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Errors in GEV analysis of wind epoch maxima from Weibull parents

R. I. Harris†

RWDI-Anemos Ltd, Unit 4, Lawrence Way, Dunstable LU6 1BD, UK
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Abstract. Parent wind data are often acknowledged to fit a Weibull probability distribution, implying that wind epoch maxima should fall into the domain of attraction of the Gumbel (Type I) extreme value distribution. However, observations of wind epoch maxima are not fitted well by this distribution and a Generalised Extreme Value (GEV) analysis leading to a Type III fit empirically appears to be better. Thus there is an apparent paradox. The reasons why advocates of the GEV approach seem to prefer it are briefly summarised. This paper gives a detailed analysis of the errors involved when the GEV is fitted to epoch maxima of Weibull origin. It is shown that the results in terms of the shape parameter are an artefact of these errors. The errors are unavoidable with the present sample sizes. If proper significance tests are applied, then the null hypothesis of a Type I fit, as predicted by theory, will almost always be retained. The GEV leads to an unacceptable ambiguity in defining design loads. For these reasons, it is concluded that the GEV approach does not seem to be a sensible option.

Keywords: extreme wind speeds; parent wind speeds; GEV; Weibull.

1. Introduction

Galambos (1978) has shown that for serially correlated data drawn from a population with a cumulative probability, $P(V)$, the maximum value in an epoch of T years itself has a cumulative probability distribution given by $[P(V)]^{rT}$. Here r is the annual rate parameter. Often for wind data $P(V)$, or more usually r, is not known with sufficient precision for this result to be used to predict epoch extremes and hence design loads. Instead it becomes necessary to use methods known generically as Extreme Value Statistics. The work of Fisher and Tippett (1928), which was expanded in the text by Gumbel (1958), showed that as $rT \to \infty$, $[P(V)]^{rT}$ tends to one of three asymptotes, called by Gumbel the Types I, II and III. Which of these is obtained depends on the behaviour of $P(V)$ for large V.

Von Mises (1936) showed that these three classical asymptotes could be represented by one common analytical form which has subsequently become known as the Generalized Extreme Value distribution, or GEV :-

$$\Psi_T\{\hat{V}_T < v\} = \exp[-\{1 - \alpha_T k(v - U_T)\}^{1/k}] \tag{1}$$

† Consultant, E-mail: riharris@anemos.co.uk

Here $\hat{V}_T$ denotes the maximum value in an epoch T, α_T and U_T are respectively dispersion and location parameters for that epoch, and k is the shape parameter. It is sometimes stated that the GEV represents a family of distributions. The fact is that every GEV for which $k > 0$ is a classical Type III asymptote (or Reversed Weibull distribution). For $k < 0$ it is a classical Type II asymptote (or Frechet distribution), while the transitional case corresponding to the limit of Eq. (1) as $k \to 0$ is the classical Type I asymptote (or Gumbel distribution).

There are over 400 references to the use of the Weibull distribution for wind speeds in the Scirus database of peer-reviewed journals. Many of these demonstrate good fits of data to a parent cumulative probability distribution of the Weibull form :-

$$P(V) = 1 - \exp[-(V/V_0)^w] \tag{2}$$

It is generally now accepted that the wind climate of many sites is made up of contributions from a number of independent physical mechanisms, e.g., depressions, thunderstorms, etc. which may be assumed to act exclusively. In some cases this results in a mixed climate for extremes: for instance, in parts of Australia the annual extreme of wind speed may be generated by a cyclone, a thunderstorm or an extra-tropical depression. In temperate latitudes, for instance the UK, the wind climate for annual extremes is designated as 'simple' because all the annual extremes are generated by one physical mechanism – temperate depressions. However, even for such a 'simple' climate, the totality of wind speeds will derive from many other causes – thunderstorms, sea-breezes, etc. Thus to fit a parent distribution, it is necessary first to separate the various components, which can often be done synoptically.

Cook, *et al*. (2003) demonstrated this for some Australian sites and found that the individual components were each a good fit to the Weibull form but, of course, with different parameters. More recently, Cook and Harris have used the Jenkinson-Lamb index of UK weather types to extract the dominant cyclonic component from 30 years' hourly mean wind data recorded at Boscombe Down in the UK. The resultant excellent fit to the Weibull form of the cumulative probability, irrespective of wind direction, is shown in Fig. 1.

Maxima from a Weibull parent lie in the domain of attraction of the Type I asymptote, thus, on

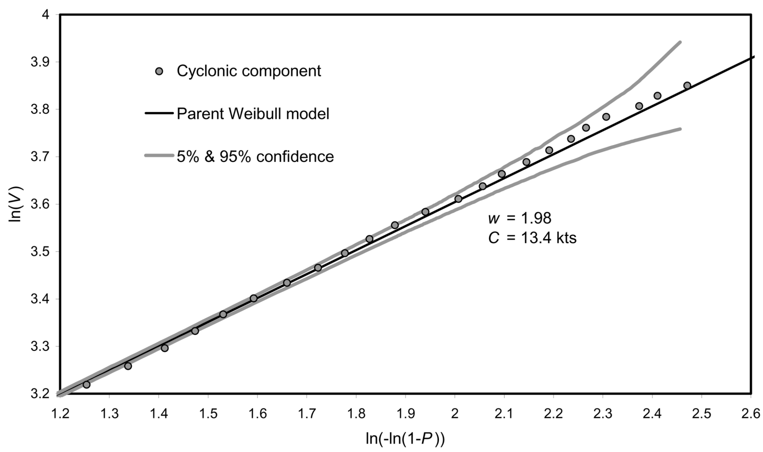

Fig. 1 Upper tail of Weibull plot of cyclonic component of parent mean wind speed (>25 kts) with bootstrap confidence limits

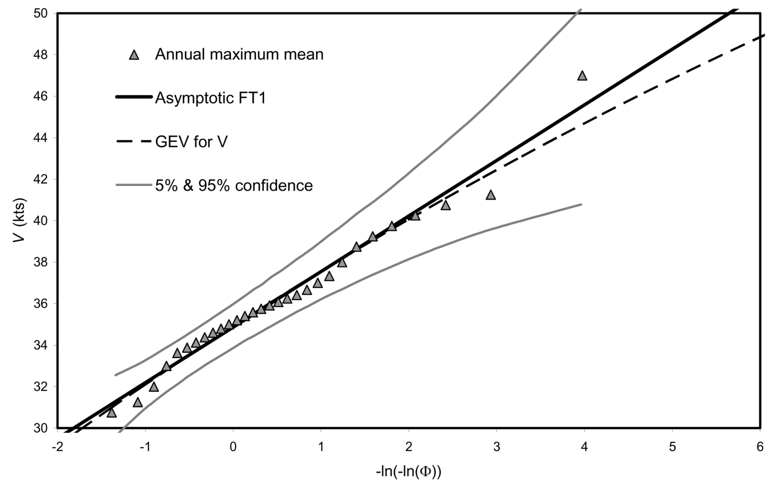

Fig. 2 Gumbel plot for annual maximum hourly mean wind speeds from observed annual maxima at Boscombe Down

the basis of Fig. 1 the annual extremes from Boscombe Down should be converging towards the Type 1 asymptote. The 30 annual maxima from the Boscombe Down data were extracted and subjected to extreme value analysis using either the Type I asymptote or the GEV approach, and the results are shown in Fig. 2. Empirically, the GEV with a positive value of k (~0.054) produces the better fit to the data. This implies a Type III asymptotic fit with an upper bound to the wind speed of 85 knots.

Thus there is an apparent paradox. Analysis of the parent data, including the 30 annual maxima in the upper tail, suggests a Weibull fit which implies a Type I asymptote, unlimited to the right. If, however, the 30 annual maxima are extracted and analysed separately by the GEV, then a Type III asymptote with an implied upper limit, is indicated. These two propositions are incompatible, so it becomes a question of which one is to be believed.

Those who favour the Type III outcome may do so for some of the following reasons:−

(i) There is continual commercial pressure to reduce design loads. As illustrated by Fig. 2, the Type III fit to a data set always predicts lower design values than the Type I fit.

(ii) There is a belief that there must be a natural upper limit to wind speeds.

(iii) Fitting the GEV avoids assuming to which domain of attraction the parent belongs … the extreme value data should be allowed "to speak for itself".

(iv) As the GEV represents all three classical asymptotes, in the limit predictions based on it will converge to the correct value, and it is possible to place error bounds on predictions from finite samples.

The following comments are appropriate: −

Firstly, because of the implications for public safety, any reductions in design loads must be rigorously justified.

Secondly, the existence of an upper limit is not sufficient grounds for assuming a Type III fit, Harris (2004). A Type III fit also requires that at the limit point the first n derivatives must also vanish, where n is the largest integer $< 1/k$. With typically reported values of $k \sim 0.1$, this implies ~10 derivatives, which in turn requires something fairly dramatic in the physics. One possible limit point is the speed of sound, but this is well above the limits reported in the literature. There have been attempts to establish limits for hurricanes and thunderstorms by applying classical equilibrium thermodynamics. These are based on the limit to the production of mechanical energy by the difference between the temperatures of the hot and cold reservoirs when the atmosphere behaves as a heat engine. Apart from any doubts as to whether equilibrium thermodynamics is applicable to the obviously non-equilibrium situation, the difficulty in applying these ideas to temperate depressions is that the thermodynamic efficiency should be about the same for any depression. The putative limit of 85 kts at Boscombe Down is likely to be exceeded by observations elsewhere in the UK in storms produced by the same physical mechanisms, suggesting that this ‘limit’ is an artefact of the GEV analysis, rather than a real physical limit.

Thirdly, inspection of the GEV form Eq. (1) reveals that fitting it to observed data can never yield $k = 0$ because this corresponds to a singularity in the index. Thus, far from leaving the domain of attraction open, a GEV fit rules out what, on the basis of evidence such as Fig. 1, is the most likely option. If the data “speaks for itself” the listener must have a “critical ear”! A corollary is that any value of k obtained from a GEV fit must be subjected to a significance test to decide whether a non-zero value is more likely than not to be an artefact of sampling error.

Fourthly, it is not obvious that it is possible to place realistic bounds on GEV results without some knowledge of the parent distribution.

From first principles the fit of the parent data to a Weibull distribution, as exemplified by Fig. 1, which is based on several hundred data points (as reflected in the tight error bounds) ought to be more reliable than the fit (as in Fig. 2) which is based on just 30 data points. There is, of course, no guarantee that the parent shown in Fig. 1 will not depart from the Weibull form for larger values of wind speed, but no evidence of that can be deduced from the values shown, which lie within an excellent fit to the Weibull form.

In recent papers, Harris (2004), Cook and Harris (2004) have shown that the paradox arises because asymptotic forms valid when $rT \to \infty$ are being applied in situations where for temperate storms $rT \sim 150$, for thunderstorm downbursts $rT \sim 50$, and for tropical cyclones and hurricanes $rT \sim 1$ – this last value being so very low as to invalidate any conventional extreme value analysis.

The ultimate asymptotic FT1 line in Fig. 2 fails to follow the observations because with rT only ~ 150, the penultimate form (which is a curve) should be used. When it is used, a fit at least as good as the GEV is produced, Cook and Harris (2004).

The better fit of the GEV in Fig. 2 arises solely because it has three disposable parameters instead of just two, which allows it to follow the curve of the data. Castillo (1988) and Gomes (1984) note that this is the only argument that supports the use of the Type III distribution as an approximation for un-converged distributions in the domain of attraction of Type I. This usage brings with it unwanted side-effects in the form of an apparent upper limit and a systematic error in extrapolation to long return periods. This extrapolation is already difficult enough because of random sampling errors.

The objective of this paper is to explore in some detail the errors which arise in applying a GEV fit to epoch maxima drawn from a Weibull parent. If these account in a rational way for the effects observed when the GEV is used with practical data, then the paradox of Figs. 1 and 2 can thereby be resolved.

2. Errors in applying GEV analysis to practical data

For most engineers, the seminal paper on GEV analysis is that by Hosking, *et al.* (1985). This leads on to the use of the method of Probability Weighted Moments (PWM), but also contains comparisons with the Maximum Likelihood (ML) method favoured by statisticians, which show that

PWM performs as well as ML. Most engineers seem to prefer the PWM method, thus, when it is necessary to adopt a particular method in this paper, PWM will be used.

The main errors in applying GEV analysis can be divided into three distinct categories:

(A) Convergence errors … These arise because asymptotic results which depend on $rT \to \infty$ for their validity are being applied in situations where rT is too low. The feature of these errors is that they cannot be removed by increasing the size of the data sample unless rT is also increased[1]. These errors can be calculated analytically from the parent distribution without sampled data being involved, if the parent distribution is known.

(B) Sampling error … Any parameter calculated from sampled data will have a random error due to the departures of the individual ranked values from their mean values.

(C) Bias errors … These arise when GEV parameters are estimated from data samples. Almost all estimation methods involve ranking the data and it is well known from the theory of order statistics that the distributions of the uppermost samples about their mean positions are strongly skewed, and the variances are systematically larger. When ranked samples are combined to calculate a GEV parameter, the skewness and behaviour of the variance combine to produce a biased result.

The paper by Hosking, *et al.* (1985) contains a quite extensive treatment of (B) and (C), but is silent on (A).

3. Convergence errors

The parent data is assumed to have a Weibull distribution, so that : –

$$P(V) = 1 - \exp[-(V/V_0)^w] \tag{3}$$

For any distribution the PWM of order $n-1$ is defined by: –

$$\beta_{n-1} = E\{V[P(V)]^{n-1}\} \qquad n = 1, 2, 3, \ldots \tag{4}$$

where E is the expectation operator. It follows for a Weibull parent: –

$$\beta_{n-1} = \int_0^\infty wrT[1 - \exp[-(V/V_0)]^w]^{nrT-1}(V/V_0)^w \exp[[-(V/V_0)]^w]dV \tag{5}$$

and on introducing the substitution $z = (V/V_0)^w$, this becomes : –

$$\beta_{n-1} = V_0 \int_0^\infty rTz^{1/w}[1 - e^{-z}]^{nrT-1} e^{-z} dz \tag{6}$$

Hosking, *et al.* (1985) gave the following relationship between the shape factor k and the β PWMs : –

$$\frac{2\beta_1 - \beta_0}{3\beta_2 - \beta_0} = \frac{1 - 2^{-k}}{1 - 3^{-k}} \tag{7}$$

Note that Eq. (7) does not involve V_0, the scale parameter of the Weibull parent, because it cancels

[1]However, larger data sets give observations further into the right hand tail, improving the ability of the fit to determine the curvature and, in the case of a penultimate FT1, reducing the value of the GEV shape parameter and therefore the degree of error.

out on the l.h.s., and hence k depends only on w and rT.

From the formulae given in Hosking, *et al.* (1985), it is simple to derive an expression for the upper limiting wind speed L : –

$$\frac{L}{\check{V}} = \frac{\beta_0}{\check{V}} + \frac{(2\beta_1 - \beta_0)/\check{V}}{1 - 2^{-k}} \tag{8}$$

Here $\check{V}$ is the **characteristic largest value** which for a Weibull parent is given by : –

$$\check{V} = V_0[\ln(rT)]^{1/w} \tag{9}$$

and the ratio $L/\check{V}$ is also independent of V_0.

Thus in principle, β_0, β_1, and β_2 can be evaluated from Eq. (6). Then Eq. (7) can be solved for k, and the value of k substituted into Eq. (8) to obtain $L/\check{V}$. Unfortunately, it does not appear possible to evaluate the integral in Eq. (6) analytically in closed form, leaving two options available – an asymptotic solution for large rT, and a numerical solution to the problem.

3.1. Asymptotic solution

In the Appendix it is shown that for $rT > 100$ and practical values of w, i.e., $1.2 \le w \le 2.5$, Eq. (7) reduces to: –

$$k \approx \frac{2}{\ln(3/2)}\left[\left(\frac{(2\beta_1 - \beta_0)/\ln 2}{(3\beta_2 - \beta_0)/\ln 3}\right) - 1\right] \tag{10}$$

the resulting error in determining k being less than 1/2%.

If rT is large, then: –

$$nrT - 1 \approx nrT \qquad n = 1, 2, 3 \ldots \tag{11}$$

and hence Eq. (6) becomes: –

$$\beta_{n-1}/V_0 \approx rT\int_0^{\infty} z^{1/w}[1 - e^{-z}]^{nrT} e^{-z} dz \tag{12}$$

An asymptotic treatment of the integral on the r.h.s of Eq. (12) valid for large rT can be developed and leads to the results: –

$$k \approx \frac{(w-1)}{w\ln(rT)} + O\left(\frac{1}{\ln(rT)}\right)^2 \tag{13}$$

and: –

$$\frac{L}{\check{V}} \approx \frac{w}{w-1} + O\left(\frac{1}{\ln(rT)}\right) \tag{14}$$

These results are not sufficiently accurate for computation for practical values of $rT \le 150$, because of the very slow convergence of the series (as an inverse power of $\ln(rT)$). However they serve to illustrate that for data drawn from a Weibull parent, if $w \ne 1$, even for very large values of rT both k and L are still some way from their ultimate asymptotic values (which are 0 and ∞, respectively).

3.2. Numerical solution

The integral Eq. (6) for β_{n-1} was solved numerically. A finite upper limit was selected to replace the infinite one, based on choosing a value beyond which there was no measurable contribution (to machine accuracy) to the value of the integral. Integration was then performed by the Romberg method, Press, *et al.* (1989), which provides automatic exit from the integration process when a specified accuracy has been attained. For these calculations, double-length arithmetic was used, the exit accuracy being 10^{-10}. The resulting values of β_0, β_1, and β_2 were then used to solve Eq. (7) for k using a modified Regula Falsi, Press, *et al.* (1989), which involves successive interpolation along the chord, with trial values such that the true solution is always bracketed. The k values were calculated for $w = 0.5$ (0.02) 3.0 and for $\log_{10}(rT) = 1$ (0.02) 4, i.e., for rT between 10 and 10,000 at equal intervals of the common logarithm. The accuracy specified for the calculation of k was 10^{-8}. The results of the calculation are shown as contours of k in Fig. 3. For practical values of w and rT quite large positive values of k are obtained.

In the same program the results for $k > 0$ were passed to a routine that calculated $L/\check{V}$ from Eq. (8) and the results for this ratio are shown in Fig. 4. Note that the results confirm the tendency for $L/\check{V}$ to become constant for a given w at large rT, as indicated by the asymptotic result Eq. (14).

4. Sampling errors

Hosking, *et al.* (1985) showed that the distribution of values of k obtained by the PWM method differed insignificantly from a Normal distribution, provided the number of samples N (in this case the number of epoch maxima) is greater than 25. For $N > 25$ the standard deviation of the values of k is given by: –

$$\sigma = (0.5633/N)^{1/2} \tag{15}$$

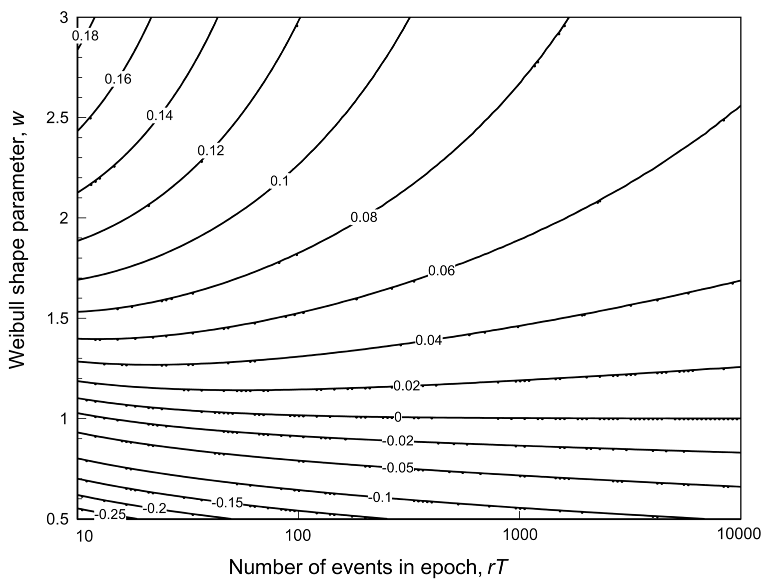

Fig. 3 GEV shape parameter, k

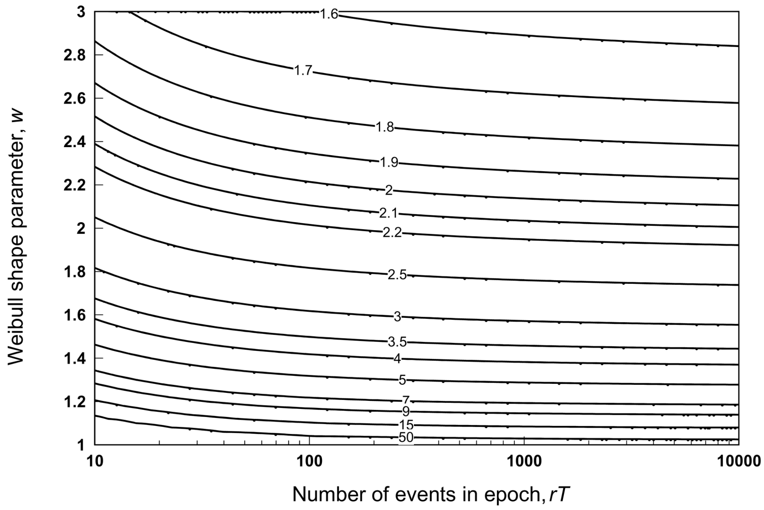

Fig. 4 GEV limit ratio, $V_{\mathrm{lim}}/V_{\mathrm{mode}}$

The results in Hosking, *et al*. (1985) referred to samples taken from a GEV parent distribution. Harris (2004) obtained by computer simulation results for the standard deviation of k for data drawn from a number of other parent distributions. The results agreed well with the predictions of Eq. (15). It may therefore be concluded that the value of σ is not sensitive to the parent distribution from which the epoch maxima are drawn, but depends primarily on the sample size. Therefore Eq. (15) can be used as a predictor of σ for epoch maxima drawn from a Weibull parent, and the distribution assumed to be Normal if $N > 25$.

5. Bias errors

Hosking, *et al*. (1985) also investigated the bias error produced when k is estimated by PWM from a sample of limited size. As bias error is produced only when k is estimated by sampled data, it is best studied by computer simulation, where the simulation process is made to replicate the acquisition procedures as closely as possible. Hosking, *et al*. (1985) presented results based on samples drawn from GEV parents. The question which is relevant to this paper is whether these results are significantly altered when the data samples are epoch maxima drawn from a Weibull parent.

For a range of practical values of w and rT a computer simulation was carried out as follows. Batches of rT rectangularly distributed random numbers in the range $0 < P < 1$ were generated, using the RAN0 algorithm, Press, *et al*. (1989) to remove any possible serial correlation, and the batch maximum extracted. The process was repeated until 40 such maxima had been generated. Each maximum was then subjected to the transform: –

$$V = 10\ [-\ln(1-P)]^{1/w} \tag{16}$$

and then rounded to the nearest integer. The resulting values of V thus represent 40 epoch maxima generated from a Weibull distribution with index w and scale parameter $V_0 = 10$ kts, and rounded to the nearest knot in accordance with UK Met Office practice. The β_0, β_1, and β_2 PWMs were then

Table 1 Bias error for GEV fit to extremes from a Weibull parent

	w					
rT	0.5	1.0	1.5	2.0	2.5	3.0
25	0.021	0.012	0.009	0.008	0.008	0.007
50	0.021	0.012	0.010	0.008	0.008	0.008
75	0.020	0.013	0.009	0.008	0.009	0.008
100	0.019	0.013	0.010	0.009	0.009	0.008
125	0.019	0.012	0.011	0.008	0.009	0.008
150	0.019	0.012	0.010	0.009	0.009	0.009

estimated from the data by the methods recommended by Hosking, *et al.* (1985). Their method was also used to estimate the value of k.

This entire procedure was repeated until an ensemble of 10^5 values of k had been generated. The ensemble average was then calculated and, from it, the convergence error for the chosen values of rT and w was subtracted, this having been determined as described in Section 3. The resulting difference represents the bias error. It was necessary to use such a large ensemble because, based on the results given in Hosking, *et al.* (1985), it was anticipated that the bias error from samples of 40 epoch maxima could be as low as ~0.01, whereas based on Eq. (15) the standard deviation of the sampling error is ~0.12, i.e., an order of magnitude greater. Thus a large ensemble is necessary to reduce the standard deviation of the sampling error of the ensemble mean to a level at which it will not mask the bias error.

The results of this investigation are summarised in Table 1. Note that in spite of the large ensemble used, the third decimal place still behaves erratically. These results show that the bias error is always positive, i.e., it tends to reduce the value predicted for the false upper limit. The values are roughly the same size as found by Hosking, *et al.* (1985). For practical values of rT ($\leq$150) the bias error is small compared with the systematic convergence error (Section 3), unless the Weibull index, $w \approx 1$. For practical sizes of samples of epoch maxima, say $N \leq 40$, the bias error is very small compared with the standard deviation of the sampling error.

6. Analysis and discussion

Science and engineering advance by modifying and extending the existing knowledge base. There is a large body of papers in peer-reviewed journals which refer to the use of the Weibull distribution for parent wind speeds. These papers include studies aimed at both building and structural engineering and for applications such as wind power generation. Many of these papers demonstrate good Weibull fits to observations. This collective body of knowledge cannot be abandoned or ignored, unless and until it is shown to be incorrect and/or a superior alternative is demonstrated. It is conceded that many of these observations do not have sufficient data at the low probability end to accurately define the parent distribution in that region. There are, however, good quality observations which show tight Weibull fits. For a thirty-year record the parent data will consist of several hundred thousand observations, whilst the annual maxima comprise only 30 values. This means that above the level of the lowest annual maximum, the parent data has many more observations and, of course, includes all the annual maxima. Moreover, since the parent population is so much larger, the error bounds on the fit of the ordered data are much tighter – this is well illustrated by the data from Boscombe Down shown in Figs. 1 & 2. It can be argued that in such a case, the evidence of the fit of the parent is more compelling than that of the extremes. Of course, it is neither possible to prove or disprove whether observations which might be collected in the future over a much longer observation period, lying beyond the range of present observations, would still conform to the Weibull model. The present position is that wind extremes may well be drawn from a Weibull parent. This requires that any method used for extreme value analysis must perform satisfactorily on epoch maxima drawn originally from such a Weibull parent. It also seems obligatory that the null hypothesis $k = 0$ should be adopted in any significance tests for sampling error. In the interest of public safety, it follows that design loads should be reduced on the basis that $k > 0$ if, and only if, this null hypothesis is disproved.

Therefore, in this paper the performance of the method of E.V. analysis which involves fitting the GEV to a set of epoch maxima has been examined. It has been shown that the GEV, which is an asymptotic form which depends for its validity on $rT \to \infty$, leads to a systematic error in k due to poor convergence for practical analyses where $rT \leq 150$. For practical values of the Weibull index w for wind speeds, this error can be quite large, ~0.1. Thus in the absence of sampling error, applying a GEV analysis to data derived from a Weibull parent will always produce a Type III fit ($k > 0$) and an artificial upper limit, both influencing predictions for long return periods.

The bias error from processing samples is small (of the same order as those found by Hosking, *et al.* 1985) and is always positive, so tending to reduce the value of the artificial limit.

For practical sizes of samples of epoch maxima $30 \leq N \leq 40$, there is a large sampling error. This paper recommends using the result of Hosking, *et al.* (1985) that the sampling error is normally distributed with a standard deviation of $\sqrt{0.5633/N}$, provided $N > 25$. This sampling error does not appear to depend on the parent distribution from which the maxima were drawn, but only on N. However, the mid-point of the normal distribution is centred not on zero, but on the sum of the systematic convergence and bias errors. Since the convergence error depends on both w and rT, this makes it difficult to set error bounds on k without assuming values for these quantities.

A further illustration of the problem can be obtained by considering two examples: –

1) Temperate storms. As typical values, take $w = 2.0$, $rT = 150$ & $N = 40$. From the work in Section 3, the convergence error is $k = 0.083$. To this must be added a bias error of +0.009, making for a value of $k = 0.092$ in the absence of any sampling error, confirming that a Type III fit is clearly indicated.

For a sample size of 40, from Eq. (15), the standard deviation of the sampling error is 0.119 and, if the sampling error is Normally distributed, then when a GEV analysis is applied to a large ensemble of such data sets:

22% of the time $k < 0$, indicating Type II behaviour, 78% of the time $k > 0$, indicating Type III behaviour, and 5% of the time $k > 0.288$, exceeding the 95% confidence value.

In the context of the analysis of a single sample data set, these percentages are interpreted as the likelihood that the sample k will deviate either side of the true value $k = 0$.

As noted earlier, the presence of a large sampling error, combined with the inability of the GEV method to return a value of $k = 0$, and the evidence leading to a presumption of a null hypothesis $k = 0$, all require that the result of the analysis be subjected to a significance test. Given the implications for public safety of a false rejection of the null hypothesis, it seems appropriate to demand at least the usual industrial standard of 95% confidence. On this basis, it requires a value of $k > 0.288$ from the GEV before the null hypothesis $k = 0$ can be rejected – a figure not attained in any of the published work on temperate storms of which the writer is aware.

2) Thunderstorm downbursts. As typical values Cook, *et al.* (2003), Cook and Harris (2001), take $w = 1.5$, $rT = 50$ & $N = 30$. From the work in Section 3, the convergence error is $k = 0.064$. To this must be added a bias error of +0.010, making for a value of $k = 0.074$ in the absence of any sampling error. For a sample size of 30, Eq. (15) indicates a sampling error with a standard deviation of 0.137. Thus, if a large ensemble of such data sets is analysed by the GEV, then: −

29% of the time $k < 0$, indicating Type II behaviour, 71% of the time $k > 0$, indicating Type III behaviour, and 5% of the time $k > 0.299$.

Thus, in this case, when the significance test is performed, the null hypothesis $k = 0$ will not be rejected unless $k > 0.299$. Again, this figure is not exceeded in published work (except where either rT is far too low, or mixed climates have not been separated before analysis).

A more serious defect of the GEV analysis can be illustrated by this second example. Wind engineers require extreme values of wind speeds in order to derive loads which are proportional to the square of wind speed. There are two possible routes to find such a load. In the first route, a wind speed of a given probability is found by EV analysis, and this value is squared to get the load. Historically, this has been the favoured route, possibly because engineers originally relied on the meteorological service to do the EV analysis for them. A second, equally valid route, is to square all the wind speed data first to derive epoch maximum loads, and then perform the EV analysis. In the thunderstorm example, as above, this first route gives $k = 0.074$ in the absence of sampling error and, without a significance test, this would be taken to indicate Type III behaviour and an upper limit value.

Squaring the data set converts the parent into a Weibull distributed variable with $w = 0.75$, $rT = 50$ & $N = 30$. Now from Section 3, the convergence error is −0.073, the bias error will now be +0.016, and thus a value of $k = -0.057$ will be obtained. Again, without a significance test, this would be taken to indicate Type II behaviour and no upper bound. By virtue of the way design codes are referenced by Building Regulations, these design values have legal implications and must be capable of being defended, if necessary, in a Court of Enquiry. This example shows that GEV analysis is not able to provide an unambiguous definition of design value, which is unacceptable as the basis of a Code of Practice.

To summarise this discussion, GEV analysis of epoch maximum wind data cannot define design values unambiguously, which is an axiomatic requirement. Given the available size of samples, currently at best about 40 years of annual maxima, there is a very large random sampling error. This, combined with the inability of the method to yield $k = 0$, demands a significance test of the result. If such a test is carried out, then the null hypothesis $k = 0$ will almost never be rejected.

Contrary to what is sometimes claimed, it does not appear possible to put sensible bounds on a GEV result. This is because, given that the rate parameter is too low, there will be a systematic error which depends on the value of that rate parameter and also on the parameter(s) of the parent distribution, all of which the GEV analysis presumes to be unknown. For the Weibull parent used in this paper as a test case, the relevant parameters are the rate parameter and the index (shape parameter) of the Weibull parent. In any case, a design code has to be written in terms of a design value, not a design range.

Given that the null hypothesis $k = 0$ will always be confirmed and the GEV analysis cannot yield parameters appropriate for $k = 0$, the implication is that a Type I re-analysis may be required, otherwise design loads will be derived from parameters just deemed more likely than not to be artefacts of the analysis method. This may not be essential if the usual 1:50 year value is required since, with 30 to 40 years of epoch maxima, this represents only a mild extrapolation of the data and results given by several different methods have been shown to give substantially the same answers, Cook and Harris (2004). It is, however, a different matter when it comes to 1:10,000 year values and the existence (or not) of an upper limit, Holmes (2003). Longer return period design values are generally demanded for safety-critical structures, such as in the nuclear industry, which are those for which poor design values can be least tolerated. Fortunately a new Type I method. Cook and Harris (2004) is now available, which eliminates (or at least minimises) convergence errors, provides at least as good a fit to the data as the GEV, and provides unambiguous design values regardless of whether wind speeds or loads are analysed.

7. Conclusions

In this paper, the application of GEV analysis to epoch maxima drawn originally from a Weibull parent has been examined. It has been shown that for practical values of the relevant parameters there are large errors, both systematic and random sampling errors. Historical data suggests a Weibull parent for wind speeds and, consequently, this has to be taken as the null hypothesis. The paper shows that GEV analysis performs badly on such data and, when a significance test is carried out, the result of the GEV analysis will almost always be rejected in favour of the null hypothesis $k = 0$. Given also that the GEV cannot produce unambiguous design loads, it must be concluded that the use of GEV analysis on wind epoch maxima is not a sensible option.

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References

Castillo, E. (1988), *Extreme Value Theory in Engineering*, Academic Press, London.

Cook, N.J. and Harris, R.I. (2001), "Discussion on application of the generalized pareto distribution to extreme value analysis in wind engineering", by Holmes, J.D. and Moriarty, W.W. *J. Wind Eng. Ind. Aerodyn.*, **89**, 215-224.

Cook, N.J. and Harris, R.I. (2004), "Exact and general penultimate distributions of extreme wind speeds drawn from tail-equivalent Weibull parents", *Structural Safety*, **26**, 391-420.

Cook, N.J. and Harris, R.I., "Postscript to exact and general FT1 penultimate distributions of extreme wind speeds drawn from tail-equivalent Weibull parents", *Offered to Structural Safety*.

Cook, N.J., Harris, R.I., and Whiting, R. (2003), "Extreme wind speeds in mixed climates revisited", *JWEIA*, **91**, 403-422.

Fisher, R.A. and Tippett, L.M.C. (1928), "Limiting forms of the frequency distributions of the largest or smallest number in a sample", *Proc. Camb. Phil. Soc.*, **24**, 180-190.

Galambos, J. (1978), *The Asymptotic Theory of Extreme Order Statistics*, Wiley, New York.

Gomes, M.I.L. (1984), "Penultimate limiting forms in extreme value theory", *Ann. Inst. Statist. Math.*, **36**, 71-85.

Gumbel, E.J. (1958), *Statistics of Extremes*, Columbia University Press, New York.

Harris, R.I. (2004), "Extreme value analysis of epoch maxima-convergence and choice of asymptote", *J. Wind Eng. Ind. Aerodyn.*, **92**, 897-918.

Holmes, J.D. (2003), "Discussion on generalized extreme gust wind speeds", *J. Wind Eng. Ind. Aerodyn.*, **91**, 965-967.

Hosking, J.R.M., Wallis, J.R., and Wood, E.F. (1985), "Estimation of the generalized extreme value distribution by the method of probability weighted moments", *Technometrics*, **27**, 251-261.
Press, W.H., *et al*. (1989), *Numerical Recipies*, Cambridge University Press, Cambridge.
Von Mises, R. (1936), "La distribution de la plus grandes des n valeurs", *Rev. Math. De l'Union Interbalkanique*, Athens, 1, 1.

Appendix

$$\frac{1-2^{-k}}{1-3^{-k}} = \frac{1-\exp[-k\ln 2]}{1-\exp[-k\ln 3]} = \frac{k\ln 2 - 1/2(k\ln 2)^2 + 1/6(k\ln 2)^3 - \dots}{k\ln 3 - 1/2(k\ln 3)^2 + 1/6(k\ln 3)^3 - \dots}$$

$$= \frac{\ln 2}{\ln 3}[1 - 1/2k\ln 2 + 1/6(k\ln 2)^2 - \dots] \times [1 - 1/2k\ln 3 + 1/6(k\ln 3)^2 - \dots]^{-1}$$

$$= \frac{\ln 2}{\ln 3}\left[1 - \frac{k}{2}\ln(3/2) + \frac{k^2}{12}\{2(\ln 2)^2 - 3\ln 2\ln 3 + (\ln 3)^2\}\right] + O(k^3)$$

Evaluating the coefficients in [ ] to 4 D.P. gives: –

$$\frac{\ln 3}{\ln 2}\left(\frac{1-2^{-k}}{1-3^{-k}}\right) = 1 + 0.2027k(1 - 0.0479k) + O(k^3)$$

Now if $rT > 100$, for the practical range of w, i.e., $1.2 \le w \le 2.5$, k is at most ~0.1. Hence from Eq. (7), to an accuracy better than 1/2%: –

$$k = \frac{2}{\ln(3/2)}\left[\left\{\frac{(2\beta_1 - \beta_0)/\ln 2}{(3\beta_2 - \beta_0)/\ln 3}\right\} - 1\right]$$

GS