Analytical analysis of the interfacial shear stress in RC beams strengthened with prestressed exponentially-varying properties plate

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Abstract. In this paper, a closed-form rigorous solution for interfacial shear stress in simply supported beams strengthened with bonded prestressed E-FGM plates and subjected to an arbitrarily positioned single point load, or two symmetric point loads is developed using linear elastic theory. This improved solution is intended for application to beams made of all kinds of materials bonded with a thin plate, while all existing solutions have been developed focusing on the strengthening of reinforced concrete beams, which allowed the omission of certain terms. The theoretical predictions are compared with other existing solutions. Finally, numerical results from the present analysis are presented to study the effects of various parameters of the beams on the distributions of the interfacial shear stresses. The results of this study indicated that the E-FGM plate strengthening systems are effective in enhancing flexural behavior of the strengthened RC beams.

Keywords: concrete beam; prestressed composite plate; interfacial stresses; strengthening; adherend shear deformation

1. Introduction

Analysis of interfacial shear stress in beams with bonded composite plates has been performed by several researchers Tounsi (2006), Benyoucef (2006), Roberts (1989), Smith and Teng (2001), Shen (2001), Yang (2007), Rabahi (2016) and Bouakaz (2014). The analyses provided closed-form formulas for the calculation of interfacial shear and peeling stress in beams with bonded non-prestressed plates or laminates. However, the problem of interfacial stress when prestressed laminates are used in strengthening and repair has treated only by Al-Emrani (2006), Rabahi (2018), Ashraful (2018), Attia (2018), Benchohra (2018), Chedad (2018), Belabed (2018), Hassaine Daouadji (2013b), Benferhat (2016), Awad Jodooe (2018), Guenaneche (2014), Krour (2014), Fu B. (2018), Touati (2015), Kongjian (2018), Adim (2018), Belifa (2017), Wensu (2018),

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Zidani (2015), Xu Yang (2018) and Jian yang (2010). In this investigation, only interfacial shear stress was studied and the analyzed beam was not loaded.

In this paper, a general new solution is developed to predict both shear interfacial stress in simply supported beams strengthened with bonded prestressed E-FGM plate. The considered beam is subjected to an arbitrarily positioned single point load, or two symmetric point loads. Hence, compared with the existing solutions such as presented by Hassaine Daouadji (2015, 2017), El Mahi (2014), Tounsi (2008), Adim (2016b), Hachemi (2017) and Mouffoki (2017) the present model is general in nature and it is applicable to more general loads cases. With the escalating use of this strengthening scheme, there is a great need for calculation models that can be used to predict the magnitude of maximum interfacial shear stress at the end of the prestressed E-FGM plate (Abualnour 2018, Benchohra 2018, Belabed 2018, Adim 2016a, Mokhtar 2018, Bouhadra 2018, Khelifa 2018, Abdelhak 2016, Fourn 2018, Attia 2018, Benferhat 2015, Hadji 2015, Elhaina 2017, Menasria 2017, Abdelaziz 2017). There is also some lack of knowledge today regarding how material and geometric properties of the strengthening system should be chosen in order to minimize the magnitude of these interfacial shear stresses and ensure sufficient strength of the strengthening system without need for expensive and complicated mechanical anchorage devices. Finally, the adopted improved model describes better the actual response of the EFGM-RC hybrid beams and permits the evaluation of the adhesive stresses, the knowledge of which is very important in the design of such structures. It is believed that the present results will be of interest to civil and structural engineers and researchers.

2. Analytical approach

The present analysis takes into consideration the transverse shear stress and strain in the beam and the plate but ignores the transverse normal stress in them. One of the analytical approaches proposed by Hassaine Daouadji *et al*. 2016 for concrete beam strengthened with a bonded FGM Plate (Fig. 1) was used in order to compare it with a finite element analysis. The analytical approach (Hassaine Daouadji *et al*. 2016) is based on the following assumptions:
- Elastic stress strain relationship for concrete, FGM and adhesive;
- There is a perfect bond between the composite plate and the beam;
- The adhesive is assumed to only play a role in transferring the stresses from the concrete to the composite plate reinforcement;
- The stresses in the adhesive layer do not change through the direction of the thickness.

Since the functionally graded materials is an orthotropic material. In analytical study (Hassaine Daouadji *et al*. 2016), the classical plate theory is used to determine the stress and strain behaviours of the externally bonded composite plate in order to investigate the whole mechanical performance of the composite-strengthened structure.

2.1 Basic equation of elasticity

Fig. 1 shows a schematic sketch of the steps involved in strengthening a beam with a bonded prestressed laminate. P_0 is the initial prestressing force in the laminate. P_1 is the residual prestressing force in the laminate upon removing the prestressing device. The loss of prestressing force in the laminates is thus

$$\Delta P_1 = P_0 - P_2 \tag{1}$$

And equilibrium requires that

$$P_1 = -P_2 \tag{2}$$

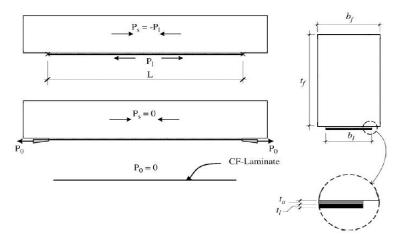Where P is the compression force in the beam due to prestressing.

Fig. 1 A reinforced concrete beam strengthened with bonded prestressed E-FGM plate
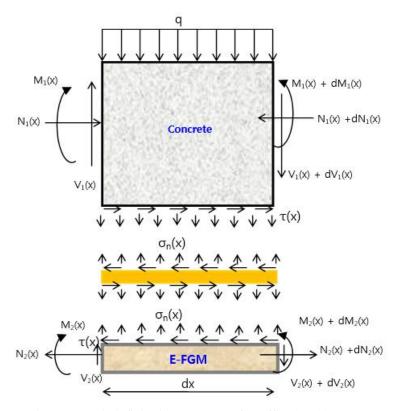

Fig. 2 Forces in infinitesimal element of a soffit-plated beam

A differential segment of a plated beam is shown in Fig. 2, where the interfacial shear and normal stresses are denoted by $\tau(x)$ and $\sigma(x)$, respectively. Fig. 2 also shows the positive sign convention for the bending moment, shear force, axial force and applied loading.

Shear stress in the adhesive layer is directly related to the difference in deformation between the laminate and lower flange of the steel beam

$$\tau(x) = \frac{G_a}{t_a} \big[u_2(x) - u_1(x) \big] \tag{3}$$

Where G_a, t_a, u_l and u_s denote the shear modulus, the thickness of the adhesive layer, the displacement of the RC beam at the bottom of lower flange and the displacement of the externally bonded prestressed FGM plate at the boundary of the bond, respectively. Eq. (3) can be expressed in terms of the mechanical strain of the RC beam, $\varepsilon_2(x)$ and the prestressed FGM plate $\varepsilon_1(x)$ after differentiating the equation with respect to x.

$$\frac{d\tau(x)}{dx} = \frac{G_a}{t_a} \big[\varepsilon_2(x) - \varepsilon_1(x) \big] \tag{4}$$

$$\frac{du_1(x)}{dx} = \varepsilon_1(x) \qquad \frac{du_2(x)}{dx} = \varepsilon_2(x) \tag{5}$$

Tensile strain at the bottom of the beam is induced by two basic stress components:
- The tensile stress induced by the bending moment $M_1(x)$ in the beam,
- The axial stress induced by the adhesive shear stress at the bond interface.

Therefore, Eq. (5) can be written as

$$\varepsilon_1(x) = \frac{du_1(x)}{dx} = \frac{y_1}{E_1 I_1} M_1(x) + \frac{N_1}{E_1 A_1} + \frac{t_1}{4 G_1} \frac{d\tau_a}{dx} \tag{6}$$

The change in axial strain in the laminate due to the deformability of the RC beam can be related to the loss in the prestressing force as follows

$$\varepsilon_2(x) = \frac{du_2(x)}{dx} = A_{11}' \frac{(N_2(x) + P_0)}{b_2} - D_{11}' \frac{y_2}{b_2} M_2(x) - \frac{5 t_2}{12 G_2} \frac{d\tau_a}{dx} \tag{7}$$

Where   and   are the horizontal displacements of the concrete beam and the FGM plate respectively. And   are respectively the bending moments applied to the concrete beam and the FGM plate; E_1 is Young's modulus of concrete; I_1 the moment of inertia and   are the axial forces applied to the concrete and the FGM plate respectively and   are the width and thickness of the reinforcing plate. The Young's modulus, which varies as a function of (z), is expressed as follows:

The material properties of FGM plates are assumed to vary continuously through the thickness. For the exponential distribution E-FGM, the Young's modulus is given as Hassaine daouadji (2013a)

$$E(z) = E_m . e^{\frac{1}{h} \ln\left(\frac{E_c}{E_m} \right)\left(z + \frac{h}{2} \right)} \tag{8}$$

Where E_m is the Young's modulus of the homogeneous plate; E_m denote Young's modulus of the bottom (as metal) and top E_c (as ceramic) surfaces of the FGM plate, respectively; E_m is Young's modulus of the homogeneous plate

The linear constitutive relations of an FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{9}$$

where $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yx}, \tau_{yx})$ and $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{yx})$ are the stress and strain components, respectively.

The computation of the elastic constants Q_{ij} in the plane stress reduced elastic constants, defined as

$$Q_{11} = Q_{22} = \frac{E(z)}{1-v^2}; \quad Q_{12} = \frac{v\,E(z)}{1-v^2} \qquad \text{and} \qquad Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+v)} \tag{10}$$

where A_{ij} and D_{ij} are the plate stiffness, defined by

$$A_{ij} = \int_{-h/2}^{h/2} Q_{ij}\,dz \qquad\qquad D_{ij} = \int_{-h/2}^{h/2} Q_{ij}\,z^2\,dz \tag{11}$$

where A'_{11} and D'_{11} are defined as

$$A'_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2} \qquad\qquad D'_{11} = \frac{D_{22}}{D_{11}D_{22} - D_{12}^2} \tag{12}$$

2.2 Shear stress distribution along the FFGM–concrete interface

The governing differential equation for the interfacial shear stress (Hassaine Daouadji *et al.* 2016) is expressed as

$$\frac{d^2\tau(x)}{dx^2} - K_1\left(A'_{11} + \frac{b_2}{E_1 A_1} + \frac{(y_1+y_2)(y_1+y_2+t_a)}{E_1 I_1 D'_{11} + b_2} b_2 D'_{11}\right)\tau(x) + K_1\left(\frac{(y_1+y_2)}{E_1 I_1 D'_{11} + b_2} D'_{11}\right)V_T(x) = 0 \tag{13}$$

Where

$$K_1 = \frac{1}{\left(\dfrac{t_a}{G_a} + \dfrac{t_1}{4G_1} + \dfrac{5t_2}{12G_2}\right)} \tag{14}$$

For simplicity, the general solutions presented below are limited to loading which is either concentrated or uniformly distributed over part or the whole span of the beam, or both. For such loading, $d^2V_T(x)/dx^2 = 0$ and the general solution to Eq. (13) is given by

$$\tau(x) = B_1\cosh(\lambda x) + B_2\sinh(\lambda x) + m_1 V_T(x) \tag{15}$$

Where

$$\lambda^2 = K_1 \left(A_{11}^{'} + \frac{b_2}{E_1 A_1} + \frac{(y_1 + y_2)(y_1 + y_2 + t_a)}{E_1 I_1 D_{11}^{'} + b_2} b_2 D_{11}^{'} \right) \qquad (16)$$

And

$$m_1 = \frac{K_1}{\lambda^2} \left(\frac{(y_1 + y_2)}{E_1 I_1 D_{11}^{'} + b_2} D_{11}^{'} \right) \qquad (17)$$

And B_1 and B_2 are constant coefficients determined from the boundary conditions. In the present study, a simply supported beam was investigated which is subjected single point load and for two point loads. The interfacial shear stress for this load case at any point is written as (Hassaine Daouadji *et al*. 2016)

$$\tau(x) = B_1 \cosh(\lambda x) + B_2 \sinh(\lambda x) + m_1 q \left(\frac{L}{2} - x - a \right) \qquad 0 \le x \le L_p \quad (18)$$

The constants of integration need to be determined by applying suitable boundary conditions (fig. 3).

2.3 Interfacial shear stress for a single point load

The general solution for the interfacial shear stress for this load case is

$$a < b: \qquad \tau(x) = \begin{cases} \dfrac{m_2}{\lambda} Q a \left(1 - \dfrac{b}{L} \right) e^{-\lambda x} \dfrac{q}{\lambda} + m_1 Q \left(1 - \dfrac{b}{L} \right) - m_1 Q \cosh(\lambda x) e^{-k} & 0 \le x \le (b\text{-}a) \\[4mm] \dfrac{m_2}{\lambda} Q a \left(1 - \dfrac{b}{L} \right) e^{-\lambda x} \dfrac{q}{\lambda} - m_1 \dfrac{Qb}{L} + m_1 Q \sinh(k) e^{-\lambda x} & (b\text{-}a) \le x \le Lp \end{cases} \qquad (19)$$

$$a > b: \qquad \tau(x) = \frac{m_2}{\lambda} Q b \left(1 - \frac{a}{L} \right) e^{-\lambda x} - m_1 \frac{Qb}{L} \qquad 0 \le \mathrm{x} \le \mathrm{Lp} \qquad (20)$$

where Q is the concentrated load and $k = \lambda(b{-}a)$. The expression of m_1, m_2 and, takes into consideration the shear deformation of adherends.
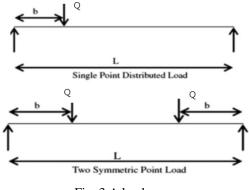

Fig. 3 A load cases

2.4 Interfacial shear stress for two point loads

The general solution for the interfacial shear stress for this load case is

$$a < b: \qquad \tau(x) = \begin{cases} \dfrac{m_2}{\lambda} Qae^{-\lambda x} + m_1 Q - m_1 Q \cosh(\lambda x)e^{-k} & 0 \le x \le (b\text{-}a) \\[2ex] \dfrac{m_2}{\lambda} Qae^{-\lambda x} + m_1 Q \sinh(k)e^{-\lambda x} & (b\text{-}a) \le x \le \dfrac{Lp}{2} \end{cases} \qquad (21)$$

$$a > b: \qquad \tau(x) = \frac{m_2}{\lambda} Qbe^{-\lambda x} \qquad\qquad 0 \le x \le Lp \qquad (22)$$

Table 1 Geometric and material properties

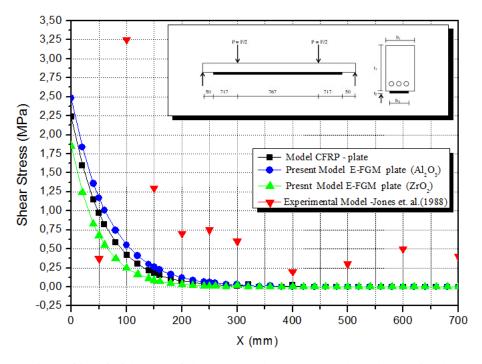| Component	Width (mm)	Depth (mm)	Young's modulus (MPa)	Poisson's ratio	Shear modulus (MPa)
RC beam	$b_1 = 200$	$t_1 = 300$	$E_1 = 30000$	0.18	-
Adhesive layer RC beam	$ba = 200$	$ta = 4$	$Ea = 3000$	0.35	-
CFRP plate (bonded RC beam)	$b_2 = 200$	$t_2 = 4$	$E_2 = 140000$	0.28	$G_{12} = 5000$
FGM (Al_2O_3) plate (bonded RC beam)	$b_2 = 200$	$t_2 = 4$	$E_c = 380000$ $E_m = 70000$	0.3	$G_{12} = 5000$
FGM (ZrO_2) plate (bonded RC beam)	$b_2 = 200$	$t_2 = 4$	$E_c = 200000$ $E_m = 70000$	0.3	$G_{12} = 5000$

Fig. 4 Comparison of interfacial shear of the GFRP and E-FGM plated RC beam with the experimental results
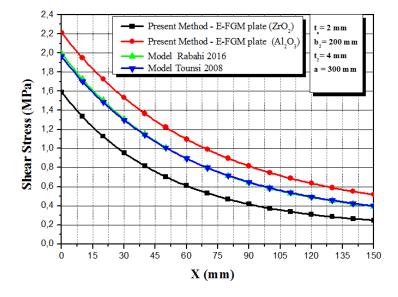
Fig. 5 Comparison of shear interfacial stress in RC beam bonded with prestressed E-FGM plate $P_0=0$

3. Numerical verification and discussions

3.1 Material used

The material used for the present studies is an RC beam bonded with a GFRP or EFGM plate. The beams are simply supported and subjected to a different type of loading (a single point distributed load and a Two symmetric point load). A summary of the geometric and material properties is given in Table 1.

3.2 Comparison with experimental results

To validate the present method, a rectangular section is used here. One of the tested beams bonded with steel plate by Jones (1988), beams F31, is analyzed here using the present improved solution. The beam is simply supported and subjected to four-point bending, each at the third point. The geometry and materials properties of the specimen are summarized in the table 1. The interfacial shear stress distributions in the beam bonded with a soffit steel plate under the applied load 180 kN, are compared between the experimental results and those obtained by the present method. As it can be seen from figure 4, the predicted theoretical results are in reasonable agreement with the experimental results.

3.3 Comparison with other solutions

To verify the analytical model, the present predictions are compared firstly with those with approximate solution available in the literature. These include Tounsi (2008) and Rabahi (2016); the beams are simply supported and subjected to a different type of loading (a single point distributed load and a Two symmetric point load) loads in the case of the absence of the prestressing force and in the second time the present method is compared with that developed by

Table 2 Comparison of interfacial shear stress

Comparison of interfacial shear stress (MPa) Reinforced Concrete Beam bonded with a thin plate						
Model	RC beam with CFRP plate		RC beam E-FGM plate (Al$_2$O$_3$)		RC beam E-FGM plate (ZrO$_2$)	
Model	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load
Present Model E-FGM	1.962	2.235	2.210	2.482	1.588	1.853
Model Rabahi 2016	1.998	2.283	2.250	2.535	1.618	1.893
Model Tounsi 2008	1.962	2.235	2.210	2.482	1.588	1.853

Comparison of interfacial shear stress (MPa) Reinforced Concrete Beam bonded with a prestressed " P = 20 kN " thin plate						
Model	RC beam with CFRP plate		RC beam E-FGM plate (Al$_2$O$_3$)		RC beam E-FGM plate (ZrO$_2$)	
Model	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load
Present Model E-FGM	0.631	0.905	1.082	1.354	-0.142	0.122
Model Rabahi 2016	0.639	0.924	1.098	1.383	-0.149	0.124
Model Tounsi 2008	0.631	0.905	1.082	1.354	-0.142	0.122

Comparison of interfacial shear stress (MPa) Reinforced Concrete Beam bonded with a prestressed " P = 40 kN " thin plate						
Model	RC beam with CFRP plate		RC beam E-FGM plate (Al$_2$O$_3$)		RC beam E-FGM plate (ZrO$_2$)	
Model	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load
Present Model E-FGM	-0.698	-0.424	-0.045	0.227	-1.874	-1.609
Model Rabahi 2016	-0.719	-0.433	-0.053	0.344	-1.918	-1.643
Model Tounsi 2008	-0.698	-0.424	-0.045	0.227	-1.874	-1.609

Comparison of interfacial shear stress (MPa) Reinforced Concrete Beam bonded with a prestressed " P = 50 kN " thin plate						
Model	RC beam with CFRP plate		RC beam E-FGM plate (Al$_2$O$_3$)		RC beam E-FGM plate (ZrO$_2$)	
Model	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load	Single Point Distributed Load	Two Symmetric Point Load
Present Model E-FGM	-1.363	-1.089	-0.609	-0.336	-2.740	-2.475
Model Rabahi 2016	-1.398	-1.113	-0.629	-0.344	-2.803	-2.528
Model Tounsi 2008	-1.363	-1.089	-0.609	-0.336	-2.740	-2.475

Rabahi (2016) in the case where only the prestressing force is applied. A comparison of the interfacial shear and normal stresses from the different existing closed – form solutions and the present solution is undertaken in this section. An undamaged beams bonded with CFRP, E-FGM (Al$_2$O$_3$) and E-FGM (ZrO$_2$) plate soffit plate is considered. The results of the peak interfacial shear stress are given in table 2 for the beams strengthened by bonding CFRP, E-FGM (Al$_2$O$_3$) and E-FGM (ZrO$_2$) plate. As it can be seen from the results, the peak interfacial stresses assessed by the present theory are smaller compared to those given by Tounsi (2008) and Rabahi (2016) solutions. This implies that adherend shear deformation is an important factor influencing the adhesive interfacial stresses distribution.
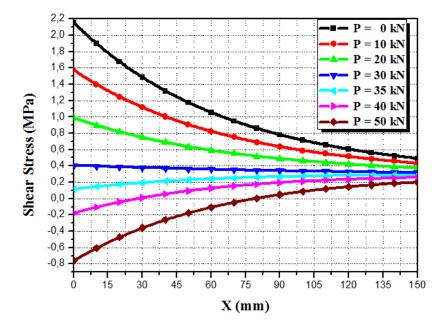
Fig. 6 Adhesive shear stress along the bond line for prestressed E-FGM (Al$_2$O$_3$) strengthened

Fig. 5 plots the interfacial shear stress near the plate end for the example RC beam bonded with an E-FGM plate for the uniformly distributed load case. Overall, the predictions of the different solutions agree closely with each other. Hence, it is apparent that the adherend shear deformation reduces the interfacial stresses concentration and thus renders the adhesive shear distribution more uniform. The interfacial normal stress is seen to change sign at a short distance away from the plate end.

3.4 Adhesive stresses without prestressing force (P0 = 0)

A comparison of the edge interfacial shear stress from the different closed-form solutions reviewed earlier is undertaken in this section figure 5. Two example problems are considered. In the problem, the beam is simply supported and subjected to a different type of loading (a single point distributed load and a Two symmetric point load). The results of the peak interfacial shear stress are given in Table 2. From the presented results, it can be seen that the present solution agrees closely with the other methods.

3.5 Effect of the prestressing force (P0) on adhesive stress

In this section, analytical results of the present solution are presented to study the effect of the prestressing force P_0 on the distribution of interfacial stress in a steel beam strengthened with bonded prestressed FRP plate. Seven value of P_0 are considered in this study ($P_0 = 0$; 10, 20, 30, 35, 40 and 50 kN). Fig. 6 plot the interfacial shear stress for the steel beam strengthened with bonded prestressed FRP plate for the mid-point load case, from these results, one can observe: Maximum stress occurs at the ends of adhesively bonded plates and the shear, or peeling, stress disappears at
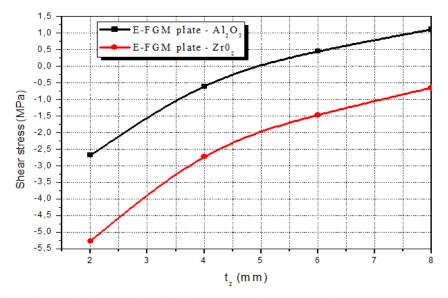
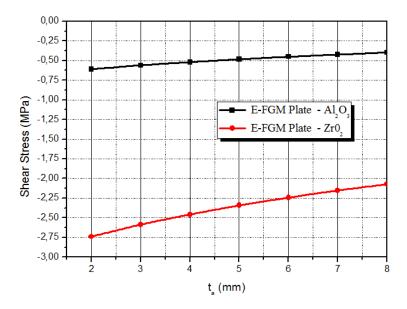Fig. 7 Effect of plate thickness on interfacial shear stress in prestressed E-FGM strengthened RC beam

Fig. 8 Effect of adhesive layer thickness on interfacial shear stress in prestressed E-FGM strengthened RC beam

around 20 mm from the end of the plates. It is seen that increasing the value of prestressing force P_0 leads to high stress concentrations.

3.6 Effect of the laminate thickness

The thickness of the E-FGM plate is an important design variable in practice. Fig. 7 shows the
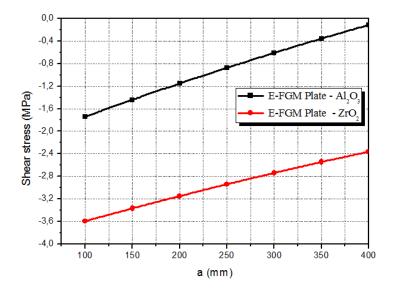
Fig. 9 Effect of length of unstrengthened region on interfacial stress in prestressed E-FGM strengthened RC beam

effect of the thickness of the FRP plate on the interfacial stresses. Here, four values of the thickness, 2, 4, 6 and 8 mm, are considered. It is shown that the level and concentration of interfacial stress are influenced considerably by the thickness of the E-FGM plate. The interfacial stresses increase as the thickness of FRP plate increases. Generally, the thickness of E-FGM plates used in practical engineering is small. Therefore, the fact of the smaller interfacial stress level and concentration should be one of the advantages of retrofitting by E-FGM plate compared with a steel plate.

3.7 Effect of adhesive layer thickness

Fig. 8 show the effects of the thickness of the adhesive layer on the interfacial shear stress. Increasing the thickness of the adhesive layer leads to a significant reduction in the peak interfacial shear stress. Thus using thick adhesive layer, especially in the vicinity of the edge, is recommended. In addition, it can be shown that these stresses decrease during time, until they become almost constant after a very long time.

3.8 Effect of length of unstrengthened region "a"

The influence of the length of the ordinary-beam region (the region between the support and the end of the composite strip on the edge stresses) appears in fig. 9. It is seen that, as the plate terminates further away from the supports, the interfacial shear stress increases significantly. This result reveals that in any case of strengthening, including cases where retrofitting is required in a limited zone of maximum bending moments at midspan, it is recommended to extend the strengthening strip as possible to the lines. This result reveals that in any case of strengthening, including cases where retrofitting is required in a limited zone of maximum bending moments at midspan, it is recommended to extend the strengthening strip as possible to the lines.

5. Conclusions

This paper has presented an interfacial shear stress analysis for RC beams strengthened with bonded prestressed exponentially-varying properties E-FGM plate under to a subjected to an arbitrarily positioned single point load, or two symmetric point loads. In this analysis, the partial use of the plane section assumption in the previous work is avoided. The obtained solution could serve as a basis for establishing simplified E-FGM theories or as a benchmark result to assess other approximate methodologies. Compared with the existing solutions, the present solution is applicable to more general loads cases. The results show that there exists a high concentration of shear and peeling stress at the ends of the prestressed exponentially-varying properties plate. This research is helpful for the understanding on mechanical behavior of the interface and design of the EFGM–RC hybrid structures. The new solution is general in nature and may be applicable to all kinds of materials.

References

Abdelaziz, H.H., Meziane, M.A.A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct.*, **25**(6), 693-704.

Abdelhak, Z., Hadji, L., Hassaine Daouadji, T. and Adda, B. (2016), "Thermal buckling response of functionally graded sandwich plates with clamped boundary conditions", *Smart Struct. Syst.*, **18**(2), 267-291.

Abualnour, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

Adim, B., Hassaine Daouadji, T., Abbas, B. and Rabahi, A. (2016a), "Buckling and free vibration analysis of laminated composite plates using an efficient and simple higher order shear deformation theory", *J. Mech. Industry*, **17**(5), 512.

Al-Emrani, M. and Kliger, R. (2006), "Analysis of interfacial shear stresses in beams strengthened with bonded prestressed laminates", *J. Compos. Part B*, **37**(4-5), 265-272.

Ashraful, A. and Al Riyami, K. (2018), "Shear strengthening of reinforced concrete beam using natural fibre reinforced polymer laminates", *Construct. Build. Mater.*, **162**, 683-696.

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2018), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations", *Struct. Eng. Mech.*, **65**(4), 453-464.

Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct.*, **14**(2), 103-115.

Belkacem, A. and Hassaine Daouadji, T. (2016b), "Effects of thickness stretching in FGM plates using a quasi-3D higher order shear deformation theory", *Adv. Mater. Res.*, **5**(4), 223-244.

Belkacem, A., Hassaine Daouadji, T., Abderrezak, R., Amine, B.M., Mohamed, Z. and Boussad, A. (2018),

"Mechanical buckling analysis of hybrid laminated composite plates under different boundary conditions", *Struct. Eng. Mech.*, **66**(6), 761-769.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct.*, **25**(3), 257-270.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Bedia, E.A.A. and Hassan, S. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech.*, **65**(1), 19-31.

Benferhat, R., Hassaine Daouadji, T. and Mansour, M.S. (2015), "A higher order shear deformation model for bending analysis of functionally graded plates," *Trans. Indian Inst. Met.*, **68**(1), 7-16.

Benferhat, R., Hassaine Daouadji, T. and Mansour, M.S. (2016), "Free vibration analysis of FG plates resting on the elastic foundation and based on the neutral surface concept using higher order shear deformation theory", *Comptes Rendus Mecanique*, **344**(9), 631-641.

Benyoucef, S., Tounsi, A., Meftah, S.A. and Adda-Bedia, E.A. (2006), "Approximate analysis of the interfacial stress concentrations in FRP-RC hybrid beams", *Compos. Interfaces*, **13**(7), 561-571.

Bouakaz, K., Hassaine Daouadji, T., Meftah, S.A., Ameur, M., Tounsi, A. and Bedia, E.A. (2014), "A Numerical analysis of steel beams strengthened with composite materials", *Mech. Compos. Mater.*, **50**(4), 685-696.

Bouhadra, A., Tounsi, A., Bousahla, A.A., Benyoucef, S. and Mahmoud, S.R. (2018), "Improved HSDT accounting for effect of thickness stretching inadvanced composite plates", *Struct. Eng. Mech.*, **66**(1), 61-73.

Chaded, A., Hassaine Daouadji, T., Rabahi, A., Adim, B., Abbes, B., Benferhat, R. and Fazilay, A. (2018), "A high-order closed-form solution for interfacial stresses in externally sandwich FGM plated RC beams", *Adv. Mater. Res.*, **6**(4), 317-328.

Chen, W., Pham, T.M., Sichembe, H., Chen, L. and Hao, H., (2018), "Experimental study of flexural behaviour of RC beams strengthened by longitudinal and U-shaped basalt FRP sheet", *Compos. Part B Eng.*, **134**, 114-126

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech.*, **63**(5), 585-595.

Fourn, H., Atmane, H.A., Bourada, M., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates", *Steel Compos. Struct.*, **27**(1), 109-122.

Fu, B., Teng, J.G., Chen, G.M., Chen, J.F. and Guo, Y.C. (2018), "Effect of load distribution on IC debonding in FRP-strengthened RC beams: Full-scale experiments", *Compos. Struct.*, **188**, 483-496.

Guenaneche, B., Tounsi, A., Bedia, E.A.A. (2014), "Effect of shear deformation on interfacial stress analysis in plated beams under arbitrary loading", *Adhesion Adhesives*, **48**, 1-13.

Hachemi, H., Kaci, A., Houari, M.S.A., Bourada, M., Tounsi, A. and Mahmoud, S.R. (2017), "A new simple three-unknown shear deformation theory for bending analysis of FG plates resting on elastic foundations", *Steel Compos. Struct.*, **25**(6), 717-726.

Hadji, L., Hassaine Daouadji, T. and Bedia, E.A.A. (2015), "A refined exponential shear deformation theory for free vibration of FGM beam with porosities", *Geomech. Eng.*, **9**(3), 361-372.

Hassaine Daouadji, T. (2013b), "Analytical analysis of the interfacial stress in damaged reinforced concrete beams strengthened by bonded composite plates", *Strength Mater.*, **45**(5), 587-597.

Hassaine Daouadji, T. and Belkacem, A. (2017), "Mechanical behaviour of FGM sandwich plates using a quasi-3D higher order shear and normal deformation theory", *Struct. Eng. Mech.*, **61**(1), 49-63.

Hassaine Daouadji, T. and Hadji, L. (2015), "Analytical solution of nonlinear cylindrical bending for functionally graded plates", *Geomech. Eng.*, **9**(5), 631-644.

Hassaine Daouadji, T., Rabahi, A., Abbes, B. and Adim, B. (2016), "Theoretical and finite element studies of interfacial stresses in reinforced concrete beams strengthened by externally FRP laminates plate", *Adhes. Sci. Technol.*, **30**(12), 1253-1280.

Hassaine Daouadji, T., Tounsi, A. and Bedia, E.A.A. (2013a), "Analytical solution for bending analysis of

functionally graded plates", *Scientia Iranica, Trans. B Mech. Eng.*, **20**(3), 516-523.

Jadooe, A., Al-Mahaidi, R. and Abdouka, K. (2018) "Performance of heat-damaged partially-insulated RC beams strengthened with NSM CFRP strips and epoxy adhesive", *Construct. Build. Mater.*, **159**, 617-634.

Khelifa, Z., Hadji, L., Hassaine Daouadji, T. and Bourada, M. (2018), "Buckling response with stretching effect of carbon nanotube-reinforced composite beams resting on elastic foundation", *Struct. Eng. Mech.*, **67**(2), 125-130.

Krour, B., Bernard, F. and Tounsi, A. (2014) "Fibers orientation optimization for concrete beam strengthened with a CFRP bonded plate: A coupled analytical-numerical investigation", *Eng. Struct.*, **56**, 218-227.

Mahi, B.E., Benrahou, K.H., Belakhdar, K. Tounsi, A. and Bedia, E.A.A. (2014), "Effect of the tapered end of a FRP plate on the interfacial stresses in a strengthened beam used in civil engineering applications", *Mech. Compos. Mater.*, **50**(4), 465-474.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct.*, **25**(2), 157-175.

Mokhtar, Y., Heireche, H., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst.*, **21**(4), 397-405.

Mori, T. and Tanaka, K. (1973), "Average stress in matrix and average elastic energy of materials with misfitting inclusions", *Acta Metallurgica*, **21**(5), 571-574.

Mouffoki, A., Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383.

Rabahi, A., Hassaine Daouadji, T., Abbes, B. and Adim, B. (2016), "Analytical and numerical solution of the interfacial stress in reinforced-concrete beams reinforced with bonded prestressed composite plate", *J. Reinf. Plast. Comp.*, **35**(3), 258-272.

Rabahi, A., Hassaine Daouadji, T., Benferhat, R. and Adim, B. (2018), "Elastic analysis of interfacial stress concentrations in CFRP-RC hybrid beams: Effect of creep and shrinkage", *Adv. Mater. Res.*, **6**(3), 257-278.

Roberts, T.M. (1989), "Approximate analysis of shear and normal stress concentrations in the adhesive layer of plated RC beams", *Struct. Eng.*, **67**(12), 229-233.

Shen, H.S., Teng, J.G. and Yang, J. (2001), "Interfacial stresses in beams and slabs bonded with thin plate", *J. Eng. Mech.*, **127**(4), 399-406.

Shen, K., Wan, S., Mo, Y.L. and Jiang, Z. (2018), "Theoretical analysis on full torsional behavior of RC beams strengthened with FRP materials", *Compos. Struct.*, **183**, 347-357.

Smith, S.T. and Teng, J.G. (2001), "Interfacial stresses in plated beams", *Eng. Struct.*, **23**(7), 857-871.

Touati, M., Tounsi, A. and Benguediab, M. (2015), "Effect of shear deformation on adhesive stresses in plated concrete beams: Analytical solutions", *Comput. Concrete*, **15**(3), 141-166.

Tounsi, A. (2006), "Improved theoretical solution for interfacial stresses in concrete beams strengthened with FRP plate", *J. Solids Struct.*, **43**(14-15), 4154-4174.

Tounsi, A., Hassaine Daouadji, T., Benyoucef, S. and Bedia, E.A.A. (2008), "Interfacial stresses in FRP-plated RC beams: Effect of adherend shear deformations", *J. Adhesion Adhesives*, **29**(4), 343-351.

Wattanasakulpong, N. and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol.*, **32**(1), 111-120.

Yang, J. and Wu, Y.F. (2007) "Interfacial stresses of FRP strengthened concrete beams: Effect of shear deformation", *Compos. Struct.* **80**(3), 343-351.

Yang, J. and Ye, J. (2010), "An improved closed-form solution to interfacial stresses in plated beams using a two-stage approach", *J. Mech. Sci.*, **52**(1), 13-30.

Yang, J., Ye, J. and Niu, Z. (2007), "Interfacial shear stress in FRP-plated RC beams under symmetric loads", *Cement Concrete Compos.*, **29**(5), 421-432.

Yang, X., Gao, W.Y., Dai, J.H., Lu, Z.D. and Yu, K.Q. (2018), "Flexural strengthening of RC beams with CFRP grid-reinforced ECC matrix", *Compos. Struct.*, **189**, 9-26.

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Zidani, M.B., Belakhdar, K., Tounsi, A. and Bedia, E.A.A. (2015), "Finite element analysis of initially damaged beams repaired with FRP plates", *Compos. Struct.*, **134**, 429-439.

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