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# Effect of inclined load on transversely isotropic magneto thermoelastic rotating solid with time harmonic source

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**Abstract.** The present research deals with the time harmonic deformation in transversely isotropic magneto thermoelastic solid with two temperature (2T), rotation and without energy dissipation due to inclined load. Lord-Shulman theory has been formulated for this mathematical model. The entire thermo-elastic medium is rotating with a uniform angular velocity. The Fourier transform techniques have been used to find the solution to the problem. The displacement components, stress components and conductive temperature distribution with the horizontal distance are computed in the transformed domain and further calculated in the physical domain using numerical inversion techniques. The effect of time harmonic source and rotation is depicted graphically on the resulting quantities.

**Keywords:** time harmonic sources; transversely isotropic thermoelastic; rotation; inclined load; magneto thermoelastic solid

#### 1. Introduction

A lot of research and attention has been given to deformation and heat flow in a continuum using thermoelasticity theories during the past few years. When sudden heat/external force is applied in a solid body, it transmits time harmonic wave by thermal expansion. The change at some point of the medium is beneficial to detect the deformed field near mining shocks, seismic and volcanic sources, thermal power plants, high-energy particle accelerators, and many emerging technologies. The study of time harmonic source is one of the broad and dynamic areas of continuum dynamics.

Chen *et al.* (Chen and Gurtin 1968, Chen *et al.* 1968, 1969) formulated a two temperature thermoelasticity of deformable bodies for the conduction of heat depending on two types of temperatures. Ailawalia and Narah (2009) had studied the deformation of a rotating generalized thermoelastic solid beneath the impact of gravity with a superimposing infinite thermoelastic fluid due to different forces acting along the interface. Ailawalia *et al.* (2010) had studieda rotating generalized thermoelastic medium with two temperatures beneath hydrostatic stress and gravity with different types of sources using integral transforms. Marin (1997a) had proved the Cesaro means of the kinetic and strain energies of dipolar bodies with finite energy. Sharma *et al.* (2010) presented the propagation of Rayleigh waves in a generalized thermoelastic half-space with voids.

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The surface chosen is stress-free and thermally insulated. They detected the elliptical paths during the Rayleigh wave motion without rotation. Abd-Alla *et al.* (2012) investigated the Rayleigh waves propagation in a homogeneous orthotropic elastic medium with impact of rotation, initial stress and gravity field by Lame's potentials and governing equations.

Mahmoud (2012) had considered the impact of rotation, relaxation times, magnetic field, gravity field and initial stress on Rayleigh waves and attenuation coefficient in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lame's potential techniques. Abd-alla *et al.* (2015) had discussed the influence of magnetic field and rotation on plane waves in transversely isotropic thermoelastic medium under the GL theory in presence of two relaxation times to show the presence of three quasi plane waves in the medium. Marin *et al.* (2013) has modelled a micro stretch thermoelastic body with two temperatures and eliminated divergences among the classical elasticity and research.

Sharma et al. (2015) investigated the 2-D deception in a transversely isotropic homogeneous thermoelastic solids in presence of two temperatures in GN-II theory with an inclined load (linear combination of normal load and tangential load). Kumar et al. (2016a, b) investigated the impact of Hall current in a transversely isotropic magnetothermoelastic in presence and absence of energy dissipation due to normal force. Kumar et al. (2016c) studied the conflicts caused by thermomechanical sources in a transversely isotropic rotating homogeneous thermoelastic medium with magnetic effect as well as two temperature and applied to the thermoelasticity Green-Naghdi theories with and without energy dissipation using thermomechanical sources. Lata et al. (2016) studied two temperature and rotation aspect for GN-II and GN-III theory of thermoelasticity in a homogeneous transversely isotropic magnetothermoelastic medium for the case of the plane wave propagation and reflection. Ezzat et al. (2017a) proposed a mathematical model of electrothermoelasticity for heat conduction with memory-dependent derivative. Kumar et al. (2017) analyzed the Rayleigh waves in a transversely isotropic homogeneous magnetothermoelastic medium in presence of two temperature, with Hall current and rotation. Marin et.al. (2017) studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability. Lata (2018) studied the impact of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with two temperature, rotation and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity. Ezzat and El-Bary (2017a) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space of based on memory-dependent derivative.

Abo-Dahab (2018) analyzed the wave propagation in a microstretch elastic medium with GN theory with impact of gravity. Othman *et al.* (2019) discussed the deformation in rotating infinite microstretch generalized thermoelastic medium. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1996, 2009, 2010), Marin and Baleanu (2016), Ezzat *et al.* (2016), Marin (1997b, 2008, 2016) Ezzat *et al.* (2012, 2015, 2017b), Marin and Stan (2013), Ezzat and AI-Bary (2016), Marin and Nicaise (2016), Marin and Öchsner (2017), Ezzat and El-Bary (2017b), Othman and Marin (2017), Chauthale and Khobragade (2017), Marin (1998, 2009, 2010), Kumar *et al.* (2018), Marin *et al.* (2017), Lata and Kaur (2019a, b, c) and Lata and Kaur (2019d, e).

Irrespective of these, not much work has been carried out in magneto-thermoelastic transversely isotropic solid with rotation, time harmonic source for inclined load with two temperature in generalized thermoelasticity without energy dissipation. In this paper, we have attempted to study the deformation in transversely isotropic magneto thermoelastic solid with the combined effects of rotation for inclined load with two temperature by considering the

disturbances harmonically time-dependent. The expressions of displacement components, conductive temperature and stress components due to time harmonic sources are calculated in transformed domain by using the Fourier transformation. Numerical inversion technique is used to find the resulting quantities in the physical domain and effects of frequency at different values have been represented graphically.

#### 2. Basic equatios

For a considered transversely isotropic thermoelastic medium, the constitutive equation (Green and Naghdi 1992) is given by

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T. \tag{1}$$

and equation of motion as described by Schoenberg and Censor (1973) for a uniformly rotating medium with an angular velocity and Lorentz force which governs the dynamic displacement u is

$$t_{ij,j} + F_i = \rho \{ \ddot{u}_i + (\Omega \times (\Omega \times \mathbf{u})_i + (2\Omega \times \dot{u})_i \},$$
<sup>(2)</sup>

where

 $\Omega = \Omega \hat{n}$ , n is a unit vector representing the direction of axis of rotation, The term  $\Omega \times (\Omega \times u)$ is the additional centripetal acceleration due to the time-varying motion only, and the term  $2\Omega \times \dot{u}$  is the Coriolis acceleration.

$$F_i = \mu_0(\vec{j} \times \vec{H}_0).$$

The heat conduction equation without energy dissipation using Lord-Shulman model (1967) is

$$K_{ij}\varphi_{,ij} + \rho(Q + \tau_0\dot{Q}) = \beta_{ij}T_0(\dot{e}_{ij} + \tau_0\ddot{e}_{ij}) + \rho C_E(\dot{T} + \tau_0\ddot{T}),$$
(3)

where

$$\beta_{ij} = C_{ijkl} \,\alpha_{ij}, \tag{4}$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad i, j = 1, 2, 3.$$

$$T = \varphi - a_{ij} \varphi_{,ij}$$

$$\beta_{ij} = \beta_i \delta_{ij}, \quad K_{ij} = K_i \delta_{ij}, \quad i \text{ is not summed.}$$
(5)

Here  $C_{ijkl}$  ( $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$ ) are elastic parameters and having symmetry due to homogeneous transversely isotropic medium. The basis of these symmetries of  $C_{ijkl}$  is due to

- (i) The stress tensor is symmetric, which is only possible if  $(C_{ijkl} = C_{jikl})$
- (ii) If a strain energy density exists for the material, the elastic stiffness tensor must satisfy  $C_{ijkl} = C_{klij}$

From stress tensor and elastic stiffness tensor symmetries infer ( $C_{ijkl} = C_{ijlk}$ ) and  $C_{ijkl} =$  $C_{klij} = C_{jikl} = C_{ijlk}$ 

#### 3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic magnetothermoelastic medium, permeated by an initial magnetic field  $\vec{H}_0 = (0, H_0, 0)$  acting along y-axis. The rectangular Cartesian coordinate system (x, y, z) having origin on the surface (z = 0) with z-axis pointing vertically into the medium is introduced. The surface of the half-space is subjected to an inclined load acting at z = 0.

In addition, we consider that

$$\mathbf{\Omega}=(0,\Omega,0).$$

From the generalized Ohm's law

 $J_2 = 0$ .

The density components  $J_1$  and  $J_3$  are given as

$$J_1 = -\varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2},\tag{6}$$

$$J_3 = \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}.$$
 (7)

In addition, the equations of displacement vector (u, v, w) and conductive temperature  $\varphi$  for transversely isotropic thermoelastic solid in presence of two temperature and without energy dissipation are

$$u \equiv u(x, z, t), v = 0, w \equiv w(x, z, t) and \varphi \equiv \varphi(x, z, t).$$
(8)

Now using the proper transformation on Eqs. (1)-(3) with the aid of (8), following Slaughter (2002) are as under

$$C_{11}\frac{\partial^{2}u}{\partial x^{2}} + C_{13}\frac{\partial^{2}w}{\partial x\partial z} + C_{44}\left(\frac{\partial^{2}u}{\partial z^{2}} + \frac{\partial^{2}w}{\partial x\partial z}\right) - \beta_{1}\frac{\partial}{\partial x}\left\{\varphi - \left(a_{1}\frac{\partial^{2}\varphi}{\partial x^{2}} + a_{3}\frac{\partial^{2}\varphi}{\partial z^{2}}\right)\right\} - \mu_{0}J_{3}H_{0}$$

$$= \rho\left(\frac{\partial^{2}u}{\partial t^{2}} - \Omega^{2}u + 2\Omega\frac{\partial w}{\partial t}\right),$$
(9)

$$(C_{13} + C_{44})\frac{\partial^2 u}{\partial x \partial z} + C_{44}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x^2} + a_3\frac{\partial^2 \varphi}{\partial z^2}\right)\right\} - \mu_0 J_1 H_0$$

$$= \rho \left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right),$$
(10)

$$K_{1}\frac{\partial^{2}\varphi}{\partial x^{2}} + K_{3}\frac{\partial^{2}\varphi}{\partial z^{2}} + \rho\left(Q + \tau_{0}\dot{Q}\right)$$

$$= \rho C_{E}(\dot{T} + \tau_{0}\ddot{T}) + T_{0}\frac{\partial}{\partial t}\left\{\beta_{1}\left(1 + \tau_{0}\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} + \beta_{3}\left(1 + \tau_{0}\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}\right\},$$
(11)

and

$$t_{11} = C_{11}e_{11} + C_{13}e_{13} - \beta_1 T, \qquad (12)$$

$$t_{33} = C_{13}e_{11} + C_{33}e_{33} - \beta_3 T, \tag{13}$$

$$t_{13} = 2C_{44}e_{13},\tag{14}$$

where

$$T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2}\right),$$
  
$$\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3,$$
  
$$\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3.$$

We consider that medium is initially at rest. Therefore, the preliminary and symmetry conditions are given by

$$u(x, z, 0) = 0 = \dot{u}(x, z, 0),$$
  

$$w(x, z, 0) = 0 = \dot{w}(x, z, 0),$$
  

$$\varphi(x, z, 0) = 0 = \dot{\varphi}(x, z, 0) \text{ for } z \ge 0, -\infty < x < \infty,$$
  

$$u(x, z, t) = w(x, z, t) = \varphi(x, z, t) = 0 \text{ for } t > 0 \text{ when } z \to \infty.$$

Assuming the time harmonic behaviour as

$$(u, w, \varphi)(x, z, t) = (u, w, \varphi)(x, z)e^{i\omega t}.$$
(15)

To facilitate the solution, following dimensionless quantities are introduced

$$x' = \frac{x}{L}, \quad z' = \frac{z}{L}, \quad t' = \frac{c_1}{L}t, \quad u' = \frac{\rho c_1^2}{L\beta_1 T_0}u, \quad w' = \frac{\rho c_1^2}{L\beta_1 T_0}w, \quad T' = \frac{T}{T_0},$$
  

$$t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \quad t'_{33} = \frac{t_{33}}{\beta_1 T_0}, \quad t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0},$$
  

$$a'_1 = \frac{a_1}{L^2}, \quad a'_3 = \frac{a_3}{L^2}, \quad h' = \frac{h}{H_0}, \quad \Omega' = \frac{L}{C_1}\Omega.$$
(16)

Making use of (15) in Eqs. (9)-(11), after suppressing the primes, yield

$$\frac{\partial^{2} u}{\partial x^{2}} + \delta_{4} \frac{\partial^{2} w}{\partial x \partial z} + \delta_{2} \left( \frac{\partial^{2} u}{\partial z^{2}} + \frac{\partial^{2} w}{\partial x \partial z} \right) - \frac{\partial}{\partial x} \left\{ \varphi - \left( a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} + a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right) \right\}$$

$$= \left( \frac{\varepsilon_{0} \mu_{0}^{2} H_{0}^{2}}{\rho} + 1 \right) (-\omega^{2} u) - \Omega^{2} u + 2\Omega i \omega w,$$
(17)

$$\delta_{1} \frac{\partial^{2} u}{\partial x \partial z} + \delta_{2} \frac{\partial^{2} w}{\partial x^{2}} + \delta_{3} \frac{\partial^{2} w}{\partial z^{2}} - \frac{\beta_{3}}{\beta_{1}} \frac{\partial}{\partial z} \left\{ \varphi - \left( a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} + a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right) \right\}$$

$$= \left( \frac{\varepsilon_{0} \mu_{0}^{2} H_{0}^{2}}{\rho} + 1 \right) (-\omega^{2} w) - \Omega^{2} w + 2\Omega i \omega u,$$
(18)

$$\frac{\partial^{2} \varphi}{\partial x^{2}} + \frac{K_{3}}{K_{1}} \frac{\partial^{2} \varphi}{\partial z^{2}} + \rho \left( 1 + \tau_{0} \frac{c_{1}}{L} i \omega \right) Q$$

$$= \delta_{5} \frac{\partial}{\partial t} \left( 1 + \tau_{0} \frac{c_{1}}{L} i \omega \right) \left[ \varphi - a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} - a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right] + \delta_{6} i \omega \left( 1 + \tau_{0} \frac{c_{1}}{L} i \omega \right) \left[ \beta_{1} \frac{\partial u}{\partial x} + \beta_{3} \frac{\partial w}{\partial z} \right],$$
<sup>(19)</sup>

where

$$\delta_1 = \frac{c_{13} + c_{44}}{c_{11}}, \qquad \delta_2 = \frac{c_{44}}{c_{11}}, \qquad \delta_3 = \frac{c_{33}}{c_{11}}, \qquad \delta_4 = \frac{c_{13}}{c_{11}},$$
$$\delta_5 = \frac{\rho C_E C_1 L}{K_1}, \qquad \delta_6 = -\frac{T_0 \beta_1 L}{\rho C_1 K_1}$$

Apply Fourier transforms defined by

$$\hat{f}(\xi, z, \omega) = \int_{-\infty}^{\infty} f(x, z, \omega) e^{i\xi x} dx$$
(20)

On Eqs. (17)-(19), we obtain a system of equations

$$[-\xi^{2} + \delta_{2}D^{2} + \delta_{7}\omega^{2} + \Omega^{2}]\hat{u}(\xi, z, \omega) + [\delta_{4}Di\xi + \delta_{2}Di\xi - 2\Omega i\omega]\hat{w}(\xi, z, \omega) + (-i\xi)[1 + a_{1}\xi^{2} - a_{3}D^{2}]\hat{\varphi}(\xi, z, \omega) = 0,$$
(21)

$$[\delta_{1}Di\xi + 2\Omega i\omega]\hat{u}(\xi, z, \omega) + [-\delta_{2}\xi^{2} + \delta_{3}D^{2} + \delta_{7}\omega^{2} + \Omega^{2}]\hat{w}(\xi, z, \omega) - \frac{\beta_{3}}{\beta_{1}}D[1 + a_{1}\xi^{2} - a_{3}D^{2}]\hat{\varphi}(\xi, z, \omega) = 0,$$
(22)

$$[-\delta_6\omega\delta_8\beta_1\xi]\hat{u}(\xi,z,\omega) + [\delta_6i\omega\delta_8\beta_3D]\hat{w}(\xi,z,\omega) + \left[\xi^2 - \frac{\kappa_3}{\kappa_1}D^2 + \delta_5\delta_8i\omega(1+a_1\xi^2 - a_3D^2)\right]\hat{\varphi}(\xi,z,\omega) = \rho\delta_8\hat{Q}(\xi,z,\omega),$$
(23)

where

$$\delta_7 = \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1, \ \delta_8 = 1 + \tau_0 \frac{C_1}{L} i\omega.$$

By taking  $\hat{Q}(\xi, z, s) = 0$ , i.e., no external heat is supplied the non trivial solution of (21)-(23) yields

$$(AD^{6} + BD^{4} + CD^{2} + E)(\hat{u}, \hat{w}, \hat{\varphi}) = 0, \qquad (24)$$

where

$$\begin{split} A &= \delta_2 \delta_3 \zeta_7 - \zeta_5 \delta_2 \frac{\beta_3}{\beta_1} a_3, \\ B &= \delta_3 \zeta_1 \zeta_7 - a_3 \zeta_1 \zeta_5 \frac{\beta_3}{\beta_1} + \delta_2 \delta_3 \zeta_6 + \delta_2 \zeta_7 \zeta_3 - \zeta_5 \zeta_9 \delta_2 \\ &- \zeta_8 \delta_1 i \xi \zeta_7 + \zeta_8 \zeta_4 \frac{\beta_3}{\beta_1} a_3 - a_3 \xi^2 \zeta_5 \delta_1 - a_3 \delta_3 \zeta_4 i \xi, \\ C &= \delta_3 \zeta_1 \zeta_6 + \zeta_1 \zeta_3 \zeta_7 - \zeta_1 \zeta_5 \zeta_9 + \delta_2 \zeta_6 \zeta_3 + \zeta_4 \zeta_8 \zeta_9 - \zeta_8 \delta_1 i \xi \zeta_6 \\ &- 4 \Omega^2 \omega^2 \zeta_7 + \zeta_2 \delta_1 i \xi \zeta_5 - \zeta_2 \zeta_4 \delta_3 - a_3 \zeta_4 i \xi \zeta_3, \\ E &= \zeta_3 \zeta_1 \zeta_6 - 4 \Omega^2 \omega^2 \zeta_6 - \zeta_2 \zeta_4 \zeta_3, \\ \zeta_1 &= \xi^2 + \delta_7 \omega^2 + \Omega^2, \\ \zeta_2 &= -i \xi (1 + a_1 \xi^2), \\ \zeta_3 &= -\delta_2 \xi^2 + \delta_7 \omega^2 + \Omega^2, \\ \zeta_4 &= -\delta_6 \delta_8 \omega \beta_1 \xi, \\ \zeta_5 &= \delta_6 \delta_8 i \omega \beta_3, \\ \zeta_6 &= \xi^2 + \delta_5 \delta_8 i \omega (1 + a_1 \xi^2), \\ \zeta_7 &= -\frac{K_3}{K_1} - a_3 \delta_5 \delta_8 i \omega, \\ \zeta_8 &= \delta_1 i \xi, \\ \zeta_9 &= -(1 + a_1 \xi^2) \frac{\beta_3.}{\beta_1} \end{split}$$

The roots of the Eq. (24) are  $\pm \lambda j$ , (j = 1, 2, 3), the solution of the Eq. (24) is calculated by using the radiation condition of  $\tilde{u}, \tilde{v}, \tilde{w}$  and can be written as

$$\hat{u}(\xi, z, \omega) = \sum_{j=1}^{3} A_j e^{-\lambda_j z}, \qquad (25)$$

$$\widehat{w}(\xi, z, \omega) = \sum_{j=1}^{3} d_j A_j e^{-\lambda_j z}, \qquad (26)$$

$$\hat{\varphi}(\xi, z, \omega) = \sum_{j=1}^{3} l_j A_j e^{-\lambda_j z}, \qquad (27)$$

where  $A_j(\xi, \omega), j = 1, 2, 3$  being undetermined constants and  $d_j$  and  $l_j$  are given by

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$$d_{j} = \frac{\delta_{2}\zeta_{7}\lambda_{j}^{4} + (\zeta_{7}\zeta_{1} - a_{3}\zeta_{4}i\xi + \delta_{2}\zeta_{6})\lambda_{j}^{2} + \zeta_{1}\zeta_{6} - \zeta_{4}\zeta_{2}}{\left(\delta_{3}\zeta_{7} - \frac{\beta_{3}}{\beta_{1}}a_{3}\zeta_{5}\right)\lambda_{j}^{4} + (\delta_{3}\zeta_{6} + \zeta_{3}\zeta_{7} - \zeta_{5}\zeta_{9})\lambda_{j}^{2} + \zeta_{3}\zeta_{6}}$$
$$l_{j} = \frac{\delta_{2}\delta_{3}\lambda_{j}^{4} + (\delta_{2}\zeta_{3} + \zeta_{1}\delta_{3} - \delta_{1}\zeta_{8}i\xi)\lambda_{j}^{2} - 4\Omega^{2}\omega^{2} + \zeta_{3}\zeta_{1}}{\left(\delta_{3}\zeta_{7} - \frac{\beta_{3}}{\beta_{1}}a_{3}\zeta_{5}\right)\lambda_{j}^{4} + (\delta_{3}\zeta_{6} + \zeta_{3}\zeta_{7} - \zeta_{5}\zeta_{9})\lambda_{j}^{2} + \zeta_{3}\zeta_{6}}$$

### 4. Boundary conditions

We consider a normal line load  $F_1$  per unit length acting in the positive z-axis on the plane boundary z = 0 along the y-axis and a tangential load  $F_2$  per unit length, acting at the origin in the positive x-axis. The appropriate boundary conditions are

i. 
$$t_{33}(x, z, t) = -F_1 \psi_1(x) e^{i\omega t}$$
, (28)

ii. 
$$t_{31}(x, z, t) = -F_2 \psi_2(x) e^{i\omega t}$$
, (29)

iii. 
$$\frac{\partial \varphi}{\partial z}(x, z, t) = 0,$$
 (30)

where  $F_1$  and  $F_2$  are the magnitude of the forces applied,  $\psi_1(x)$  and  $\psi_2(x)$  specify the vertical and horizontal load distribution function along *x*-axis.

Applying Fourier transform defined by (20) on the boundary conditions (28)-(30), (13)-(14) and with the help of Eqs. (25)-(27), we obtain the components of displacement, normal stress, tangential stress, and conductive temperature as

$$\hat{u} = \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[ \sum_{j=1}^3 \Gamma_{1j} e^{-\lambda_j z} \right] e^{i\omega t} + \frac{F_2 \hat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{j=1}^3 \Gamma_{2j} e^{-\lambda_j z} \right] e^{i\omega t}, \qquad (31)$$

$$\widehat{w} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{j=1}^3 d_j \Gamma_{1j} e^{-\lambda_j z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{j=1}^3 d_j \Gamma_{2j} e^{-\lambda_j z} \right] e^{i\omega t}, \qquad (32)$$

$$\hat{\varphi} = \frac{F_1 \hat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{j=1}^3 l_j \Gamma_{1j} e^{-\lambda_j z} \right] e^{i\omega t} + \frac{F_2 \hat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{j=1}^3 l_j \Gamma_{2j} e^{-\lambda_j z} \right] e^{i\omega t}, \qquad (33)$$

$$\widehat{t_{11}} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{j=1}^3 S_j \Gamma_{1j} e^{-\lambda_j z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{j=1}^3 S_j \Gamma_{2j} e^{-\lambda_j z} \right] e^{i\omega t}, \qquad (34)$$

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$$\widehat{t_{13}} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{j=1}^3 N_j \Gamma_{1j} e^{-\lambda_j z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{j=1}^3 N_j \Gamma_{2j} e^{-\lambda_j z} \right] e^{i\omega t}, \quad (35)$$

$$\widehat{t_{33}} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{j=1}^3 M_j \Gamma_{1j} e^{-\lambda_j z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{j=1}^3 M_j \Gamma_{2j} e^{-\lambda_j z} \right] e^{i\omega t}, \quad (36)$$

where

$$\begin{split} &\Gamma_{11} = -N_2 R_3 + R_2 N_3, \\ &\Gamma_{12} = N_1 R_3 - R_1 N_3, \\ &\Gamma_{13} = -N_1 R_2 + R_1 N_2, \\ &\Gamma_{13} = -N_1 R_2 + R_1 N_2, \\ &\Gamma_{21} = M_2 R_3 - R_2 M_3, \\ &\Gamma_{21} = M_2 R_3 - R_2 M_3, \\ &\Gamma_{22} = -M_1 R_3 + R_1 M_3, \\ &\Gamma_{23} = M_1 R_2 - R_1 M_2, \\ &\Gamma = -M_1 \Gamma_{11} - M_2 \Gamma_{12} - M_3 \Gamma_{13}, \\ &N_j = -\delta_2 \lambda_j + i\xi d_j, \\ &M_j = i\xi - \delta_3 d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j \left[ (1 + a_1 \xi^2) - a_3 \lambda_j^2 \right], \\ &R_j = -\lambda_j l_j, \\ &S_j = -i\xi - \delta_4 d_j \lambda_j - l_j \left[ (1 + a_1 \xi^2) - a_3 \lambda_j^2 \right]. \end{split}$$

### 5. Special cases

#### 5.1 Concentrated force

The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x), \qquad \psi_2(x) = \delta(x), \tag{37}$$

where  $\delta(x)$  is dirac delta function.

Applying Fourier transform defined by (20) on (37), we obtain

$$\hat{\psi}_1(\xi) = 1, \qquad \hat{\psi}_2(\xi) = 1.$$
 (38)

Using (38) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

#### 5.2 Uniformly distributed force

The solution due to uniformly distributed force applied on the half space is obtained by setting

$$\psi_1(x), \quad \psi_2(x) = \begin{cases} 1 \text{ if } |x| \le m\\ 0 \text{ if } |x| > m \end{cases}$$
(39)

The Fourier transforms of  $\psi_1(x)$  and  $\psi_2(x)$  with respect to the pair (x,  $\xi$ ) for the case of a uniform strip load of non-dimensional width 2 m applied at origin of co-ordinate system x = z = 0 in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2\sin(\xi m)}{\xi} \right\}, \quad \xi \neq 0.$$
 (40)

Using (40) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

#### 5.3 Linearly distributed force

The solution due to linearly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x), \quad \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} \text{ if } |x| \le m\\ 0 \text{ if } |x| > m \end{cases}$$
(41)

Here 2 m is the width of the strip load, using (15) and applying the transform defined by (20) on (41), we get

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2\{1 - \cos(\xi m)\}}{\xi^2 m} \right\}, \quad \xi \neq 0.$$
(42)

Using (42) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.



Fig. 1 Inclined load over a transversely isotropic magneto-thermoelastic solid

#### 6. Inclined load

Suppose an inclined load,  $F_0$  per unit length is acting on the y-axis and its inclination with z-axis is  $\theta$ , we have  $F_1 = F_0 \cos\theta$  and  $F_2 = F_0 \sin\theta$  (see Fig. 1),

Using Eq. (43) in Eqs. (31)-(36) and with aid of Eqs. (37)-(42) we obtain the expressions for displacements, and stresses and conductive temperature for concentrated force, uniformly distributed force and linearly distributed force on the surface of transversely isotropic magneto-thermoelastic body without energy dissipation.

#### 7. Inversion of the transformation

For obtaining the result in physical domain, invert the transforms in Eqs. (31)-(36) using

$$\tilde{f}(x,z,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi,z,\omega) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x)f_e - i\sin(\xi x)f_o| d\xi,$$

where  $f_o$  is odd and  $f_e$  is the even parts of  $\hat{f}(\xi, z, s)$  respectively.

#### 8. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of two temperature and rotation, we now present some numerical results. Following Dhaliwal and Sherief (1980), cobalt material has been taken for thermoelastic material as

 $\begin{array}{ll} c_{11}=3.07\times 10^{11}Nm^{-2}, & c_{33}=3.581\times 10^{11}Nm^{-2}, & c_{13}=1.027\times 10^{10}Nm^{-2}, \\ c_{44}=1.510\times 10^{11}Nm^{-2}, & \beta_{1}=7.04\times 10^{6}Nm^{-2}deg^{-1}, & \beta_{3}=6.90\times 10^{6}Nm^{-2}deg^{-1}, \\ \rho=8.836\times 10^{3}Kgm^{-3}, & C_{E}=4.27\times 10^{2}jKg^{-1}deg^{-1}, & K_{1}=0.690\times 10^{2}Wm^{-1}Kdeg^{-1}, \\ K_{3}=0.690\times 10^{2}Wm^{-1}K^{-1}, & T_{0}=298\ \mathrm{K}, \mathrm{H}_{0}=1\mathrm{Jm^{-1}nb^{-1}}, \\ \varepsilon_{0}=8.838\times 10^{-12}\mathrm{Fm^{-1}}, & \mathrm{L}=1. \end{array}$ 

Using the above values, the graphical representations of displacement component u, normal displacement w, conductive temperature  $\varphi$ , stress components  $t_{11}$ ,  $t_{13}$  and  $t_{33}$  for transversely isotropic thermoelastic medium have been investigated and the effect of inclination with two temperature has been depicted.

- (i) The black solid line with square symbols corresponds to transversely isotropic magnetothermoelastic medium with  $\Omega = 0.5$ ,  $\omega = 0.25$
- (ii) The red solid line with circle symbols corresponds to transversely isotropic magnetothermoelastic medium with  $\Omega = 0.5, \omega = 0.50$
- (iii) The green solid line with circle symbols corresponds to transversely isotropic magnetothermoelastic medium with  $\Omega = 0.5$ ,  $\omega = 0.75$
- (iv) The blue solid line with diamond symbols corresponds to transversely isotropic magnetothermoelastic medium with  $\Omega = 0.5$ ,  $\omega = 1.0$



Fig. 2 Variations of displacement component u with Fig. 3 Variations of displacement component w with distance x



Fig. 4 Variations of conductive temperature  $\varphi$  with distance x



Fig. 6 Variations of stress component  $t_{13}$  with distance x



distance x



Fig. 5 Variations of stress component  $t_{11}$  with distance x



Fig. 7 Variations of stress component  $t_{33}$  with distance x

# Case 1: Concentrated force due to inclined load and with frequency, rotation and with two temperature

Figs. 1 to 6 shows the variations of the displacement components (*u* and *w*), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magneto-thermoelastic medium with concentrated force and with combined effects of rotation, time harmonic source for inclined load with two temperature in generalized thermoelasticity without energy dissipation respectively. The displacement components (*u* and *w*), Conductive temperature $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) illustrate the same pattern but having different magnitudes for different value of frequency. These components varies (increases or decreases) during the initial range of distance near the loading surface of the time harmonic source and follow small oscillatory pattern for rest of the range of distance. Low value of time harmonic source frequency shows more stress near loading surface.



Fig. 8 Variations of displacement component *u* with distance *x* 



Fig. 10 Variations of conductive temperature  $\varphi$  with distance *x* 



Fig. 9 Variations of displacement component *w* with distance *x* 



Fig. 11 Variations of stress component  $t_{11}$  with distance x



Fig. 12 Variations of stress component  $t_{13}$  with distance x



Fig. 13 Variations of stress component  $t_{33}$  with distance x

## Case 2: Linearly distributed force due to inclined load and with frequency, rotation and with two temperature

Figs. 8 to 13 shows the variations of the displacement components (*u* and *w*), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magneto-thermoelastic medium with linearly distributed force and with combined effects of rotation, time harmonic source for inclined load with two temperature in generalized thermoelasticity without energy dissipation respectively. As the value of frequency increase displacement component *u* increase, normal displacement *w* decrease, conductive temperature  $\varphi$  increasenear the loading surface rest remain same for transversely isotropic magneto-thermoelastic medium. However, as the value of frequency increase stress components  $t_{11}$  increase for  $\omega = 0.25 t_{13}$  and  $t_{33}$  decrease for transversely isotropic magneto-thermoelastic medium also decrease near the loading surface rest remain same. Low value of time harmonic source frequency shows more stress near loading surface.



Fig. 14 Variations of displacement component u with distance x



Fig. 15 Variations of displacement component *w* with distance *x* 

# Case 3: Uniformly distributed force due to inclined load and with frequency, rotation and with two temperature

Figs. 14 to 19 expresses the variations of the displacement components (u and w), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magneto-thermoelastic medium with uniformly distributed force and with combined effects of rotation, time harmonic source for inclined load with two temperature in generalized thermoelasticity without energy dissipation respectively. As the value of frequency increase displacement component u oscillates, normal displacement w decrease, conductive temperature  $\varphi$  increasenear the loading surface rest remain same for transversely isotropic magneto-thermoelastic medium. As the value of frequency increase, stress components  $t_{11}$  show a lot of variation increase for  $\omega = 1.0$  while  $t_{13}$  and  $t_{33}$  decrease for transversely isotropic magneto-thermoelastic medium also decrease near the loading surface rest remain same. Low value of time harmonic source frequency shows more stress near loading surface.



Fig.16 Variations of conductive temperature  $\varphi$  with distance *x* 



Fig. 18 Variations of stress component  $t_{13}$  with distance x



Fig. 17 Variations of stress component  $t_{11}$  with distance *x* 



Fig. 19 Variations of stress component  $t_{33}$  with distance *x* 

#### 9. Conclusions

From above investigation, it is observed that time harmonic source plays a key role for the oscillation of physical quantities both close to the point of use of source as well as just as far from the source. Moreover, the magnetic effect of two temperature, rotation as well as the angle of inclination of the applied load plays a key part in the deformation of all the physical quantities. The physical quantities amplitude differ (i.e., either rise or fall) with change in frequency of time harmonic source. In presence of two temperature and inclined load, the displacement components and stress components show an oscillatory nature with respect to x. The result gives an inspiration to study magneto-thermoelastic materials as an innovative domain of applicable thermoelastic solids. The shape of curves shows the impact of frequency  $\omega$  on the body and fulfils the purpose of the study. The outcomes of this research are extremely helpful in the 2-D problem with dynamic response of time harmonic sources in transversely isotropic magneto-thermoelastic medium with rotation and two temperature which beneficial to dissect the deformation field such as geothermal engineering; advanced aircraft structure design, thermal power plants, composite engineering, geology, high-energy particle accelerators and in real life as in geophysics, auditory range, geomagnetism etc. The proposed model in this research is relevant to different problems in thermoelasticity and thermodynamics.

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CC

### Nomenclature

| $\delta_{ij}$          | Kronecker delta,                      |
|------------------------|---------------------------------------|
| $C_{ijkl}$             | Elastic parameters,                   |
| $eta_{ij}$             | Thermal elastic coupling tensor,      |
| Т                      | Absolute temperature,                 |
| $T_0$                  | Reference temperature,                |
| $\varphi$              | conductive temperature,               |
| t <sub>ij</sub>        | Stress tensors,                       |
| <i>e</i> <sub>ij</sub> | Strain tensors,                       |
| u <sub>i</sub>         | Components of displacement,           |
| ρ                      | Medium density,                       |
| $C_E$                  | Specific heat,                        |
| a <sub>ij</sub>        | Two temperature parameters,           |
| $\alpha_{ij}$          | Linear thermal expansion coefficient, |
| K <sub>ij</sub>        | Materialistic constant,               |
| $K_{ij}^*$             | Thermal conductivity,                 |
| ω                      | Frequency,                            |
| $	au_0$                | Relaxation Time,                      |
| Ω                      | Angular Velocity of the Solid,        |
| $F_i$                  | Components of Lorentz force,          |
| $\vec{H}_0$            | Magnetic field intensity vector,      |
| Ĵ                      | Current Density Vector,               |
| ū                      | Displacement Vector,                  |
| $\mu_0$                | Magnetic permeability,                |
| $\varepsilon_0$        | Electric permeability,                |
| $\delta(x)$            | dirac delta function.                 |