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Simulation of stationary Gaussian stochastic wind velocity field

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Abstract. An improvement to the spectral representation algorithm for the simulation of wind velocity fields on large scale structures is proposed in this paper. The method proposed by Deodatis (1996) serves as the basis of the improved algorithm. Firstly, an interpolation approximation is introduced to simplify the computation of the lower triangular matrix with the Cholesky decomposition of the cross-spectral density (CSD) matrix, since each element of the triangular matrix varies continuously with the wind spectra frequency. Fast Fourier Transform (FFT) technique is used to further enhance the efficiency of computation. Secondly, as an alternative spectral representation, the vectors of the triangular matrix in the Deodatis formula are replaced using an appropriate number of eigenvectors with the spectral decomposition of the CSD matrix. Lastly, a turbulent wind velocity field through a vertical plane on a long-span bridge (span-wise) is simulated to illustrate the proposed schemes. It is noted that the proposed schemes require less computer memory and are more efficiently simulated than that obtained using the existing traditional method. Furthermore, the reliability of the interpolation approximation in the simulation of wind velocity field is confirmed.

Keywords: simulation; spectral representation; stationary; multivariate; wind velocity.

1. Introduction

One important aspect in the design of structures such as tall buildings, long-span cable-stayed or suspension bridges, towers, etc., is the analysis of their aerodynamic responses because these structures are prone to wind-induced vibration. In general, a time-domain approach based on the Monte Carlo method is more appropriate in predicting the dynamic response including some important nonlinear factors for large structures. In order to conduct aerodynamic response analysis in the time domain, simulation of the time histories of stochastic wind loads on a structure will be required, which are generally represented as functions of fluctuating wind velocity components $u(x,y,z,t)$, $v(x,y,z,t)$, and $w(x,y,z,t)$. The fluctuating wind velocities at given points in the space are usually modeled as normal stationary stochastic processes, characterized by their power spectral density (PSD) functions. The problem of simulating the discretized wind

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velocity field will be concentrated on the simulation of one-dimensional, multivariate stationary stochastic processes.

Although many methods are now available to simulate stochastic fields, the most common representations are: ARMA and AR models, filtered white noise (SDE), hot noise and filtered Poisson white noise, covariance decomposition, Karhunnn-Loéve (KL) and polynomial chaos expansion, spectral representation and wavelets representation (Spanos and Zeldin 1998). Among these various methods, the spectral representation methods appear to be most widely used because they are fast and conceptually straightforward. They are discussed in this paper.

The basic method for simulating one-dimensional, one-variate Gaussian process appeared in the 1950s. To our knowledge, Shinozuka (1971, 1972) first applied the spectral representation method for simulation purposes including multidimensional, multivariate, and non-stationary cases. Yang (1972) showed that the FFT technique could be used to significantly improve the computational efficiency of the spectral representation algorithm, and proposed a formula to simulate random envelope processes. Shinozuka (1974) extended the application of the FFT technique to multidimensional cases. Deodatis and Shinozuka (1989) then extended the application of the spectral representation method to the analysis of stochastic waves. Yamazaki and Shinozuka (1988) developed an iterative procedure to simulate non-Gaussian stochastic fields. Moreover, several reviewed papers on the subject of simulation using the spectral representation method were written by Shinozuka (1987), and Shinozuka and Deodatis (1991). In 1996, Deodatis (1996) further extended the spectral representation method to generate stochastic ergodic sample functions, each of which fulfills exactly the requirement of a target power spectrum.

Shinozuka (1990) applied the spectral representation method in wind engineering. For the simulation of wind velocity processes, Kovacs, *et al*. (1992) and later Mann (1998) proposed a simple formula based on Shinozuka and Jan's method, respectively. Di Paola (1998) suggested using eigenvectors with the spectral decomposition instead of the vectors of the triangular matrix with the Cholesky decomposition and discussed the physical significance of the eigenquantities. Introducing an explicit expression of the Cholesky decomposition of the special CSD matrix, Yang, *et al*. (1997) and Cao, *et al*. (2000) greatly improved Shinozuka's and Deodatis' method on simulating the wind velocity along the span-wise horizontal axis of bridge decks, respectively. Other researchers have developed various formulae for the simulation of wind velocity fields (Li and Kareem 1993, Solari and Carassale 2000, Grigoriu 2000, Li, *et al*. 2004). However, the number of wind velocity processes to be simulated will be very large in the response analysis of large scale structures, e.g., long-span bridges, when the stochastic wind loads on all major structural components such as deck, cable, tower, etc., are taken into consideration. Due to the cause of computational efforts, it is very difficult to do the simulation using the existing traditional methods.

In this paper, the algorithm of the spectral representation method is improved, which is considered suitable for the simulation of wind velocity fields on large scale structures. The method proposed by Deodatis (1996) serves as the basis of the improved algorithm.

2. Simulation of stationary stochastic processes

Consider a set of n one-dimensional stationary stochastic processes $\{f_j^0(t)\}(j = 1, 2, \ldots, n)$ with zero as their mean value, where the superscript 0 denotes the target function. The two-side target cross-spectral density matrix $\mathbf{S}^0(\omega)$ is given by

$$\mathbf{S}^0(\omega) = \begin{bmatrix} S_{11}^0(\omega) & S_{12}^0(\omega) & \cdots & S_{1n}^0(\omega) \\ S_{21}^0(\omega) & S_{22}^0(\omega) & \cdots & S_{2n}^0(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1}^0(\omega) & S_{n2}^0(\omega) & \cdots & S_{nn}^0(\omega) \end{bmatrix} \tag{1}$$

According to Deodatis (1996) and Shinozuka (1972), the stochastic process $f_j^0(t)$, $j = 1, 2, \ldots, n$, can be simulated by the following series

$$f_j(t) = 2\sqrt{\Delta\omega}\sum_{m=1}^{j}\sum_{l=1}^{N}\left|H_{jm}(\omega_{ml})\right|\cos(\omega_{ml}t - \theta_{jm}(\omega_{ml}) + \phi_{ml}) \tag{2}$$

where N = sufficiently large number; $\Delta\omega = \omega_{up}/N$ = frequency increment, ω_{up} = upper cutoff frequency, with the condition that, when $\omega > \omega_{up}$, the value of $\mathbf{S}^0(\omega)$ is trivial; ϕ_{ml} = sequences of independent random phase angles, uniformly distributed over the interval $[0, 2\pi]$; ω_{ml} = double-indexing frequency

$$\omega_{ml} = (l-1)\Delta\omega + \frac{m}{n}\Delta\omega, \qquad l = 1, 2, \ldots, N \tag{3}$$

and $H_{jm}(\omega)$ is a typical element of matrix $\mathbf{H}(\omega)$, which is defined with Cholesky decomposition of cross-spectral density matrix $\mathbf{S}^0(\omega)$; thus

$$\mathbf{S}^0(\omega) = \mathbf{H}(\omega)\,\mathbf{H}^{T*}(\omega) \tag{4}$$

$$\mathbf{H}(\omega) = \begin{bmatrix} H_{11}(\omega) & 0 & \cdots & 0 \\ H_{21}(\omega) & H_{22}(\omega) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ H_{n1}(\omega) & H_{n2}(\omega) & \cdots & H_{nn}(\omega) \end{bmatrix}$$

$\theta_{jm}(\omega)$ = complex angle of $H_{jm}(\omega)$ and is given by

$$\theta_{jm}(\omega) = \tan^{-1}\left\{\frac{\mathrm{Im}[H_{jm}(\omega)]}{\mathrm{Re}[H_{jm}(\omega)]}\right\} \tag{5}$$

where $\mathrm{Im}[H_{jm}(\omega)]$ and $\mathrm{Re}[H_{jm}(\omega)]$ are the imaginary and real parts of complex function $H_{jm}(\omega)$ respectively. It has been proved that both ensemble and temporal auto-/cross-correlation functions of any sample function by Eq. (2) will approach to the target auto-/cross-correlation functions as $\Delta\omega \to 0$ and $N \to \infty$.

In order to avoid aliasing, the time step Δt has to obey the condition

$$\Delta t \le \frac{2\pi}{2\omega_{up}} \tag{6}$$

The period of the sample functions expressed by Eq. (2) is

$$T_0 = \frac{2\pi n}{\Delta\omega} = \frac{2\pi n N}{\omega_{up}} \tag{7}$$

The ergodicity of the results of (2) has been proved by Deodatis (1996). It is clear that the stationary one-dimensional, multivariate, Gaussian stochastic process can be simulated quite well by means of expression (2), when the cross-spectral density matrix has been given and values of the parameters N, ω_{up}, and $\Delta\omega$ have been properly chosen.

3. Improved spectral representation method

Since $\mathbf{H}(\omega)$ is functions of ω, it can be seen from the structure of (2) that the Cholesky decomposition has to be conducted separately for every frequency ω_{ml}. Thus the computational effort and memory expense are enormous. The approach obviously becomes computationally prohibitive when a large number of wind velocity time histories are to be simulated, which is quite often the case for the time domain aerodynamic analysis of a large scale structure.

When the quadrature spectrum of wind velocity can be neglected, the cross-spectral density matrix $\mathbf{S}^0(\omega)$ is real and symmetric. Then the elements of the lower triangular matrix $\mathbf{H}(\omega)$ with Cholesky decomposition are also real numbers. It is noted that each element of $\mathbf{H}(\omega)$ varies continuously with the frequency of the wind spectra. Therefore, as long as $\mathbf{H}(\omega)$ at some appropriate frequency points are computed, $\mathbf{H}(\omega)$ at other frequency points can be obtained by an interpolation approach. There are many different ways to interpolate the functions, here a cubic Lagrangian interpolation is utilized for the purpose as shown in Fig. 1.

$$\tilde{H}_{jk}(\omega) = \sum_{l=i-1}^{i+2} H_{jk}(\omega_l) L_l(\omega) \tag{8}$$

where $L_l(\omega)$ = the cubic Lagrangian interpolation function.

Introducing the interpolation of $\mathbf{H}(\omega)$, the formula for simulating the fluctuating wind velocity becomes

$$f_j(t) = 2\sqrt{\Delta\omega} \sum_{m=1}^{j} \sum_{l=1}^{N} \tilde{H}_{jm}(\omega_{ml}) \cos(\omega_{ml} t + \phi_{ml}) \tag{9}$$

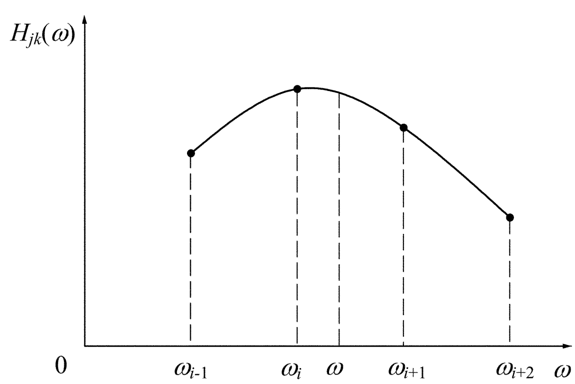

Fig. 1 Cubic Lagrangian interpolation of $H_{jk}(\omega)$

The ergodicity of Eq. (9) can be proved to be similar to Deodatis' approach and is omitted here for brevity.

The process of digitally generating sample functions of the simulated stochastic vector can be significantly simplified using the FFT technique (Yang 1972, Deodatis 1996). To take advantage of the FFT technique, Eq. (9) is rewritten explicitly in the following form:

$$f_j(p\Delta t) = Re\left\{\sum_{m=1}^{j} C_{jm}(q\Delta t)\exp\left[i\left(\frac{m\Delta\omega}{n}\right)(p\Delta t)\right]\right\} \tag{10}$$

$$p = 0,1,\ldots,\quad M\times n-1,\quad j=1,2,\ldots,n$$

where $q = 0,1,\ldots, M-1$, is the remainder of p/M; $M \ge 2N$; and $C_{jm}(q\Delta t)$ is given by

$$C_{jm}(q\Delta t) = \sum_{l=0}^{M-1} B_{jm}(l\Delta\omega)\exp\left(ilq\frac{2\pi}{M}\right) \tag{11}$$

where, in the current paper, $B_{jm}(l\Delta\omega)$ is expressed with the following equation

$$B_{jm}(l\Delta\omega) = \begin{cases} 2\sqrt{\Delta\omega}\tilde{H}_{jm}\left(l\Delta\omega + \frac{m\Delta\omega}{n}\right)\exp(i\phi_{ml}) & 0 \le l < N \\ 0 & N \le l < M \end{cases} \tag{12}$$

It can be seen from Eqs. (11) and (12) that $C_{jm}(q\Delta t)$ is the Fourier transformation of $B_{jm}(l\Delta\omega)$ and can thus be computed with the FFT technique.

4. Alternative spectral representation

In fact, the above-mentioned algorithm for simulating stochastic wind field is suitable for any orthogonal decomposition of the cross-spectral density matrix $\mathbf{S}^0(\omega)$, and there are infinite ways for the orthogonal decomposition of the matrix $\mathbf{S}^0(\omega)$. Di Paola (1998) has demonstrated the significance of the spectral (modal) decomposition of $\mathbf{S}^0(\omega)$ in his simulation method.

Since the CSD matrix $\mathbf{S}^0(\omega)$ in the simulation of stochastic wind velocity field is real and symmetric, the spectral decomposition of matrix $\mathbf{S}^0(\omega)$ can be written as follows:

$$\mathbf{S}^0(\omega)\mathbf{\Phi}(\omega) = \mathbf{\Phi}(\omega)\mathbf{\Lambda}(\omega)$$

$$\mathbf{\Phi}^T(\omega)\mathbf{\Phi}(\omega) = I \tag{13}$$

where $\mathbf{\Lambda}(\omega)$ = a diagonal matrix listing the eigenvalues of the matrix $\mathbf{S}^0(\omega)$; I = unit matrix. The eigenvectors $\phi_k(\omega)$ are real and orthogonal.

Eq. (13) can be rewritten as

$$\mathbf{S}^0(\omega) = \mathbf{\Phi}(\omega)\mathbf{\Lambda}(\omega)\mathbf{\Phi}^T(\omega) \tag{14}$$

Denoting $\mathbf{G}(\omega) = \mathbf{\Phi}(\omega)\sqrt{\mathbf{\Lambda}(\omega)}$, then

$$\mathbf{S}^0(\omega) = \mathbf{G}(\omega)\mathbf{G}^T(\omega) \tag{15}$$

After the vectors of the triangular matrix with the Cholesky decomposition are replaced using the eigenvectors with the spectral decomposition, the above formula for simulating fluctuating wind velocities becomes

$$f_j(t) = 2\sqrt{\Delta\omega}\sum_{m=1}^{K}\sum_{l=1}^{N}|G_{jm}(\omega_{ml})|\cos(\omega_{ml}t + \phi_{ml}) \tag{16}$$

where K is the number of eigenvectors.

By comparing Eqs. (16) and (2) using the eigenvectors instead of the vectors of the triangular matrix, the full-mode spectral decomposition does not seem to be a good choice for digital simulation purposes. In fact, the full-mode spectral decomposition (13) requires a greater computational effort with respect to that necessary for the computation of the matrix $\mathbf{H}(\omega)$. Fortunately, only few spectral modes exhibit significant power, one can truncate the spectral modal matrix retaining only the significant waves, i.e., $K \ll n$, in this way we only need to evaluate the first K eigenvectors. Therefore, the spectral decomposition of the CSD matrix on the basis of eigenvectors becomes very competitive with the classical Cholesky decomposition in the simulation of large wind velocity vectors.

In Fig. 2, a typical sample of eigenvalues and eigenvectors of the CSD matrix is plotted as a function of the wind spectra frequency. It is noted from these figures and also Di Paola's illustration (1998) that both eigenvalues and eigenvectors are regular functions with the wind spectra frequency. Therefore, in order to reduce the computational effort in the simulation processes, the above-mentioned interpolation approximation can be utilized. When the concerned eigenvalues and eigenvectors at some frequency points are evaluated, $\mathbf{G}(\omega)$ at other frequency points can be made available by a linear interpolation as follows

$$\tilde{G}_{jk}(\omega) = \frac{\omega_{i+1} - \omega}{\omega_{i+1} - \omega_i}G_{jk}(\omega_i) + \frac{\omega - \omega_i}{\omega_{i+1} - \omega_i}G_{jk}(\omega_{i+1}) \tag{17}$$

and then the alternative formula for simulating fluctuating wind velocities becomes

$$f_j(t) = 2\sqrt{\Delta\omega}\sum_{m=1}^{K}\sum_{l=1}^{N}\tilde{G}_{jm}(\omega_{ml})\cos(\omega_{ml}t + \phi_{ml}) \tag{18}$$

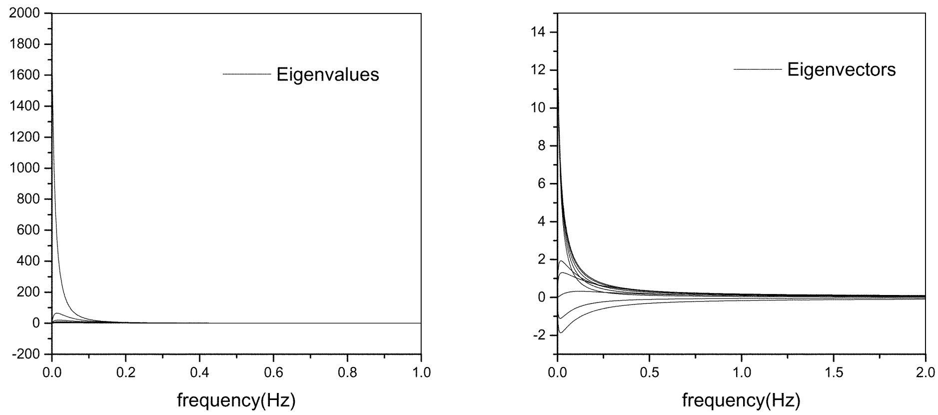

Fig. 2 Eigenvalues and eigenvectors of the CSD matrix versus the frequency

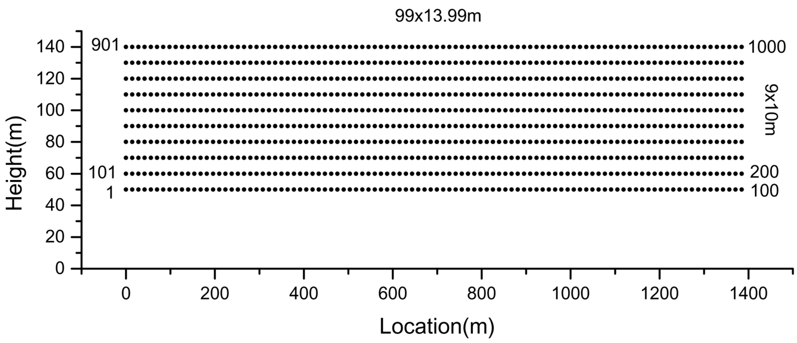

Fig. 3 Simulation of horizontal wind velocity on 1000 spatial points

the double-indexing frequency becomes

$$\omega_{ml} = (l-1)\Delta\omega + \frac{m}{K}\Delta\omega, \qquad l = 1, 2, \ldots, N \tag{19}$$

where K = the number of applied eigenvectors. Similarly, the efficiency of simulation can also be greatly enhanced by utilizing the FFT technique.

5. Numerical example

An artificial horizontal wind velocity field through a vertical plane on a long-span bridge (spanwise) is simulated using the proposed schemes. The simulated region comprising 1000(= 100 × 10) points is shown in Fig. 3. The main data of the simulating conditions are as follows:

Total size of simulated region: Length = 1385 m, Height = 90 m
Height of the lowest simulated location above water level: z = 50 m
Mean wind velocity at a height of z = 50 m: $U(50)$ = 40.0 m/s
Ground roughness length: $z_0 = 0.03$
Upper cutoff frequency: $\omega_{up} = 4\pi$ rad/s
Dividing number of frequency: N = 2048
Time interval: Δt = 0.25 s
Period: T = 1024000 s

Kaimal's spectrum formula for horizontal turbulent wind velocity is defined as

$$\frac{nS_u(\omega)}{u_*^2} = \frac{200f}{(1+50f)^{5/3}} \tag{20}$$

where

$$f = \frac{nz}{U(z)}, \quad u_* = \frac{KU(z)}{\ln\left(\frac{z}{z_0}\right)}, \quad K = 0.4$$

Davenport's coherence function is computed by

$$\mathrm{coh}(r,\omega) = \exp\left(-\frac{\omega}{2\pi}\frac{\left[C_z^2(z_1-z_2)^2 + C_y^2(y_1-y_2)^2\right]^{1/2}}{\frac{1}{2}\left[U(z_1)+U(z_2)\right]}\right) \tag{21}$$

and C_z and C_y are taken as 10 and 16, respectively (Simiu and Scanlan 1986).

To illustrate the schemes proposed by the authors, the simulation of the turbulent wind velocity components has been carried out on a personal computer with a Pentium-IV CPU. The comparison of several different schemes, taking advantage of the FFT technique, is listed in Table 1. The Scheme 1 and Scheme 2 are simulated using the improved spectral representation method with the Cholesky decomposition and the alternative spectral decomposition technique, respectively. Both the

Table 1 Comparison of several different schemes for wind velocity simulation

Methods	Memory expense (Mb)	Time elapsed (min)
Existing method (estimated)	4104.1	244.8
Proposed method (Scheme 1)	260.3	37.3
Proposed method (Scheme 2)	55.2	18.2

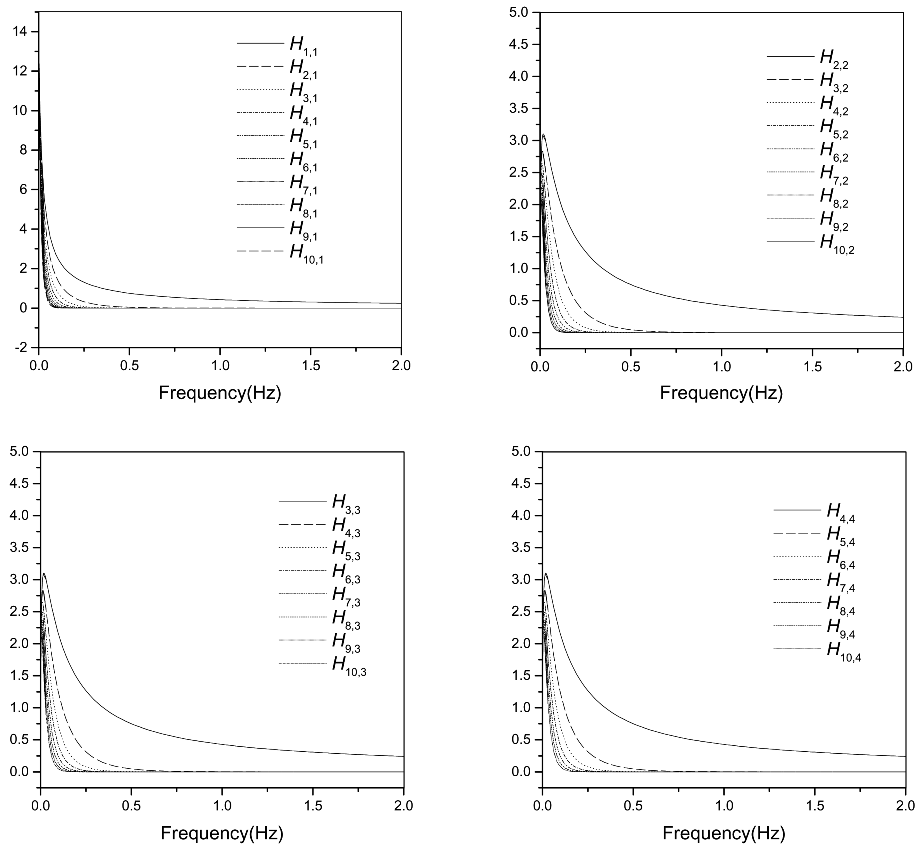

Fig. 4 Several typical vectors of the matrix $H(\omega)$ versus the frequency

Cholesky decomposition and the alternative spectral decomposition with 100 modes of the CSD matrix are only conducted on 148 frequency points. It is noted that both schemes require less computer memory and are more efficiently simulated than that obtained using the existing method. It can be seen that the simulation of Scheme 2 with the alternative spectral decomposition is very competitive on the computational aspect when the number of modes is appropriately selected.

In Fig. 4 several typical vectors of the lower triangular matrix with the Cholesky decomposition of the CSD matrix are plotted as a function of wind spectra frequency as an example. It is noted that each component of the lower triangular matrix $\mathbf{H}(\omega)$ varies continuously with the frequency. Thus, the interpolation approximation applied to the element of $\mathbf{H}(\omega)$ is considered suitable for the simulation of wind velocity field.

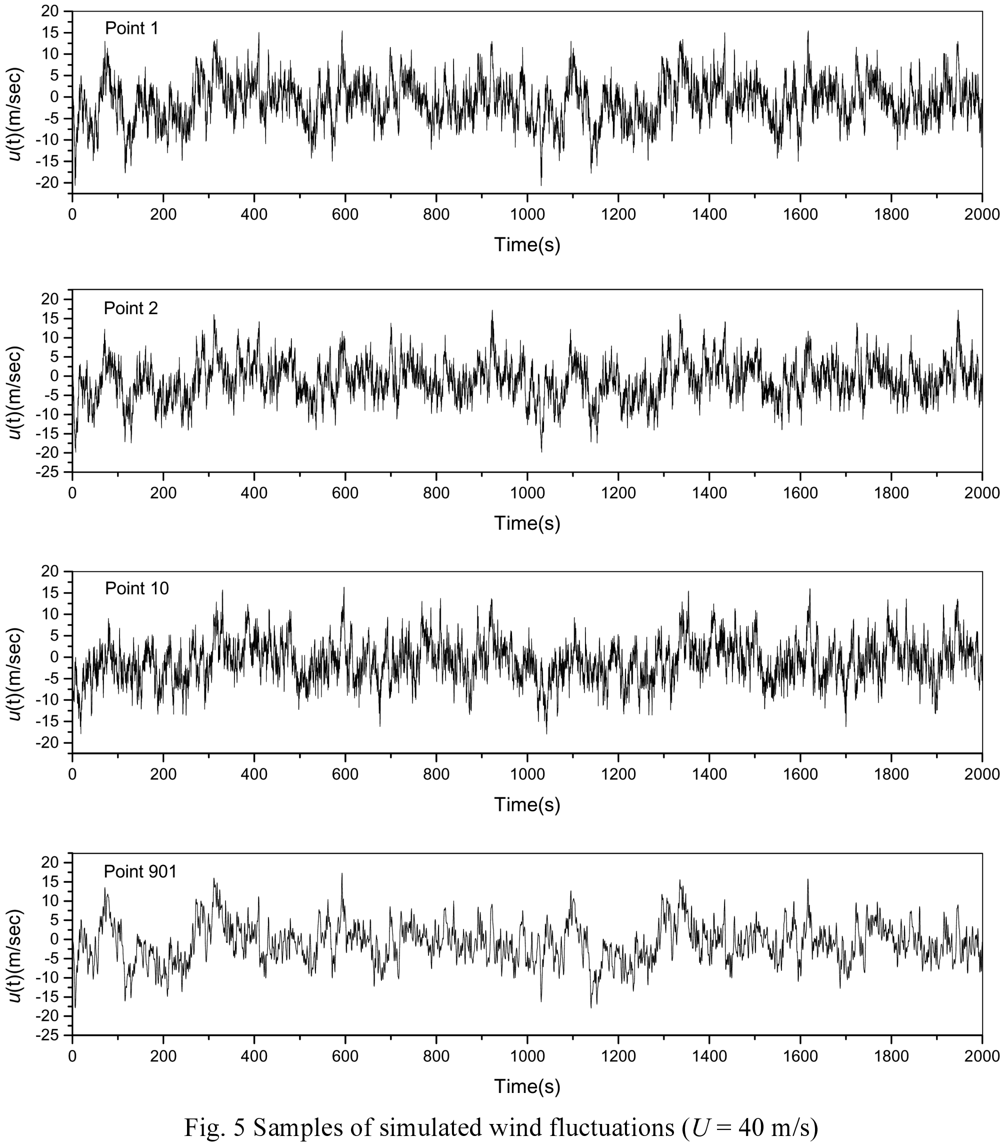

Fig. 5 Samples of simulated wind fluctuations (U = 40 m/s)

Fig. 5 shows the first 2000 sec samples of the simulated wind velocities at points 1, 2, 10, 901. It can be seen that the wind velocities at points 1 and 2 have a quite strong correlation between them, since they are close to each other. However, there is a considerable loss of coherence between the wind velocities of points 1 and 10, since the two points are 125 meters apart.

Using the Scheme 1 with the Cholesky decomposition, the temporal auto- and cross-correlation functions of simulated wind velocities at some typical points are shown as Fig. 6. It is seen that the temporal auto- and cross-correlation functions of simulated wind velocities have good agreement with their targets, which are indicated with a superscript 0. Therefore, the reliability of the interpolation approximation applied to the element of $\mathbf{H}(\omega)$ in the simulation of wind velocity field is confirmed.

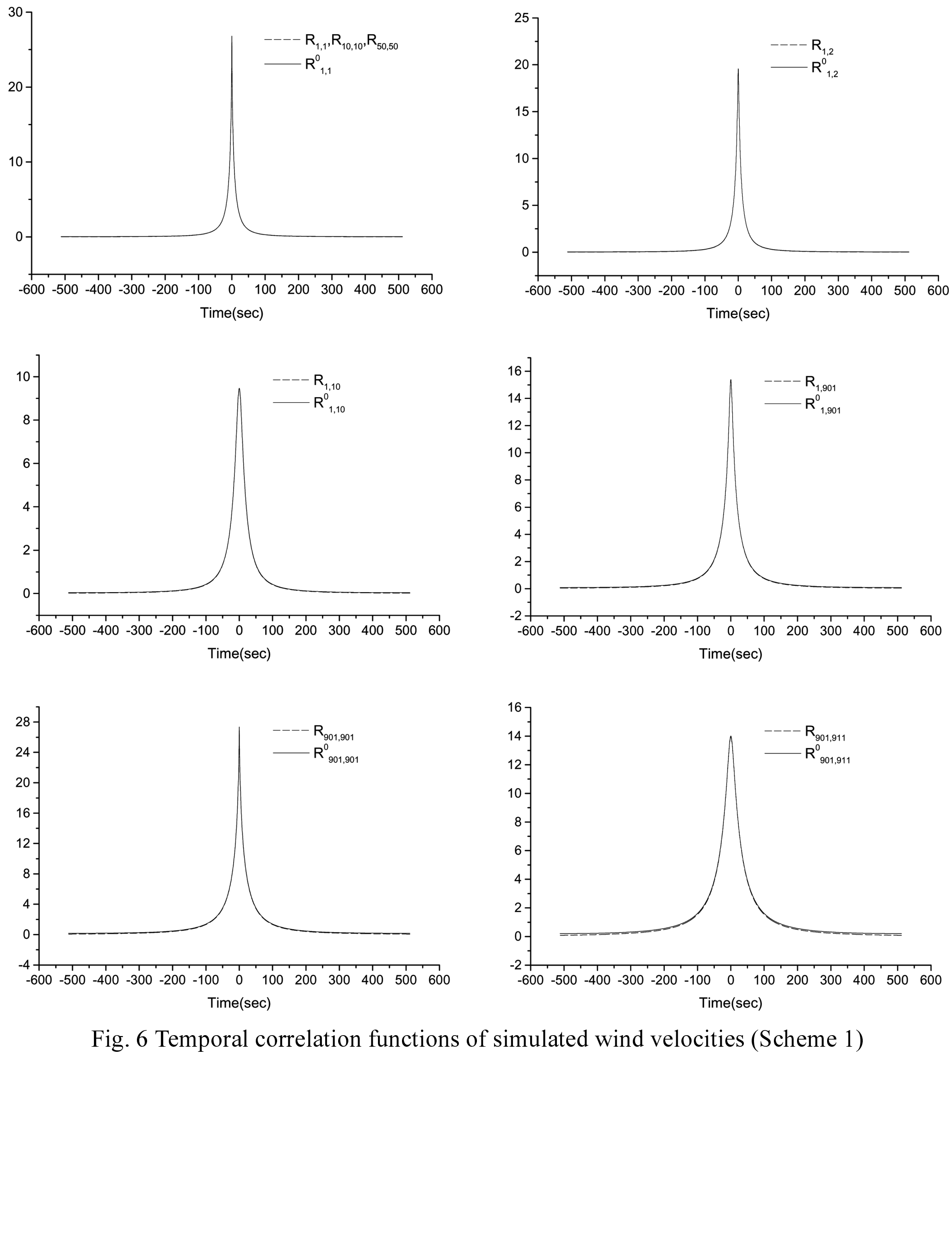

Fig. 6 Temporal correlation functions of simulated wind velocities (Scheme 1)

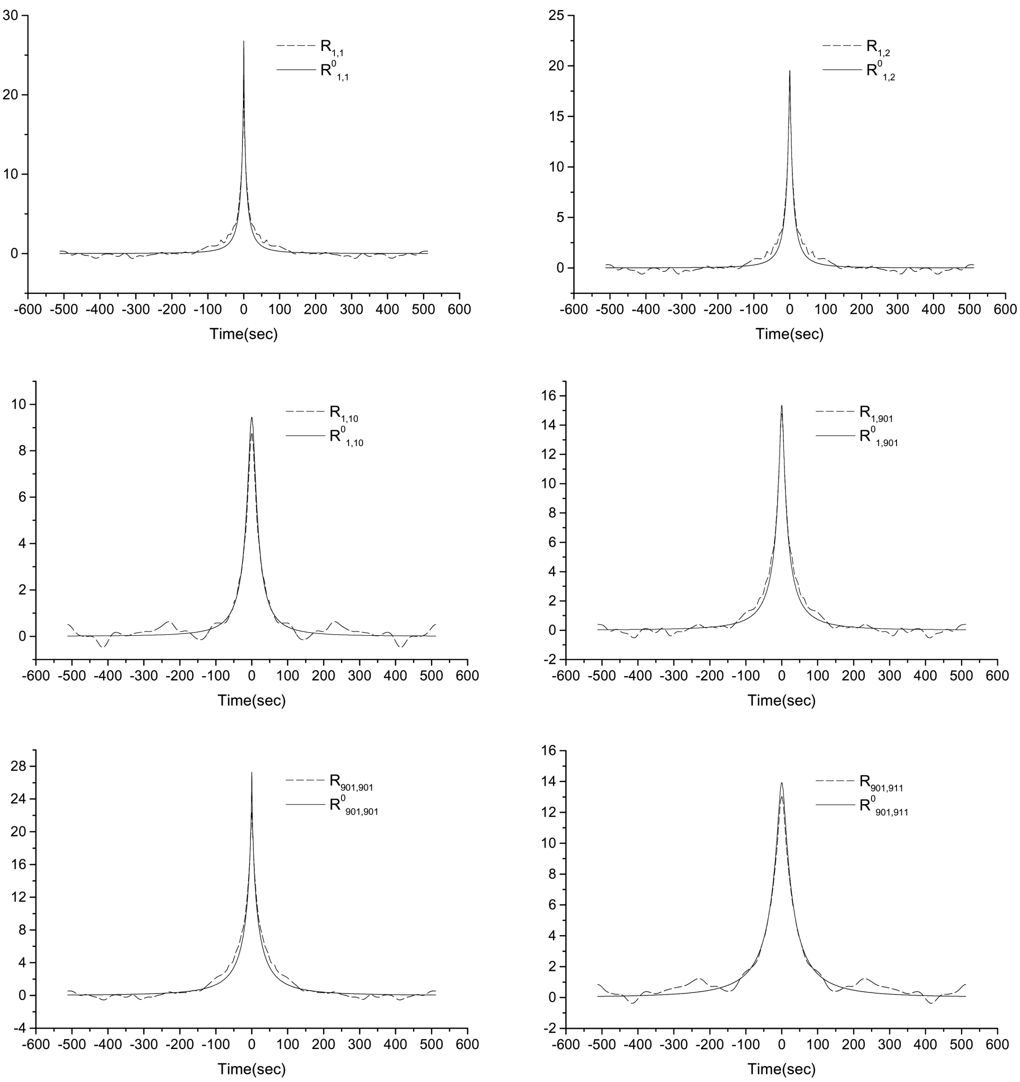

Fig. 7 Temporal correlation functions of simulated wind velocities (Scheme 2)

Fig. 7 shows the corresponding temporal auto- and cross-correlation functions of simulated wind velocities using the Scheme 2 with 100 modes. It is noted that there are general agreements between the simulated auto- and cross-correlation functions of wind velocities and their targets. Although the truncation of the modes has slightly affected the accuracy, the simulation results are still considered satisfactory.

6. Concluding remarks

The algorithm of the spectral representation method proposed by Deodatis has been improved for the simulation of wind velocity fields on large scale structures in this paper. An interpolation approximation is introduced to simplify the computation of both the vectors of the lower triangular matrix with the Cholesky decomposition and the eigenquantities with the alternative spectral decomposition of the CSD matrix. FFT technique are used to further enhance the efficiency of computation in each case.

A turbulent wind velocity field through a vertical plane on a long-span bridge is simulated using the proposed schemes. It is noted that the proposed schemes require less computer memory and are more efficiently simulated than that obtained using the existing method. It is also noted that the simulation using the alternative spectral decomposition is very competitive on the computational aspect when the number of modes is appropriately selected.

It is seen that the temporal auto- and cross-correlation functions of simulated wind velocities with the Cholesky decomposition have good agreement with their targets. It is also seen that there are general agreements between the simulated auto- and cross-correlation functions and the target of wind velocities using the 100-mode spectral decomposition. Furthermore, the reliability of the interpolation approximation in the simulation of wind velocity field is confirmed.

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