Wind and Structures, Vol. 7, No. 2 (2004) 89-106
DOI: http://dx.doi.org/10.12989/was.2004.7.2.089

Effects of viscoelastic memory on the buffeting response of tall buildings

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(Received March 5, 2003, Accepted November 28, 2003)

Abstract. The response of tall buildings to gust buffeting is usually evaluated assuming that the structural damping is of a viscous nature. In addition, when dampers are incorporated in the design to mitigate the response, their effect is allowed for increasing the building modal damping ratios by a quantity corresponding to the additional energy dissipation arising from the presence of the devices. Even though straightforward, this procedure has some degree of inaccuracy due to the existence of a memory effect, associated with the damping mechanism, which is neglected by a viscous model. In this paper a more realistic viscoelastic model is used to evaluate the response to gust buffeting of tall buildings provided with energy dissipation devices. Both cases of viscous and hysteretic inherent damping are considered, while for the dampers a generic viscoelastic behaviour is assumed. The Laguerre Polynomial Approximation is used to write the equations of motion and find the frequency response functions. The procedure is applied to a 25-story building to quantify the memory effects, and the inaccuracy arising when the latter is neglected.

Keywords: gust buffeting; alongwind response; viscoelastic damping; viscoelastic memory; Laguerre Polynomial Approximation.

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1. Introduction

For wind engineering applications, it is customary to model flexible structures as linear dynamic systems provided with viscous damping. In the case in which Energy Dissipation Devices (EDDs) are added, it is often assumed that the viscous behaviour of the system is not changed. The effects of the dampers are, then, accounted for increasing each modal damping ratio by a quantity termed *effective damping*. The latter represents the additional viscous modal damping to be provided to the plain structure in order to make it experience the same steady state response, under a given excitation, as the structure with the dampers would.

For the above approach to be consistent, two conditions have to be met: (a) the structure inherent damping has to be viscous, and (b) the devices have to be viscous dashpots rigidly connected to the main structural system. In real life, however, both conditions quite often fail to be satisfied. Structural systems in many cases exhibit a damping mechanism which, even though linear for small amplitude oscillations, proves to be weakly dependent on frequency (Nashif, Jones and Henderson 1985, Sun and Lu 1995). A linear hysteretic model (viscoelastic), therefore, provides a more accurate description of their behaviour than a viscous model. On the other hand, dampers exhibit a variety of types of behaviour, which in many cases are far away from being of a linear viscous nature. For the mitigation of the wind induced response viscoelastic dampers are usually adopted (Soong and Dargush 1997), as their damping capacities, even though of a lower magnitude with respect to metal yielding dampers used in seismic applications, do not suffer from fatigue damage. Moreover, also in the case in which viscous dashpots are used, their global behaviour turns out to be viscoelastic, when not rigidly connected to the main structure (e.g., in the case in which flexible braces are used for their connection). Based on the two above observations, it is clear that a viscoelastic model, rather than a viscous one, would be adequate to describe the behaviour of wind exposed flexible structures provided with damping devices.

One of the principal characteristics of a viscoelastic system is that of having a memory behaviour. This means that knowledge of displacement and velocity at a particular time instant, together with the external excitation, does not allow prediction of the system evolution. Indeed, knowledge of the whole previous displacement history is required, meaning that displacement and velocity do not fully describe the state of the system. When a viscous approximation is used to model a system featuring a viscoelastic behaviour, the memory effect is lost, and this affects the accuracy of the analyses, and the loss of accuracy depends on the characteristics of the viscoelastic memory and of the excitation.

In this paper a procedure is presented for the analysis of tall buildings subjected to gust buffeting, including the viscoelastic memory of both main structure and additional dampers. The Laguerre Polynomial Approximation method (Palmeri, *et al*. 2003, De Luca, *et al*. 2002) will be applied to write the equations of motions and derive the frequency response functions of the system. Finally, the procedure is used for the analysis of the response of a 25-story moment resisting frame building, without and with the addition of viscous dashpots connected to the main structure through elastic braces.

2. Equations of motion for buildings with viscoelastic behaviour including viscoelastic dampers

2.1. Equations of motion in Lagrangian coordinates

The motion of buildings featuring linear viscous damping is described through the equation:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{L_f}\mathbf{f}(t) \quad (1)$$

where $\mathbf{x}(t)=[x_1(t) \ ... \ x_n(t)]^T$ is the array listing the time histories of the n Lagrangian coordinates of the system, **M**, **C** and **K** are the mass, damping and stiffness matrices, respectively, $\mathbf{f}(t)=[f_1(t) \ ... \ f_r(t)]^T$ is the array listing the time histories of the r external loads, and $\mathbf{L_f}$ is its influence matrix.

In the case in which the building is provided with EDDs, the damping matrix in Eq. (1) is the sum of two terms, one accounting for the inherent damping, the other accounting for the additional damping arising from the presence of the EDDs.

Eq. (1) can be solved in the frequency domain as:

$$\begin{aligned} &\mathbf{X}(\omega)=\mathbf{H}(\omega)\mathbf{F}(\omega) \\ &\mathbf{H}(\omega)=[\mathbf{K}-\omega^2\mathbf{M}+\mathrm{j}\omega\mathbf{C}]^{-1}\mathbf{L_f} \end{aligned} \quad (2)$$

where $\mathbf{X}(\omega)$ and $\mathbf{F}(\omega)$ are the Fourier transforms of the response $\mathbf{x}(t)$ and of the excitation $\mathbf{f}(t)$, respectively, and $\mathbf{H}(\omega)$ is the $n \times r$ frequency response matrix of the system in Lagrangian coordinates.

In Eq. (1) the displacement $\mathbf{x}(t)$ together with the velocity $\dot{\mathbf{x}}(t)$ fully define the state of the system; as a consequence, the memory effect is neglected. The system described through Eq. (1) is of Kelvin-Voigt type, and is the only viscoelastic system without memory.

More generally, the equations of motion for a building featuring a linear viscoelastic behaviour including memory can be written as (Fig. 1(a)):

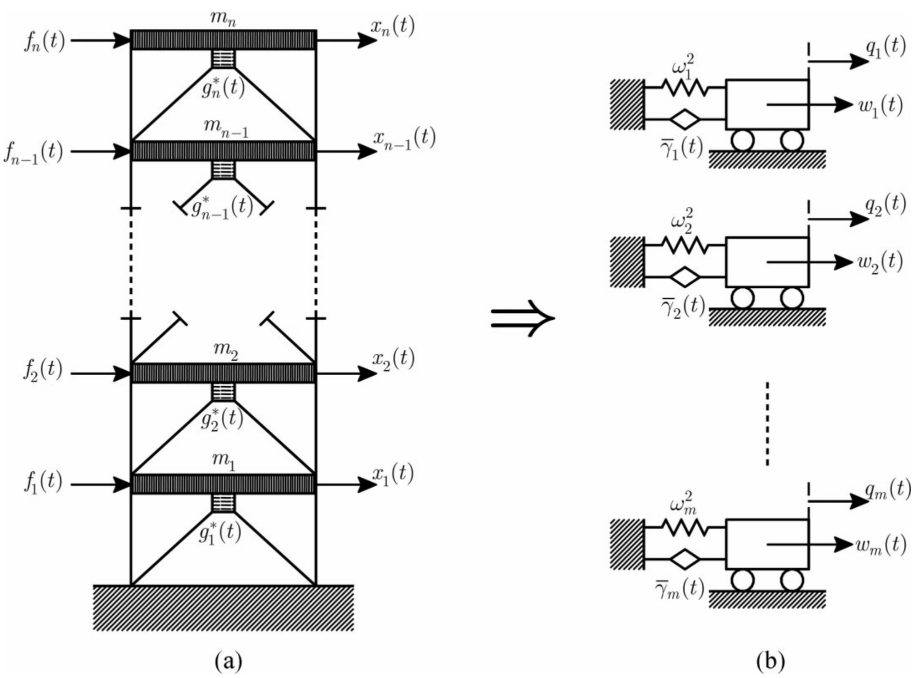


Fig. 1 Schematic of building with viscoelastic dampers (a) and decoupled modal representation (b)

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{r}(t) + \mathbf{L}_\mathrm{p}\,\mathbf{p}(t) = \mathbf{L}_\mathbf{f}\,\mathbf{f}(t) \\ \mathbf{r}(t) = \int_0^t \mathcal{K}(t-\tau)\dot{\mathbf{x}}(\tau)\mathrm{d}\tau \\ \mathbf{p}(t) = \int_0^t \mathcal{G}(t-\tau)\mathbf{V}\dot{\mathbf{x}}(\tau)\mathrm{d}\tau \end{cases} \tag{3}$$

where $\mathbf{r}(t)=[r_1(t) \ldots r_n(t)]^\mathrm{T}$ is the array listing the internal viscoelastic forces, $\mathbf{p}(t)=[p_1(t) \ldots p_l(t)]^\mathrm{T}$ is the array of the viscoelastic reactions of the additional EDDs, $\mathcal{K}(t)$ and $\mathcal{G}(t)$ are the relaxation matrices of the structure and of the l EDDs, respectively, $\mathbf{V}$ is the $l \times n$ matrix associating the relative displacements at the EDDs to the building displacements, and $\mathbf{L_p}$ is the $n \times l$ matrix associating the building displacements to the relative displacements at the EDDs. Eqs. (3) hold for $t \geq 0$, and it is assumed that the structure is at rest for $t < 0$.

The internal viscoelastic forces can be expressed as the sum of n terms, each associated with one particular Degree of Freedom (DoF) of the building. Therefore the second of Eqs. (3) takes the expression:

$$r_i(t) = \sum_{j=1}^{n} \int_0^t k_{i,j}(t-\tau)\dot{x}_j(\tau)\mathrm{d}\tau \qquad i=1, \ldots, n \tag{4}$$

in which $k_{i,j}(t) \equiv k_{j,i}(t)$ is the time history of the viscoelastic force in the i-th DoF, associated with a unit step displacement in the j-th DoF at $t=0$.

The function $k_{i,j}(t)$ is termed a *relaxation function*, and can be expressed as the sum of a constant portion $k_{i,j}^{\infty} = k_{i,j}(\infty)$, representing the purely elastic part of the viscoelastic reaction, and a time-varying portion $k_{i,j}^{*}(t) = k_{i,j}(t) - k_{i,j}^{\infty}$, which takes into account the viscoelastic memory. Eq. (4) becomes:

$$r_i(t) = \sum_{j=1}^{n} k_{i,j}^{\infty} x_j(t) + \sum_{j=1}^{n} \int_0^t k_{i,j}^{*}(t-\tau)\dot{x}_j(\tau)\mathrm{d}\tau \qquad i=1, \ldots, n \tag{5}$$

or, in a matrix form:

$$\mathcal{K}(t) = \mathbf{K}^{\infty} + \mathbf{K}^{*}(t) \tag{6}$$

where:

$$\mathbf{K}^{\infty} = \begin{bmatrix} k_{1,1}^{\infty} & k_{1,2}^{\infty} & \cdots & k_{1,n}^{\infty} \\ & k_{2,2}^{\infty} & \cdots & k_{2,n}^{\infty} \\ \text{sym} & & \ddots & \vdots \\ & & & k_{n,n}^{\infty} \end{bmatrix}; \quad \mathbf{K}^{*}(t) = \begin{bmatrix} k_{1,1}^{*}(t) & k_{1,2}^{*}(t) & \ldots & k_{1,n}^{*}(t) \\ & k_{2,2}^{*}(t) & \ldots & k_{2,n}^{*}(t) \\ \text{sym} & & \ddots & \vdots \\ & & & k_{n,n}^{*}(t) \end{bmatrix} \tag{7}$$

which brings the second of Eqs. (3) to be rewritten as:

$$\mathbf{r}(t) = \mathbf{K}^{\infty}\mathbf{x}(t) + \int_0^t \mathbf{K}^{*}(t-\tau)\dot{\mathbf{x}}(\tau)\mathrm{d}\tau \tag{8}$$

In Eq. (7), $\mathbf{K}^{\infty}$ is the long term stiffness matrix of the plain building, playing the same role as the stiffness matrix **K** in Eq. (1).

Following the same approach, the viscoelastic reactions of the additional EDDs can be expressed as the sum of n terms, each associated with one DoF of the building. The third of Eqs. (3) takes the expression (similar to Eq. (5)):

$$p_i(t) = \sum_{j=1}^{n} g_i^{\infty} V_{i,j} x_j(t) + \sum_{j=1}^{n} \int_0^t g_i^{*}(t-\tau) V_{i,j} \dot{x}_j(\tau)\mathrm{d}\tau \qquad i = 1, \ldots, l \tag{9}$$

g_i^{∞} and $g_i^{*}(t)$ being the constant and the time-varying parts of the relaxation function of the i-th viscoelastic EDD.

In a matrix form (similar to Eq. (6)):

$$\mathcal{G}(t) = \mathbf{G}^{\infty} + \mathbf{G}^{*}(t) \tag{10}$$

where $\mathbf{G}^{\infty} = \mathrm{diag}\{g_i^{*}(t);\ i = 1, \ldots, l\}$ and $\mathbf{G}^{*}(t) = \mathrm{diag}\{g_i^{*}(t);\ i = 1, \ldots, l\}$ are matrices listing the constant and the time-varying parts of the relaxation functions of the viscoelastic EDDs. Finally, the third of Eqs. (3) can then be rewritten as (similar to Eq. (8)):

$$\mathbf{p}(t) = \mathbf{G}^{\infty}\mathbf{V}\mathbf{x}(t) + \int_0^t \mathbf{G}^{*}(t-\tau)\mathbf{V}\dot{\mathbf{x}}(\tau)\mathrm{d}\tau \tag{11}$$

2.2. Equations of motion in modal coordinates

Eqs. (3) can be expressed in modal coordinates, defined through the transformation:

$$\mathbf{x}(t) = \mathbf{\Phi}\,\mathbf{q}(t) = \sum_{i=1}^{m} \phi_i q_i(t) \tag{12}$$

where $\mathbf{\Phi}=[\phi_1\ \ldots\ \phi_m]$ is the modal matrix of the plain building evaluated for $t \to \infty$ (that is, accounting only for the long term stiffness matrix) whose columns are the first $m \le n$ structural modes, and where $\mathbf{q}(t)=[q_1(t)\ \ldots\ q_m(t)]^{\mathrm{T}}$ is the array of the modal coordinates.

Substitution of Eqs. (8) and (11) into Eqs. (3) provides the equations of motion in modal coordinates:

$$\begin{cases} \ddot{\mathbf{q}}(t) + \boldsymbol{\Omega}^2\mathbf{q}(t) + \int_0^t \boldsymbol{\Gamma}(t-\tau)\dot{\mathbf{q}}(\tau)\mathrm{d}\tau + \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathbf{p}(t) = \boldsymbol{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathbf{f}}\,\mathbf{f}(t) \\ \mathbf{p}(t) = \mathbf{G}^{\infty}\mathbf{V}\boldsymbol{\Phi}\mathbf{q}(t) + \int_0^t \mathbf{G}^{*}(t-\tau)\mathbf{V}\boldsymbol{\Phi}\dot{\mathbf{q}}(\tau)\mathrm{d}\tau \end{cases} \tag{13}$$

where $\boldsymbol{\Omega}=\mathrm{diag}\{\omega_i\ ;\ i=1, ..., m\}$, $\omega_1 \leq ... \leq \omega_m$, is the matrix containing the natural circular frequencies of the plain building, obtained as solutions of the eigenproblem $\mathbf{K}^{\infty}\phi_i = \mathbf{M}\phi_i\omega_i^2$, and where $\boldsymbol{\Gamma}(t)$ is the matrix containing the time-varying parts of the building modal relaxation functions:

$$\boldsymbol{\Gamma}(t)=\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{K}^{*}(t)\boldsymbol{\Phi} \tag{14}$$

The equation governing the i-th modal coordinate is:

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) + \sum_{j=1}^{m}\int_0^t \gamma_{i,j}(t-\tau)\dot{q}_j(\tau)\mathrm{d}\tau + u_i(t) = w_i(t) \tag{15}$$

where $w_i(t) = \phi_t^{\mathrm{T}}\mathbf{L}_{\mathbf{f}}\,\mathbf{f}(t)$ is the i-th modal excitation, $u_i(t) = \phi_i^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\,\mathbf{p}(t)$ is the projected EDD reaction on the i-th mode, and where $\gamma_{i,j}(t) = \phi_i^{\mathrm{T}}\boldsymbol{\Gamma}(t)\phi_j \equiv \gamma_{j,i}(t)$ is the projected time-varying part of the structure relaxation functions on the i-th mode, i.e. the time history of the i-th modal force due to a unit step displacement in the j-th mode.

For buildings with viscous damping, it is often assumed that the eigenvectors are orthogonal with respect not only to the mass and stiffness matrices, but also with respect to the damping matrix; as consequence the equations of motion in modal coordinates are decoupled (Fig. 1(b)). This result can be extended to the case of viscoelastic damping, assuming that at any time instant the eigenvectors are orthogonal with respect to the time-varying part of the relaxation matrix $\mathbf{K}^{*}(t)$. As a consequence Eq. (14) becomes:

$$\boldsymbol{\Gamma}(t)=\mathrm{diag}\{\gamma_i(t)\ ;\ i=1, ..., m\} \tag{16}$$

where for simplicity $\gamma_{i,i}(t)$ has been written as $\gamma_i(t)$. Under this hypothesis, Eq. (15) becomes:

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) + \int_0^t \gamma_i(t-\tau)\dot{q}_i(\tau)\mathrm{d}\tau + u_i(t) = w_i(t) \qquad i = 1, ..., m \tag{17}$$

where the only coupling derives from the viscoelastic forces exerted by the EDDs:

$$u_i(t) = \phi_i^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\left\{\mathbf{G}^{\infty}\mathbf{V}\sum_{j=1}^{m}\phi_j q_j(t) + \sum_{j=1}^{m}\int_0^t \mathbf{G}^{*}(t-\tau)\mathbf{V}\phi_j\dot{q}_j(\tau)\mathrm{d}\tau\right\} \tag{18}$$

In the following section, Eq. (17) will be written for the particular cases of buildings with either linear viscous damping or linear hysteretic damping.

3. Equations of motion for buildings with viscous or hysteretic inherent damping

3.1. Buildings with viscous inherent damping

The case in which the structural damping is of a viscous nature (i.e., without memory) can now be seen as a particular case of Eq. (17). The *i*-th relaxation function $\gamma_i^{\mathrm{V}}(t)$ of the building with viscous damping is (Lockett 1972):

$$\gamma_i^{\mathrm{V}}(t) = 2\zeta_i\omega_i\delta(t) \tag{19}$$

where, ζ_i is the *i*-th viscous damping ratio, and where $\delta(t)$ is the Dirac delta function. Substitution of Eq. (19) into Eq. (17) brings:

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) + 2\zeta_i\omega_i\dot{q}_i(t) + u_i(t) = w_i(t) \qquad i = 1, ..., m \tag{20}$$

Eqs. (20) are a set of m equations coupled by the terms $u_i(t)$, which in a matrix form can be written:

$$\ddot{\mathbf{q}}(t) + \mathbf{\Xi}^{\mathrm{V}}\dot{\mathbf{q}}(t) + (\mathbf{\Omega}^2 + \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathbf{G}^{\infty}\mathbf{V\Phi})\mathbf{q}(t) + \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\int_0^t \mathbf{G}^*(t-\tau)\mathbf{V\Phi}\dot{\mathbf{q}}(\tau)\mathrm{d}\tau = \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathbf{f}}\,\mathbf{f}(t) \tag{21}$$

where:

$$\Xi^{\mathrm{V}} = 2\mathrm{diag}\{\zeta_i\omega_i\ ;\ \mathrm{i} = 1, ..., m\} \tag{22}$$

The frequency domain solution of Eq. (21) is:

$$\mathbf{Q}(\omega) = \tilde{\mathbf{H}}^V(\omega)\mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathbf{f}}\mathbf{F}(\omega) \tag{23}$$

$\mathbf{Q}(\omega)$ being the Fourier transform of the modal response $\mathbf{q}(t)$, and $\tilde{\mathbf{H}}^V(\omega)$ being the modal frequency response matrix of the building with viscous inherent damping and viscoelastic EDDs:

$$\tilde{\mathbf{H}}^{\mathrm{V}}(\omega) = \{(\mathbf{\Omega}^2 + \bar{\mathbf{\Omega}}^2) - \omega^2\mathbf{I}_m + \mathrm{j}\omega[\mathbf{\Xi}^{\mathrm{V}} + \bar{\mathbf{\Xi}}(\omega)]\}^{-1} \tag{24}$$

where $\mathbf{I}_m$ is the identity matrix of order m, and where the overbar indicates the terms related to the viscoelastic behaviour of the EDDs:

$$\bar{\mathbf{\Omega}}^2 = \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathbf{G}^{\infty}\mathbf{V\Phi}$$

$$\bar{\mathbf{\Xi}}(\omega) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathcal{F}\langle\mathbf{G}^*(t)\rangle\mathbf{V\Phi} \tag{25}$$

in which $\mathcal{F}\langle\cdot\rangle$ stands for the Fourier transform operator. The elements of the matrix $\mathcal{F}\langle\mathbf{G}^*(t)\rangle$ are related to the dynamic stiffness of the EDDs, which can be directly measured by means of sinusoidal tests on the dampers.

In general, neither matrices defined through Eqs. (25) are diagonal, which brings the frequency response matrix $\tilde{\mathbf{H}}(\omega)$ to be sparse. Nevertheless, if the distribution of the EDDs in the structure is almost homogeneous, then it is expected that the off-diagonal terms of the $\tilde{\mathbf{H}}(\omega)$ matrix are negligible. In the latter case, Eqs. (25) become:

$$\begin{aligned} \bar{\mathbf{\Omega}}^2 &\cong \mathrm{diag}\{\bar{\omega}_i^2;\ i=1,\ldots,m\} \\ \bar{\mathbf{\Xi}}(\omega) &\cong \mathrm{diag}\{\mathcal{F}\langle\bar{\gamma}_i(t)\rangle;\ i=1,\ldots,m\} \end{aligned} \tag{26}$$

with:

$$\begin{aligned} \bar{\omega}_i^2 &= \phi_i^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathbf{G}^{\infty}\mathbf{V}\phi_i \\ \bar{\gamma}_i(t) &= \phi_i^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathbf{G}^*(t)\mathbf{V}\phi_i \end{aligned} \tag{27}$$

Eqs. (26) decouple Eqs. (20), which become:

$$\ddot{q}_i(t) = (\omega_i^2+\bar{\omega}_i^2)q_i(t) + 2\zeta_i\omega_i\dot{q}_i(t) + \int_0^t \bar{\gamma}_i(t-\tau)\dot{q}_i(\tau)\,\mathrm{d}\tau = w_i(t) \qquad i=1,\ldots,m \tag{28}$$

Then, the solution of Eq. (28) in the frequency domain is:

$$Q_i(\omega) = \tilde{H}_i^{\mathrm{V}}(\omega)W_i(\omega) \qquad i=1,\ldots,m \tag{29}$$

where $Q_i(\omega) = \mathcal{F}\langle q_i(t)\rangle$ and $W_i(\omega) = \mathcal{F}\langle w_i(t)\rangle = \phi_i^{\mathrm{T}}\mathbf{L}_{\mathbf{f}}\mathcal{F}\langle\mathbf{f}(t)\rangle$, while:

$$\tilde{H}_i^{\mathrm{V}}(\omega) = \{\omega_i^2+\bar{\omega}_i^2-\omega^2+\mathrm{j}\omega[2\zeta_i\omega_i+\mathcal{F}\langle\bar{\gamma}_i(t)\rangle]\}^{-1} \tag{30}$$

3.2. Buildings with hysteretic inherent damping

If compared to the viscous model, in many cases the linear hysteretic model has been found to provide a better description of the real damping of buildings (Nashif, Jones and Henderson 1985, Sun and Lu 1995), as for a wide range of structural materials the energy loss per cycle appears to be almost frequency independent. The analytical model commonly used to describe the linear hysteretic damping in the time domain involves the Hilbert transform operator (Bracewell 1986). This was proved to be a pathologic model (Inaudi and Kelly 1995, Inaudi and Makris 1996) because it does not meet the causality requirement. In two recent papers, Makris and Zhang (2001) and Spanos and Tsavachidis (2001) suggested that the Biot model (Biot 1958) be used to approximate linear hysteretic damping. This model is causal and physically realisable, and brings a closed form time domain representation. Using the Biot model, the time-varying relaxation function in the *i*-th mode of a building with hysteretic damping, i.e., the *i*-th term in Eq. (16) is written as:

$$\gamma_i^{\mathrm{H}}(t) = -\frac{2}{\pi}\omega_i^2\eta_i \mathrm{Ei}(-\varepsilon_i t) \tag{31}$$

where η_i is the loss factor, ε_i is a parameter to be calibrated based on the natural frequency of the structure and on the frequency content of the excitation, and where $\mathrm{Ei}(\cdot)$ is the exponential integral function defined as (Gradshteyn and Ryzhik 1994):

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^{\xi}}{\xi}\mathrm{d}\xi \qquad x<0 \tag{32}$$

Substitution of Eq. (31) into Eq. (17) gives the equation of motion in the *i*-th mode for a building with inherent hysteretic damping:

$$\ddot{q}_i(t) + \omega_i^2\left\{q_i(t) - \frac{2\eta_i}{\pi}\int_0^t \mathrm{Ei}[-\varepsilon_i(t-\tau)]\dot{q}_i(\tau)\mathrm{d}\tau\right\} + u_i(t) = w_i(t) \qquad i = 1, ..., m \tag{33}$$

which plays the same role as Eq. (20) does for buildings with viscous damping. In a matrix form (corresponding to Eq. (21)):

$$\ddot{\mathbf{q}}(t) = \int_0^t \mathbf{\Gamma}^{\mathrm{H}}(t-\tau)\dot{\mathbf{q}}(\tau)\mathrm{d}\tau + [\mathbf{\Omega}^2 + \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\mathbf{G}_{\infty}\mathbf{V}\mathbf{\Phi}]\mathbf{q}(t)$$
$$+ \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{p}}\int_0^t \mathbf{G}^*(t-\tau)\mathbf{V}\mathbf{\Phi}\dot{\mathbf{q}}(\tau)\mathrm{d}\tau = \mathbf{\Phi}^{\mathrm{T}}\mathbf{L}_{\mathrm{f}}\mathbf{f}(t) \tag{34}$$

in which:

$$\mathbf{\Gamma}^{\mathrm{H}}(t) = -\frac{2}{\pi}\mathrm{diag}\{\omega_i^2\eta_i\mathrm{Ei}(-\varepsilon_i t);\ i = 1, ..., m\} \tag{35}$$

The modal frequency response matrix has an expression similar to that found for structures with viscous damping (Eq. (24)):

$$\tilde{\mathbf{H}}^{\mathrm{H}}(\omega) = \{(\mathbf{\Omega}^2 + \bar{\mathbf{\Omega}}^2) - \omega^2\mathbf{I}_m + \mathrm{j}\omega[\mathbf{\Xi}^{\mathrm{H}}(\omega) + \bar{\mathbf{\Xi}}(\omega)]\}^{-1} \tag{36}$$

where:

$$\mathbf{\Xi}^{\mathrm{H}}(\omega) = \mathcal{F}\langle\mathbf{\Gamma}^{\mathrm{H}}(t)\rangle$$
$$= \frac{2}{\pi\omega}\mathrm{diag}\left\{\omega_i^2\eta_i\left[\arctan\left(\frac{\omega}{\varepsilon_i}\right) - \mathrm{j}\ln\sqrt{1+\left(\frac{\omega}{\varepsilon_i}\right)^2}\right];\ i = 1, ..., m\right\} \tag{37}$$

and where $\bar{\mathbf{\Omega}}^2$ and $\bar{\mathbf{\Xi}}(\omega)$ have been defined through Eqs. (25).

Finally, under the hypothesis of homogeneously distributed EDDs, Eqs. (33) become:

$$\ddot{q}_i(t) + (\omega_i^2 + \bar{\omega}_i^2)q_i(t) + \int_0^t \left\{ \frac{2\eta_i}{\pi} \mathrm{Ei}[-\varepsilon_i(t-\tau)] + \bar{\gamma}_i(t-\tau) \right\} \dot{q}_i(\tau)\,\mathrm{d}\tau = w_i(t)$$

$$i = 1, \ldots, m \tag{38}$$

and the i-th modal frequency response function is:

$$\tilde{H}_i^{\mathrm{H}}(\omega) = \left\{ \omega_i^2 + \bar{\omega}_i^2 - \omega^2 + \frac{2}{\pi}\omega_i^2 \eta_i \left[\ln\sqrt{1 + \left(\frac{\omega}{\varepsilon_i}\right)^2} + \mathrm{j}\arctan\left(\frac{\omega}{\varepsilon_i}\right) \right] + \mathrm{j}\,\omega \mathcal{F}\langle \bar{\gamma}_i(t) \rangle \right\}^{-1} \tag{39}$$

4. The Laguerre Polynomial Approximation for linear viscoelastic systems

In section 3 the decoupled modal integrodifferential equations of motion have been derived for buildings featuring either viscous or hysteretic damping, and provided with an homogeneous distribution of EDDs. The equations can be directly solved in the frequency domain, and it has been shown that the building response is the superposition of the responses of m SDoF modal oscillators, featuring a linear viscoelastic memory (Fig. 1). For each oscillator, the modal relaxation function $\bar{\gamma}_i(t)$ fully defines the memory behaviour.

The solution of Eqs. (28) and (38), however, is not an easy task, and approximated models are usually adopted to handle the viscoelastic memory. Common models are the generalized Maxwell model and generalized Kelvin-Voigt model (Bland 1960), both based on a spring-dashpot representation of the system. Recently, Palmeri *et al.* (2003) proposed a new method to evaluate the dynamic response of a linear viscoelastic SDoF oscillator. The method is based on an approximated form of the relaxation function, in which a linear combination of Laguerre polynomials is used to modulate the relaxation function of a Maxwell element. This approach, termed Laguerre Polynomial Approximation (LPA), brings the introduction of a number of Additional Internal Variables to account for the memory of the system. The LPA method proves to be computationally effective, and its parameters can be directly evaluated from relaxation tests. This approximation was originally developed to be applied in time domain analyses; however, for frequency domain analyses it has the advantage of providing a closed form expression for the dynamic stiffness $\mathrm{j}\,\omega\mathcal{F}\langle\gamma(t)\rangle$ appearing in the Frequency Response Function (FRF). In the following the main features of the LPA method will be briefly outlined.

The equation of motion for a SDoF oscillator made of a mass M connected to a linear viscoelastic support (Fig. 2), with relaxation function $\mathcal{K}(t)$, at rest for $t<0$, is:

$$M\ddot{x}(t) + \int_0^t \mathcal{K}(t-\tau)\dot{x}(\tau)\,\mathrm{d}\tau = f(t) \tag{40}$$

where $x(t)$ is the displacement of the mass and $f(t)$ the external excitation. If the elastic portion K

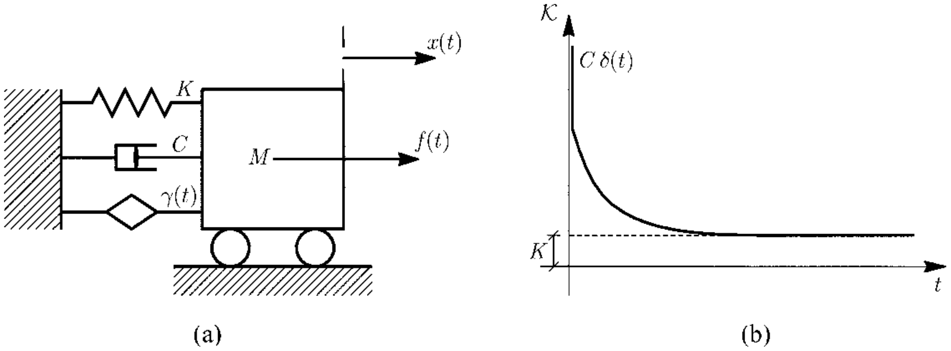


Fig. 2 SDoF viscoelastic system (a) and relaxation function (b)

and the viscous portion $C\delta(t)$ of the reaction are removed from the relaxation function, then Eq. (40) becomes:

$$\ddot{x}(t) + 2\zeta_0\omega_0\dot{x}(t) + \omega_0^2 x(t) + \int_0^t \gamma(t-\tau)\dot{x}(\tau)\mathrm{d}\tau = \frac{1}{M}f(t) \tag{41}$$

where $\omega_0 = \sqrt{M^{-1}K}$ is the natural circular frequency, $\zeta_0 = M^{-1}C/(2\omega_0)$ is the viscous damping ratio and where $\gamma(t)$ is the memory kernel, such that $\mathcal{K}(t) = K + C\delta(t) + M\gamma(t)$.

Eq. (41) coincides with Eqs. (28) or (38) if one sets $\omega_0 = \sqrt{\omega_i^2 + \overline{\omega}_i^2}$, $x(t) = q_i(t)$, $f(t) = M w_i(t)$, and either $\zeta_0 = \zeta_i\omega_i/\omega_0$ and $\gamma(t) = \overline{\gamma}_i(t)$, or $\zeta_0 = 0$ and $\gamma(t) = (2\eta_i/\pi)\mathrm{Ei}(-\varepsilon_i t) + \overline{\gamma}_i(t)$.

The memory kernel is expressed in the approximated form:

$$\gamma_N(t) = \exp\left(-\frac{t}{t_0}\right)p_N(t) \tag{42}$$

in which $p_N(t)$ is an $(N-1)$-order polynomial and t_0 is a characteristic relaxation time, to be chosen based on a linear regression of experimental data (Palmeri, *et al.* 2003).

The role of the polynomial $p_N(t)$ is that of modulating the exponential function $\exp(-t/t_0)$, which is the relaxation function of a Maxwell element with unit stiffness. The polynomial $p_N(t)$ is conveniently expressed as a linear combination of the first N Laguerre polynomials (Gradshteyn and Ryzhik 1994):

$$p_N(t) = \sum_{i=0}^{N-1} a_i \mathrm{L}_i\left(\frac{t}{t_0}\right) \tag{43}$$

where the Laguerre polynomial $\mathrm{L}_i(\cdot)$ can be evaluated through the formulae:

$$\mathrm{L}_0(\xi) = 1$$
$$\mathrm{L}_1(\xi) = 1-\xi$$
$$\mathrm{L}_{i+1}(\xi) = \frac{2i+1-\xi}{i+1}\mathrm{L}_i(\xi) - \frac{i}{i+1}\mathrm{L}_{i-1}(\xi) \qquad i = 2, ..., N-2, ... \tag{44}$$

Substitution of Eq. (43) into Eq. (42) brings:

$$\gamma_N(t) = \sum_{i=0}^{N-1} a_i \theta_i(t)$$
$$\theta_i(t) = \exp\left(-\frac{t}{t_0}\right) \mathrm{L}_i\left(\frac{t}{t_0}\right) \quad i = 0, \dots, N-1 \tag{45}$$

where $\gamma_N(t)$ is the N-order approximation of the relaxation function $\gamma(t)$ ($\gamma_N(t) \to \gamma(t)$ as $N \to \infty$).

Upon substitution of Eq. (45) into Eq. (41), one obtains:

$$\ddot{x}(t) + 2\zeta_0 \omega_0 \dot{x}(t) + \omega_0^2 x(t) + \sum_{i=0}^{N-1} a_i \lambda_i(t) = \frac{1}{M} f(t) \tag{46}$$

where a_i and $\lambda_i(t)$ are termed the i-th Laguerre stiffness and the i-th Laguerre strain, respectively. The Laguerre stiffnesses can be evaluated from the memory kernel as:

$$a_i = \frac{1}{t_0} \int_0^{+\infty} \gamma(t) \mathrm{L}_i\left(\frac{t}{t_0}\right) \mathrm{d}t \tag{47}$$

while the i-th Laguerre strain are defined as:

$$\lambda_i(t) = \int_0^t \theta_i(t-\tau) \dot{x}(\tau) \mathrm{d}\tau \tag{48}$$

By differencing Eq. (48) one obtains the state equations for the Laguerre strains:

$$\dot{\lambda}_i(t) = \dot{x}(t) - \frac{1}{t_0} \sum_{j=0}^{i} \lambda_i(t) \qquad i = 0, \dots, N-1 \tag{49}$$

Eqs. (49) and (46) form a set of linear differential equation that approximate the original integrodifferential equation of motion. The solution in the time domain can be computed using any standard numerical technique. As an alternative, the solution of Eqs. (46) and (49) can be obtained in the frequency domain as:

$$H_N(\omega) = \frac{1}{\omega_0^2 - \omega^2 + \mathrm{j}\omega \xi_N(\omega)}$$
$$\xi_N(\omega) = 2\zeta_0 \omega_0 + \frac{1}{\mathrm{j}\omega} \sum_{i=0}^{N-1} a_i \left(\frac{t_0^2 \omega^2 + \mathrm{j} t_0 \omega}{t_0^2 \omega^2 + 1}\right)^{i+1} \tag{50}$$

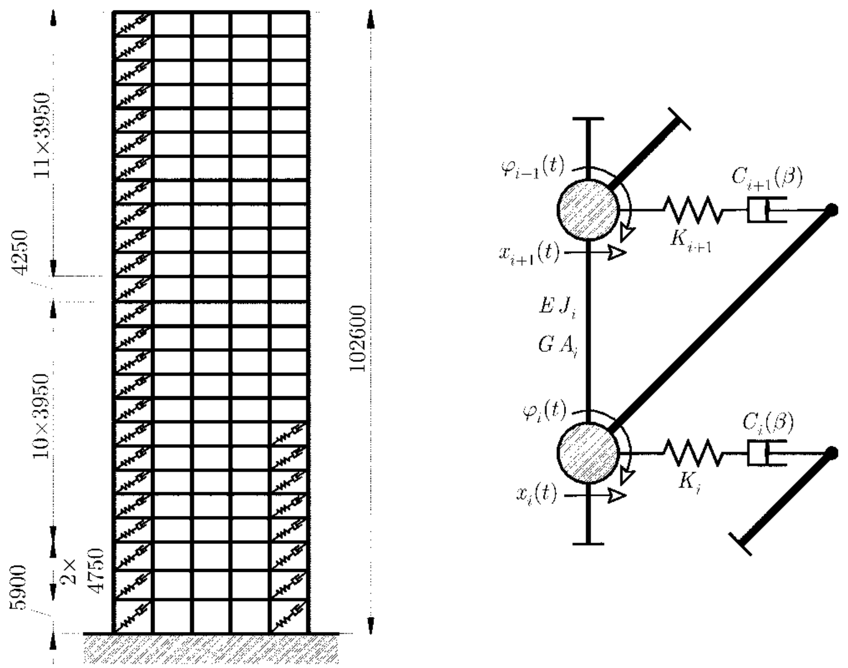
where $X(\omega) = \mathcal{F}\langle x(t) \rangle$ and $F(\omega) = \mathcal{F}\langle f(t) \rangle$, while $H_N(\omega)$ is the approximated FRF. The functions $\xi_N(\omega)$ describe the frequency-dependent damping properties of the system. Particular cases are the undamped oscillator, for which $\xi_N(\omega)=0$, the oscillator with viscous damping, for which $\xi_N(\omega)=2\zeta_0\omega_0$, and the oscillator with hysteretic damping, for which $\xi_N(\omega) \to \eta\omega_0/|\omega|$.

5. Numerical example

The procedure presented in the previous sections was implemented in a Mathematica® 4.0 (1999) code to investigate the effects of viscoelastic memory on the alongwind buffeting response of a 25-story building.

The building (Fig. 3(a)), with a rectangular plan of $44.20\ \mathrm{m} \times 32.00\ \mathrm{m}$, 103 m high (Niwa, *et al.* 1995, Hatada, *et al.* 2000) has a mass of about 160×10^6 kg. The analyses were carried out on a 2-dimensional model of the longitudinal frames, and the DoFs considered in the analyses are the story drifts. The first three natural circular frequencies are $\omega_1=1.87$, $\omega_2=5.61$ and $\omega_3=9.77$ rad/s. The inherent damping is assumed to be of Rayleigh type, with $\zeta_1=0.02$ and $\zeta_3=0.10$.

In a first stage, the response of the plain building (i.e., without additional dampers) in the two cases of viscous and hysteretic inherent damping, are compared. In particular, the hysteretic damping is approximated using the Biot model, with $\eta_i=2\zeta_i$ and $\varepsilon_i=\omega_i/10$ (Makris and Zhang 2000). In Fig. 4 the modulus $|\tilde{H}_i(\omega)|$ and the phase $\measuredangle\tilde{H}_i(\omega)$ of the first three modal FRFs are presented, as evaluated through Eq. (30) and Eq. (39), respectively. The figure shows that globally the difference between the models increases with increasing modal damping. In particular, the



(a) (b)

Fig. 3 25-story building with viscous damping devices (a) and schematic representation of the spring-dashpot medel (b)

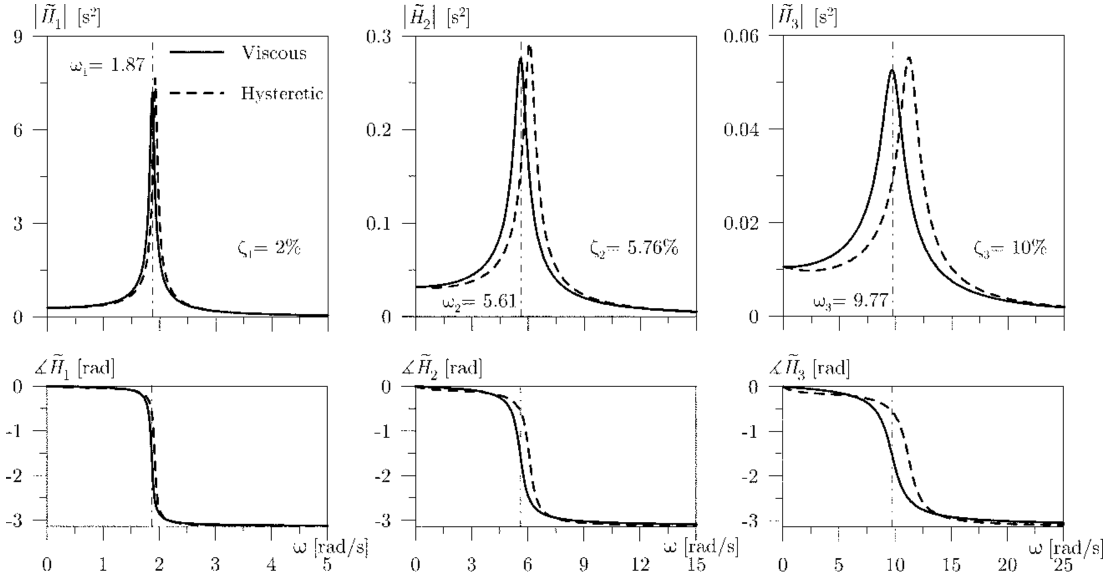


Fig. 4 Comparison between the Frequency Response Function of the building with viscous and hysteretic damping

resonant peak for the hysteretic damping case is at higher frequencies than that of the viscous damping case.

To increase the system damping capacity, at each level viscous damper are installed, connected to the structure through inverted V-shaped brace. Due to the brace axial deformability, the final behaviour of each device is that of a dashpot in series with a spring (Maxwell element), and some memory effect is expected (Fig. 3(b)). The relaxation function of the i-th EDD is $g_i(t)=K_i\exp[-t/(\beta t_i)]$, where K_i is the stiffness of the i-th spring (axial stiffness of the brace), t_i is the time constant of the i-th EDD, and β is a parameter used to simultaneously control the relaxation times of all the devices. $C_i(\beta)=\beta K_i t_i$ is the viscous coefficient of the i-th dashpot. Different values of the time constants t_i in the range of 0.181 s to 0.251 s were selected at the different levels. In addition, values of the β parameter in the range of 0 to 10 were considered, which allowed to assess the influence on the response of the ratio of the average relaxation time to the system first natural period. For β=0, the system is without memory, and therefore behaves as a Kelvin-Voigt system. For β=10, the ratio of the average relaxation time to the system first natural period is about 2/3. The Fourier transform of the relaxation function is. $\mathcal{F}(g_i(t))=K_i/[(\beta \dot{t}_i)^{-1}+\mathrm{j}\omega]$, then one can use Eq. (30) to evaluate the i-th modal FRF, $\tilde{H}_i^{\mathrm{V}}(\omega)$ for the case of viscous inherent damping and assuming that the EDDs are almost homogeneously distributed in the structure. For the purpose of comparison, the FRF of an equivalent Kelvin-Voigt SDoF oscillator (without memory) was also considered:

$$\tilde{H}_i^{\mathrm{V,KV}}(\omega) = [(\omega_i+\Delta\omega_i)^2-\omega^2+2\mathrm{j}\omega(\zeta_i+\Delta\zeta_i)(\omega_i+\Delta\omega_i)]^{-1} \tag{51}$$

where the quantities $\Delta\omega_i$ and $\Delta\zeta_i$ are computed such that the moduli of $\tilde{H}_i^{\mathrm{V}}(\omega)$ and of $\tilde{H}_i^{\mathrm{V,KV}}(\omega)$ have the same zero- and second-order moments:

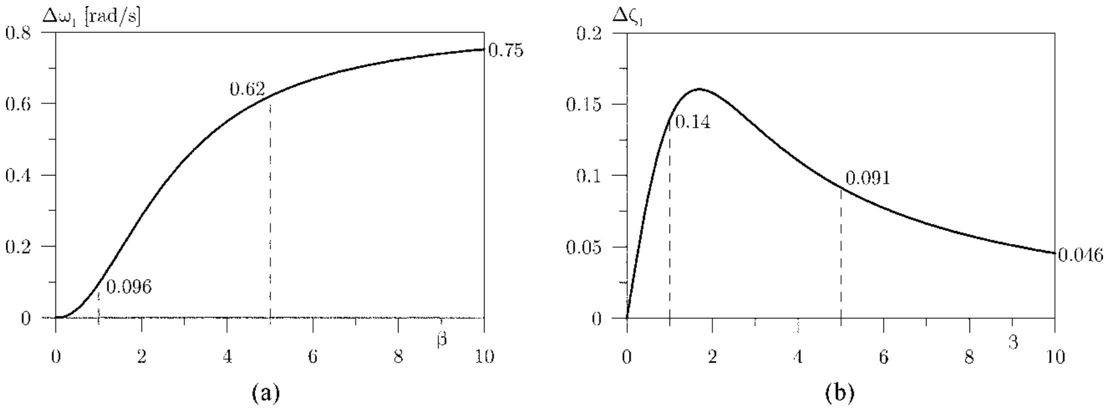


Fig. 5 $\Delta\omega_i$ and $\Delta\zeta_i$ parameters for the calibration of the equivalent Kelvin-Voigt system

$$\Delta\omega_i = \sqrt{\frac{\mu_{2,\,i}}{\mu_{0,\,i}}} - \omega_i$$

$$\Delta\zeta_i = \frac{\pi}{4(\omega_i + \Delta\omega_i)\mu_{2,\,i}} - \zeta_i \tag{52}$$

being:

$$\mu_{j,\,i} = \int_0^{+\infty} \left|\tilde{H}_i^{\mathrm{V}}(\omega)\right| \omega^j \mathrm{d}\omega \tag{53}$$

The values obtained for $\Delta\omega_i$ and $\Delta\zeta_i$, such that the variance of the displacement and velocity response to a white noise are the same for the two systems, are related to the relaxation time, and therefore depend on β. As an example the variation of $\Delta\omega_1$ and $\Delta\zeta_1$ is shown in Fig. 5. $\Delta\omega_1$ monotonically increases in the whole range of β, while $\Delta\zeta_1$ has a peak for $\beta \cong 1.7$.

To check the inaccuracy associated with the use of an equivalent Kelvin-Voigt model, in Fig. 6 the real and imaginary parts of the first modal dynamic stiffness $\tilde{K}_1^{\mathrm{V}}(\omega)$, together with the modulus and phase of the first modal FRF $\tilde{H}_1^{\mathrm{V}}(\omega)$ are shown, and compared with those obtained on the equivalent Kelvin-Voigt model. The real part of the dynamic stiffness (*storage modulus*) of the system with memory increases with increasing frequency, and intersects the constant value pertaining to the Kelvin-Voigt model at the natural frequency of the system with memory, evaluated for $t \to \infty$ (indicated with a dot-dashed line). Also the imaginary part of the dynamic stiffness (*loss modulus*) intersects the value (linear with ω) pertaining to the Kelvin-Voigt model, and the frequency of intersection is the natural frequency of the system with memory, evaluated for $t = 0$. Comparison of the FRFs in Fig. 6 suggests that the memory effect in the EDDs is not negligible in the case of wind excitation, as large discrepancies are found between the FRF of the system with memory and that of the equivalent Kelvin-Voigt system, both at the background and resonant frequencies. In Fig. 6 the agreement between the results obtained through Eq. (30) and through a LPA of order 2 (Eq. (50)) is shown, which proves quite satisfactory.

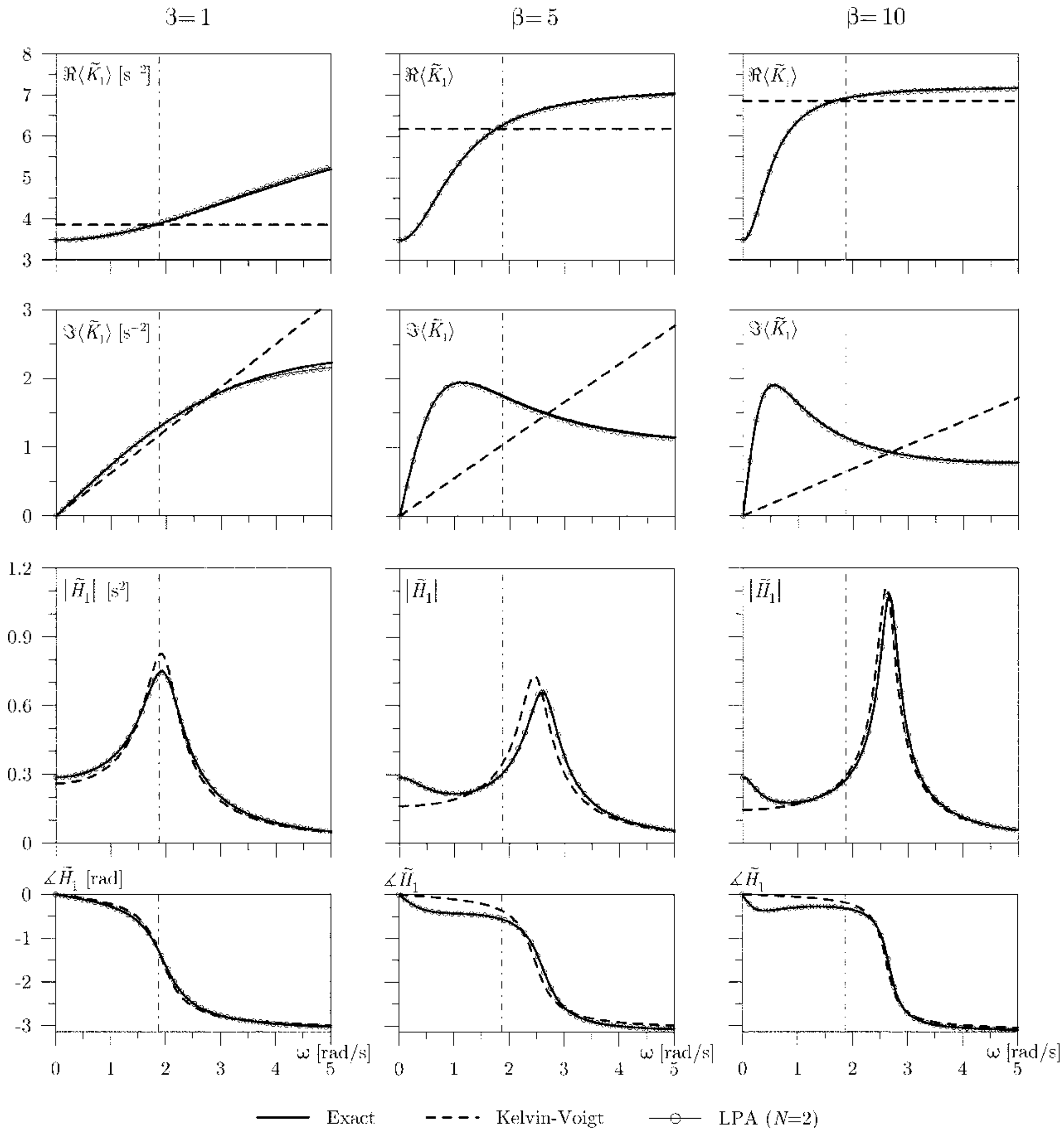


Fig. 6 Dynacim stiffness and Frequency Response Funciton for the building including memory and for the equivalent Kelvin-Voigt model

Finally, the building alongwind buffeting response was calculated assuming a logarithmic mean velocity profile with a roughness length of 0.50 m and a reference wind speed of 15 m/s at 10 m of elevation. The longitudinal component of turbulence was modelled using the Kaimal spectrum, together with the Davenport coherence function with a vertical decay coefficient $C_z = 10$. A drag coefficient $C_D = 1.3$ was used, for a mean wind direction orthogonal to 44.20 m face of the building. The air density was set equal to 1.25 Kgm^{-3}.

In Fig. 7 the spectra of the building tip displacement Y and acceleration $\ddot{Y}$ are shown in a logarithmic scale. Three different cases are considered: (a) three coupled modes including memory,

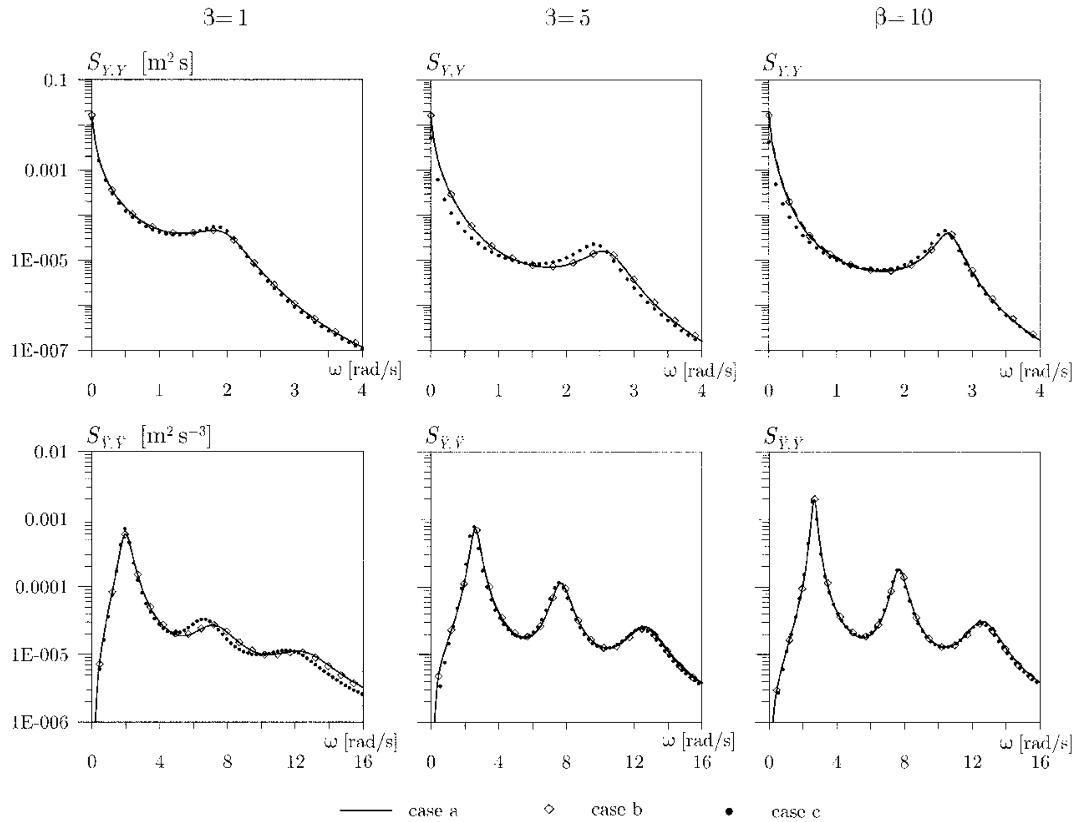


Fig. 7 Tip displacement and acceleration spectra for the building including memory and for the equivalent Kelvin-Voigt model

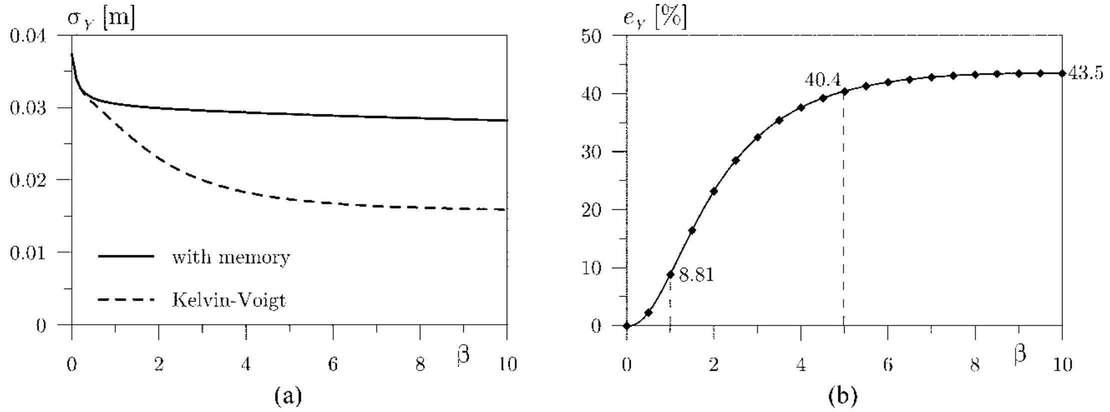


Fig. 8 RMS tip displacement (a) and percentage error (b) for the building including memory and for the equivalent Kelvin-Voigt model

(b) three uncoupled modes including memory, and (c) three uncoupled modes neglecting memory. Comparison of cases (a) and (b) tells that the use of coupled equations of motion is unjustified, as it brings the same result as would be obtained using uncoupled equations of motion. Comparison of cases (b) and (c) shows that the use of a viscous model allows a rather accurate evaluation of the building accelerations, but brings an inaccurate estimate of the building displacements.

To globally quantify the effect of the viscoelastic memory on the building response, in Fig. 8(a) the RMS tip displacement and in Fig 8(b) percent error in the prediction of the tip displacement associated with the use of a Kelvin-Voigt model, are plotted as a function of β. As expected, the error increases with increasing the relaxation time, and is larger than 40% for $\beta \geq 5$.

6. Conclusions

In this paper a mathematical model for the evaluation of the buffeting response of buildings including memory effects associated with viscoelastic memory, has been presented. The model has been implemented using an approximated procedure called Laguerre Polynomial Approximation, which allows writing the system equation of motion in differential, rather than integrodifferential, form.

An application to a 25-story building has shown the magnitude of the errors associated with the use of an equivalent model featuring viscous damping (Kelvin-Voigt). In particular it was shown that, while a Kelvin-Voigt model almost accurately predicts the building accelerations, it tends to underestimate the displacements. The error depends on the relaxation time, and can be as high as 40%.

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