Advances in Aircraft and Spacecraft Science, *Vol. 6, No. 3 (2019) 189-206*
DOI: https://doi.org/10.12989/aas.2019.6.3.189

A novel first order refined shear-deformation beam theory for vibration and buckling analysis of continuously graded beams

Ahmed Bekhadda[1], Abdelmadjid Cheikh[1], Ismail Bensaid[*1], Abdelhamid Hadjoui[1] and Ahmed A. Daikh[2]

[1]IS2M Laboratory, Faculty of Technology, Mechanical engineering Department, University Abou Beckr Belkaid (UABT), Tlemcen, Algeria
[2]Mechanics of structures and solids Laboratory, Department of Mechanical Engineering, Faculty of Technology, PO Box 89, University of Sidi Bel Abbes, Algeria

(Received July 31, 2018, Revised February 13, 2019, Accepted February 18, 2019)

Abstract. In this work, a novel first-order shear deformation beam theory is applied to explore the vibration and buckling characteristics of thick functionally graded beams. The material properties are assumed to vary across the thickness direction in a graded form and are estimated by a power-law model. A Fourier series-based solution procedure is implemented to solve the governing equation derived from Hamilton's principle. The obtained results of natural frequencies and buckling loads of functionally graded beam are checked with those supplied in the literature and demonstrate good achievement. Influences of several parameters such as power law index, beam geometrical parameters, modulus ratio and axial load on dynamic and buckling behaviors of FGP beams are all discussed.

Keywords: vibration; buckling; functionally graded beams; refined first shear deformation theory, coupled effect

1. Introduction

Normally, due to their specific properties, composite materials have found very wide applications in various applications into modern industrial construction such as mechanical engineering, aerospace, transportation industries and so on. However, in extreme conditions as a use in the high-temperature environments, these classical composite materials represent some deficits and fail to preserve their integrity. To surmount these drawbacks, a new sort of advanced composite materials or functionally graded materials (FGMs) have been designed (Koizumi 1997). Usually these materials are made from a mixture of a ceramic and a metal with volume fractions which are varied continuously as a function of location depending on some dimension(s) of the structure to reach a required function. The FGMs have been applied in several hi-tech industrial applications of for defense industries, aerospace, aircrafts, automobile, shipbuilding industries, and further engineering structures. Presenting these notable advantages, studies of structures made of these types of materials have attracted the worldwide interest by numerous researchers (Chakraborty *et al*. 2003, Li 2008, Sina *et al*. 2009, Attia *et al*. 2015, Tounsi *et al*. 2013). Sankar *et*

∗Corresponding author, Ph.D., E-mail: bensaidismail@yahoo.fr

http://www.techno-press.org/?journal=aas&subpage=7 ISSN: 2287-528X (Print), 2287-5271 (Online)

al. (2001) developed an elasticity solution on the basis of functionally graded Euler-Bernoulli beam theory; the FG beam was subjected to static transverse loads and assuming that Young's modulus of the beam changes exponentially across the thickness. Aydogdu and Taskin (2007) analyzed the dynamic behavior of a simply supported FG beam by employing various beam theory like Euler-Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation model. Zhong and Yu (2007) used a two-dimensional elasticity theory and analytical solution for a cantilever FG beam with arbitrary graded variations of material property distribution. Li (2008) analyzed the static and dynamic behaviors of FG beams by using a new unified approach, the rotary inertia and shear deformation have been included. Sallai *et al*. (2009) contributed to the bending responses of a sigmoid FG thick beam by different higher order beam theories. Simsek (2010a) researched the free vibration characteristic of an FG beam via different higher order beam theories. In another one, Simsek (2010b) has investigated the dynamic deflections and the stresses of an inhomogeneous FG beam subjected to a moving mass by utilizing Euler-Bernoulli, Timoshenko and the parabolic shear deformation beam theory. Thai and Vo (2012) studied the impacts of using various shear deformation beam theories on the bending and vibration responses of thick FG beams. Bouremana *et al*. (2013) developed a new first shear deformation beam theory model based on neutral surface position for FG beams. Ould Larbi *et al*. (2013) used an efficient shear deformation beam theory and the concept of neutral surface position for bending and free vibration of functionally graded beams. Nguyen *et al*. (2013) investigated by employing a novel first order shear deformation beam model the static bending and free vibration of axially loaded FG beams in which, an improved transverse shear stiffness has been introduced without using shear correction factor. Vo *et al*. 2014 used a new developed finite element model based on refined shear deformation theory to examine the Static and vibration analysis of functionally graded beams. Meradjah *et al*. (2015) proposed a novel shear deformation beam model including the stretching effect for studying the flexural and free vibration responses of functionally graded beams. Vo *et al*. (2015) employed a finite element model to examine the dynamic vibration and buckling of FG sandwich beams via the novel quasi-3D theory in which both shear deformation and thickness stretching effects are incorporated. Bourada *et al*. (2015) developed a new simple shear and normal deformations theory for functionally graded beams in which the numbers of unknowns has been reduced to optimize the calculation time. Bensaid *et al*. (2017) contributed to static deflection and dynamic behavior of higher-order hyperbolic shear deformable compositionally graded beams, in this study the authors made a further supposition in the beam kinematic to construct a new simple and efficient shear deformation beam model. Zidi *et al*. 2017 proposed a novel simple two-unknown hyperbolic shear deformation theory to investigate bending and free vibration analysis of functionally graded (FG) beams. Recently, Vo *et al*. (2017) investigated the free vibration of axially loaded composite beams by employing a four-unknown shear and normal deformation theory. More recently, Kaci *et al*. (2018) studied the postbuckling response of laminated composite beams via the new developed two unknowns shear deformation beam model.

In addition, a significant progression in the use of structural elements such as beams and plates at nano scales after the invention of carbon nanotubes (CNTs) by Iijima (1991), due to providing outstanding mechanical, chemical, and electronic characteristics compared to the conventional structural materials. Recently, lots of studies have been also performed to investigate mechanical responses of the nano structures subjected to multi field load; one can cite the works of Reza Barati and his co-workers (Ebrahimi and Barati 2016b, c, d, 2017, a, b, Barati *et al*. 2016, Barati and Shahverdi 2016, Barati and Shahverdi 2017a, b, c, d, e) they used an improved analytical

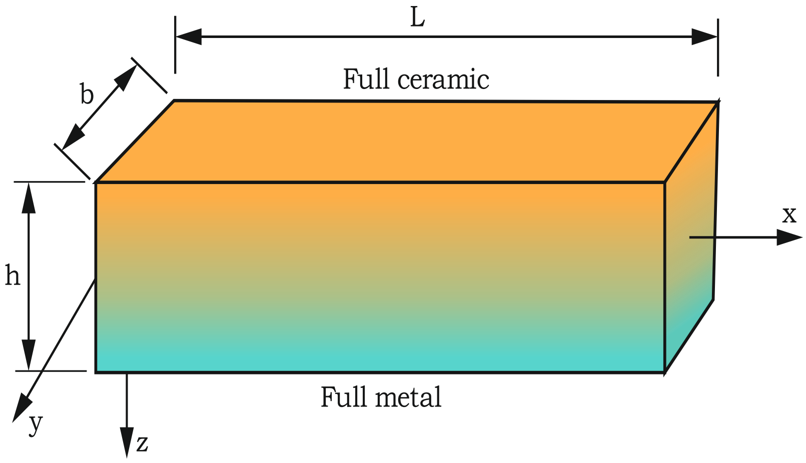

Fig. 1 Schematic arrangement of axially loaded functionally graded beam

models in their investigations. They also discussed the effects of various physical fields and small scale parameter on the buckling, wave propagation and dynamic behavior of functionally graded nano beams and plates. Also, and in the recent years, the concept of functionally graded materials (FGMs) has been integrated in the design of smart materials and structures, several researchers examined mechanical properties of structural elements made from the well-known magneto-electro-elastic functionally graded (MEEFG) materials. Some of researchers in recent years have analyzed mechanical behaviors of FGM macro/nano beams and plates based on various plate shear deformation plate theories (Ebrahimi and Barati 2016e, f).

In view of above, some works cited were based on the classical first order shear deformation theory which sometimes involves many unknowns in the governing equations of motion which is highly computational cost. Recently, a new FSDT which involves only four unknowns by making a further assumption in the plate kinematic was developed by Thai *et al*. (2014). A number of investigations have been recently provided on the basis of this model. Nguyen (2015) employed this supposition to develop a higher-order hyperbolic shear deformation plate model for analysis of functionally graded materials. Hadji *et al*. (2016) performed a dynamic analysis of functionally graded beam using a novel first-order shear deformation theory. Bellifa *et al*. (2016) utilized this simple NFSDT to explore the static bending and free vibration analysis of functionally graded plates based on the concept the neutral surface position. It is apparent that few of published papers on FG beams have explored the vibration and buckling of graded beam by using the new first order shear deformation theory discussed earlier. It is mentioned that the mechanical attitude of FG beams is notably influenced when taking into account the coupling between the vibration and axial load.

Present research is devoted to study free vibration and buckling of axially loaded simply supported FG beams by employing a refined first shear deformation beam theory. The materials properties of the FG beam are presumed to be graded across the thickness direction based on power-law model. By using the Hamilton's principle, the governing equations are extracted and then solved in the framework of an analytical procedure. To confirm the validity of the proposed theory, the obtained results are compared with the existing solutions. A variety of graphical and tabulated results reveal the remarkable effect of length to thickness ratios, power-law exponent, Young's modulus ratio, axial loadings, on the vibration frequencies and buckling loads of FGP beams.

2. Mathematical development

Let assume a functionally graded beam having length L and uniform rectangular cross section $b\times h$, with b represents the width and h the thickness which its coordinates is depicted in Fig. 1. The beam is fabricated of elastic and isotropic material with material properties changing graded and smoothly in the z thickness direction.

2.1 Material properties

The volume fraction of the ceramic constituent of the FG beam is supposed to be given by

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{1}$$

In which k is a variable parameter that dictates material variation profile through the thickness and z is the distance from the mid-plane of the FG beam. When k is set to be zero represents a fully ceramic beam, whereas infinite k indicates a fully metallic beam. Material characteristics of a functionally graded beam may be acquired by means of the Voigt rule of mixture (Suresh and Mortensen 1998, Bourada *et al*. 2012, Simsek and Yurtcu 2013, Nguyen *et al*. 2013, Chien *et al*. 2016, Ebrahimi and barati 2016a, Bensaid and Bekhadda 2018) as

$$P_f = P_c V_c + P_m V_m \tag{2}$$

where P_m, P_c, V_m and V_c are the corresponding material properties and the volume fractions of the metal and the ceramic constituents related by

$$V_c + V_m = 1 \tag{3}$$

Hence, Young's modulus (E) and mass density (ρ), of the FG beam can be described by the following power-law distribution as

$$E(z) = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_m \tag{4}$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + \rho_m \tag{5}$$

2.2 Kinematic relations

Until now, various beam theories have been developed for modeling and analysis of beams (Ebrahimi and Barati 2016b, e, g, Barati and Zenkour 2017). So, based on the new proposed refined First order shear deformation theory, the displacement field of the present beam model can be written in a simpler form as follow (Thai *et al*. 2014, Nguyen 2015, Bellifa *et al*. 2016)

$$u(x,z,t) = u_0(x,t) - z\frac{\partial \phi}{\partial x}$$
$$w(x,z,t) = w(x,t) \tag{6}$$

It is mentioned that the displacement field of the recent refined or simplified existing FSDT theories (Thai and Choi 2013a, b) are obtained by splitting the transverse displacement into bending and shear parts, in which the number of unknowns are reduced in the general equations of motion, the procedure is shown by the following equation, (Bouremana *et al.* 2013, Malikan 2017):

$$w = w(bending) + w(shear) \tag{7}$$

In addition, the φ in Timoshenko beam theory (TBT) parameter was also supposed as follow:

$$\theta_x = -\frac{dw_b}{dx} \tag{8}$$

Therefore, by making another assumption to existing ones as $\theta_x = -d\phi/dx$ and without splitting the transverse displacement w into two components, the displacement field also generates fewer unknown variables and the governing equations of motion resulting in this study will be completely different compared with those cited above.

In Eq. (6), u, w are displacements in the x, z directions, u_0 is the mid surface displacements. ϕ is function of coordinates x and time t which represents the shear transverse displacement.

The nonzero strains combined with the displacements in Eq. (6), there must be

$$\varepsilon_x = \varepsilon_x^0 + zk_x \tag{9a}$$

$$\gamma_{xz} = \gamma_{xz}^s \tag{9b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \qquad k_x = -\frac{\partial^2 \phi}{\partial x^2} \tag{10a}$$

$$\gamma_{xz}^s = \frac{\partial w}{\partial x} - \frac{\partial \phi}{\partial x} \tag{10b}$$

Supposing that the material of FG beam obeys Hooke’s law, the constitutive relations can be given as

$$\sigma_x = Q_{11}(z)\varepsilon_x \;\; and \;\; \tau_{xz} = k_s Q_{55}(z)\gamma_{xz}; \tag{11a}$$

In which, k_s is a shear correction factor which is similar to shear correction factor proposed by Mindlin (1951). Employing the material properties defined in Eq. (2), stiffness coefficients, Q_{ij} can be given as

$$Q_{11}(z) = \frac{E(z)}{1-\nu^2} \;\; and \;\; Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \tag{11b}$$

2.3 Variational formulation

To derive the governing equations of motion, Hamilton's principle is employed, (Reddy 2002)

$$\delta\int_0^T \left(U+V-K\right)dt=0 \tag{12}$$

By which δU is the difference of the strain energy; δV represents the work done by external forces due to in-plane load; and the variation of the kinetic energy is given by δK. The variation of the strain energy of the beam can be declared by the following form

$$\begin{aligned}\delta U &= \int_0^L \int_{\frac{-h}{2}}^{\frac{h}{2}} \left(\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}\right) dzdx \\ &= \int_0^L \left(N_x \frac{d\delta u_0}{dx} - M_x \frac{d^2\delta w_0}{dx^2} + Q_{xz}\frac{d\delta\left(w-\phi\right)}{dx}\right)dx\end{aligned} \tag{13}$$

where N, M and Q are the stress resultants defined as

$$\left(N_x, M_x\right) = \int_{\frac{-h}{2}}^{\frac{h}{2}} (1,z)\sigma_x dz \quad and \quad Q_{xz} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \tau_{xz} dz \tag{14}$$

The variation of work done by externally transverse load q and the in-plane $\bar{N}$, which can be expressed as

$$\delta V = -\int_0^L \left[q\delta w + \bar{N}\frac{dw}{dx}\frac{d(\delta w)}{dx}\right]dx \tag{15}$$

where (q and $\bar{N}$) are the transverse and axial loads, respectively. Since this paper is devoted to the study of the free vibration and buckling of FG beams, the transverse load q will be omitted, because of the absence of the investigation on the bending behavior.

The variation of the kinetic energy can be expressed as

$$\begin{aligned}\delta K &= \int_0^L \int_{\frac{-h}{2}}^{h/2} \rho(z)\left[\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}\right] dzdx \\ &= \int_0^L \left\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + (\dot{w})(\delta\dot{w})\right] - I_1\left(\dot{u}_0\frac{d\delta\dot{\phi}}{dx} + \frac{d\dot{\phi}}{dx}\delta\dot{u}_0\right)\right. \\ &\quad \left. + I_2\left(\frac{d\dot{\phi}}{dx}\frac{d\delta\dot{\phi}}{dx}\right)\right\}dx\end{aligned} \tag{16}$$

where, the dot-superscript sign indicates the differentiation with sense to the time variable t; ρ is the mass density; and (I_0, I_1, I_2) are the mass inertias expressed as

$$\left(I_0, I_1, I_2\right) = \int_{\frac{-h}{2}}^{\frac{h}{2}} \left(1, z, z^2\right)\rho(z)dz \tag{17}$$

Substituting the expressions for δU, δV, and δK from Eqs. (13), (15), and (16) into Eq. (12) and integrating by parts, and collecting the coefficients of δu_0, δw_0 and $\delta \varphi$, the following equations of motion of the FG beam are obtained

$$\delta u_0 : \frac{dN_x}{dx} = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{\phi}}{dx} \tag{18a}$$

$$\delta\phi : \frac{d^2 M_x}{dx^2} - \frac{dQ_{xz}}{dx} = I_1 \frac{d\ddot{u}_0}{dx} - I_2 \frac{d\ddot{\phi}}{dx^2} \tag{18b}$$

$$\delta w : \frac{dQ_{xz}}{dx} - \bar{N} \frac{d^2 w}{dx^2} = I_0 \ddot{w} \tag{18c}$$

Eqs. (18) can be expressed in terms of displacements (u_0, w_0 and ϕ) by using Eqs. (6), (9), (11) and (14) as follows

$$A_{11} \frac{d^2 u}{dx^2} - B_{11} \frac{d^3 \phi}{dx^3} = I_0 \ddot{u} - I_1 \frac{d\ddot{\phi}}{dx} \tag{19a}$$

$$B_{11} \frac{d^3 u}{dx^3} - D_{11} \frac{d^4 \phi}{dx^4} - A_{55}^s \frac{d^2}{dx^2}(w - \phi) = I_1 \frac{d\ddot{u}}{dx} - I_2 \frac{d\ddot{\phi}}{dx^2} \tag{19b}$$

$$A_{55}^s \frac{d^2}{dx^2}(w - \phi) - \bar{N} \frac{d^2 w}{dx^2} = I_0 \ddot{w} \tag{19c}$$

where A_{11}, B_{11}, D_{11}, etc., are the beam stiffness, defined by

$$(A_{11}, B_{11}, D_{11}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{11}(1, z, z^2)\, dz \tag{20a}$$

and

$$A_{55}^s = k_s \int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{E(z)}{2(1+\nu)} dz \tag{20b}$$

3. Solution method

In this part, by employing analytical solution (Navier's method), the governing equations of motion for free vibration and buckling of simply-supported FG beam has been solved. The displacement functions are provided as product of non-unknowns coefficients and known trigonometric functions to assure the boundary conditions at x=0, $x=L$, the next displacements functions are estimated to be of the formed

$$\begin{Bmatrix} u_0 \\ \phi \\ w \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ \psi_m \sin(\lambda x) e^{i\omega t} \\ W_m \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \tag{21}$$

where (Um, ψm, Wm) are the unknown Fourier coefficient that will be determined for each value of m, and $\lambda = m\pi / L$.

Substituting the expansions of u_0, ϕ and w from Eqs. (21), into the equations of motion Eq. (19), the analytical solutions can be obtained from the following Eqs. (22a) to (22c)

$$\left(-A_{11}\left(\frac{m\pi}{L}\right)^2 - I_0\omega_m^2\right)U_m - \begin{pmatrix} B_{11}\left(\frac{m\pi}{L}\right)^3 \\ -I_1\left(\frac{m\pi}{L}\right)\omega_m^2 \end{pmatrix}\psi_m = 0 \tag{22a}$$

$$\begin{pmatrix} -D_{11}\left(\frac{m\pi}{L}\right)^4 + A_{55}^s\left(\frac{m\pi}{L}\right)^2 - \\ I_2\left(\frac{m\pi}{L}\right)^2 \omega_m^2 \end{pmatrix}\psi_m + \left(A_{55}^s\left(\frac{m\pi}{L}\right)^2\right)W_m = 0 \tag{22b}$$

1

$$\left(A_{55}^s\left(\frac{m\pi}{L}\right)^2\right)\psi_m + \begin{pmatrix} -A_{55}^s\left(\frac{m\pi}{L}\right)^2 + \bar{N}\left(\frac{m\pi}{L}\right)^2 \\ -I_0\omega_m^2 \end{pmatrix} = 0 \tag{22c}$$

By making the determinant of the coefficient matrix of the beyond equations and setting this polynomial to zero, we can obtain natural frequencies ω_m and buckling load $\bar{N}$.

4. Numerical result and discussions

Through this section, a variety of numerical examples are presented and discussed to verify the correctness of the present theory in predicting the free vibration and buckling responses of simply supported FG beams. The new results are obtained from the new simple and accurate first-order beam theory.

In this study, the FG beam is selected to be built of aluminum and alumina with the following material properties:

Ceramic (P_C: Alumina, Al2O3): $E_c = 380$ GPa; $\nu = 0.3$; ρ_c 3960 kg/m^3.

Metal (P_m: Aluminium, Al): $E_m = 70$ GPa; $\nu = 0.3$; $\rho_m = 2707$ kg/m^3.

For convenience, the following non-dimensional forms are used during this investigation.

$$\bar{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}} \tag{23}$$

$$\bar{N}_{cr} = N_{cr}\frac{12L^2}{E_m h^3} \tag{24}$$

as well as Young's modulus ratio:

$$E_{ratio} = \frac{E_c}{E_m} \tag{25}$$

Table 1 Comparison of the first three non-dimensional natural frequencies of simply-supported FG beams

L/h	Mode	Source	p				
			0	1	2	5	10
5	1	Present	5.1524	3.9806	3.6223	3.4225	3.3091
		Ref[(a)]	5.1525	3.9902	3.6344	3.4312	3.3135
		Ref[(b)]	5.1525	3.9902	3.6344	3.4312	3.3134
		Ref[(c)]	5.1527	3.9904	3.6264	3.4012	3.2816
	2	Present	17.8710	13.9109	12.5974	11.7388	11.2710
		Ref[(a)]	17.8711	14.0030	12.7120	11.8157	11.3073
		Ref[(c)]	17.8812	14.0100	12.6405	11.5431	11.0240
	3	Present	34.1449	26.7934	24.1759	22.2619	21.2311
		Ref[(a)]	34.1439	27.0525	24.4970	22.4642	21.3219
		Ref[(c)]	34.2097	27.0979	24.3152	21.7158	20.5561
20	1	Present	5.4603	4.2043	3.8358	3.6501	3.5412
		Ref[(a)]	5.4603	4.2051	3.8368	3.6509	3.5416
		Ref[(b)]	5.4603	4.2051	3.8368	3.6509	3.5416
		Ref[(c)]	5.4603	4.2051	3.8361	3.6485	3.5390
	2	Present	21.5731	16.6230	15.1571	14.4004	13.9599
		Ref[(a)]	21.5732	16.6344	15.1715	14.4110	13.9653
		Ref[(c)]	21.5732	16.6344	15.1619	14.3746	13.9263
	3	Present	47.5921	36.7138	33.4462	31.6983	30.6930
		Ref[(a)]	47.5921	36.7673	33.5135	31.7473	30.7173
		Ref[(c)]	47.5930	36.7679	33.4689	31.578	30.5369

Ref[(a)] Nguyen *et al*. (2013)
Ref[(b)] Simsek (2010)
Ref[(c)] Thai and Vo (2012)

in which $I = bh^3/12$ and the shear correction factor is taken as $k = 5/6$ for simply supported, boundary conditions, respectively.

4.1 Validation of vibration analysis

To confirm the validity of the present model, the obtained results are compared with the other works that exist in the literature for the first five natural frequencies of FG beams. For L/h = (5, 20) respectively and several values of gradient index (p), the results are showed in Table 1. It is observed that the results match well with those presented by (Simsek 2010, Thai and Vo 2012, Nguyen *et al*. 2013) which demonstrate the efficient of the present model. Also, it is apparent that an increase in the values of power-law exponent leads to a reduction in the stiffness of porous FG beam and consequently natural frequency decreases. Further, at bigger values of length-to-thickness ratio, the present model provides higher frequencies, and vice versa for lower values of slenderness ratio.

Table 2 Validation of the critical buckling loads of simply-supported FG beams

L/h	Source		*p*			
			0	1	2	5
5	Present	48.5903	24.5813	19.1616	15.9417	14.3444
	Ref[(a)]	48.835	24.687	19.245	16.024	14.427
	Ref[(b)]	48.835	24.687	19.245	16.024	14.427
10	Present	52.2373	26.1406	20.3924	17.1700	15.5879
	Ref[(a)]	52.308	26.171	20.416	17.194	15.612
	Ref[(b)]	52.309	26.171	20.416	17.192	15.612

Ref[(a)] Nguyen *et al*. (2013)
Ref[(b)] Li and Batra (2013)

Table 3 the first three non-dimensional critical buckling loads of simply-supported FG beams

Mode	Source			*p*		
		0	1	2	5	10
1	Present	5.1524	3.9806	3.6223	3.4225	3.3091
2	Present	151.9318	78.3841	61.7401	49.5790	43.4982
3	Present	250.6507	135.2048	104.9102	81.3765	69.7502

4.2 Validation for buckling analysis

Tables 2 and 3 compare the numerical results obtained from the presented refined model concerning the non-dimensional buckling loads ($\bar{N}$) with some works available in the same field (Li and Batra 2013, Nguyen *et al*. 2013) for the purpose of validation. The geometric ratios L/h are fixed for two values 5 and 20, in addition to various gradient indexes and mode numbers were selected. One can see that the current results are in good agreement with results the published cited previously. The buckling loads parameters obtained in the current analysis are in about close enough to the results supplied in these works and thus confirms the proposed process of solution. Likewise, an increase in the power law exponent leads to a reduction in critical buckling loads. Moreover, it is found that FG beams with superior values of slenderness ratio have greater critical buckling loads. In addition, a rise in mode number yields to an increment of buckling loads.

4.3 Numerical results for various materials and geometrical properties

Effect of length to thickness ratio on vibration frequency and buckling load of FG beam with respect to power law exponent (p) is showed in Figures 2 and 3 based on the novel FSDT. It is to be noticed that FG beams with higher values of length to thickness ratio have larger values on both critical loads and vibration frequencies, due to the stiffening phenomenon. Indeed, minimizing the thickness of FG beam leads to lower critical frequencies. Also, an increase in the gradient index generates a reduction in both non-dimensional values. So, it can be mentioned that composition of two parameters of FG beams has a main role on their vibration and buckling behaviors in such cases.

Dimensionless frequency and buckling load of FG beam vs. length to thickness ratios (L/h) for

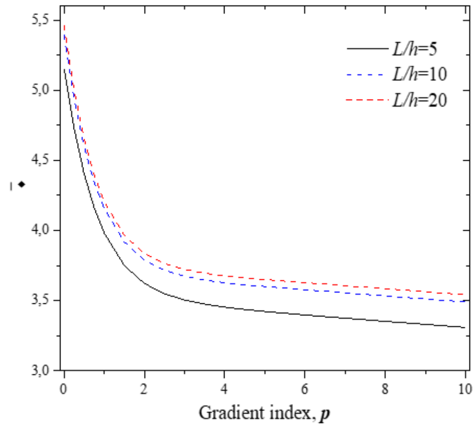

Fig. 2 Influence of the power law index p on the dimensionless frequency ϖ of FG beam with various Length to thickness ratio *L/h*

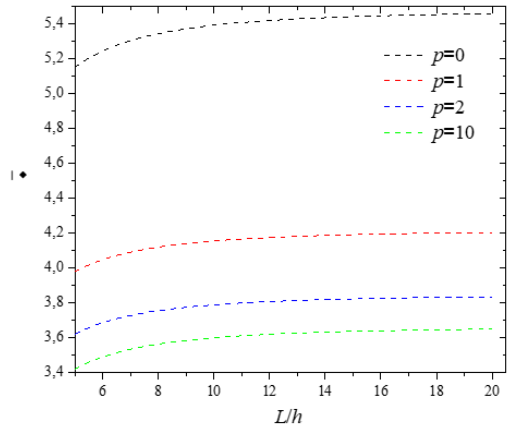

Fig. 3 Influence of the slenderness ratios *L/h* on the dimensionless frequency ϖ of FG beam with various power law index p

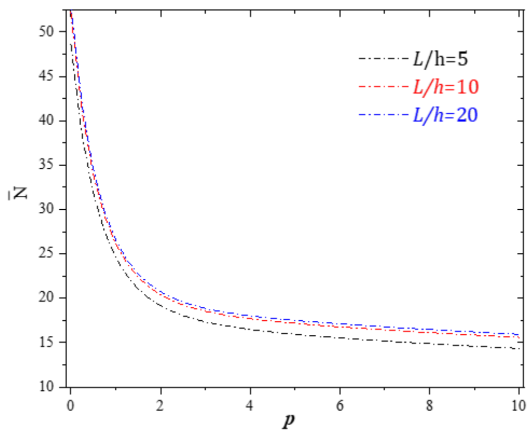

Fig. 4 Variation of the dimensionless critical buckling load $\overline{N}$ with respect to the power law index p for different values of slenderness ratios *L/h*

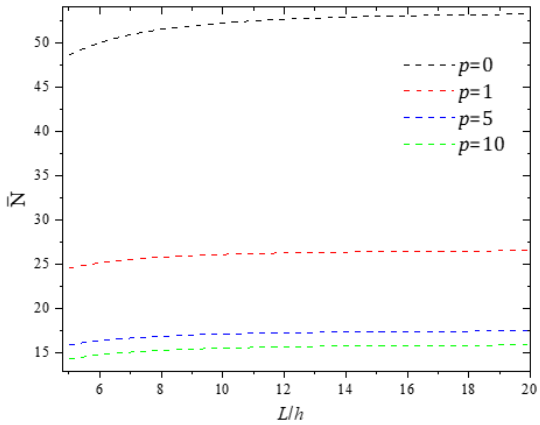

Fig. 5 Variation of the dimensionless critical buckling load $\overline{N}$ versus slenderness ratios L/h for different values of the power law index p

Table 4 Impacts of Young's modulus ratio on non-dimensional frequencies of S-S FG beams with different values of power-law index

L/h	E_{ratio}	0	1	2	5	6
5	0.25	1.1057	1.7966	2.0008	2.2576	2.4160
	0.50	1.5637	2.0526	2.1991	2.3782	2.4894
	1.0	2.2114	2.4111	2.4899	2.5772	2.6206
	2.0	3.1274	2.8994	2.8754	2.8692	2.8354
	4.00	4.4228	3.5857	3.3634	3.2416	3.1461
	6.00	5.4168	4.1256	3.7149	3.4832	3.3651
20	0.25	1.1717	1.8956	2.1085	2.3809	2.5517
	0.50	1.6571	2.1718	2.3243	2.5136	2.6331
	1.00	2.3435	2.5552	2.6396	2.7329	2.7788
	2.00	3.3142	3.0711	3.0523	3.0537	3.0177
	4.00	4.6871	3.7907	3.5658	3.4575	3.3620
	6.00	5.7405	4.3562	3.9322	3.7143	3.6023

diverse power law index (p) are depicted in Figs. 4 and 5 respectively. As seen previously, a rise in the power-law index power leads to reduce in the values of vibration frequencies and critical loads. Because that by growing the value of power-law index (p), the percentage of metal phase will rise, thus causes such FG beams to be less rigid. A further remark is that vibration and buckling behaviors of FG beams are considerably controlled by the slenderness ratio. In fact, at a constant gradient index, a decrease length to thickness (L/h) leads to lower frequencies and buckling loads by introducing a decreased rigidity on FG beam structure.

The vibration frequencies of simply-supported FG beams are presented in Table 4 and Fig. 6 to demonstrate the impact of the Young's modulus ratio. One can see that for a fixed value of power-law index, the natural frequency increases with increasing E_{ratio}. Vice versa, for a specifiedvalue of E_{ratio}. Further, a rise in the power-law exponent, it causes opposing responses on the vibration, which is augmented when $E_{ratio} < 1$ and decreased when $E_{ratio} > 1$, when $E_{ratio} = 1$ the vibration of

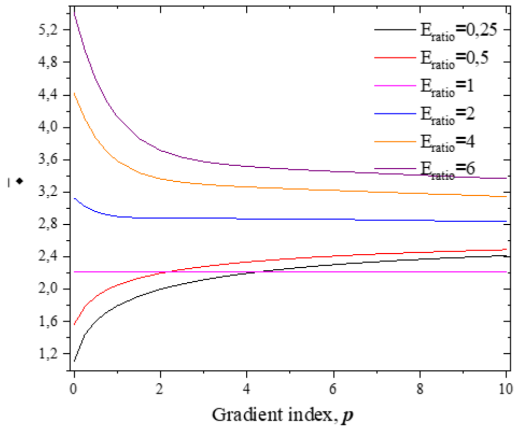

Fig. 6 Impacts of Young's modulus ratio on the dimensionless fundamental natural frequencies of S-S FG beam with respect to power-law index (L/h = 5)

Table 5 Impacts of Young's modulus ratio on dimensionless buckling loads of S-S FG beams with diverse values of power-law index

L/h	E_{ratio}	0	1	2	5	6
5	0.25	2.2377	4.9785	5.7869	6.8734	7.6110
	0.50	4.4754	6.4868	6.9824	7.6234	8.0790
	1.0	8.9508	8.9508	8.9508	8.9508	8.9508
	2.0	17.9017	12.9737	11.9682	11.1088	10.4824
	4.00	35.8034	19.9143	16.4694	14.2493	12.9363
	6.00	53.7051	26.4183	20.1751	16.5353	14.8479
20	0.25	2.4056	5.3084	6.1575	7.3220	8.1289
	0.50	4.8113	6.9561	7.4706	8.1522	8.6501
	1.00	9.6226	9.6226	9.6226	9.6226	9.6226
	2.00	19.2453	13.9123	12.8693	12.0000	11.3318
	4.00	38.4907	21.2336	17.5999	15.3897	14.0502
	6.00	57.7360	28.0710	21.4404	17.7865	16.1324

the FG beam takes a values of isotropic beam and remain constant (Fig. 6). It also proves another time some dynamic responses cited in the previous examples.

Table 5 and Fig. 7 exhibit the effect of Young's modulus ratio on the critical buckling loads of simply supported FG beams. It can be observed that the critical loads enhance sequentially with the rise of E_{ratio} for every values of power-law index considered. The effect of this ratio is more prominent for min values of gradient index than big ones (Table 5). In addition, if we take E_{ratio} equal to unity, similar behavior of vibration analysis can be seen again for buckling investigation. It is also observable, with the increase in power-law index, the buckling load increases when E_{ratio} < 1, and a reduction is noticed when E_{ratio} > 1.

Finally, as seen in previously as the beam becomes homogeneous, E_{ratio} = 1, the critical buckling is independent of the power-law index and its value remains constant (Fig. 7).

Finally, the impacts of the axial force on the natural frequencies are explored. Table 6 shows the

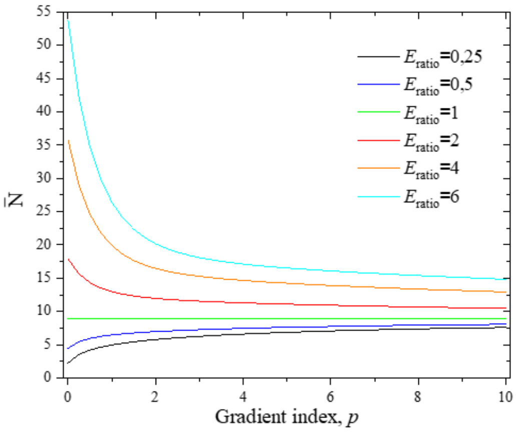

Fig. 7 Impacts of Young's modulus ratio on the dimensionless buckling loads of S-S FG beam with respect to power-law index (L/h = 5)

Table 6 Effect of the axial force on the dimensionless natural frequencies of simply-supported FG beams

L/h	p	N_{cr}	N_0=-0.5N_{cr}	No preload	N_0=0.5N_{cr}
	0	2.2675	5.5473	5.1524	4.7247
	1	1.1471	4.2855	3.9806	3.6502
5	2	0.8942	3.8998	3.6223	3.3216
	5	0.8439	3.6847	3.4225	3.1385
	10	0.6694	3.5897	3.3091	3.0023
	0	0.0388	5.4476	5.3932	5.3383
	1	0.0193	4.1974	4.1557	4.1136
20	2	0.0151	3.8273	3.7892	3.7508
	5	0.0127	3.6362	3.6001	3.5635
	10	0.0116	3.5252	3.4899	3.4543

first three natural frequencies with and without the impact of the axial force. The variation of the natural frequencies owing to the axial force is important for all values of power-law exponent. Moreover it is obvious from this table that fundamental frequencies come down as the axial force switches from tension to compression, this is due to that the tension force imposes a stiffening effect while the compressive force has a softening effect on the natural frequencies parameters.

5. Conclusions

A novel first order shear deformation beam theory is proposed for studying the dynamic and buckling behavior of FG beams. The present theory is able to provide a lower number of variables in generalized cases which facilitate engineering design computations times, also respects the zero traction boundary conditions on the upper and lower surfaces of the FG beam with using a shear correction factor. Material's properties of the FG beam are supposed to be changed along the thickness direction and are evaluated according to the power-law model. The governing equations are derived by applying Hamilton's principle, based on a newly first order beam theory and a Navier based analytical approach is implemented to resolve these equations. It has been shown that the proposed model based on the novel FSDT approach can offer precise frequency and buckling results of the FG beams as checked to analytical results cases that available in the literature and a good agreement has been revealed. At last, a parametric study was done, and the numerical results show the significant influences of several parameters such as material property gradient exponent, length to thickness ratio, Young's modulus ratio, and axial load on the fundamental frequencies and critical buckling loads of FG beams. Unlike the conventional first shear deformation theory, the proposed first shear deformation theory contains only four unknowns in the general case. In conclusion, it can be said that the adopted improved model NFBT is not only accurate but also efficient in predicting the static buckling and dynamic behaviors of functionally graded beams, which is of great interest, especially in the design of various real engineering problems by facilitating engineering computations.

References

Attia, A., Tounsi, A., Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct.*, **18**(1), 187-212. https://doi.org/10.12989/scs.2015.18.1.187.

Aydogdu, M. and Taskin, V. (2007), "Free vibration analysis of functionally graded beams with simply supported edges", *Mater. Des.*, **28**(5), 1651-1656. https://doi.org/10.1016/j.matdes.2006.02.007.

Barati, M.R. and Shahverdi, H. (2016), "A four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions", *Struct. Eng. Mech.*, **60**(4), 707-727. http://doi.org/10.12989/sem.2016.60.4.707.

Barati, M.R. and Shahverdi, H. (2017a), "An analytical solution for thermal vibration of compositionally graded nanoplates with arbitrary boundary conditions based on physical neutral surface position", *Mech. Adv. Mater. Struct.*, **24**(10), 840-853. https://doi.org/10.1080/15376494.2016.1196788.

Barati, M.R. and Shahverdi, H. (2017b), "Aero-hygro-thermal stability analysis of higher-order refined supersonic FGM panels with even and uneven porosity distributions", *J. Flui Struct*, **73**, 125-136. https://doi.org/10.1016/j.jfluidstructs.2017.06.007.

Barati, M.R. and Shahverdi, H. (2017c), "Small-scale effects on the dynamic response of inhomogeneous nanobeams on elastic substrate under uniform dynamic load", *Eur. Phys. J. Plus*, **132**(4), 167. https://doi.org/10.1140/epjp/i2017-11441-9.

Barati, M.R. and Shahverdi, H. (2017d), "Hygro-thermal vibration analysis of graded double-refined nanoplate systems using hybrid nonlocal stress-strain gradient theory", *Compos. Struct.*, **176**, 982-995. https://doi.org/10.1016/j.compstruct.2017.06.004.

Barati, M.R. and Shahverdi, H. (2017e), "Vibration analysis of porous functionally graded nanoplates", *J. Eng. Sci.*, **120**, 82-99. https://doi.org/10.1016/j.ijengsci.2017.06.008.

Barati, M.R. and Zenkour, A.M. (2017g), "Post-buckling analysis of refined shear deformable graphene platelet reinforced beams with porosities and geometrical imperfection", *Compos. Struct.*, **181**, 194-202. https://doi.org/10.1016/j.compstruct.2017.08.082.

Barati, M.R., Zenkour, A.M. and Shahverdi, H. (2016), "Thermo-mechanical buckling analysis of embedded nanosize FG plates in thermal environments via an inverse cotangential theory", *Compos. Struct.*, **141**, 203-212. https://doi.org/10.1016/j.compstruct.2016.01.056.

Barati, M.R., Zenkour, A.M. and Shahverdi, H. (2017f), "Electro-mechanical vibration of smart piezoelectric FG plates with porosities according to a refined four-variable theory", *Mech. Adv. Mater. Struct.*, **24**(12), 987-998. https://doi.org/10.1080/15376494.2016.1196799.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position”, *J. Braz. Soc. Mech. Sci. Eng*., **38**, 265-275. https://doi.org/10.1007/s40430-015-0354-0.

Bensaid, I. and Bekhadda, A. (2018), “Thermal stability analysis of temperature dependent inhomogeneous size-dependent nano-scale beams”, *Adv. Mater. Res*., **7**(1), 363-378. http://doi.org/10.12989/amr.2018.7.1.001.

Bensaid, I., Cheikh, A., Mangouchi, A. and Kerboua, B. (2017), “Static deflection and dynamic behavior of higher-order hyperbolic shear deformable compositionally graded beams”, *Adv. Mater. Res*., **6**(1), 13-26. https://doi.org/10.12989/amr.2017.6.1.013.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), “A new simple shear and normal deformations theory for functionally graded beams”, *Steel Compos. Struct*., **18**(2), 409-423. http://doi.org/10.12989/scs.2015.18.2.409.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), “A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates”, *J. Sandw. Struct. Mater*., **14**(1), 5-33. https://doi.org/10.1177/1099636211426386.

Bouremana, M, Houari, M.S.A., Tounsi, A., Kaci, A. and Adda Bedia, E.A. (2013), “A new first shear deformation beam theory based on neutral surface position for functionally graded beams”, *Steel Compos. Struct*., **15**(5), 467-479. http://doi.org/10.12989/scs.2013.15.5.467.

Chakraborty, A., Gopalakrishnan, S. and Reddy, J.N. (2003), “A new beam finite element for the analysis of functionally graded materials”, *Int. J. Mech. Sci*., **45**(3), 519-539. https://doi.org/10.1016/S0020-7403(03)00058-4.

Chian Thai, H., Zenkour, A.M., Abdel Wahab, M. and Nguyen-Xuan, H. (2016), “A simple four-unknown shear and normal deformations theory for functionally graded isotropic and sandwich plates based on isogeometric analysis”, *Compos. Struct*., **139**, 77-95. https://doi.org/10.1016/j.compstruct.2015.11.066.

Ebrahimi, F. and Barati, M.R. (2016a), “A nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams”, *Arab. J. Sci. Eng*., **41**(5), 1679-1690. https://doi.org/10.1007/s13369-015-1930-4.

Ebrahimi, F. and Barati, M.R. (2016b), “A unified formulation for dynamic analysis of nonlocal heterogeneous nanobeams in hygro-thermal environment”, *Appl. Phys. A*., **122**(9), 792. https://doi.org/10.1007/s00339-016-0322-2.

Ebrahimi, F. and Barati, M.R. (2016c), “Vibration analysis of nonlocal beams made of functionally graded material in thermal environment”, *Eur. Phys. J. Plus*, **131**(8), 279. https://doi.org/10.1140/epjp/i2016-16279-y.

Ebrahimi, F. and Barati, M.R. (2016d), “Wave propagation analysis of quasi-3D FG nanobeams in thermal environment based on nonlocal strain gradient theory”, *Appl. Phys. A*, **122**(9), 843. https://doi.org/10.1007/s00339-016-0368-1.

Ebrahimi, F. and Barati, M.R. (2016e), “A nonlocal higher-order refined magneto-electro-viscoelastic beam model for dynamic analysis of smart nanostructures”, *Int. J. Eng. Sci*., **107**, 183-196. https://doi.org/10.1016/j.ijengsci.2016.08.001.

Ebrahimi, F. and Barati, M.R. (2016f), “Electromechanical buckling behavior of smart piezoelectrically actuated higher-order size-dependent graded nanoscale beams in thermal environment”, *J. Smart Nano Mater*., **7**(2), 69-90. https://doi.org/10.1080/19475411.2016.1191556.

Ebrahimi, F. and Barati, M.R. (2016g), “Buckling analysis of nonlocal third-order shear deformable functionally graded piezoelectric nanobeams embedded in elastic medium”, *J. Brazil. Soc. Mech. Sci. Eng*., **39**(3), 1-16. https://doi.org/10.1007/s40430-016-0551-5.

Ebrahimi, F. and Barati, M.R. (2017a), “A nonlocal strain gradient refined beam model for buckling analysis of size-dependent shear-deformable curved FG nanobeams”, *Compos. Struct*., **159**, 174-182. https://doi.org/10.1016/j.compstruct.2016.09.058.

Ebrahimi, F. and Barati, M.R. (2017b), “Hygrothermal effects on vibration characteristics of viscoelastic FG nanobeams based on nonlocal strain gradient theory”, *Compos. Struct*., **159**, 433-444. https://doi.org/10.1016/j.compstruct.2016.09.092.

Hadji, L., Daouadji, T.H Ait Amar Meziane, M., Tlidji, Y. and Bedia, E.A. (2016), “Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech.*, **57**(2), 315-325. http://doi.org/10.12989/sem.2016.57.2.315.

Kaci, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech.*, **65**(5), 621-631. http://doi.org/10.12989/sem.2018.65.5.621.

Koizumi, M. (1997), "FGM activities in Japan", *Compos. Part B Eng.*, **28**(1-2), 1-4. https://doi.org/10.1016/S1359-8368(96)00016-9.

Li, X.F. (2008), "A unified approach for analyzing static and dynamic behaviors of functionally graded Timoshenko and Euler–Bernoulli beams", *J. Sound Vib.*, **318**(4-5), 1210-1229. https://doi.org/10.1016/j.jsv.2008.04.056.

Malikan, M. (2017), "Electro-mechanical shear buckling of piezoelectric nanoplate using modified couple stress theory based on simplified first order shear deformation theory", *Appl. Math. Mode.*, **48**, 196-207. https://doi.org/10.1016/j.apm.2017.03.065.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct.*, **18**(3), 793-809. http://doi.org/10.12989/scs.2015.18.3.793.

Nguyen, T.K. (2015), "A higher-order hyperbolic shear deformation plate model for analysis of functionally graded materials", *J. Mech. Mater. Des.*, **11**(2), 203-219. https://doi.org/10.1007/s10999-014-9260-3.

Nguyen, T.K., Vo, T.P. and Thai, H.T. (2013), "Static and free vibration of axially loaded functionally graded beams based on the first-order shear deformation theory", *Compos. Part B Eng.*, **55**, 147-157. https://doi.org/10.1016/j.compositesb.2013.06.011.

Ould, L.L., Kaci, A., Houari, M.S.A. and Tounsi, A. (2013), "An efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams", *Mech. Bas. Des. Struct.*, **41**(4), 421-433. https://doi.org/10.1080/15397734.2013.763713.

Pradhan, K.K. and Chakraverty, S.S. (2013), "Free vibration of Euler and Timoshenko functionally graded beams beams by Rayleigh–Ritz method", *Compos. Part B Eng.*, **51**, 175-184. https://doi.org/10.1016/j.compositesb.2013.02.027.

Reddy, J.N. (2002), *Energy Principles and Variational Methods in Applied Mechanics*, John Wiley & Sons Inc., New York, U.S.A.

Sallai, B.O., Tounsi, A., Mechab, I., Bachir, B.M., Meradjah, M. and Adda Bedia, E.A. (2009), "A theoretical analysis of flexional bending of Al/Al2O3 S-FGM thick beams", *Comput. Mater. Sci.*, **44**(4), 1344-1350. https://doi.org/10.1016/j.commatsci.2008.09.001.

Sankar, B.V. (2001), "An elasticity solution for functionally graded beams", *Compos. Sci. Technol.*, **61**(5), 689-696. https://doi.org/10.1016/S0266-3538(01)00007-0.

Simsek, M. (2010a), "Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories", *Nucl. Eng. Des.*, **240**(4), 697-705. https://doi.org/10.1016/j.nucengdes.2009.12.013.

Şimşek, M. (2010b), "Vibration analysis of a functionally graded beam under a moving mass by using different beam theories", *Compos. Struct.*, **92**(4), 904-917. https://doi.org/10.1016/j.compstruct.2009.09.030.

Şimşek, M. and Yurtcu, H.H. (2013), "Analytical solutions for bending and buckling of functionally graded nanobeams based on the nonlocal Timoshenko beam theory", *Compos. Struct.*, **97**, 378-386. https://doi.org/10.1016/j.compstruct.2012.10.038.

Sina, S.A., Navazi, H.M. and Haddadpour, H. (2009), "An analytical method for free vibration analysis of functionally graded beams", *Mater. Des.*, **30**(3), 741-747. https://doi.org/10.1016/j.jsv.2011.11.020.

Suresh, S. and Mortensen, A. (1998), *Fundamentals of Functionally Graded Materials*, IOM Communications Ltd., London, U.K.

Thai, H.T. and Choi, A. (2013a), "A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates", *Compos. Struct.*, **101**, 332-340. https://doi.org/10.1016/j.compstruct.2013.02.019.

Thai, H.T. and Choi, A. (2013b), "A simple first-order shear deformation theory for laminated composite plates", *Compos. Struct.*, **106**, 754-763. https://doi.org/10.1016/j.compstruct.2013.06.013.

Thai, H.T. and Vo, T.P. (2012), "Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories", *J. Mech. Sci.*, **62**(1), 57-66. https://doi.org/10.1016/j.ijmecsci.2012.05.014

Thai, H.T., Nguyen, T.K., Vo, T.P. and Lee, J. (2014), "Analysis of functionally graded sandwich plates using a new first-order shear deformation theory", *Eur. J. Mech.*, **45**, 211-225. https://doi.org/10.1016/j.euromechsol.2013.12.008.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013a), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220. https://doi.org/10.1016/j.ast.2011.11.009.

Vo, T.P., Thai, H.T. and Aydogdu, M. (2017), "Free vibration of axially loaded composite beams using a four-unknown shear and normal deformation theory", *Compos. Struct.*, **178**, 406-414. https://doi.org/10.1016/j.compstruct.2017.07.022

Vo, T.P., Thai, H.T., Nguyen, T.K. and Inam, F. (2014), "Static and vibration analysis of functionally graded beams using refined shear deformation theory", *Meccan.*, **49**, 155-168. https://doi.org/10.1007/s11012-013-9780-1.

Vo, T.P., Thai, H.T., Nguyen, T.K., Inam, F. and Lee, J. (2015), "A quasi-3D theory for vibration and buckling of functionally graded sandwich beams", *Compos. Struct.*, **119**, 1-12. https://doi.org/10.1016/j.compstruct.2014.08.006.

Zhong, Z. and Yu, T. (2007), "Analytical solution of a cantilever functionally graded beam", *Compos. Sci. Tech.*, **67**(3-4), 481-488. https://doi.org/10.1016/j.compscitech.2006.08.023.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017), "A novel simple two unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech.*, **64**(2), 145-153. http://doi.org/10.12989/sem.2017.64.2.145.

CC