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A parametric analysis of the flutter instability for long span suspension bridges

M. Como[†], S. Del Ferraro[‡] and A. Grimaldi[‡†]

Department of Civil Engineering, University of Tor Vergata, Via del Politecnico 1, 00133 Rome, Italy

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Abstract. A simplified analysis able to point out the most relevant geometrical and aerodynamic parameters that can influence the flutter of long span modern bridges is the aim of the paper. With this goal, by using a continuous model of the suspension bridge and by a quasi stationary approach, a simple formula of the combined vertical/torsional flutter wind speed is given. A good agreement is obtained comparing the predictions from the proposed formula with the flutter speeds of three modern suspension or cable stayed bridges: the Great Belt East Bridge, the Akashi and Normandie bridges. The paper ends with some comments and comparisons with the well known Selberg formula.

Keywords: flutter of bridges, aerodynamic instability, long span bridges.

1. Introduction

Performance and reliability of long span bridges are strongly influenced by aerodynamic loads. As the span increases, wind actions become more critical and for the longest spans flutter becomes the most relevant technical limit (Bruno, *et al*. 2001). Among the various aerodynamic instabilities, the combined flutter, originated by the interaction between the wind and the vertical-torsion oscillations, is the most complex. In spite of the wide numerical-experimental results at disposal, there is still today a lack of simple results able to grasp the problem pointing out the different roles played by the numerous mechanical and aerodynamic parameters involved and give useful suggestions for more accurate studies. In this framework, after the first pioneering studies of Bleich (1948) that used the thin airfoil theory, only the semi-empirical Selberg formula (1961) seems able to give some immediate indications on the order of magnitude of the flutter wind speed of a suspension bridge once that its inertial and dynamical parameters are defined. On the other hand, to apply the Selberg formula the aerodynamic properties of the deck section can be taken into account only by using further empirical corrective factors. In this framework aim of the paper is to give a contribution to a simpler description of the bridge flutter problem.

The aerodynamic loads acting on the bridge will be evaluated by means of the so called Scanlan (1971) aerodynamic derivatives A_i^*, H_i^* $(i=1, 2, 3)$ that describe the self - excited forces acting on the deck section oscillating in the wind. The Scanlan derivatives are strongly dependent on the

† Professor of Structural Engineering, E-mail: como@ing.uniroma2.it
‡ Ph. D. Student, E-mail: del.ferraro@ing.uniroma2.it
‡† Professor of Structural Engineering, E-mail: grimaldi@ing.uniroma2.it

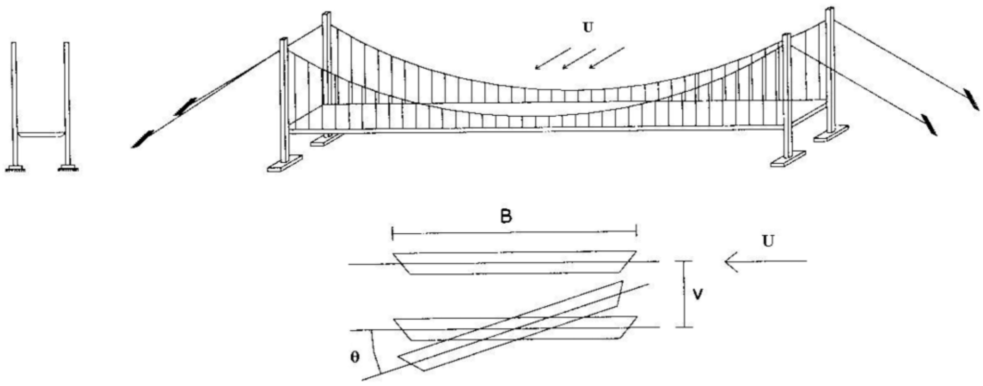


Fig. 1 The examined scheme of suspension bridge

bridge deck section shape and can be evaluated or testing the single girder cross section by wind tunnels techniques or by numerical computations.

For a modern long span bridge the flutter will occur at large wind speeds. Thus, to obtain a simple description of the flutter, as it will be shown, an asymptotic expansion of the flutter derivatives in the range of large values of the reduced velocity U_R can be very useful.

2. Vertical / torsional flutter of long span bridges

Let us consider a long span bridge deck oscillating in an air stream. The wind speed, acting in transversal direction to the girder axis, is U. The deck oscillating motion, according to the given mode, has circular frequency ω and vertical and torsional components $v(z, t)$ and $\theta(z, t)$, where t represents the time and z the girder axis. The equations of the elastic combined vertical/torsional oscillation motion in the wind of the suspension bridge are well known: they can be written in a simplified version

$$EIv^{IV} - H_c v^{II} + \left(\frac{q_g}{H_c}\right)^2 \frac{EA}{L}\int_0^L v(z)dz = -\mu_g \ddot{v} + L_{se}(z, t) \tag{1}$$

$$C_1\theta^{IV} - (C_2 + H_c b^2)\theta^{II} + \left(\frac{q_g}{H_c}\right)^2 b^2 \frac{EA}{L}\int_0^L \theta(z)dz = -I_0\ddot{\theta} + M_{se}(z, t) \tag{2}$$

where:

- EI, C_1, C_2 and EA are the flexural, warping, torsional and extensional stiffnesses of the girder;
- q_g and μ_g are the unit length weight and mass of the bridge, inclusive of the unit length mass μ_c of the cables and of the hangers;
- H_c the tension force of the cables due to the dead loads q_g,
- I_0, the polar moment of inertia of the girder section, included the mass of the cables and of the hangers, is given by

$$I_0 = I_{0G} + \mu_c(B'/2)^2 \tag{3}$$

with I_{0G} the polar moment of inertia of the girder section;
- B' the distance between the curtains of hangers and cables;
- $b = B'/2$ the semidistance between the cables;
- L_{se}, M_{se} are the lift and the moment for unit girder length of the self - excited forces.

These last, according to Scanlan (1971) are given by

$$L_{se} = 1/2\rho_a U^2(2B)\left[KH_1^*\frac{\dot{v}}{U} + KH_2^*\frac{B\dot{\theta}}{U} + K^2H_3^*\theta\right] \tag{4}$$

$$M_{se} = 1/2\rho_a U^2(2B^2)\left[KA_1^*\frac{\dot{v}}{U} + KA_2^*\frac{B\dot{\theta}}{U} + K^2A_3^*\theta\right] \tag{4a}$$

Quantities H_i^*, $A_i^*(i = 1, 2, 3)$ in Eqs. (4) and (4a), the so called aerodynamic derivatives of Scanlan (1971) referred to the whole girder section width B, are functions of the reduced frequency

$$K = \frac{B\omega}{U} \tag{5}$$

or of the reduced velocity

$$U_R = \frac{2\pi}{K} = \frac{UT}{B} = \frac{U}{nB} \tag{5a}$$

Eqs. (1) and (2) are simplified forms of the full equations of motion, which include the horizontal motions of the deck, and as many as eighteen different aeroelastic derivatives, corresponding to all possible motion-induced forces. Many of these terms are small, however, and can be neglected. In Eqs. (4), (4a) ρ_a is the air density (at the atmospheric pressure and at the temperature of 0°C) and in Eq. (5) and in Eq. (5a) ω is the circular frequency, T the period and n the frequency of the motion.

Parameter functions $K^2H_3^*$ and $K^2A_3^*$ represent aerodynamic stiffnesses while KH_1^*, KH_2^*, KA_1^*, KA_2^* are aerodynamic dampings. Generally, according to Scanlan (1971), the vertical damping coefficient H_1^* for bridge decks is, as a rule, negative and decreasing with U_R; A_2^* is negative but, in some cases, can change sign by varying U_R; the torsional stiffness A_3^* is, as a rule, positive and increasing with U_R. Particularly, for modern bridge sections, in the neighborhood of the critical value of U_R, corresponding to the incipient flutter and where the reduced velocity takes values of the order of magnitude of 10^1, $|A_i^*|<|H_i^*|$. It is further important to observe that structural damping is neglected in the formulation corresponding to Eqs. (1) and (2). The influence of the structural damping in the evaluation of the flutter speed is usually not relevant. This approximation allows to develop a simple analytical model that gives a lower bound for the flutter speed.

Solutions of Eqs. (1) and (2) represent the vertical and torsional oscillations of the bridge in the wind stream. They can be obtained by separating the variables with the positions

$$v(z, t) = A\bar{v}(z)e^{st}, \quad \phi(z, t) = D\bar{\theta}(z)e^{st} \tag{6}$$

where A and D are arbitrary constants, $\bar{v}(z)$ and $\bar{\theta}(z)$ suitable functions satisfying the boundary conditions and approximating the oscillation modes of the bridge. These functions can be represented by the vertical and the torsional components of the considered oscillation mode in still air. In Eq. (6) s is the characteristic exponent of the motion, as a rule, taking the complex form

$$s = \alpha + i\omega \tag{7}$$

with α and ω its real and imaginary parts, both functions of the wind speed U. The circular frequency ω of the motion, together with the deck width B and the wind speed U, define the reduced frequency (5). With a vanishing wind speed U the solutions (6) overlap the corresponding oscillation modes of the bridge in still air: for instance, the first symmetric or the first antisymmetric vertical-torsional mode. When $U \to 0$, in fact, Eqs. (1) and (2) decouple and describe the free vertical and torsional oscillations of the bridge in still air. In this case the exponent s takes the form

$$s = i\omega_{0v}, \quad s = i\omega_{0\theta} \tag{8}$$

where ω_{0v} and $\omega_{0\theta}$ are the corresponding circular vertical and torsional frequencies. The functions $\bar{v}(z)$ and $\bar{\theta}(z)$ will satisfy these decoupled equations together with the boundary conditions at the sections $z = 0$ and $z = L$ of the girder: for instance the conditions

$$v = \theta = v'' = \theta'' = 0, \qquad z = 0, \quad z = L \tag{9}$$

are typical of the girder with hinged end sections. Thus, if we consider the first symmetric or the first antisymmetric oscillation mode, we can assume

$$\bar{v}(z) = \sin\frac{\pi z}{L}, \quad \bar{\theta}(z) = \sin\frac{\pi z}{L}$$

or

$$\bar{v}(z) = \sin\frac{2\pi z}{L}, \quad \bar{\theta}(z) = \sin\frac{2\pi z}{L} \tag{10}$$

The frequency equation of the oscillations of the bridge in the wind stream can be properly obtained by using the Galerkin procedure. We obtain (Como 2002)

$$\begin{aligned}&\sigma^4 - \sigma^3\beta\Omega(\gamma A_2^* + H_2^*) + \sigma^2\{1 + \Phi^2 - \beta\gamma\Omega^2[A_3^* - \beta(A_2^*H_1^* - A_1^*H_2^*)]\}\\ &-\sigma\beta\Omega[(\Phi^2 H_1^* + \gamma A_2^*) - \beta\gamma\Omega^2(A_3^*H_1^* - A_1^*H_3^*)] + (\Phi^2 - \beta\gamma\Omega^2 A_3^*) = 0\end{aligned} \tag{11}$$

where

$$\sigma = \frac{s}{\omega_{ov}};\ \Phi = \frac{\omega_{o\theta}}{\omega_{ov}};\ \Omega = \frac{\omega}{\omega_{ov}};\ \beta = \frac{\rho_a B^2}{\mu_g};\ \gamma = \frac{\mu_g B^2}{I_o} \tag{12}$$

Thus the modal shapes affect the characteristic equation by means of the non dimensional parameters σ, Ω, Φ. The geometrical, inertial, structural and the aerodynamical properties of the bridge are thus represented by the parameters β, γ, ω_{ov}, $\omega_{o\theta}$ and A_i^*, H_i^* $(i = 1, 2, 3)$. It is not difficult to show (Como 2002) that Eq. (11) can control the flutter response as of suspension as of cable-stayed bridges.

The frequency Eq. (11) can be strongly simplified taking into account the order of magnitude as of the derivatives A_i^*, H_i^* as of the parameters β, γ, Φ^2 and Ω^2. The mass parameter β is very small, of the order of 10^{-2} and the parameter γ is as large as 10^1 so that the product $\beta\gamma$ is of the order of magnitude of 10^{-1}: this estimate is consistent, for instance, with the numerical values of β and γ

corresponding to the girder sections of the bridges of Table 1; the quantity Ω^2, on the other hand, is as large as Φ^2, i.e., much larger than 1. In the factors of σ^2 and σ, included in the Eq. (11), terms as $\beta\gamma\Omega^2(A_3^* H_1^* - A_1^* H_3^*)$ and $\beta(A_2^* H_1^* - A_1^* H_2^*)$ can be thus neglected with respect to $(\Phi^2 H_1^* + \gamma A_2^*)$ and A_3^*, as it has been possible to verify, by numerical computations for numerous bridges girder sections. Thus Eq. (11) simplifies and becomes

$$\sigma^4 - \sigma^3\beta\Omega(\gamma A_2^* + H_1^*) + \sigma^2(1 + \Phi^2 - \beta\gamma\Omega^2 A_3^*) - \sigma\beta\Omega(\Phi^2 H_1^* + \gamma A_2^*) + (\Phi^2 - \beta\gamma\Omega^2 A_3^*) = 0 \quad (13)$$

3. Stability

At the incipient flutter the characteristic exponent of the motion takes the form

$$\sigma_c = \pm i\Omega_c, \qquad \Omega_c = \frac{\omega_c}{\omega_{ov}} \quad (14)$$

if ω_c indicates the circular frequency at the flutter. Substitution of position (14) into the Eq. (13) yields the following two algebraic equations in the non dimensional critical frequency Ω_c that have to be simultaneously satisfied

$$\Omega_c^4 - \Omega_c^2[1 + \Phi^2 - \beta\gamma\Omega_c^2 A_3^*] + (\Phi^2 - \beta\gamma\Omega_c^2 A_3^*) = 0 \quad (15)$$

$$\Omega_c^2(H_1^* + \gamma A_2^*) - (\Phi^2 H_1^* + \gamma A_2^*) = 0 \quad (16)$$

where the aerodynamic derivatives A_2^* and A_3^* are evaluated at the critical reduced velocity U_{RC}. Now the Eq. (15) admits the two frequency solutions

$$\Omega_c^2 = 1, \qquad \Omega_c^2 = \frac{\Phi^2}{1 + \beta\gamma A_3^*} \quad (17)$$

while the Eq. (16) gives

$$\Omega_c^2 = \frac{\Phi^2 H_1^* + \gamma A_2^*}{H_1^* + \gamma A_2^*} \quad (18)$$

Since at the flutter both Eqs. (15) and (16) have to be satisfied, each of the two solutions (17) has to be coupled to the solution (18). Thus we get the two different conditions

$$1 = \frac{\Phi^2 H_1^* + \gamma A_2^*}{H_1^* + \gamma A_2^*} \quad (19)$$

or

$$\frac{\Phi^2}{1 + \beta\gamma A_3^*} = \frac{\Phi^2 H_1^* + \gamma A_2^*}{H_1^* + \gamma A_2^*} \quad (20)$$

By assuming $\Phi^2 \neq 1$, if $H_1^* \neq 0$, i.e., non considering the vertical flutter, Eq. (19) does not admit solution. If, on the contrary $\Phi^2 = 1$, the Eq. (19) has a solution for any value of the reduced velocity. From conditions (18) and (20), on the contrary, we get the critical frequency Ω_c

$$\Omega_c^2 = \frac{\Phi^2}{1 + \beta\gamma A_3^*} \tag{21}$$

and the combined vertical/torsional flutter equation

$$A_3^*(\Phi^2 H_1^* + \gamma A_2^*) = \frac{1}{\beta} A_2^*(\Phi^2 - 1) \tag{22}$$

Numerical solution of Eq. (22) yields the reduced flutter velocity U_{RC} and, by using the critical flutter frequency, the flutter velocity of the wind.

A concise formula of the flutter velocity on the other hand can be obtained from Eqs. (21) and (22) in the framework of the quasi-steady assumption, as it will show in the next Section.

4. Asymptotic expansion of the flutter derivatives in the range of large values of the reduced velocity U_R

At the sharp corners of the girder section the flow separates and creates vortices that depend on the reduced frequency and the magnitude of the oscillations of the section. At large wind speeds, with the usual values of the frequency n and the girder width B, the corresponding values of the reduced velocity U_R are >> 1. Thus the distance covered by the air particles during the oscillation period T is large compared to the girder width B. The flow thus is not influenced by the girder oscillations and follows the section in its motion remaining almost the same as the flow corresponding to the fixed section. Thus at large wind speeds the aerodynamic loads produced on the section with a good approximation can be represented by the steady flow loads that do not depend on the reduced frequency K. Consequently according to Eq. (4) and (4a), for large values of U_R the derivatives $A_i^*(K)$, $H_i^*(K)(i = 1, 2)$ become proportional to U_R, while the derivatives A_3^* and H_3^* become proportional to U_R^2 and we can write

$$\begin{aligned} &H_1^* \cong H_{1lin} = -h_1 U_R; \qquad A_2^* \cong A_{2lin} = -a_2 U_R; \\ &A_3^* \cong A_{3quad} = a_3 U_R^2 \end{aligned} \tag{23}$$

The quantities h_1, a_2, a_3, positive constants, will be evaluated by inspection of the diagrams of the aerodynamic functions H_1^*, A_2^*, A_3^* obtained by wind tunnel tests or, as pointed out by Cremona, *et al*. (2002), by using the same steady state lift and moment coefficients $C_L(\alpha)$ and $C_M(\alpha)$ as functions of the angle of attack α of the wind flow.

Damping derivatives A_2^* and H_1^* directly proportional to U_R can be observed in the aerodynamic behaviour of many girder sections of modern bridges for sufficiently large values of the reduced velocity U_R. Figs. 2, 3 and 4 show the good agreement between the derivatives A_2^*, A_3^*, H_1^* and their linear and quadratic expansions A_{2lin}, A_{3quad}, H_{1lin} given by Eq. (23), for three important long span bridges.

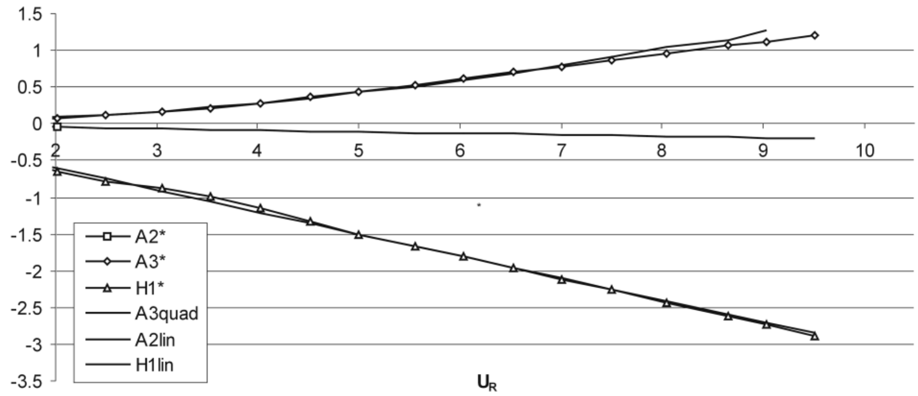


Fig. 2 The A_{2lin}, A_{3quad}, H_{1lin} derivatives of the Normandie Bridge girder section

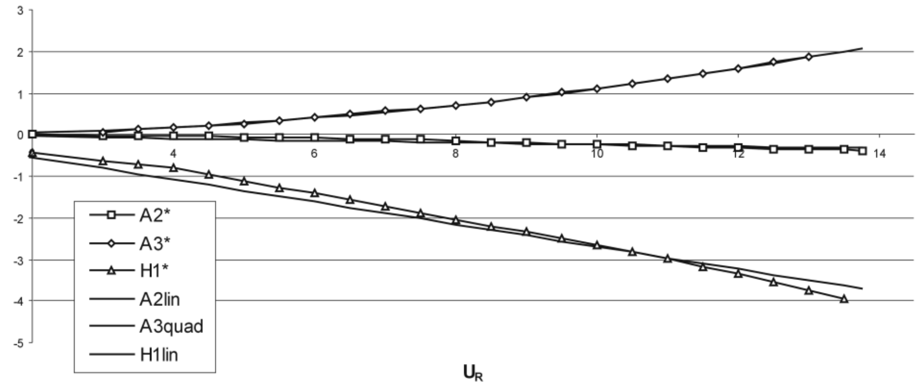


Fig. 3 The A_{2lin}, A_{3quad}, H_{1lin} derivatives of the Great Belt East Bridge girder section

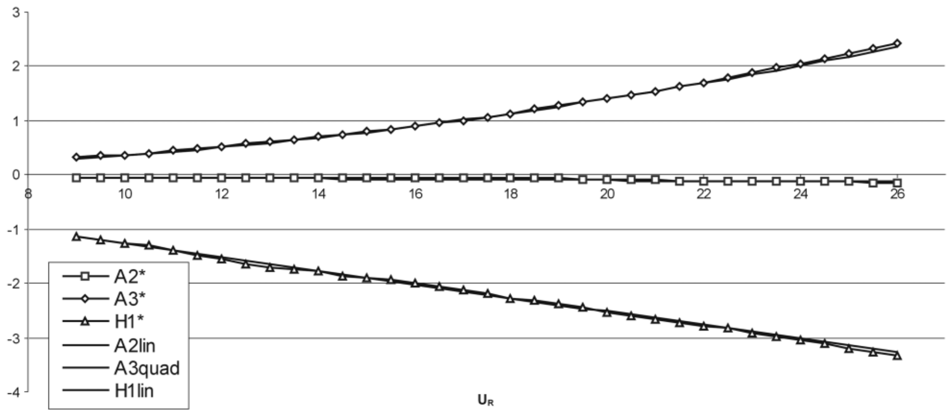


Fig. 4 The A_{2lin}, A_{3quad}, H_{1lin} derivatives of the Akashi Bridge girder section

5. The flutter wind speed

The above defined asymptotic expansion of the aerodynamic derivatives is now applied to the evaluation of the flutter wind speed of long span bridges. By means of the third of the positions (23) the flutter Eq. (22) gives

$$a_3 U_{RC}^2 [\Phi^2 H_1^*(U_{RC}) + \gamma A_2^*(U_{RC})] = \frac{(\Phi^2 - 1)}{\beta} A_2^*(U_{RC}) \tag{24}$$

and the following first expression of the reduced velocity U_{RC} at the flutter yields

$$U_{RC} = (\Phi^2 - 1) \frac{A_2^*(U_{RC})}{a_3 \beta [\Phi^2 H_1^*(U_{RC}) + \gamma A_2^*(U_{RC})]} \tag{25}$$

Further information on the critical speed U_F can be obtained by considering the first and the second of the positions (23), i.e., the linear approximations of the torsional and vertical damping coefficients A_2^* and H_1^*. Thus substitution of these positions into the Eq. (25) gives

$$U_{RC}^2 = \frac{\Phi^2 - 1}{\beta \gamma a_3 (1 + \Phi^2 h_1 / \gamma a_2)} \tag{26}$$

On the other hand, taking into account that the critical reduced flutter velocity can be expressed as

$$U_{RC} = \frac{U_F}{n_c B} \tag{27}$$

where

$$n_c = \Omega_c \frac{\omega_{ov}}{2\pi} \tag{21a}$$

is the frequency of the bridge at the flutter and Ω_c is the non dimensional circular frequency at flutter given by Eq. (21), the flutter speed can be written as

$$U_F = \Omega_c \frac{\omega_{ov}}{2\pi} B U \tag{28}$$

Thus substituting Eq. (21) into (28) and using Eq. (26) we obtain the following formula of the flutter wind speed U_F

$$U_F = \frac{B}{T_{o\theta}} \sqrt{\frac{\Phi^2 - 1}{\Phi^2} \frac{1}{\beta a_3 (\gamma + h_1 / a_2)}} \tag{29}$$

Expression (29) shows the dependence of U_F on :

- the mechanical parameters β and γ, representative of the geometry and the mass distribution of the bridge;
- the dynamical parameter Φ, the ratio between the torsional and vertical frequencies of the considered mode of the bridge oscillating in still air;
- the aerodynamic torsional stiffness coefficient a_3 and the ratio h_1/a_2 between the aerodynamic vertical and torsional damping coefficients.

A more explicit expression of the flutter wind speed can be obtained for long span bridges. By increasing the central span length L, in fact, the cable stiffness prevails on the girder stiffness. For instance, with reference to the first antisymmetric mode, the period $T_{o\theta}$ can be approximately evaluated as

$$T_{o\theta} \approx L\sqrt{\frac{4I_o}{H_c B'^2}} \quad \text{where} \quad H_c = \frac{q_g L^2}{8f} \tag{30}$$

is the tension cable under the action of the dead loads $q_g = \mu_g g$, with g the gravity acceleration, f the cables sag, I_o the central moment of inertia of the girder section, given by Eq. (3) and inclusive of the contribution of the mass of the cables and of the vertical suspenders. Hence

$$T_{o\theta} \approx 2\sqrt{L}\sqrt{\frac{8f}{L}\frac{I_o}{q_g B'^2}} \tag{31}$$

and the flutter speed for long span bridges takes the following form

$$U_F = \frac{B'}{B}\sqrt{\frac{q_g}{4\rho_a L}}\sqrt{\frac{L}{8f}}\sqrt{\frac{\Phi^2 - 1}{\Phi^2}\frac{\gamma}{a_3(\gamma + h_1/a_2)}} \tag{32}$$

The geometrical ratio L/f is nearly constant for suspension bridges. Thus Eq. (32) shows the strict dependence of the flutter speed also on the ratio $q_g/4\rho_a L$.

6. Sensitivity analysis of the combined vertical-torsion flutter speed

A sensitivity analysis of the critical speed on the various involved parameters is now developed. Formula (29) shows the influence of these parameters on the flutter speed: all the mechanical parameters are represented by the quantities β, γ and Φ while that aerodynamic ones by a_3 and the ratio h_1/a_2. All these parameters can be considered varying into suitable intervals. More specifically, the parameters β and γ are assumed to vary in the intervals

$$0.02 < \beta < 0.08, \quad 4 < \gamma < 10$$

The ratio Φ, larger than the unity, varies as

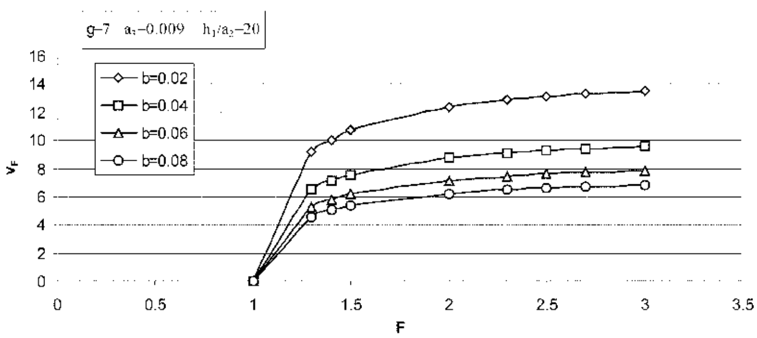
$$1 < \Phi \le 3$$

On the basis of the inspection of the behaviour of deck sections of numerous existing bridges (Scanlan 1971), the aerodynamic parameters a_3 and h_1/a_2 have been assumed to vary as

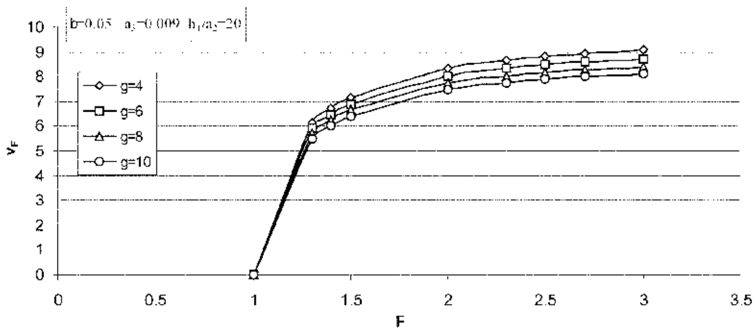
$$0.002 \le a_3 \le 0.016, \quad 5 \le h_1/a_2 \le 35$$

Values of the non dimensional flutter speed v_F obtained by varying the various parameters in the above defined intervals, are plotted in Figs. 5(a),(b),(c) and (d). The numerical analysis shows the strong dependence of v_F on the ratio Φ: we remark that v_F vanishes when Φ approaches to the unity. Diagrams of Figs. 5 show that as β as a_3 and h_1/a_2 play a relevant influence on the non dimensional flutter wind speed

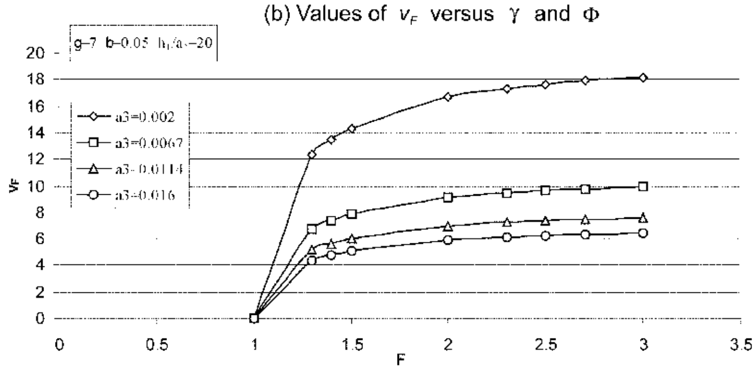
$$v_F = \frac{U_F T_{o\theta}}{B} \tag{29a}$$



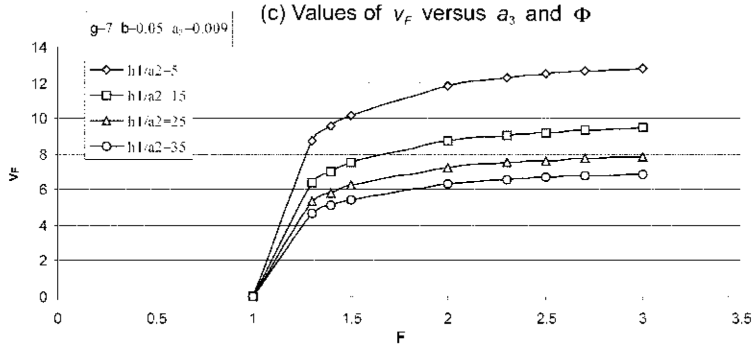
(a) Values of v_F versus β and Φ



(b) Values of v_F versus γ and Φ



(c) Values of v_F versus a_3 and Φ



(d) Values of v_F versus h_1/a_2 and Φ

Fig. 5 Values of v_F versus Φ, β, γ, a_3, h_1/a_2

Table 1 Data involved in the computation of the flutter velocity according to Eq. (29) for the Great Belt East Bridge, the Akashi and the Normandie bridges

Bridge	Geometrical data			
	B m	H m	L m	I_o tm^2/m
Great Belt East Bridge	31	4.4	1624	2470
Akashi	35.5	14	1990	9826
Normandie	21.2	3	856	288.53

Bridge	Aerodynamic data							v_F	U_F (m/s)
	$T_{o\theta}$ (sec)	T_{ov} (sec)	Φ	β	γ	a_3	h_1/a_2		
Great Belt East Bridge	3.6	10	2.78	0.055	8.84	0.011	11.25	8.46	73.1
Akashi	6.66	15.62	2.34	0.037	5.6	0.0035	25.2	14.31	76.3
Normandie	2	4.503	2.25	0.0447	9	0.014	14.15	7.44	78.9

7. Applications

The formula (29) has been applied to evaluate the flutter speed of some modern long span bridges. Table 1 gives the geometrical and the aerodynamic properties of two suspension bridges, the Great Belt East Bridge and the Akashi Bridge, according to the data quoted respectively by Larsen (1993) and by Jones, Scanlan, *et al.* (Jones, *et al*. 1998), and of the cable stayed bridge, the Normandie bridge, by Virlogeux (1992). For these bridges the flutter speeds are known: a good agreement is obtained by comparing these speeds with the corresponding values predicted by the proposed formula (29).

The flutter velocity U_F, obtained by applying Eq. (29) to the Great Belt East Bridge, is $U_F = 73.1$ m/s. This result is in good agreement with the flutter speed of the bridge that ranges between $70 \div 75$ m/s, according to wind tunnel tests and numerical computations.

Likewise, the value predicted by formula (29) for the flutter velocity of the Akashi bridge, $U_F = 76.3$ m/s, is only a bit lower than the values ranging between $78, 8 \div 81, 3$ m/s evaluated by numerical analysis; higher values, ranging between $84 \div 90$ m/s were obtained by wind tunnel tests (Jones, *et al*. 1998).

As far as the Normandie bridge is concerned, many studies have shown that flutter speed, even if never exactly evaluated, is considered much larger than the reference wind velocity, established in 44 m/s (Virlogeux 1992). Eq. (29), applied to this bridge, gives $U_F = 78.9$ m/s.

8. Comparisons with the Selberg formula

The semi-empirical formulation of the flutter speed, proposed by Selberg (1961), is given by

$$U_F = 0.44\chi B\sqrt{(\omega_{o\theta}^2 - \omega_{ov}^2)\frac{\sqrt{v}}{\sigma}} \tag{33}$$

with

$$v = 8\frac{r^2}{B^2} \qquad \sigma = \pi\frac{\rho_a B^2}{\mu_g} \tag{34}$$

where r is the radius of gyration of the cross section inclusive of all the various masses, given by $I_0 = \mu_g r^2$, and χ is an empirical factor depending on the aerodynamic and mass properties of the girder section, that becomes equal to the unity when the girder section approaches the thin airfoil.

Carrying back the Selberg parameters σ and v to the previous defined parameters β and γ from Eq. (33) we obtain

$$v_{vF} = 5.24 \frac{\chi}{\sqrt[4]{\gamma}} \sqrt{\frac{\Phi^2 - 1}{\Phi^2} \frac{1}{\beta}} \tag{35}$$

Expression (35) has a structure very similar to the formula (29) as far as the dependence of v_F on the parameters Φ and β is concerned. The empirical parameter χ includes both the effects of the aerodynamic constant a_3, h_1, a_2 and of the transversal mass distribution parameter γ.

9. Conclusions

The proposed formula predicts with good approximation the flutter velocity of long span bridges in the framework of the quasi-stationary approach, i.e., in the range of large values of the reduced velocity U_R. The flutter speeds evaluated according to the proposed formula for three important long span bridges as the Great Belt East Bridge, the Akashi Bridge and the Normandie bridge, fit very well the values of the flutter speeds obtained for these bridges by wind tunnel techniques or by numerical computations.

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