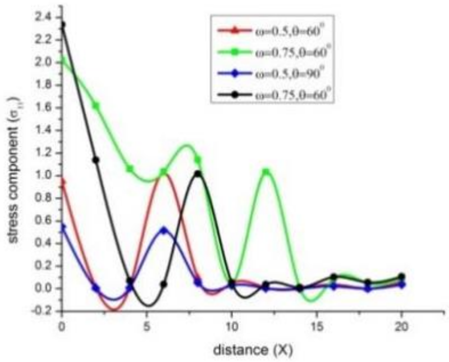

Fig. 13 Variation of stress component σ_{11} with distance x (linearly distributed force)

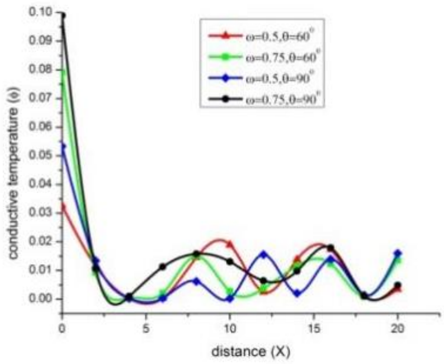

Fig. 14 Variation of conductive temperature ϕ with distance x (linearly distributed force)

8.2 Linearly distributed force

In linearly distributed force, Figs. 9 to 15 as in case of uniformly distributed force shows the

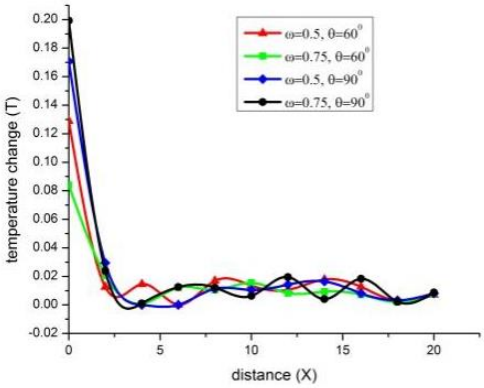

Fig. 15 Variation of temperature change T with distance x (linearly distributed force)

variation of transverse displacement, normal displacement, normal stresses, tangential stress, conductive temperature and temperature change with distance x corresponding to $\omega = 0.5, 0.75$ and $\theta = 60°, 90°$ respectively. In Figs. 9-10, we see that for $\omega = 0.5, 0.75$ and $\theta = 60°, 90°$ in the initial range the value of displacements u and w increases then decreases and moves in oscillatory manner. Also all the curves intersect each other when distance x approaches to its maximum value. Similarly as we discussed above in the case of uniformly distributed force the behaviour of normal and tangential stresses, conductive temperature and temperature change follow an oscillatory pattern i.e., the trends are oscillatory with different magnitudes in all the cases. However, we concluded from the graphs that the time harmonic source with non-dimensional frequency $\omega = 0.5, 0.75$ tends to move in oscillatory manner with different magnitudes and amplitudes.

9. Conclusions

In the present investigation, we studied the effect of non-dimensional frequency on the orthotropic thermoelastic solid in the context of fractional order theory with rotation and two-temperature. It can be observed that the time harmonic source (uniformly and linearly distributed) has great impact on the displacement components, stress components, temperature change and conductive temperature. Moreover, an inclined load in both the directions (normal and tangential) plays a key role in the deformation of an orthotropic thermoelastic body. We noticed that the value of both the normal and tangential displacements increases near the boundary and falls down away from it and oscillates (either rise or fall) with different amplitudes. The similar behaviour is noticed i.e., oscillatory for all the stress components, conductive temperature and temperature change with increasing value of distance. The outcomes of this research are helpful in the 2-D problems with dynamic response of time harmonic sources in orthotropic thermoelastic media with rotation and two-temperature. The purposed model in this research is relevant to different problems in solid mechanics and thermoelasticity. The results obtained in this article give an inspiration to study the thermoelastic solids together with fractional order theory.

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