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Determination of flutter derivatives by stochastic subspace identification technique

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Abstract. Flutter derivatives provide the basis of predicting the critical wind speed in flutter and buffeting analysis of long-span cable-supported bridges. In this paper, one popular stochastic system identification technique, covariance-driven Stochastic Subspace Identification(SSI in short), is firstly presented for estimation of the flutter derivatives of bridge decks from their random responses in turbulent flow. Secondly, wind tunnel tests of a streamlined thin plate model and a Π type blunt bridge section model are conducted in turbulent flow and the flutter derivatives are determined by SSI. The flutter derivatives of the thin plate model identified by SSI are very comparable to those identified by the unifying least-square method and Theodorson's theoretical values. As to the Π type section model, the effect of turbulence on aerodynamic damping seems to be somewhat notable, therefore perhaps the wind tunnel tests for flutter derivative estimation of those models with similar blunt sections should be conducted in turbulent flow.

Keywords: flutter derivative; modal parameter identification; stochastic subspace identification (SSI); wind-induced vibration; bridge structure.

1. Introduction

Flutter and buffeting are two momentous forms of wind-induced vibrations for long-span cable-supported bridges. To avoid the destructive flutter and suppress the violent buffeting, the flutter derivatives, which measure the potential of vibrating bridges to absorb energy from wind, are of the most importance.

In most of the current studies flutter derivatives are estimated by the traditional deterministic system identification techniques, or by some numerical techniques, e.g., Computational Fluid Dynamics (CFD in short). The deterministic system identification techniques involved in flutter derivative estimations can be grouped under two heads, i.e., forced vibration methods (Chen and Yu 2002) and free vibration methods (Gu, *et al*. 2000, Gu, *et al*. 2001, Sarkar 1994, Song 2003). Forced vibration methods are somewhat expensive since they involve sizeable equipments and considerable time and work. Furthermore, the forced vibration of bridges is very different from their kinetic characteristics in the natural wind. Free vibration testing seems to be more tractable than forced vibration testing. However, at high reduced wind speeds, the vertical bending motion of the

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structure will decay rapidly due to the effect of positive aerodynamic damping, and thus the length of time history available for system identifications will decrease, which therefore add more difficulties to the system identifications.

The aforementioned problems of the forced testing and free testing can be readily settled if stochastic vibration methods are utilized to estimate flutter derivatives. For stochastic methods, the turbulent flow is regarded as excitation, hence the sizable exciting equipments are avoided (Jakobsen 1995). Moreover, as opposed to deterministic methods, the effects of turbulent flow are no longer noise in this case, so the Signal-to-Noise Ratio (SNR in short) is not affected by wind speed, and the flutter derivatives at high reduced wind speeds can thus be available. These two aspects give the stochastic system identification methods an advantage over the deterministic methods in estimating the flutter derivatives of bridge decks.

Many stochastic system identification methods have been developed during the past decades, among which the Stochastic Subspace Identification (Overschee 1991, Peters 1999) (SSI in short) technique has proven to be a method very appropriate for civil engineering. The merit points of SSI are: (1) the assumptions of inputs are congruent with the practical wind-induced aerodynamic forces, i.e., stationary and independent on the outputs; (2) identified modes are given in frequency stabilization diagram, from which the operator can easily distinguish structural modes from the computational ones; (3) since the maximum order of the model is changeable for the operator, a relatively large model order will give an exit for noise, which in some cases can dramatically improve the quality of the identified modal parameters; (4) mode shapes are simultaneously available with the poles, without requiring a second step to identify them. There are two kinds of SSI methods, one is data-driven, and the other is covariance-driven.

In this paper, the covariance-driven SSI is applied to the determination of flutter derivatives of bridge decks in turbulent flow. The method is theoretically formulated at first. Secondly, wind tunnel tests of a streamlined thin plate model and the Π type blunt section model of Hong-guang Bridge, a cable-supported bridge with a main span of 380 m in the southwestern China, are conducted in TJ-1 wind tunnel at Tongji University, and the random buffeting responses are measured. From these responses, the flutter derivatives of the models are estimated. Finally, the identified flutter derivatives of the thin plate model are compared with its theoretical values, as well as the results of similar models. Also the flutter derivatives of Hong-guang Bridge are compared to those estimated in smooth flow.

2. Theoretical formulation of SSI

The dynamic behavior of a bridge deck with two Degrees-Of-Freedom (DOF in short), i.e., h (bending) and α (torsion), in turbulent flow can be described by the following differential equations (Scanlan 1977).

$$\begin{aligned} m[\ddot{h}(t) + 2\xi_h\omega_h\dot{h}(t) + \omega_h^2 h(t)] &= L_{se}(t) + L_b(t) \\ I[\ddot{\alpha}(t) + 2\xi_\alpha\omega_\alpha\dot{\alpha}(t) + \omega_\alpha^2\alpha(t)] &= M_{se}(t) + M_b(t) \end{aligned} \tag{1}$$

where m and I are the modal mass and mass moment of inertia of the deck distributed per unit span, respectively. ω_i is the natural frequency, and ξ_i is the modal damping ratio (i=h, α). L_{se} and M_{se} are the self-excited lift and moment, while L_b and M_b are the aerodynamic lift and moment. The self-excited lift and moment are given as follows (Scanlan 1978)

$$L_{se} = \rho U^2 B\left[K_h H_1^*(K_h)\frac{\dot{h}}{U} + K_\alpha H_2^*(K_\alpha)\frac{B\dot{\alpha}}{U} + K_\alpha^2 H_3^*(K_\alpha)\alpha\right]$$
$$M_{se} = \rho U^2 B^2\left[K_h A_1^*(K_h)\frac{\dot{h}}{U} + K_\alpha A_2^*(K_\alpha)\frac{B\dot{\alpha}}{U} + K_\alpha^2 A_3^*(K_\alpha)\alpha\right] \quad (2)$$

where ρ is air mass density, B is the width of the bridge deck, U is the mean wind speed, and $K_i=\omega_i B/U$ is the reduced frequency ($i=h$, α). H_i^* and A_i^* (i=1, 2, 3) are the so-called flutter derivatives, which can be regarded as the implicit functions of the deck's modal parameters. The aerodynamic lift and moment can be defined as (Simiu and Scanlan 1986)

$$L_b(t) = \frac{1}{2}\rho U^2 B\left[2C_L\frac{u(t)}{U} + (\dot{C}_L + C_D)\frac{w(t)}{U}\right]$$
$$M_b(t) = \frac{1}{2}\rho U^2 B^2\left[2C_M\frac{u(t)}{U} + \dot{C}_M\frac{w(t)}{U}\right] \quad (3)$$

where C_L, C_D and C_M are the steady-state force coefficients referring to the deck width, $\dot{C}_L$ and $\dot{C}_M$ are the derivatives of C_L and C_M with respect to the attack angle. $u(t)$ and $w(t)$ are the longitudinal and vertical fluctuations of wind speed respectively.

By moving L_{se} and M_{se} to the left side, and merging the similar terms into column vectors or matrices, Eq. (1) can be rewritten as follows

$$M\{\ddot{y}(t)\} + C^e\{\dot{y}(t)\} + K^e\{y(t)\} = \{f(t)\} \quad (4)$$

where $\{y(t)\}=\{h(t)\ \ \alpha(t)\}^T$ is the generalized buffeting response, $\{f(t)\}=\{L_b(t)\ \ M_b(t)\}^T$ is the generalized aerodynamic force. M is the mass matrix, C^e is the gross damping matrix, i.e., the sum of the mechanical and aerodynamic damping matrices, and K^e is the gross stiffness matrix.

The fluctuations of wind speed $u(t)$ and $w(t)$ in Eq. (3) are random functions of time, so the identification of flutter derivatives of bridge decks in turbulent flow can be simplified as a typical inverse problem in the theory of random vibration, and thus can be solved by stochastic system identification techniques.

Let

$$A_c = \begin{bmatrix} \Phi & I \\ -M^{-1}K^e & -M^{-1}C^e \end{bmatrix} \quad (5)$$
$$C_c = [I \ \ \Phi]$$

and

$$\{x\} = \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} \quad (6)$$

thus Eq. (4) is transformed into the following stochastic state equations

$$\begin{cases} \{\dot{x}\} = A_c\{x\} + \{n\} \\ \{y\} = C_c\{x\} + \{m\} \end{cases} \tag{7}$$

where

$$\{n\} = \begin{Bmatrix} \Phi \\ M^{-1}f \end{Bmatrix}$$

The discrete form of Eq. (7) can be written as

$$\begin{cases} \{x_{k+1}\} = A\{x_k\} + \{n_k\} \\ \{y_k\} = C\{x_k\} + \{m_k\} \end{cases} \tag{8}$$

where A, C and $\{x\}$ are known as state matrix, output shape matrix and state vector, respectively. $\{m_k\}$ is the measuring noise sequence. Subscript $*_k$ denotes the value of $*$ at time $k\Delta t$, where Δt means the sampling interval. Φ and I are the zero and identity matrices, respectively.

It is common to assume that $\{x_k\}$, $\{n_k\}$ and $\{m_k\}$ in Eq. (8) are mutually independent and hence

$$E[x_k n_k^T] = \Phi \qquad E[x_k m_k^T] = \Phi \tag{9}$$

Defining

$$\begin{aligned} &\Sigma = E[x_k x_k^T] && Q = E[n_k n_k^T] \\ &\Lambda_i = E[y_{k+i} y_k^T] && R = E[m_k m_k^T] \\ &G = E[x_{k+1} y_k^T] && S = E[n_k m_k^T] \end{aligned} \tag{10}$$

then we get the following Lyapunov equations for the state and output covariance matrices

$$\begin{aligned} \Sigma &= A\Sigma A^T + Q \\ \Lambda_0 &= C\Sigma C^T + R \\ G &= A\Sigma C^T + S \end{aligned} \tag{11}$$

From Eq. (8) and Eq. (9), it can be deduced

$$\Lambda_1 = E[\{y_{k+1}\}\{y_k\}^T] = E[(C\{x_{k+1}\} + \{v_{k+1}\})\{y_k\}^T] = E[C\{x_{k+1}\}\{y_k\}^T] = CG \tag{12}$$

$$\begin{aligned} \Lambda_2 &= E[\{y_{k+2}\}\{y_k\}^T] = E[(C\{x_{k+2}\}+\{v_{k+2}\})\{y_k\}^T] = E[C\{x_{k+2}\}\{y_k\}^T] = CE[\{x_{k+2}\}\{y_k\}^T] \\ &= CE[(A\{x_{k+1}\}+\{w_k\})\{y_k\}^T] = CE[A\{x_{k+1}\}\{y_k\}^T] = CAE[\{x_{k+1}\}\{y_k\}^T] = CA^{2-1}G \end{aligned} \tag{13}$$

and

$$\Lambda_i = CA^{i-1}G \tag{14}$$

Defining a block Toeplitz $T_{1|i}$ as

$$T_{1|i} = \begin{bmatrix} \Lambda_i & \Lambda_{i-1} & \cdots & \Lambda_1 \\ \Lambda_{i+1} & \Lambda_i & \cdots & \Lambda_2 \\ \vdots & \vdots & \vdots & \vdots \\ \Lambda_{2i-1} & \Lambda_{2i-2} & \cdots & \Lambda_i \end{bmatrix} \tag{15}$$

then one can infer from the definition of covariance matrix that $T_{1|i}$ can be expressed as the product of two block Hankel matrices Y_f and Y_p

$$T_{1|i} = Y_f Y_p^T \tag{16}$$

where Y_f and Y_p are composed of the 'Future' and 'Past' measurements, respectively.

$$Y_f = \frac{1}{\sqrt{j}} \begin{bmatrix} y_i & y_{i+1} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+j} \\ \vdots & \vdots & \vdots & \vdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+j-2} \end{bmatrix} \qquad Y_p = \frac{1}{\sqrt{j}} \begin{bmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ \vdots & \vdots & \vdots & \vdots \\ y_{i-1} & y_i & \cdots & y_{i+j-2} \end{bmatrix} \tag{17}$$

The meanings of i and j can be inferred from Eq. (15) and Eq. (17), i.e., i denotes the number of covariance points utilized in the identification, and j shows the number of original data points utilized in the estimation of covariance.

In a manner similar to the classical Eigensystem Realization Algorithm (ERA in short) (Juang and Pappa 1985), one can find

$$A = o_i^+ T_{2|i} \varsigma_i = S_N^{-1/2} U^T T_{2|i} V S_N^{-1/2} \tag{18}$$

where N is model order, i.e., the maximum number of modes to be computed. U, S and V are matrices derived from the Singular-Value-Decomposition (SVD in short) of matrix $T_{1|i}$

$$T_{1|i} = USV^T \tag{19}$$

Thus the modal parameters can be determined by solving the eigenvalue problem of state matrix A. By now, the theoretical formulation of covariance-driven SSI has been achieved.

According to Eq. (16), Eq. (17) and Eq. (18), a different combination of i, j and N will give a different state matrix, and thus a different pair of modal parameters. Therefore modal parameters should be derived from a series of combinations, rather than a single combination. In the process of identification, N or i should be given in series for certain j to get the frequency stability chart.

Once the modal parameters are identified, the gross damping matrix C^e and the gross stiffness matrix K^e in Eq. (4) can be readily determined by the pseudo-inverse method.

Let

$$\begin{aligned} \bar{C}^e &= M^{-1}C^e \quad & \bar{K}^e &= M^{-1}K^e \\ \bar{C} &= M^{-1}C^0 \quad & \bar{K} &= M^{-1}K^0 \end{aligned} \tag{20}$$

where C^0 and K^0 are the 'inherent' damping and stiffness matrices, respectively. Thus the flutter derivatives can be extracted from the following equations

$$\begin{aligned} H_1^*(k_h) &= -\frac{m}{\rho B^2 \omega_h}(\bar{C}_{11}^e - \bar{C}_{11}) \\ A_1^*(k_h) &= -\frac{I}{\rho B^3 \omega_h}(\bar{C}_{21}^e - \bar{C}_{21}) \\ H_2^*(k_\alpha) &= -\frac{m}{\rho B^3 \omega_\alpha}(\bar{C}_{12}^e - \bar{C}_{12}) \\ A_2^*(k_\alpha) &= -\frac{I}{\rho B^4 \omega_\alpha}(\bar{C}_{22}^e - \bar{C}_{22}) \\ H_3^*(k_\alpha) &= -\frac{m}{\rho B^3 \omega_\alpha^2}(\bar{K}_{12}^e - \bar{K}_{12}) \\ A_3^*(k_\alpha) &= -\frac{I}{\rho B^4 \omega_\alpha^2}(\bar{K}_{22}^e - \bar{K}_{22}) \end{aligned} \tag{21}$$

3. Case studies

To evaluate the applicability of SSI in extracting flutter derivatives, wind tunnel tests of a streamlined thin plate model and a Π type blunt bridge section model are carried out.

3.1. Outline of wind tunnel tests

The wind tunnel tests are performed in TJ-1 wind tunnel at Tongji University in China. This tunnel is an open circuit boundary layer wind tunnel with a working section of 1.8 m(width)×1.8 m (height), and the wind speed can be continuously regulated between 0.5~25 m/s.

The models are tested in turbulent flow generated by grid, as shown in Fig. 1. The turbulence intensity and integral length (L_u^x) of the generated turbulent flow are 12% and 24.42 cm, respectively. A pitometer is applied to measure the mean wind speed of the flow, and a hot-wire anemometer is used to measure the fluctuations of wind speed. Fig. 2 gives the power-spectral-density functions of the fluctuations of the longitudinal and vertical wind speed at 5 m/s wind speed.

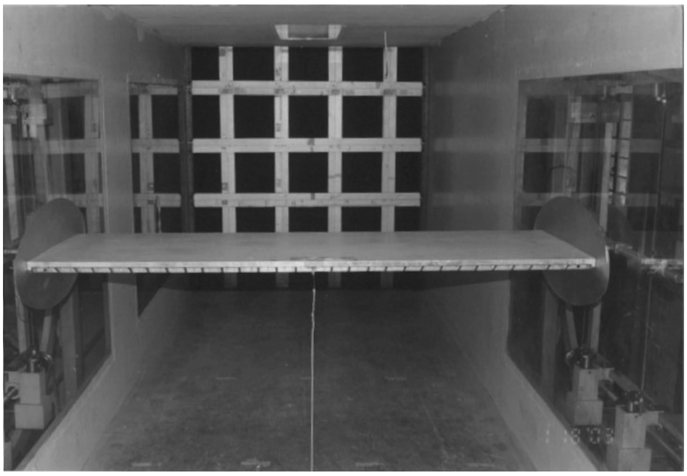

Fig. 1 Grids to generate turbulent flow

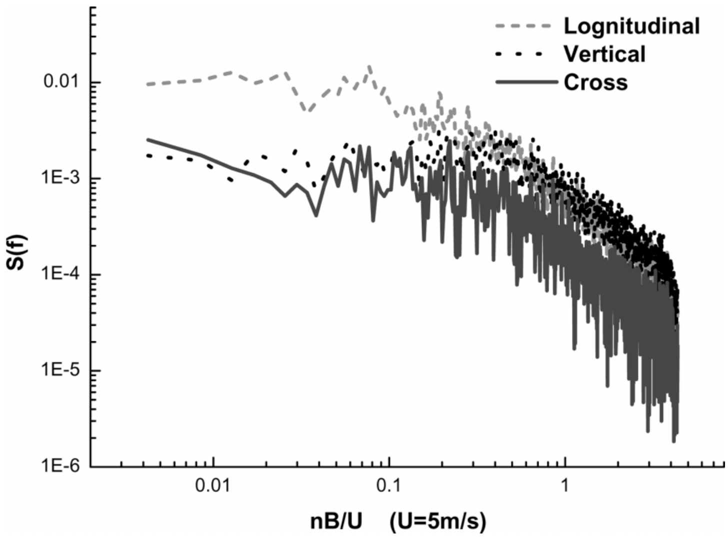

Fig. 2 Auto and cross power-spectral-density functions of the fluctuations of the longitudinal and vertical wind speed at 5 m/s wind speed

The models are suspended by eight springs outside the wind tunnel (see Fig. 3). To simulate a bridge section model with 2-DOFs, i.e., vertical bending and torsion, piano wires are used to arrest the motion of the model in longitudinal direction(see Fig. 4).

The random buffeting responses of the models are captured by three piezoelectric acceleration transducers mounted to the connecting rods outside the wind tunnel, as shown in Fig. 3. Thus the vertical and torsional responses in Eq. (1) can be obtained by

$$h = \frac{x_1 + x_3}{2} \qquad \alpha = \frac{x_1 - x_2}{2l} \tag{22}$$

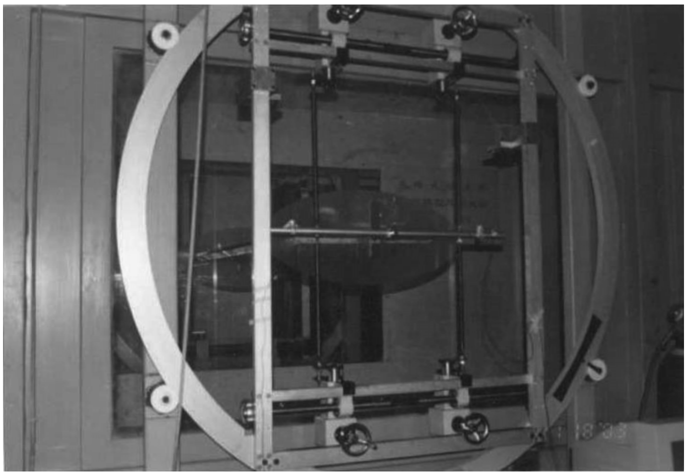

Fig. 3 Suspension device of the model

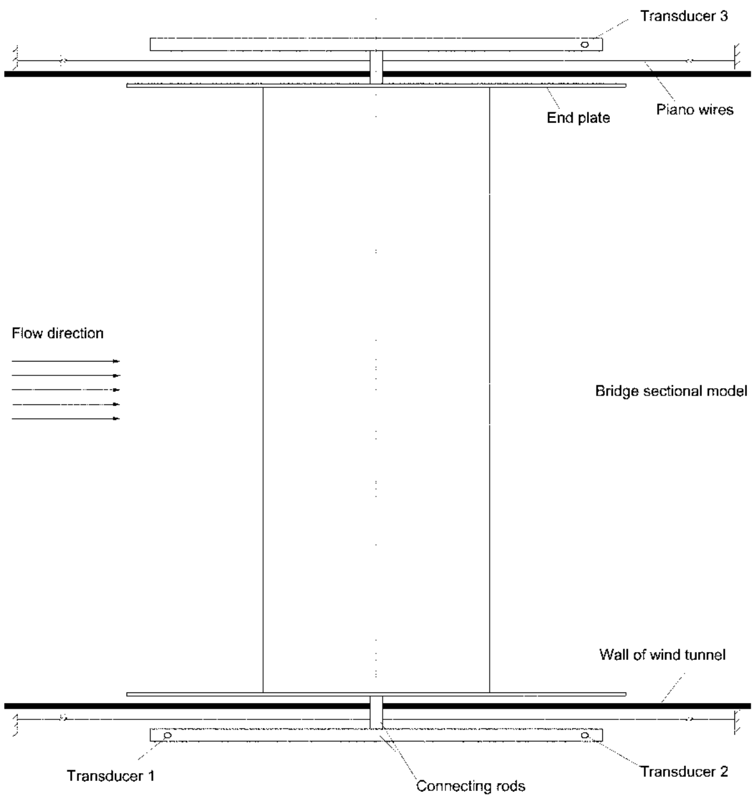

Fig. 4 Top view of the test setup

where x_1, x_2 and x_3 are the measurements of transducer 1, 2 and 3, respectively. $2l$ is the space between transducer 1 and transducer 2.

3.2. Case 1: thin plate model

A streamlined thin plate is firstly selected for wind tunnel test (see Fig. 5). The width to height (thickness) ratio of the plate is about 21, so it can be reasonably regarded as an 'ideal' thin plate, and thus its theoretical flutter derivatives can be extracted by Theodorson's functions (Li 1996).

The measured buffeting responses are sampled at a rate of 100 Hz for about 14 minutes (see Fig. 6). The covariance-driven SSI technique is used to identify modal parameters from these data, and the pseudo-inverse method is applied to estimate the stiffness and damping matrices. The natural modal parameters are extracted from the free decay responses in the no-wind case using the ERA

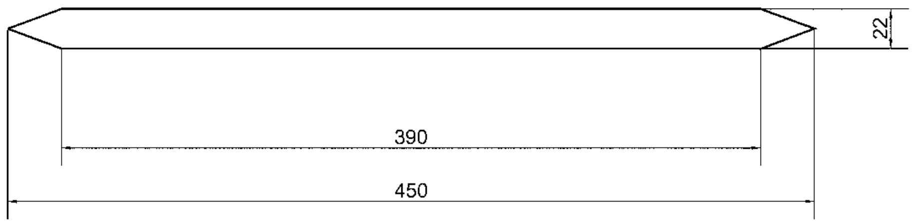

Fig. 5 Cross section of the streamlined thin plate (unit: mm)

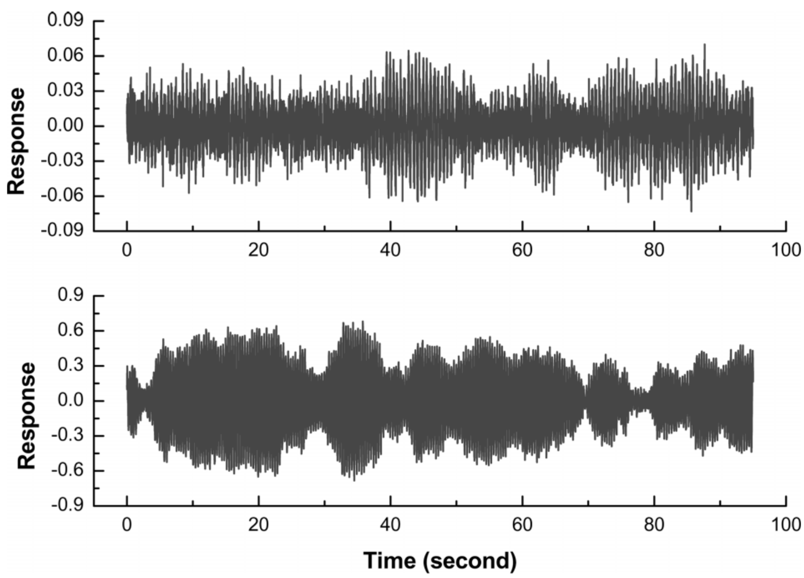

Fig. 6 A segment of random acceleration responses of the thin plate at 5 m/s wind speed (Upper: vertical Lower: torsional)

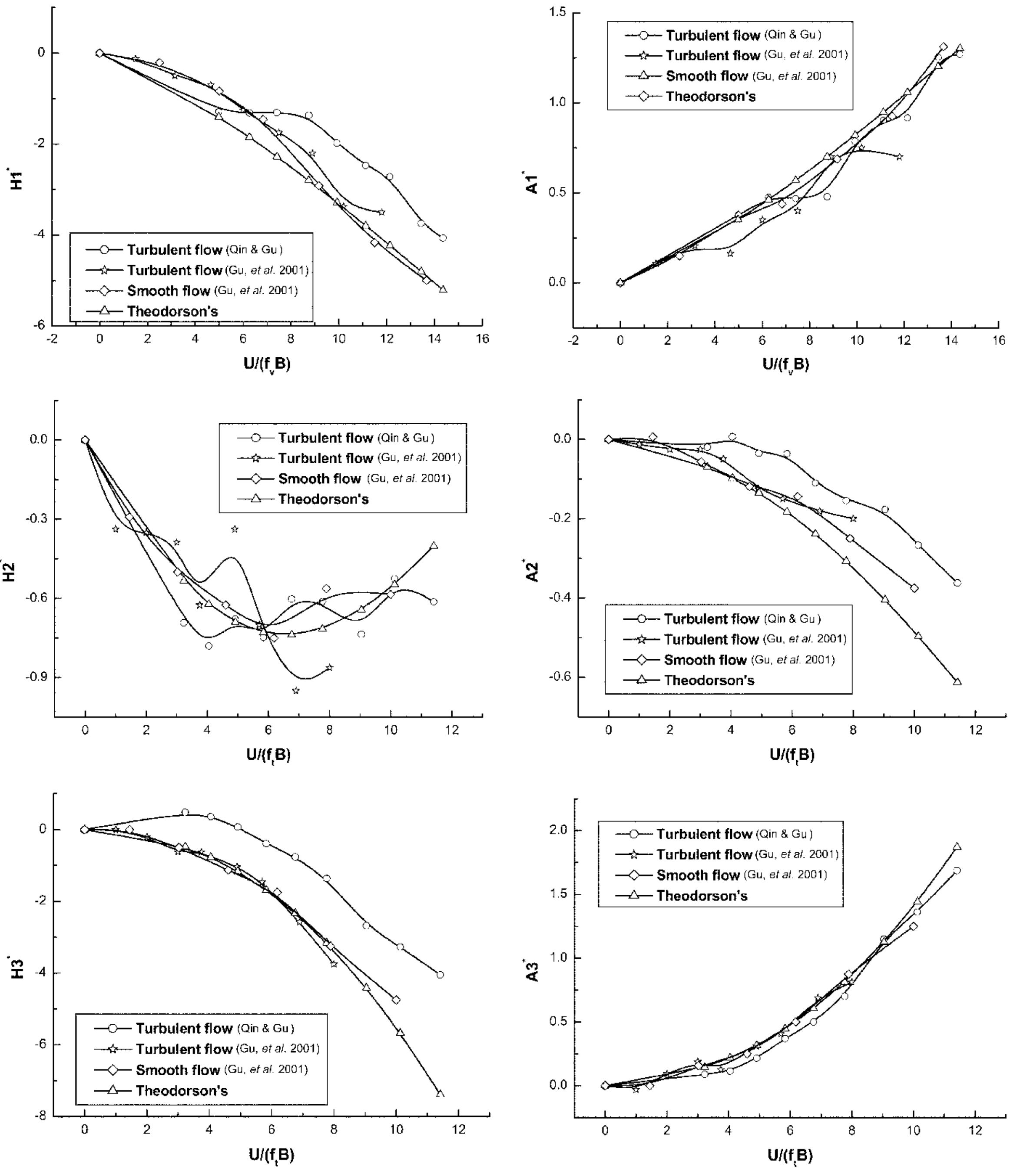

Fig. 7 The flutter derivatives of the thin plate identified by SSI in comparison with Theodorson's theoretical values and those identified by the unifying least-square method (Gu, *et al.* 2000)

technique.

The testing wind speed ranges from 4 m/s to 12 m/s. Fig. 7 shows the flutter derivatives of the

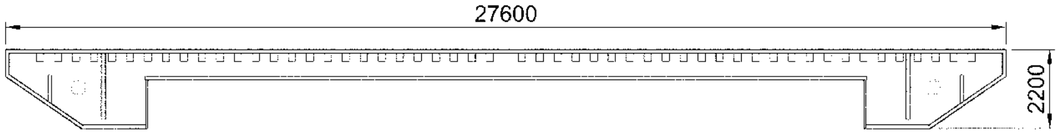

Fig. 8 Schematic cross section of Hong-guang Bridge (unit: mm)

thin plate estimated by covariance-driven SSI and those identified by the unifying least-square method (Gu, *et al*. 2000), as well as Theodorson's theoretical values.

Flutter derivative A_1^*, H_2^* and A_3^* identified by SSI match well with the theoretical values and those identified by the unifying least-square method (Gu, *et al*. 2000); and the absolute values of the flutter derivatives H_1^* and A_2^* seem to be somewhat smaller than the theoretical values; but H_3^* shows much less absolute values. The differences between the flutter derivatives identified by the present method and the corresponding derivatives by the other methods may be due to the different treatments of turbulence in incoming wind, as mentioned before.

3.3. Case 2: section model of Hong-guang bridge

Encouraged by the success in the thin plate model, we proceed to estimate the flutter derivatives of the Hong-guang Bridge, a cable-supported bridge with a Π type stiffened girder, as shown in Fig. 8.

The section model of Hong-guang Bridge is designed according to the law of similarity at a geometric scale of one to fifty. The model is tested in turbulent flow. The testing wind speed ranges from 4 m/s to 10 m/s, in steps of 1 m/s. The random acceleration responses are measured and flutter derivatives are identified by covariance-driven SSI. The identified flutter derivatives and their comparison with those of the same model in smooth flow (Song 2003) are shown in Fig. 9.

Generally speaking, the flutter derivatives of the bridge in turbulent flow identified by SSI are in agreement with those in smooth flow (Song 2003) identified by the unifying least-square method (Gu, *et al*. 2000). From Fig. 9 it can further be found that the influence of flow type on H_3^* and A_3^*, i.e., flutter derivatives related to aerodynamic stiffness, seems to be negligible, while the other four derivatives concerned with aerodynamic damping show rather notable deviations from those in smooth flow, especially at high reduced wind speeds. Such deviations may reveal the fact that for those bridges with the Π type sections similar to Hong-guang Bridge, the effects of turbulence on aerodynamic damping may be significant. Consequently, the wind tunnel tests of such bridges for flutter derivative estimation should be carried out in turbulent flow.

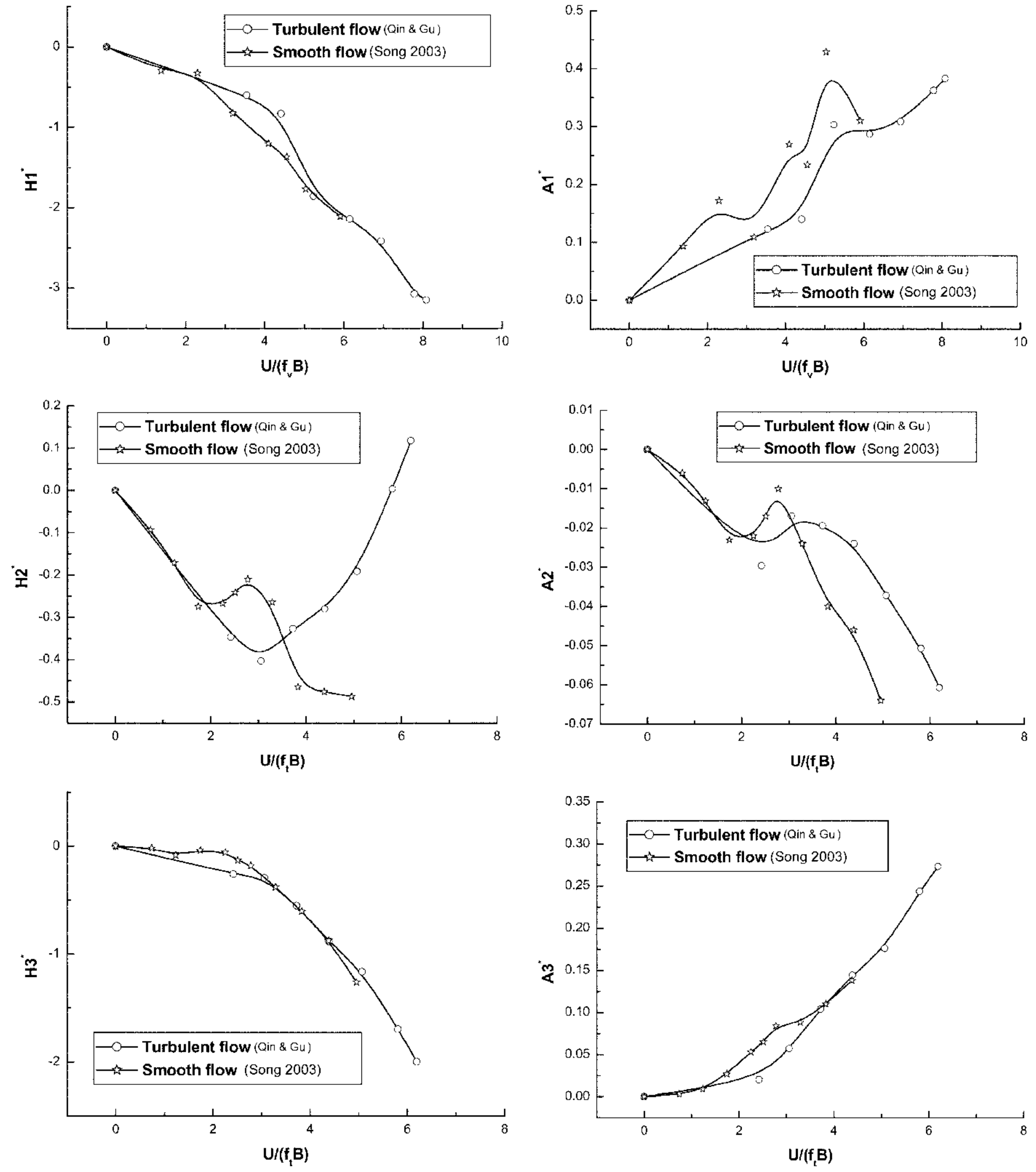

Fig. 9 The flutter derivatives of Hong-guang Bridge in turbulent flow identified by SSI in comparison with those in smooth flow identified by unifying least-square method

4. Conclusions

The covariance-driven Stochastic Subspace Identification (SSI in short) technique is applied to determination of the flutter derivatives of bridge section models in turbulent flow. Wind tunnel tests of a streamlined thin plate and the Π type blunt section model of the cable-supported Hong-guang Bridge are conducted in TJ-1 boundary layer wind tunnel in Tongji University. Random acceleration responses of the models are measured, and the flutter derivatives are estimated by the suggested covariance-driven SSI technique.

The flutter derivatives of the thin plate model identified by SSI match well with the theoretical values and those estimated by the unifying least-square method. As to the Π type section model of Hong-guang Bridge, the flutter derivatives of the bridge in turbulent flow identified by SSI are in agreement with those in smooth flow identified by the unifying least-square method. The effects of flow type on the two flutter derivatives of the bridge concerning with aerodynamic stiffness, i.e., H_3^* and A_3^*, seem to be negligible, whereas the other four derivatives related to aerodynamic damping characteristics show somewhat marked deviations from those in smooth flow, especially at high reduced wind speeds. Such deviations seem to suggest that for those bridges with the blunt sections similar to Hong-guang Bridge, the aerodynamic damping characteristics in turbulent flow may be different from those in smooth flow. Therefore it may be proper to conduct wind tunnel tests for flutter derivative estimation of this kind of bridges in turbulent flow.

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