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Analytical solution for stability analysis of joined cross-ply thin laminated conical shells under axial compression

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Abstract. The present research considers the stability and corresponding modes of two axially compressed joined cross-ply laminated conical shells. The joined conical shells are the general case of a wide area of joined structures, including cylinder-cone, cone-plate, cylinder-plate, stepped thickness cone and stepped thickness cylinder. The principle of minimum potential energy is applied to extract the equilibrium equations under the thin Donnell type shell theory assumptions. The analytical procedure is used to solve the equations by applying trigonometric and series responses in circumferential and meridional directions, respectively. To ensure from accuracy and correctness of the results, the finite element analysis is done for various stacking sequences and the analytical results are compared and validated with other literature and finite element results. Finally, the effects of some parameters including semi-vertex angles, meridional lengths, number of layers and various kinds of simply supported and clamped boundary conditions at both ends are studied.

Keywords: joined conical shells; cross-ply laminate; buckling; analytical solution

1. Introduction

Due to high capacity of load carrying, shells of revolution and joined shells find extensive uses in engineering branches like construction, mechanical and aeronautical structures. The joined shell structures, made from shell parts joined along their boundaries, experience high values of bending moment and shear force near the joining region. It causes the more instability in these structures. Therefore, the instability of the shells is one of the most important modes of failure of joined shell structures and needs more attention.

The stability and vibrational behavior of joined shells are one of the attractive subjects in recent years. Most previous investigations are focused on vibration of the joined shells (Shakouri and Kouchakzadeh 2014, Sarkheil *et al*. 2017, Bagheri *et al*. 2018, Izadi *et al*. 2018). However, there are some published papers on stability of joined shells. Flores and Godoy (1991) applied finite element analysis to study the buckling and post-buckling of cone-cylinder and sphere-cylinder shells under external pressure. A comprehensive review of recent works on joined shells is performed by Pietraszkiewicz and Konopinska (2015).

The elastic buckling and post-buckling of joined conical-cylindrical shells subjected to internal

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$$G_{3,3}(m)=(\sin\alpha(A_{11}(2R_0\cos\alpha(3A_{12}m+A_{22})-2B_{11}(m-2)m(3m-1)\sin^2\alpha$$
$$+B_{22}(1-2m)\cos 2\alpha+4B_{12}mn^2+8B_{66}mn^2+2B_{22}(m+n^2)-B_{22})$$
$$-2B_{11}(m+1)(A_{22}\sin^2\alpha+A_{66}n^2)))/(2m_{04}R_0^3(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,4}(m)=-(A_{11}\sin^2(\alpha)(2R_0\cos\alpha(3A_{12}(m-1)+2A_{22})+(m-2)\cos 2\alpha(B_{11}(m-1)^2-B_{22})$$
$$+2n^2(B_{12}(m-1)+2B_{66}(m-1)+B_{22})-(m-2)(B_{11}(m-1)^2-B_{22})))/(2m_{04}R_0^4(A_{11}D_{11}-B_{11}^2))$$

$$G_{3,5}(m)=\frac{A_{11}\sin^3\alpha\cos\alpha(A_{12}(m-2)+A_{22})}{m_{04}R_0^4(B_{11}^2-A_{11}D_{11})}$$

$$G_{3,6}(m)=(B_{11}n((A_{12}+A_{66})R_0+(B_{12}+B_{66})\cos\alpha)-A_{11}n((B_{12}+2B_{66})R_0$$
$$+(D_{12}+2D_{66})\cos\alpha))/((m+3)(m+4)R_0^2(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,7}(m)=(m+1)(n\sin\alpha(A_{11}(R_0(B_{22}-3(B_{12}+2B_{66})m)+\cos\alpha(-2D_{12}(m-1)-4D_{66}(m-1)+D_{22}))$$
$$+B_{11}(R_0(2A_{12}(m+1)+A_{66}(2m+1)-A_{22})+\cos\alpha(B_{12}m+B_{66}(m-1)-B_{22}))))/(m_{04}R_0^3(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,9}(m)=(A_{11}n\sin\alpha(4A_{22}R_0\cos\alpha-2B_{12}(m-2)(m-1)\sin^2\alpha-4B_{66}(m-2)(m-1)\sin^2\alpha$$
$$+B_{22}(-(m-3)\cos 2\alpha+m+2n^2-1)))/(2m_{04}R_0^4(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,10}(m)=\frac{A_{11}A_{22}n\sin^2\alpha\cos\alpha}{m_{04}R_0^4(B_{11}^2-A_{11}D_{11})}$$

$$G_{3,11}(m)=\frac{\sin\alpha(2A_{11}D_{11}(2m+1)-B_{11}^2(3m+4))}{(m+4)R_0(B_{11}^2-A_{11}D_{11})}$$

$$G_{3,12}(m)=(A_{11}(R_0(P\sec\alpha-4\pi B_{12}\cos\alpha)+2\pi(\sin^2\alpha(6D_{11}m^2-D_{22})-2(D_{12}+2D_{66})n^2))$$
$$+2\pi B_{11}(A_{12}R_0\cos\alpha+B_{22}\sin^2\alpha+(B_{12}+2B_{66})n^2)$$
$$-2\pi B_{11}^2(m+1)(3m+2)\sin^2\alpha)/(2\pi(m+3)(m+4)R_0^2(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,13}(m)=(A_{11}(-6\pi B_{12}mR_0\sin 2\alpha+2\pi(2m-1)\sin\alpha(\sin^2\alpha(2D_{11}(m-1)m-D_{22})$$
$$-2(D_{12}+2D_{66})n^2)+3mPR_0\tan(\alpha))+\pi B_{11}\sin\alpha(2R_0\cos\alpha(2A_{12}(m+1)-A_{22})$$
$$-B_{22}((m+1)\cos 2\alpha-m+2n^2-1)+2B_{12}mn^2+4B_{66}mn^2)$$
$$-2\pi B_{11}^2m^2(m+1)\sin^3\alpha)/(2\pi(m+2)(m+3)(m+4)R_0^3(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,14}(m)=(A_{11}/(\pi(m+1)(-4\pi\cos 2\alpha(A_{22}R_0^2+4D_{66}(m-1)^2n^2+D_{22}((m-2)m+2n^2))$$
$$+\pi(-4A_{22}R_0^2+16D_{66}(m-1)^2n^2+D_{22}(3(m-2)m-8n^4+8n^2))+$$
$$+2R_0\cos\alpha(6(m-1)m(\pi B_{12}+P)+\pi B_{22}(1-8n^2))-2\pi R_0\cos 3\alpha(6B_{12}(m-1)m+B_{22})$$
$$-8\pi D_{11}(m-2)(m-1)^2m\sin^4\alpha+\pi D_{22}(m-2)m\cos 4\alpha+16\pi D_{12}(m-1)^2n^2\sin^2\alpha$$
$$-12(m-1)mPR_0\sec\alpha)+\frac{8B_{11}R_0\sin^2\alpha\cos\alpha(A_{22}-A_{12}m)}{A_{11}}))/(8(m+2)(m+3)(m+4)R_0^4(A_{11}D_{11}-B_{11}^2))$$

$$G_{3,15}(m)=(A_{11}\tan\alpha(\pi\cos^2\alpha(4A_{22}R_0\cos\alpha+B_{22}(\cos 2\alpha+4n^2-1))-\pi B_{12}(m-2)(m-1)\sin^2 2\alpha$$
$$+(m-2)(m-1)P\sin^2(\alpha)))/(2\pi m_{04}R_0^4(B_{11}^2-A_{11}D_{11}))$$

$$G_{3,16}(m)=\frac{A_{11}A_{22}\sin^2(\alpha)\cos^2(\alpha)}{m_{04}R_0^4(B_{11}^2-A_{11}D_{11})}$$

$$m_{04}=(m+1)(m+2)(m+3)(m+4)$$