

A five-variable refined plate theory for thermal buckling analysis of composite plates

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Abstract. This research is devoted to investigate the thermal buckling analysis behaviour of laminated composite plates, by applying an analytical model based on a refined plate theory (RPT) with five independent unknown variables. The theory accounts for parabolic distribution of the transvers shear strains through the plate thickness, and satisfied the zero traction boundary condition on the surface without using shear correction factors, hence a shear correction factor is not required. The governing differential equations and associated boundary conditions are derived by employing the principle of virtual work and solved via Navier-type analytical procedure to obtain critical buckling temperature for simply supported boundary condition of symmetric and antisymmetric cross-ply and angle-ply laminated plates. MATLAB 2018 program is used to investigate the effect of thickness ratio (a/h), aspect ratio (a/b), orthogonality ratio (E_1/E_2), coefficient of thermal expansion ratio (α_2/α_1) and numbers of layers on thermal buckling of laminated plate. It can be concluded that this theory gives good results when compared with other theory.

Keywords: thermal buckling; cross & angle-ply plate; critical buckling temperature; refined plate theory

1. Introduction

Designs of airframes for high speed flight and spacecraft structures have to consider carefully the effect of the thermal environment on structural and material behavior. The plate structures are often subjected to severe thermal environments during launching and reentry and may have significant and unavoidable initial geometric imperfections. When the plate is subjected to temperature change, thermally induced compressive stresses are developed in the constraint plate due to thermoelastic properties and consequently buckling occurs. Therefore, the study of the buckling behavior of composite laminated plates under such environmental conditions is a matter of considerable importance in the design of aircraft. Thangaratnam and Ramachandran (1989) used finite element method using semiloof elements to analyse critical buckling temperature for composite laminates under thermal load. The equation of motion for critical temperature is obtained by equating the second variation of total potential energy to zero. Different boundary condition for cross-ply and angle-ply symmetric and antisymmetric plates. Chang and Leu (1991) studied thermal buckling of antisymmetric angle-ply laminated simply supported subjected to uniform thermal load using higher order deformation theory which account for transverse shear and transverse normal

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Appendix

For stiffness cross-ply:

$$\begin{aligned}
 s_{11} &= A_{11}\alpha^2 + A_{66}\beta^2, \quad s_{12} = (A_{12} + A_{66})\alpha\beta, \quad s_{13} = -B_{11}\alpha^3 - (B_{12} + 2B_{66})\alpha\beta^2, \\
 s_{14} &= -B_{11}^s\alpha^3 - (B_{12}^s + 2B_{66}^s)\alpha, \quad s_{22} = A_{66}\alpha^2 + A_{22}\beta^2, \quad s_{23} = -(B_{12} + 2B_{66})\alpha^2\beta - B_{22}\beta^3, \\
 s_{24} &= -(B_{12}^s + 2B_{66}^s)\alpha^2\beta - B_{22}^s\beta^3, \quad s_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4, \\
 s_{34} &= D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\beta^2 + D_{22}^s\beta^4, \quad s_{44} = H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 \\
 &\quad + A_{55}^s\alpha^2 + A_{44}^s\beta^2, \quad s_{45} = A_{55}^a\alpha^2 + A_{44}^a\beta^2, \quad s_{55} = A_{55}\alpha^2 + A_{44}\beta^2 \\
 A_{16} &= A_{26} = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16}^s = H_{26}^s = B_{16} = B_{26} = B_{12}^s = B_{16}^s = B_{26}^s = A_{45} \\
 &= A_{45}^a = A_{45}^s = 0
 \end{aligned}$$

For stiffness angle-ply:

$$\begin{aligned}
 s_{11} &= A_{11}\alpha^2 + A_{66}\beta^2, \quad s_{12} = (A_{12} + A_{66})\alpha\beta, \quad s_{13} = -(3B_{16}\alpha^2\beta + B_{26}\beta^3), \\
 s_{14} &= -(3B_{16}^s\alpha^2\beta + B_{26}^s)\beta^3, \quad s_{22} = A_{66}\alpha^2 + A_{22}\beta^2, \quad s_{23} = -(B_{16}\alpha^3 + 3B_{26}\alpha\beta^2), \\
 s_{24} &= -(B_{16}^s\alpha^3 + 3B_{26}^s\alpha\beta^2), \quad s_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4, \\
 s_{34} &= D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\beta^2 + D_{22}^s\beta^4, \\
 s_{44} &= H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2, \\
 s_{45} &= A_{55}^a\alpha^2 + A_{44}^a\beta^2, \quad s_{55} = A_{55}\alpha^2 + A_{44}\beta^2 \\
 A_{16} &= A_{26} = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16} = H_{26} = H_{16}^s = H_{26}^s = B_{11} = B_{12} = B_{22} = B_{66} = B_{11}^s \\
 &= B_{12}^s = B_{22}^s = B_{66}^s = 0 \\
 A_{45} &= A_{45}^a = A_{45}^s = 0
 \end{aligned}$$

The plane stress reduced stiffness Q_{ij} are Reddy (2004)

$$\begin{aligned}
 Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{11} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \\
 \alpha &= \frac{m\pi}{a}, \quad \beta = \frac{m\pi}{b}, \quad \text{and } (U_{mn}, V_{mn}, W_{bmn}, W_{bmn}, W_{bmn}) \text{ are coefficients}
 \end{aligned}$$