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Effect of magnetic field and gravity on thermoelastic fiber-reinforced with memory-dependent derivative

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Abstract. The purpose of this paper is to study the effects of magnetic field and gravitational field on fiber-reinforced thermoelastic medium with memory-dependent derivative. Three-phase-lag model of thermoelasticity (3PHL) is used to study the plane waves in a fiber-reinforced magneto-thermoelastic material with memory-dependent derivative. A gravitating magneto-thermoelastic two-dimensional substrate is influenced by both thermal shock and mechanical loads at the free surface. Analytical expressions of the considered variables are obtained by using Laplace-Fourier transforms technique with the eigenvalue approach technique. A numerical example is considered to illustrate graphically the effects of the magnetic field, gravitational field and two types of mechanical loads (continuous load and impact load).

Keywords: fiber-reinforced thermoelastic; gravity; magnetic field; memory-dependent derivative; three-phase-lag model

1. Introduction

The theory of generalized thermoelasticity has drawn the attention of researchers due to its applications in various diverse fields such as engineering, nuclear reactor design, high energy particle accelerators, etc. Actually, as is well known, the term ‘generalized’ usually refers to thermo-dynamics theories based on the hyperbolic-type (wave-type) heat equations, so that a finite speed of propagation of thermal signal is admitted. Lord and Shulman (1967) introduced a theory of generalized thermoelasticity with one relaxation time for an isotropic body. In this theory, a modified law of heat conduction, including both the heat flux and its time derivatives replaces the conventional Fourier’s law. The heat equation is associated with this theory. Hetnarski and Ignaczak (1999) introduced a theory which is known as low-temperature thermoelasticity and called (H-I) theory. This model is characterized by a system of nonlinear field equations. Green and Naghdi (1993) establish a theory of thermoelasticity that permits the propagation of thermal

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waves at a finite speed, where its evolution equations are hyperbolic. Roy Choudhuri (2007) established a mathematical model that includes the three phase-lag (3PHL) model in the heat flux vector, the temperature gradient and in the thermal displacement gradient. The three-phase-lag model is very much effective in the problems of nuclear boiling, exothermic catalytic reactions, phonon electron interactions, phonon-scattering, etc. Kumar and Chawla (2011) studied the plane wave propagation in the anisotropic medium in the context of the theory of the three-phase-lag and two-phase-lag models. Recently, Wang and Li (2011) introduced a concept of ‘‘memory dependent derivative’’, which is simply defined in an integral form of a common derivative with a kernel function on a slipping interval. In case the time delay tends to zero, it tends to the common derivative. Higher order derivatives also accord with the first order one. Many researchers discussed the problem of thermoelasticity with memory-dependent derivative, see (Othman and Song 2009, Yu *et al*. 2014, Purkait *et al*. 2017, El-Karamany and Ezzat 2016, Hendy *et al*. 2020, Othman and Lotfy 2009). Many researchers such as Marin (1997), Othman (2005), Marin (2010), Hu *et al*. (2013), Gao *et al*. (2016), Marin *et al*. (2014), Abbas and Marin (2020), Al-Basyouni *et al*. (2020), Lata and Kaur (2018), Yang *et al*. (2019), Yang (2019a, b), Yang (2020a, b), Bhatti *et al*. (2020), Xue *et al*. (2018, 2020), Fahmy (2020), Cheng *et al*. (2021), Fahmy (2021a, b, c, d, e, f), Yang (2021), Fing *et al*. (2021), Yang *et al*. (2021a, b), Yang and Liu (2021), Fahmy (2022) further studied the thermoelastic materials and the linear theory associated with them.

Some engineering materials are unsuited for the second sound propagation experiment due to their relatively high thermal damping rate. To study the propagation of thermal waves with finite speeds, scholars have found an ideal material, fiber-reinforced material. There is a wide application of fiber-reinforced composites in a variety of structures because of their low weight and high strength. The fiber-reinforced materials are presented and developed by many investigators see (Belfield *et al*. 1983, Verma and Rana 1983, Weitsman 1972, Anya and Khan 2019, Knopoff 1955, Othman and Said 2014, Lata *et al*. 2016, Kumar *et al*. 2016a, b, Kumar *et al*. 2017, Lata and Kaur 2018, Kaur *et al*. 2021, Lata and Singh 2021).

The effect of gravity is generally neglected in the classical theory of elasticity. Bromwich (1898) first considered the effect of gravity on wave propagation in an elastic solid medium. De and Sengupta (1974) investigated the problem of elastic waves and vibrations under the influence of gravity field. Othman *et al*. (2019) have studied the effect of hall current and gravity on magneto-micropolar thermoelastic medium with micro temperatures. Othman *et al*. (2019) considered a novel model of plane waves of the two-temperature fiber-reinforced thermoelastic medium under the effect of gravity with the three-phase-lag model.

In this work, the problem is a fiber-reinforced thermo-elastic medium under the influence of the magnetic field, gravity field and variable thermal conductivity. The memory-dependent derivative used instead of fractional calculus, in the four theories, the three-phase-lag model, Green-Naghdi theory without energy dissipation (G-N II), Green-Naghdi theory with energy dissipation (G-N III) and Lord-Shulman theory with one relaxation time. The matrix differential equation is formed by using Laplace and Fourier transforms into the considered equations which are solved by the eigenvalue approach, see Honig and Hirdes (1984). The effect of the magnetic field, gravity, time delay and kernel function with the considered parameters is presented graphically.

2. Formulation of the problem

A problem of a micropolar thermoelastic medium in xz −plane with micro-rotation vector $\Phi =$

$(0, \Phi_2, 0)$. The adjacent free space is assumed to be permeated by a uniform magnetic field $\mathrm{H} = (0, H_0, 0)$ which is acting parallel to the y-axis. The field equations and constitutive relations can be written as Said and Othman (2016), Said *et al.* (2020), Cheng *et al.* (2021), in the context of generalized thermoelasticity as follows.

2.1 The constitutive equations

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j - \gamma\theta\delta_{ij}. \tag{1}$$

2.2 The equations of motion

$$\rho u_{i,tt} = \sigma_{ij,j} + \mu_0(J \times H)_i + F_i, \tag{2}$$

$$\mu_0(J \times H)_1 = -\mu_0 H_0 \frac{\partial h}{\partial x} - \varepsilon_0\mu_0^2 H_0^2 \frac{\partial^2 u}{\partial t^2}, \quad \mu_0(J \times H)_2 = 0,$$
$$\mu_0(J \times H)_3 = -\mu_0 H_0 \frac{\partial h}{\partial z} - \varepsilon_0\mu_0^2 H_0^2 \frac{\partial^2 w}{\partial t^2}, \quad F_1 = \rho g \frac{\partial w}{\partial x}, F_3 = -\rho g \frac{\partial u}{\partial x}. \tag{3}$$

2.3 The heat conduction equation (Roy Choudhuri 2007, El-Karamany and Ezzat 2016)

$$K^*(1 + \tau_v D_{w_3})\nabla^2\theta + K(1 + \tau_T D_{w_2})\nabla^2\theta_{,t} = (1 + \tau_q D_{w_1} + \frac{1}{2}\tau_q^2 D^2{}_{w_1})[\rho C_E(n_0\theta_{,tt} + n_1\theta_{,t}) + \gamma T_0(n_0 e_{,tt} + n_1 e_{,t})]. \tag{4}$$

Here $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the components of strain, $a = (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fiber-direction as $a = (1,0,0)$.

D_{w_i} is the memory-dependent derivative operator is defined as El-Karamany and Ezzat (2016)

$$D_{w_i} f(t) = \frac{1}{w_i}\int_{t-w_i}^{t} L(t - \beta) f'(\beta) d\,\beta. \tag{5}$$

From Eq. (5), it can be visualized that for any real number β, the kernel $L(t-\beta)$ is a fixed function. But from the viewpoint of applications, different processes need different kernels to reflect their memory effects, so the kernel should be chosen freely. In fact, the memory effect of a real process basically occurs on a segment of time, i.e., on the delayed interval ($[t-\omega, t]$, $\omega > 0$) indicates the time delay). Enlightened by these, the novel concept of derivative was initiated as the "memory-dependent derivative" to reflect the memory effect in a distinct manner. The parameter w_i is the time-delay and $L(t-\beta)$ can be chosen freely, see Caputo and Mainardi (1971a, b) for more explanations.

$$L(t - \xi) = 1 - \frac{2b}{\varpi}(t - \beta) + a^2\frac{(t-\beta)^2}{\varpi^2}. \tag{6}$$

In the present paper, we take $L(t-\beta)=q+n(t-\beta)$, where a,b are constant. Introducing Eqs. (1) and (3) into Eqs. (2), we get Othman and Said (2019)

$$\rho\frac{\partial^2 u}{\partial t^2}=A_1\frac{\partial^2 u}{\partial x^2}+A_4\frac{\partial^2 w}{\partial x\,\partial z}+\mu_L\frac{\partial^2 u}{\partial z^2}-\gamma\frac{\partial\theta}{\partial x}-\mu_0 H_0\frac{\partial h}{\partial x}-\varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}+\rho g\frac{\partial w}{\partial x}, \tag{7}$$

$$\rho\frac{\partial^2 w}{\partial t^2}=\mu_L\frac{\partial^2 w}{\partial x^2}+A_4\frac{\partial^2 u}{\partial x\,\partial z}+A_3\frac{\partial^2 w}{\partial z^2}-\gamma\frac{\partial\theta}{\partial z}-\mu_0 H_0\frac{\partial h}{\partial z}-\varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 w}{\partial t^2}-\rho g\frac{\partial u}{\partial x}, \tag{8}$$

where, $A_1=\lambda+2\alpha+4\mu_T-2\mu_T+\beta, A_2=\lambda+\alpha,\ A_3=\lambda+2\mu_T,\ A_4=\lambda+\alpha+\mu_L,$

To facilitate the solution, the following dimensionless quantities are introduced

$$(x',z',u',w')=c_0\eta(x,z,u,w),\ \theta'=\frac{\gamma\theta}{A_3},$$
$$(t',\tau_q',\tau_\nu',\tau_T')=c_0^2\eta(t,\tau_q,\tau_\nu,\tau_T),\ h'=\frac{h}{H_0},\ \sigma_{ij}'=\frac{\sigma_{ij}}{\mu_T},\ g'=\frac{g}{\eta c_0^3},\quad \eta=\frac{\rho C_E}{K^*},\quad c_0^2=\frac{A_3}{\rho}. \tag{9}$$

We also consider the thermal conductivity defined as follows
Caputo and Mainardi (1971a)

$$K=K(\theta)=K_0(1+K_1\theta). \tag{10}$$

Where K_0 is a constant which is equal to the thermal conductivity of the material when it does not depend on the thermodynamic temperature (θ) and K_1 is a non-positive small parameter.

Using Kirchhoff transformation (Bonani and Ghione1995)

$$\psi=\frac{1}{K_0}\int_0^{\theta}K(\Phi')d\Phi'. \tag{11}$$

For linearity, then the above equation will be reduced to (see Said *et al*. 2020)

$$\frac{\partial\theta}{\partial x_i}=\frac{\partial\psi}{\partial x_i},\frac{\partial\theta}{\partial t}=\frac{\partial\psi}{\partial t}. \tag{12}$$

Eqs. (7), (8) and (4) with aid of Eqs. (9) and (12) recast into the following form

$$A_5\frac{\partial^2 u}{\partial t^2}=A_6\frac{\partial^2 u}{\partial x^2}+A_7\frac{\partial^2 w}{\partial x\,\partial z}+A_8\frac{\partial^2 u}{\partial z^2}-\frac{\partial\psi}{\partial x}+g\frac{\partial w}{\partial x}, \tag{13}$$

$$A_5\frac{\partial^2 w}{\partial t^2}=A_9\frac{\partial^2 w}{\partial z^2}+A_7\frac{\partial^2 u}{\partial x\,\partial z}+A_8\frac{\partial^2 w}{\partial x^2}-\frac{\partial\psi}{\partial z}-g\frac{\partial u}{\partial x}, \tag{14}$$

$$\begin{aligned}(1+\tau_\nu\frac{\partial}{\partial t})\psi_{,ii}+A_{10}(1+\tau_T\frac{\partial}{\partial t})\psi_{,iit}&=(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})(A_{11}\psi_{,tt}\\&+A_{13}\psi_{,t}+A_{12}e_{,tt}+A_{14}e_{,t}),\end{aligned} \tag{15}$$

where $A_5=1+\frac{\varepsilon_0\mu_0^2H_0^2}{\rho},\ A_6=\frac{A_1+\mu_0H_0^3}{\rho c_0^2},\ \ A_7=\frac{A_4+\mu_0H_0^3}{\rho c_0^2},\ \ A_8=\frac{\mu_L}{\rho c_0^2},\ \ A_9=\frac{A_3+\mu_0H_0^3}{\rho c_0^2},\ \ A_{10}=\frac{Kc_0^2\eta}{K^*},$
$A_{11}=\frac{\rho C_E n_0 c_0^2}{K^*},\quad A_{12}=\frac{\gamma^2T_0n_0c_0^2}{K^*A_3}, A_{13}=\frac{\rho C_E n_1}{\eta K^*},\quad A_{14}=\frac{\gamma^2T_0n_1}{\eta K^*A_3}.$.

3. The analytical solution of the problem

Applying the Laplace and Fourier transform defined by

$$\bar{f}(x,z,p) = \int_0^{\infty} f(x,z,t)e^{-pt}\,dt, \tag{16}$$

$$f^{*}(\zeta,z,p) = \int_{-\infty}^{\infty} \bar{f}(x,z,p)e^{i\zeta x}\,dx. \tag{17}$$

Introducing Eqs. (16) and (17) in (13)- (15), thus we get

$$D^2u^* = M_{11}u^* + M_{12}w^* + M_{13}\psi^* + M_{15}Dw^*, \tag{18}$$

$$D^2w^* = M_{21}u^* + M_{22}w^* + M_{24}Du^* + M_{26}D\psi^*, \tag{19}$$

$$D^2\psi^* = M_{31}u^* + M_{33}\psi^* + M_{35}Dw^*, \tag{20}$$

where, $M_{11} = \frac{A_6\zeta^2+A_5p^2}{A_8}$, $M_{12} = \frac{-ig\zeta}{A_8}$, $M_{13} = \frac{i\zeta}{A_8}$, $M_{15} = \frac{-i\zeta A_7}{A_8}$, $M_{21} = \frac{ig\zeta}{A_9}$, $M_{22} = \frac{A_6\zeta^2+A_5p^2}{A_9}$, $M_{24} = \frac{-iA_7\zeta}{A_9}$, $M_{26} = \frac{1}{A_9}$, $M_{31} = \frac{-iA_7\zeta}{A_{15}}$, $M_{33} = \frac{A_{16}+A_{15}\zeta^2}{A_{15}}$, $M_{35} = \frac{A_{17}}{A_{15}}$, $D = \frac{d}{dz}$. $A_{15} = 1 + G_3 + (1+G_4)pA_{10}$, $A_{16} = (A_{11}p^2 + A_{13}p)(1+G_1+G_2)$, $A_{17} = (1+G_1+G_2)(A_{12}p^2 + A_{14}p)$, $G_1 = \frac{\tau_q}{w_1}[\frac{qp+n}{p}(1-e^{-pw_1}) - nw_1e^{-pw_1}]$, $G_2 = \frac{p\tau_q^2}{2w_1}[\frac{qp+n}{p}(1-e^{-pw_1}) - nw_1e^{-pw_1}]$, $G_3 = \frac{\tau_T}{w_2}[\frac{qp+n}{p}(1-e^{-pw_2}) - nw_2e^{-pw_2}]$, $G_4 = \frac{\tau_v}{w_3}[\frac{qp+n}{p}(1-e^{-pw_3}) - nw_3e^{-pw_3}]$, w_1, w_2, w_3 are the time delay for three-phase-heat equation.

The system of Eqs. (18)-(20) can be written in a vector-matrix differential equation in the following way (Abbas and Kumar 2014, Das and Lahiri 2009)

$$DV(z) = A(\zeta,p)V(z). \tag{21}$$

Where $V(z) = [u^*, w^*, \psi^*, Du^*, Dw^*, D\psi^*]^T$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ M_{11} & M_{12} & M_{13} & 0 & M_{15} & 0 \\ M_{21} & M_{22} & 0 & M_{24} & 0 & M_{26} \\ M_{31} & 0 & M_{33} & 0 & M_{35} & 0 \end{bmatrix}. \tag{22}$$

Using the eigenvalue approach (Abbas and Kumar 2014), we now proceed to solve the vector-matrix differential Eq. (19). The characteristic equation of the matrix A is

$$\Gamma^6 - E_1\,\Gamma^4 + E_2\,\Gamma^2 - E_3 = 0. \tag{23}$$

In a similar manner

$$E_1 = M_{35}M_{26} + M_{15}M_{24} + M_{33} + M_{22} + M_{11},$$
$$E_2 = -M_{13}M_{35}M_{24} + M_{33}M_{15}M_{24} + M_{22}M_{33} + M_{26}M_{35}M_{11} - M_{15}M_{31}M_{26} + M_{11}M_{33} - M_{13}M_{31} + M_{11}M_{22} - M_{21}M_{12},$$
$$E_3 = M_{11}M_{22}M_{33} - M_{21}M_{12}M_{33} - M_{13}M_{31}M_{22},$$

Let $\Gamma_1^2, \Gamma_2^2, \Gamma_3^2$ be the roots Eq. (23) with positive real parts. The solution of Eq. (21) which bound as $x \to \infty$, is given by

$$(u^*, w^*, \Psi^*)(z) = \textstyle\sum_{n=1}^{3}(\Pi_{1n}, \Pi_{2n}, \Pi_{3n})R_n\exp(-\Gamma_n z). \tag{24}$$

$\Pi_{1n} = \Gamma_nM_{26}(M_{12} + \Gamma_nM_{15}) - (M_{22} - \Gamma_n^2)M_{13}$, $\Pi_{2n} = \Gamma_nM_{26}(\Gamma_n^2 - M_{11}) + (M_{21} + \Gamma_nM_{24})M_{13}$, $\Pi_{3n} = (\Gamma_n^2 - M_{11})(M_{22} - \Gamma_n^2) + (M_{21} + \Gamma_nM_{24})(M_{12} + \Gamma_nM_{15})$, Using Eqs. (24)- (26), we get

$$(\sigma_{zz}^{*}, \sigma_{xz}^{*})(z) = \sum_{n=1}^{3} (\Pi_{4n}, \Pi_{5n}) R_n \exp(-\Gamma_n z). \tag{25}$$

Where,

$$\Pi_{4n} = \frac{1}{\mu_T}[i\zeta A_2 \Pi_{1n} - A_3 \Gamma_n \Pi_{2n} - (\lambda + 2\mu_T)\Pi_{3n}], \quad \Pi_{5n} = \frac{\mu_L}{\mu_T}(i\zeta \Pi_{2n} - \Gamma_n \Pi_{1n}).$$

4. Application

In order to determine the parameters $R_n (n = 1,2,3)$, we need to consider the following boundary conditions at $z = 0$:

4.1 Thermal boundary condition that the surface of the half-space is subjected to an isothermal

$$\theta = 0, \tag{26}$$

4.2 Mechanical boundary condition that the surface to the half-space is subjected to mechanical force

$$\sigma_{zz} = -F_0 \delta(x) F(t). \tag{27}$$

4.3 Mechanical boundary condition that the surface to the half-space is traction free

$$\sigma_{xz} = 0. \tag{28}$$

Where F_0 is a constant, $\delta(x)$ is the Dirac-delta function and in this paper, we consider two types of loads on the plane boundary of which is as defined below

$$F(t) = \begin{cases} H(t) \text{ for continuous load} \\ \delta(t) \text{ for impact load} \end{cases} \tag{29}$$

5. Continuous load

Using the expressions of the variables considered into the above boundary conditions (Eqs. (26)-(29)), we can obtain the following equations satisfied with the parameters

$$\sum_{n=1}^{3} \Pi_{3n} R_n = 0, \ \sum_{n=1}^{3} \Pi_{4n} R_n = -\frac{F_0}{p}, \quad \sum_{n=1}^{3} \Pi_{5n} R_n = 0. \tag{30}$$

After applying the inverse of the matrix method on the Eq. (30), we have

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} \Pi_{31} & \Pi_{32} & \Pi_{33} \\ \Pi_{41} & \Pi_{42} & \Pi_{43} \\ \Pi_{51} & \Pi_{52} & \Pi_{53} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -\frac{F_0}{p} \\ 0 \end{pmatrix}. \tag{31}$$

II. Impact load

We can obtain the following equations satisfied with the parameters

$$\sum_{n=1}^{3}\Pi_{3n}R_n = 0, \sum_{n=1}^{3}\Pi_{4n}R_n = -F_0, \quad \sum_{n=1}^{3}\Pi_{5n}R_n = 0. \tag{32}$$

Solving Eq. (35), by using the inverse of the matrix method, we have the values of the three constants $R_n (n = 1,2,3)$.

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} \Pi_{31} & \Pi_{32} & \Pi_{33} \\ \Pi_{41} & \Pi_{42} & \Pi_{43} \\ \Pi_{51} & \Pi_{52} & \Pi_{53} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -F_0 \\ 0 \end{pmatrix}. \tag{33}$$

6. Inversion of the transforms

The transformed displacements, the stress components and the tangential couple stress are the functions of z and the parameters p and ζ of Laplace and Fourier transforms respectively and hence are of the form $f(z, p, \zeta)$. To obtain the solution of the problem in the physical domain, we invert the Laplace and Fourier transforms by using the method described by Kumar and Rani (2004).

7. Numerical calculations and discussion

To study the influence of a magnetic field and gravity on wave propagation, we use the following physical constants for generalized fiber-reinforced thermoelastic materials (Othman and Said 2012)

$$\lambda = 5.65 \times 10^{10}\ \mathrm{N.m^{-2}}, \mu_T = 2.46 \times 10^{10}\ \mathrm{N.m^{-2}}, x = 0.2,$$
$$\mu_L = 5.66 \times 10^{10}\ \mathrm{N.m^{-2}}, \rho = 2660\ \mathrm{kg.m^{-3}},\ T_0 = 293\ \mathrm{K},$$
$$\beta = 0.015 \times 10^{-4}\ \mathrm{N.m^{-2}}, \gamma = 0.017 \times 10^{-4}\ \mathrm{N.m^{-2}}, n = 0.5,$$
$$\alpha = 1.28 \times 10^{10}\ \mathrm{N.m^{-2}}, C_E = 0.787 \times 10^{3}\ \mathrm{kg.K^{-1}}, t = 0.02\ \mathrm{s},$$
$$\mu_0 = 1.7\ \mathrm{kg.m^{-1}.s^{-2}}, \alpha_t = 1.78 \times 10^{-4}\ \mathrm{K^{-1}}, \varepsilon_0 = 0.3, q = 0.3,$$
$$K^* = 150\ \mathrm{w.m^{-1}.K^{-1}}, \omega_1 = 0.04, \omega_2 = 0.06, \omega_3 = 0.07, \beta = 0.07, \mathrm{g} = 9.8\ \mathrm{m.s^{-2}},$$
$$\tau_q = 0.7\ s, \tau_T = 0.5\ \mathrm{s}, \tau_\upsilon = 0.3\ s, k_1 = -\ 0.4.$$

The comparisons have been made in the context of four theories of thermoelasticity, namely; (3PHL), (G-N: III), (G-N: II) and (L-S), in three situations:

(i) With and without magnetic field [$H_0 = 120$ and $H_0 = 0$].

(ii) Whether we have some gravity parameter or not [$g = 9.8$ and $g = 0$].

(iii) Two types of mechanical loads [continuous load and impact load].

6.1 The influence of the mechanical loads

Figs. 1-4 show the variations of the nondimensional displacement component w, temperature θ and stress components σ_{zz}, σ_{xz}, respectively, which demonstrate the effects of the mechanical loads (continuous load and impact load) on the variations of the considered variables when $t =$

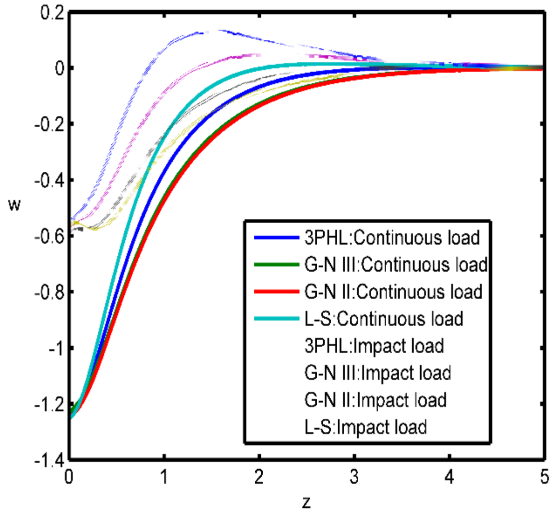

Fig. 1 Variation of displacement component w for different theories under different loads

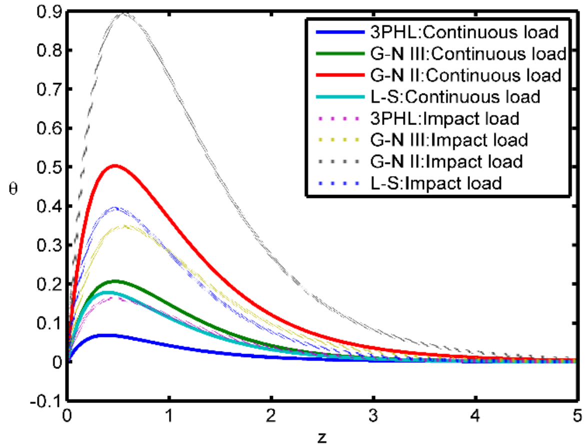

Fig. 2 Variation of temperature θ for different theories under different loads

0.02 and $F_0 = 10$. In each figure, there are eight curves predicted by the four models 3PHL, G-N: III, G-N: II and L-S considered in this work. These figures evidence that; the behavior of all models may be the same with different amplitudes. Figs. 1, 2 exhibit the variation of the displacement component and the temperature against the distance z. We notice from these figures that, the values of the displacement and temperature for the impact load are large compared to those for the continuous load in the range $0 \leq z \leq 3.5$, while the values are the same for two cases at $z \geq 3.5$. Figs. 3 and 4 study the variation of the stress components σ_{zz} and σ_{xz} versus z −axis. In the two figures, the impact load decreases the values of stress. It is observed that: in the context of the four theories, the values of the tangential stress component σ_{xz} start from a zero, which satisfies the boundary conditions.

6.2 Effect of the magnetic field

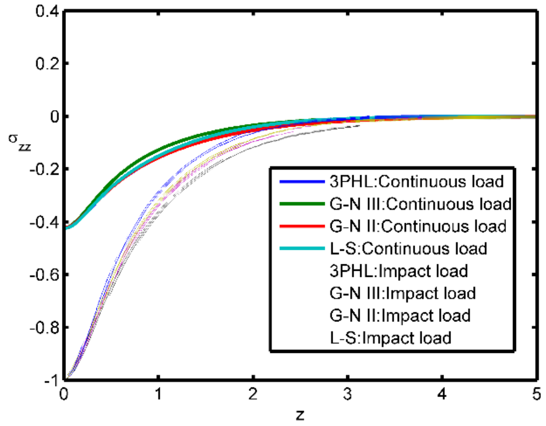

Fig. 3 Variation of stress component σ_{zz} for different theories under different loads

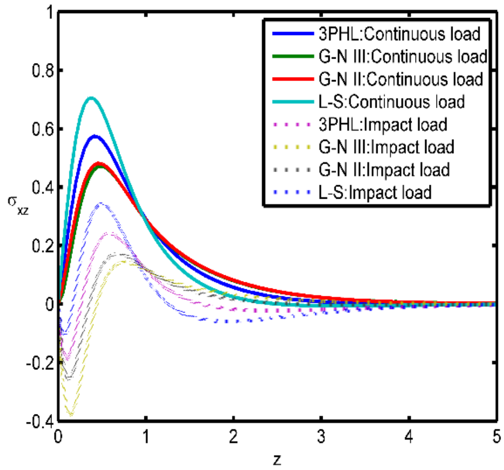

Fig. 4 Variation of stress component σ_{xz} for different theories under different loads

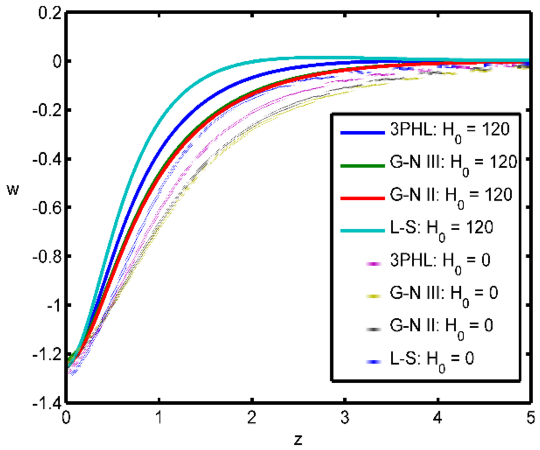

Fig. 5 Variation of displacement component w for different theories in the presence and absence the magnetic field

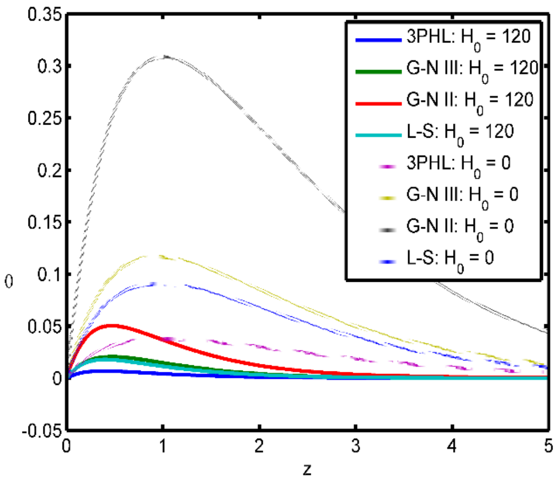

Fig. 6 Variation of temperature θ for different theories in the presence and absence the magnetic field

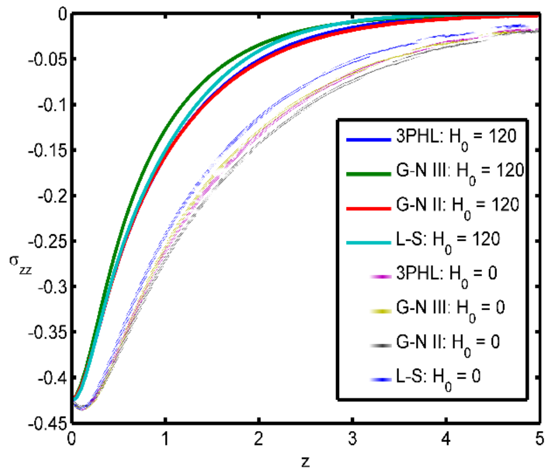

Fig. 7 Variation of stress component σ_{zz} for different theories in the presence and absence the magnetic field

Figs. 5-8 explain the effect of the magnetic field H_0 on the physical fields with respect to the z −axis, in the two cases: with a magnetic field ($H_0 = 120$) and without a magnetic field ($H_0 = 0$). The calculations are carried out for the time $t = 0.02$, the gravity $g = 10$, and the range $0 \leq z \leq 4$. In these figures, the magnetic field has a significant role in the distribution of all physical quantities in the problem, that is agree with Othman and Song (2009). Fig. 5 indicates the distribution of the displacement component w, the values of the displacement for $H_0 = 120$ are small compared to those for $H_0 = 0$. In this figure, the presence of the magnetic field increases the magnitude of the displacement components. Fig. 6 depicts the variation of the temperature θ with respect to the z-axis. It can be seen that the magnetic field shows a decreasing effect on the magnitude of temperature θ. In Figs. 7 and 8 show the variation of the dimensionless normal stress component σ_{zz} and the tangential stress component σ_{xz} according to the different magnetic field parameter ($H_0 = 120$ and $H_0 = 0$). In the two figures, the presence of the magnetic field increases

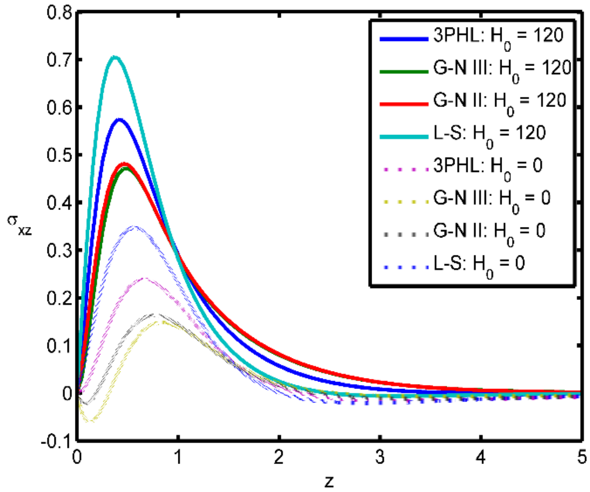

Fig. 8 Variation of stress component σ_{xz} for different theories in the presence and absence the magnetic field

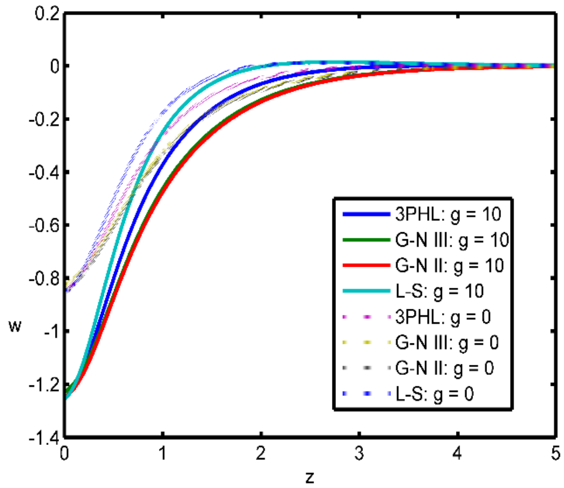

Fig. 9 Variation of displacement component w for different theories in the presence and absence the gravitational field

the magnitude of the stress components.

6.3 Effect of the gravitational field

The third categories of Figs. 9-12 illustrate the effect of the gravity parameter g on the displacement component w, temperature θ and stress components σ_{zz}, σ_{xz}, along the z-axis of the medium, respectively. These figures show the considered variables at two values of gravity parameter with gravity effect $g = 9.8$ and without gravity effect $g = 0$. The effect of gravity is much pronounced in all the resulting quantities, which agrees with Othman and Said (2019).

Fig. 9 investigates the variation of the displacement component $w$versus z. It can be seen that the magnitude of displacement is found to be large for the L-S theory and smaller for the G-N: II

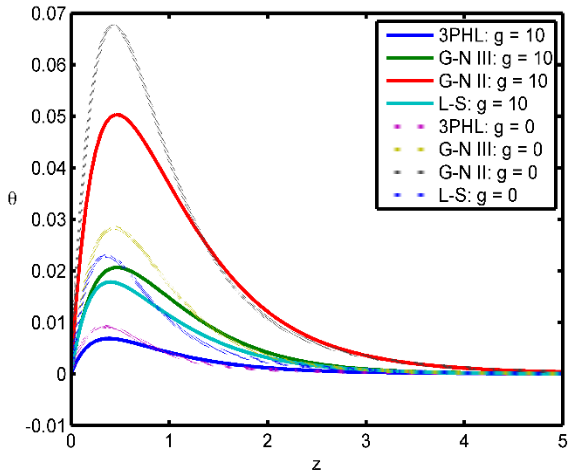

Fig. 10 Variation of temperature θ for different theories in the presence and absence the gravitational field

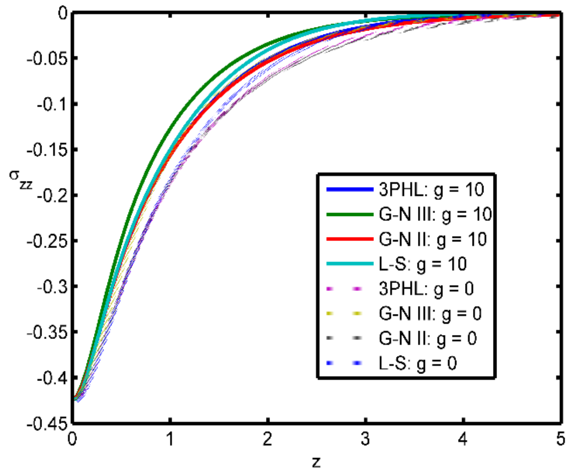

Fig. 11 Variation of stress component σ_{zz} for different theories in the presence and absence the gravitational field

theory. Also, the gravity parameter shows a decreasing effect on the magnitude of displacement. In Fig. 10, the values of the temperature for presence the gravity is small compared to those for absence the gravity. Figs. 11 and 12 investigate the variation of the normal stress component σ_{zz} and the tangential stress component σ_{xz} against the z-axis. In these figures, the presence of gravity shows an increasing effect on the magnitude of the stress components.

6.4 The 3D surface curves

Figs. 13-15 are giving 3D surface curves for some physical quantities, i.e., the displacement component w, the temperature T and the stress component σ_{zz} of the effects of the magnetic field and gravity on a fiber-reinforced thermoelastic medium under the effect of variable thermal conductivity and being enlightened by memory-dependent derivative (MDD). These figures are

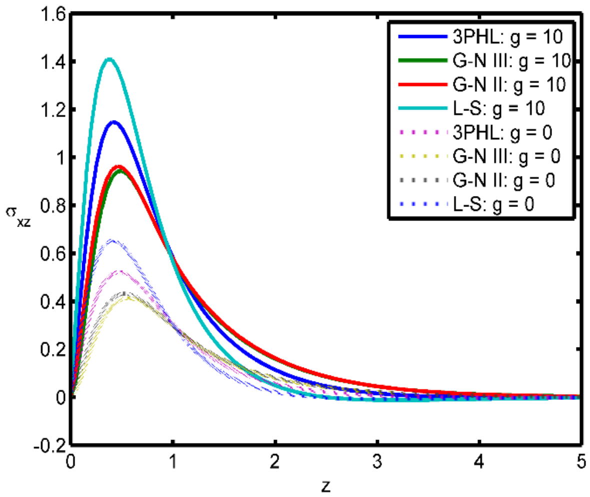

Fig. 12 Variation of stress component σ_{xz} for different theories in the presence and absence the gravitational field

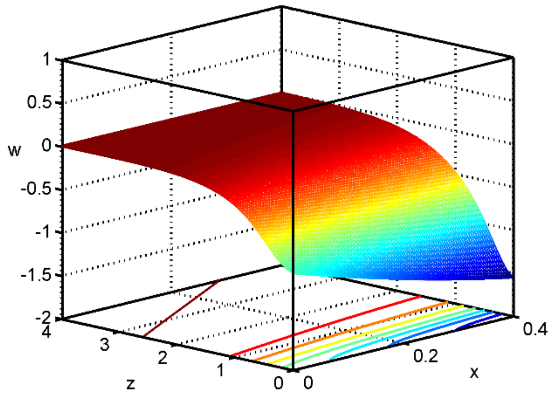

Fig. 13 (3D curve) Distribution of the displacement w versus the distances for 3PHL at $g = 9.8\ \mathrm{m \cdot s^{-2}}, H_0 = 120\ \mathrm{A^{-1} \cdot m^{-1}}$

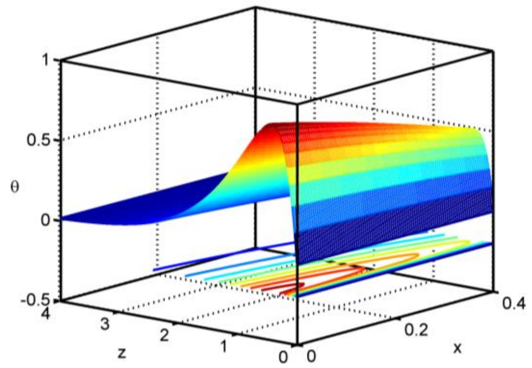

Fig. 14 (3D curve) Distribution of the temperature θ versus the distances for 3PHL at $g = 9.8\ \mathrm{m \cdot s^{-2}}, H_0 = 120\ \mathrm{A^{-1} \cdot m^{-1}}$

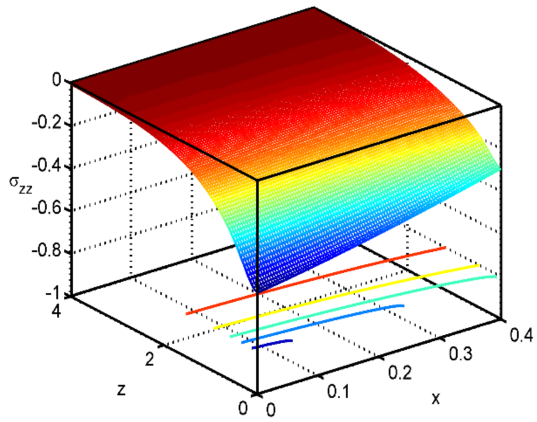

Fig. 15 (3D curve) Distribution of the stress component σ_{zz} versus the distances for 3PHL at $g = 9.8\ m.s^{-2}, H_0 = 120\ A^{-1} \cdot m^{-1}$

very important to study the dependence of these physical quantities on the vertical component of distance. The deformation of a body depends on the nature of the applied forces due to the type of boundary conditions.

7. Conclusions

From the above discussions, we obtain the following important conclusions:

• All the distributions considered have a nonzero value only in a bounded region of the half-space. At the end of this region, the values vanish identically, which means that the region has not yet felt a thermal disturbance.

• There are significant differences in the field quantities between the theories 3PHL, G-N: III, G-N: II and the L-S due to the phase-lags.

• The effect of the magnetic field is much pronounced in all the resulting quantities.

• It is noticed from the figures that the gravitational field plays a significant role in all the field quantities.

• The field quantities are very sensitive to the applied mechanical loads (continuous load and impact load).

• The method used in this article is applicable to a wide range of problems in thermodynamics.

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CC

Nomenclature

C_E	The specific heat at constant strain
e_{kk}	The dilatation
n_0, n_1	An integer
D_{w_i}	The memory-dependent derivative operator
e_{ij}	represents strain components,
K	The coefficient of thermal conductivity
K_1	A non-positive small parameter
K^*	The additional material constant
K_0	A constant
σ_{ij}	The components of stress
λ, μ	The elastic constants
$\alpha, \beta, (\mu_L - \mu_T)$,	The reinforcement parameters
α_t	The thermal expansion coefficient
T_0	The reference temperature
$\theta = T - T_0$,	where $T$is the temperature above the reference temperature T_0
τ_T	The phase-lag of temperature gradient
τ_q	The phase-lag of heat flux
τ_v	The phase-lag of thermal displacement gradient
ρ	The mass density