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Control of 3-D coupled responses of wind-excited tall buildings by a spatially placed TLCD system

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Abstract. The possible application of a spatially placed passive tuned liquid column damper system for suppressing coupled lateral-torsional responses of tall buildings is investigated in this paper. The wind loads acting on rectangular tall buildings are analytically expressed as 3-D stochastic model. Meanwhile, the 3-D responses of tall buildings may be coupled due to eccentricities between the stiffness and mass centers of the buildings. In these cases, torsional responses of the buildings are rather larger, and a TLCD system composed of several TLCD located near the sides of the buildings is more effective than the same TLCD placed at the building center in reducing both translational and torsional responses of the buildings. In this paper, extensive analytical and numerical work has been done to present the calculation method and optimize the parameters of such TLCD systems. The numerical examples show that the spatially placed TLCD system can reduce coupled along-wind, across-wind and torsional responses significantly with a fairly small mass ratio.

Key words: tall building; wind loads; coupled responses; control; TLCD

1. Introduction

Wind-induced vibration of modern tall building may cause discomfort to their occupants; hence, it is important to search for effective devices for suppressing dynamic responses, especially acceleration responses, of tall buildings under wind loads.

Since tuned liquid column damper(TLCD) as passive damping device for controlling structural vibration was introduced by Sakai *et al*. (1989), extensive research has proved that TLCD is a very effective device for suppressing wind-induced translational responses of tall buildings (Qu 1991,

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Xu *et al*. 1992). However, when the side faces of a tall building are wider and/or its lowest torsional natural frequency is approaching to its either of the lowest translational natural frequency, especially when the centers of mass and stiffness of the building are noncoincident as the result of asymmetric arrangement of structure and/or mass, the wind-induced torsional responses of the building will obviously increase and wind-induced 3-D responses will be coupled with each other in the latter case. In this paper, a spatially placed TLCD system composed of several TLCD located near the sides of a tall building is presented to suppress both translational and torsional responses, especially 3-D coupled responses of the building. A modal control method based on random vibration theory is developed to calculate 3-D coupled responses of wind-excited tall buildings with the TLCD system in frequency domain. The relationships between the reduction ratio of structural response and the parameters of the TLCD system are investigated in detail by numerical examples.

2. Fundamental theory for calculating wind induced 3-D responses of tall building with spatially placed TLCD system

2.1.The design of spatially placed TLCD system

The TLCD system is composed of several tuned liquid column dampers. In order to suppress both along-wind and across-wind vibration of a tall building, the direction of TLCD should be parallel to two horizontal principal axes of the building respectively, and their natural frequencies should be equal to or near the lowest natural frequency of the building in the corresponding direction respectively. An orifice in the tube is designed to tune the damping ratio of TLCD for obtaining a better control effect. The circular natural frequency ω_L or the natural period T_L of TLCD is given by the following equation :

$$\omega_L = \sqrt{\frac{2g}{L}} \qquad \text{or} \qquad T_L = 2\pi\sqrt{\frac{L}{2g}} \tag{1}$$

where L is the length of the liquid column. In order to increase control effect on torsional response, all TLCD of the system should be placed far off the building center; the farther the better.

2.2. Calculation of wind-induced 3-D responses of tall buildings with the TLCD system

A multi-degree-of freedom model is used in this paper, i.e., the mass of a tall building is lumped at every floor, and the motion of a rigid floor is regarded as the resultant of translations along two horizontal principal axes of the building and rotation about the mass center of the floor. The structural model of a tall building with a spatially placed TLCD system is schematic in Fig. 2, where O, S and

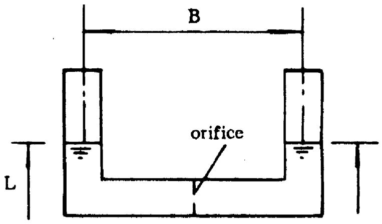

Fig. 1 Tuned liquid column damper

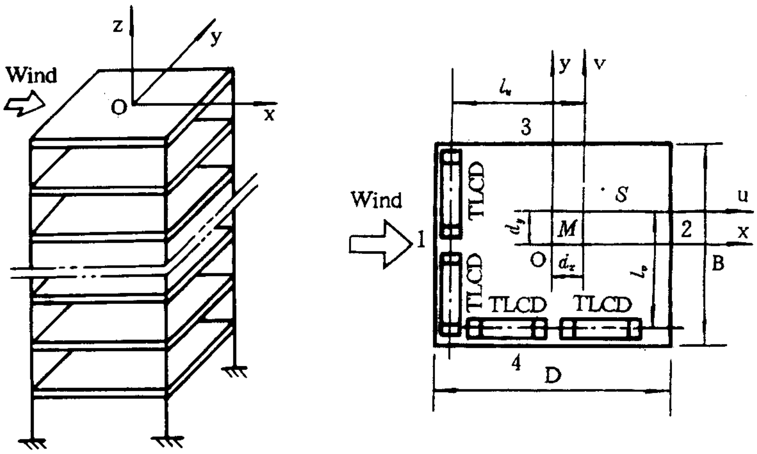

Fig. 2 The structural model of a tall building with a spatially placed TLCD system

M are geometry, stiffness and mass centers respectively.

The coupled oscillation differential equation of a tall building with the TLCD system under 3-D wind loads is as follows:

$$\begin{bmatrix} M_{xx} & 0 & 0 \\ 0 & M_{yy} & 0 \\ 0 & 0 & I_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{u} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} C_{xx} & 0 & C_{x\theta} \\ 0 & C_{yy} & C_{y\theta} \\ C_{\theta x} & C_{\theta y} & C_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{u} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} K_{xx} & 0 & K_{x\theta} \\ 0 & K_{yy} & K_{y\theta} \\ K_{\theta x} & K_{\theta y} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u \\ v \\ \theta \end{Bmatrix} = \begin{Bmatrix} P_u \\ P_v \\ P_\theta \end{Bmatrix} - \begin{Bmatrix} W_u \\ W_v \\ W_\theta \end{Bmatrix} \tag{2}$$

in which u, v, θ = vectors representing along-wind, across-wind and torsional displacement responses at the mass centers respectively; C_{xx}, C_{yy}, $C_{\theta\theta}$, $C_{x\theta}$, $C_{y\theta}$, $C_{\theta x}$, $C_{\theta y}$ = S $\times$ S submatrices of damping; K_{xx}, K_{yy}, $K_{\theta\theta}$, $K_{x\theta}$, $K_{y\theta}$, $K_{\theta x}$, $K_{\theta y}$ = S $\times$ S submatrices of stiffness, and S is the story number; diagonal submatrices M_{xx} ($M_{yy}=M_{xx}$) and $I_{\theta\theta}$ have floor mass m_i and floor moment of inertia to mass center I_i as non-zero elements repectively; $\{P\}^T=\{P_u, P_v, P_\theta\}$=3-D wind loading vector, $\{W\}^T=\{W_u, W_v, W_\theta\}$=3-D control force vector of the TLCD system. If all TLCD are located at the top floor of the building, we have

$$\{W\}^T = \{0, \cdots 0, W_{u_S}; 0 \cdots 0, W_{v_S}; 0, \cdots 0, W_{\theta_S}\} \tag{3}$$

The vibration equation of TLCD in the directions of x-and y-axes are as follows:

$$\rho A_{x_i} L_{x_i} \ddot{x}_i + \frac{1}{2}\rho A_{x_i} \xi_{x_i} |\dot{x}_i| \dot{x}_i + 2\rho A_{x_i} g x_i = -\rho A_{x_i} B_{x_i} \ddot{u}_s + \rho B_{x_i} l_{v_i} A_{x_i} \ddot{\theta}_s + \rho B_{x_i} I_{u_i} A_{x_i} \dot{\theta}_s^2 \tag{4}$$

$$\rho A_{y_i} L_{y_i} \ddot{y}_i + \frac{1}{2}\rho A_{y_i} \xi_{y_i} |\dot{y}_i| \dot{y}_i + 2\rho A_{y_i} g y_i = -\rho A_{y_i} B_{y_i} \ddot{v}_s - \rho B_{y_i} l_{u_i} A_{y_i} \ddot{\theta}_s + \rho B_{y_i} I_{v_i} A_{y_i} \dot{\theta}_s^2 \tag{5}$$

in which, ρ is the density of the liquid; A_{x_i}, A_{y_i}, B_{x_i}, B_{y_i}, L_{x_i}, L_{y_i}, ξ_{x_i} and ξ_{y_i} are, respectively, the cross-sectional area, width, length of the liquid column and damping coefficient (the coefficient of head loss) of the ith TLCD in x-and y-direction; l_{u_i}, l_{v_i}, I_{u_i}, I_{v_i} =the coordinates of the central axis of the ith TLCD in the Muv coordinate system; x_i, and y_i are the displacement of the liquid column of the ith TLCD in x-and y-direction respectively; u_s, v_s and θ_s are, repectively, the displacement of mass center in x-and y-direction and angular displacement of the sth floor on which the TLCD system is located. The last terms in Eq. (4) and Eq. (5) induced by the normal transport inertial forces are small quantities of high-order, and in practice, it is sufficiently accurate to neglect them.

By neglecting the normal transport inertial forces, the control force of the TLCD system on the sth floor of the building in u-, v-and θ - direction are, respectively, as follows:

$$W_{u_s} = \sum_{i=1}^{n} \rho A_{x_i} B_{x_i} \ddot{x}_i + \sum_{i=1}^{n} \rho A_{x_i} L_{x_i} \ddot{u}_s + \sum_{i=1}^{m} \rho A_{y_i} L_{y_i} \ddot{u}_s - \sum_{i=1}^{m} 2\rho A_{y_i} B_{y_i} \dot{\theta}_s \dot{y}_i \tag{6}$$

$$W_{v_s} = \sum_{i=1}^{m} \rho A_{y_i} B_{y_i} \ddot{y}_i + \sum_{i=1}^{n} \rho A_{x_i} L_{x_i} \ddot{v}_s + \sum_{i=1}^{m} \rho A_{y_i} L_{y_i} \ddot{v}_s + \sum_{i=1}^{n} 2\rho A_{x_i} B_{x_i} \dot{\theta}_s \dot{x}_i \tag{7}$$

$$W_{\theta_s} = \sum_{i=1}^{m} I_{y_i} \ddot{\theta}_s + \sum_{i=1}^{n} I_{x_i} \ddot{\theta}_s + \sum_{i=1}^{m} \rho A_{y_i} B_{y_i} l_{u_i} \ddot{y}_i - \sum_{i=1}^{n} \rho A_{x_i} B_{x_i} l_{v_i} \ddot{x}_i \tag{8}$$

in which n and m are the number of TLCD in parallel with x- and y-axis respectively; I_{x_i}, I_{y_i} = the inertia moment of liquid of the ith TLCD in x-, y-direction to the mass center of the sth floor. The last terms in Eq. (6) and (7) are Coriolis inertial forces of the TLCD system.

By using mode superposition method, the differential equations of generalized coordinates governing the motions of a building with the TLCD system can be deduced from Eq. (2) :

$$\ddot{q}_j + 2\zeta_j \omega_j \dot{q}_j + \omega_j^2 q_j = F_j^* - \frac{\Phi_j^T\{W\}}{M_j^*}, \quad j = 1, r \tag{9}$$

where ζ_j, ω_j and Φ_j are, respectively, the damping ratio, circular frequency and mode shape of the jth mode; r is the mode number for a precise calculation (usually $r = 3$), and $r_{\max} = 3s$; $M_j^* = \Phi_j^T[M]\Phi_j$ = the generalized mass of the jth mode; $F_j^* = \Phi_j^T\{P\}/M_j^*$ = the generalized force of the jth mode ;

$$\Phi_j^T\{W\} = \varphi_{x_s j} W_{u_s} + \varphi_{y_s j} W_{v_s} + \varphi_{\theta_s j} W_{\theta_s} \tag{10}$$

where $\ddot{u}_s$, $\ddot{v}_s$, $\ddot{\theta}_s$ and $\dot{\theta}_s$ in the expressions of W_{u_s}, W_{v_s} and W_{θ_s} are rewritten as

$$\sum_{j=1}^{r} \varphi_{x_s j} \ddot{q}_j, \; \sum_{j=1}^{r} \varphi_{y_s j} \ddot{q}_j, \; \sum_{j=1}^{r} \varphi_{\theta_s j} \ddot{q}_j \text{ and } \sum_{j=1}^{r} \varphi_{\theta_s j} \dot{q}_j .$$

By neglecting the last terms on the right sides of Eqs. (4) and (5), Eqs. (4) and (5) can be rewritten as

$$\ddot{x}_i + 2\zeta_{Lx_i} \omega_{Lx_i} \dot{x}_i + \omega_{Lx_i}^2 x_i = -\frac{B_{x_i}}{L_{x_i}} \sum_{j=1}^{r} \varphi_{x_s j} \ddot{q}_j + \frac{l_{v_i} B_{x_i}}{L_{x_i}} \sum_{j=1}^{r} \varphi_{\theta_s j} \ddot{q}_j \tag{11}$$

$$\ddot{y}_i + 2\zeta_{Ly_i} \omega_{Ly_i} \dot{y}_i + \omega_{Ly_i}^2 y_i = -\frac{B_{y_i}}{L_{y_i}} \sum_{j=1}^{r} \varphi_{y_s j} \ddot{q}_j - \frac{l_{u_i} B_{y_i}}{L_{y_i}} \sum_{j=1}^{r} \varphi_{\theta_s j} \ddot{q}_j \tag{12}$$

where equivalent damping ratio ζ_{Lx_i} and ζ_{Ly_i} can be derived by minimizing the mean square value of the equation error (Qu 1991) :

$$\zeta_{Lx_i} = \frac{\xi_{x_i}}{2\sqrt{\pi g L_{x_i}}}\sigma_{\dot{x}_i} \quad \text{and} \quad \zeta_{Ly_i} = \frac{\xi_{y_i}}{2\sqrt{\pi g L_{y_i}}}\sigma_{\dot{y}_i} \tag{13}$$

in which $\sigma_{\dot{x}_i}$ and $\sigma_{\dot{y}_i}$ are the standard deviations of the liquid column velocities of the *i*th TLCD in *x*- and *y*-direction respectively.

The last terms in Eqs. (6) and (7) induced by Coriolis inertial forces have changed the damping force of the *j*th mode in Eq. (9) and made the damping terms of all modes coupled with each other. According to our numerical analyses, the value of damping term induced by Coriolis inertial force is less than 2% of structural damping, and it can be neglected so as to simplify calculation.

By letting $\{Z\}^T=\{q_1, \cdots q_r, x_1, \cdots x_n, y_1 \cdots y_m\}$; the simultaneous equations can be derived from Eqs. (9), (11) and (12) :

$$[A]\{\ddot{Z}\} + [B]\{\dot{Z}\} + [C]\{Z\} = \{F^*\} \tag{14}$$

in which, $\{F^*\}^T = \{F_1^*, \ldots\ldots F_r^*, \underbrace{0\ldots\ldots 0}_{m+n}\}$ and $[A] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, where A_{11} is a $r\times r$ submatrix.

Its diagonal element is $A_{ii} = 1 + \mu_i(\varphi_{x_s i}^2 + \varphi_{y_s i}^2 + r_E^2\varphi_{\theta_s i}^2)$, in which $\mu_i = \left(\sum_{i=1}^{n}\rho A_{x_i}L_{x_i} + \sum_{i=1}^{m}\rho A_{y_i}L_{y_i}\right)/M_i^* =$ the generalized mass ratio of the *i*th mode; $r_E = \left[\left(\sum_{i=1}^{n} I_{x_i} + \sum_{i=1}^{m} I_{y_i}\right)/\left(\sum_{i=1}^{n}\rho A_{x_i}L_{x_i} + \sum_{i=1}^{m}\rho A_{y_i}L_{y_i}\right)\right]^{1/2}$ = the equivalent radius of gyration of the TLCD system to the mass center of the *s*th floor; the non-diagonal element in A_{11} is $A_{ij} = \mu_i(\varphi_{x_s i}\varphi_{x_s j} + \varphi_{y_s i}\varphi_{y_s j} + r_E^2\varphi_{\theta_s i}\varphi_{\theta_s j})$. A_{12} is a $r\times(m+n)$ submatrix. When $0 < j \leqslant n$, the element in A_{12} is : $A_{ij} = \{\rho A_{xj}B_{xj} / M_i^*\}(\varphi_{x_s i} - l_{vj}\ \varphi_{\theta_s i})$. When $n < j \leqslant (n+m)$, the element in A_{12} is: $A_{ij} = \{\rho A_{y_j}B_{y_j} / M_i^*\}(\varphi_{y_s i} - l_{u_j}\varphi_{\theta_s i})$. A_{21} is a $(m+n)\times r$ submatrix. When $0 < i \leqslant n$ the element in A_{21} is : $A_{ij} = \{B_{x_i}/L_{x_i}\}(\varphi_{x_s j} - l_{v_i}\varphi_{\theta_s j})$. When $n < i \leqslant (m+n)$ the elemrent in A_{21} is: $A_{ij} = \{B_{y_i}/L_{y_i}\}(\varphi_{y_s j} + l_{u_i}\varphi_{\theta_s j})$. $A_{22} = E = (n+m)\times(n+m)$ unit submatrix.

$[B]$ and $[C]$ in Eq. (14) are diagonal matrices. The non-zero elements in $[B]$ are : $2\zeta_1\omega_1, \ldots 2\zeta_r\omega_r$, $2\zeta_{Lx_1}\omega_{Lx_1}, \ldots 2\zeta_{Lx_n}\omega_{Lx_n}$, $2\zeta_{Ly_1}\omega_{Ly_1} \ldots 2\zeta_{Ly_m}\omega_{Ly_m}$, and the non-zero element in $[C]$ are $\omega_1^2 \ldots \omega_r^2$, $\omega_{Lx_1}^2$, $\ldots\omega_{Lx_n}^2$ $\omega_{Ly_1}^2 \ldots \omega_{Ly_m}^2$.

In the light of random vibration theory, the matrix of complex frequency response functions can be derived from Eq. (14):

$$[H(i\omega)] = [-\omega^2[A] + i\omega[B] + [C]]^{-1} \tag{15}$$

The RMS acceleration of the *i*th mode of the building with the TLCD system is as follows :

$$\sigma_i = \left[\int_0^\infty \omega^4 \sum_{k=1}^{r}\sum_{j=1}^{r} H_{ik}(i\omega)H_{ij}^*(i\omega)S_{F_{kj}^*}(\omega)d\omega\right]^{1/2}, i = 1, r \tag{16}$$

in which $S_{F_{kj}^*}(\omega) = \{\Phi_j^T S_{pp}(\omega)\Phi_k / M_j^* M_k^*\}$ is the generalized cross spectral density of the *j*th and *k*th mode;

$$S_{pp}(\omega) = \begin{bmatrix} S_{uu}(\omega) & 0 & S_{u\theta}(\omega) \\ 0 & S_{vv}(\omega) & S_{v\theta}(\omega) \\ S_{\theta u}(\omega) & S_{\theta v}(\omega) & S_{\theta\theta}(\omega) \end{bmatrix} \tag{17}$$

is the matrix of spectral density of 3-D wind loads. The element in submatrix $S_{uu}(\omega)$ is the cross spectral density of along-wind force of the l and mth floor. It can be expressed as :

$$S_{u_m u_l}(\omega) = C_D^2 \rho_A^2 V_{10}^2 h^2 \left(\frac{mh}{10}\right)^{\alpha} \left(\frac{lh}{10}\right)^{\alpha} S_u(\omega) \int_{-\frac{B}{2}}^{\frac{B}{2}} \int_{-\frac{B}{2}}^{\frac{B}{2}} \mathrm{coh}(y_1, y_2, z_1, z_2, \omega) dy_1 dy_2 \tag{18}$$

where C_D, ρ_A, V_{10}, h, B and α are, respectively, the drag coefficient, density of air, mean wind speed at 10 m height, story height, building width and an exponent corresponding to the surface roughness;

$$S_u(\omega) = \frac{4 Z_0 V_{10}^2 t^2}{\omega(1+t^2)^{4/3}} \tag{19}$$

is the Davenport spectral function of longitudinal turbulence, in which Z_0 is the surface drag coefficient, $t = 600\omega / \pi V_{10}$;

$$\mathrm{coh}(y_1, y_2, z_1, z_2, \omega) = \exp\left\{-\frac{\varepsilon\omega(C_y|y_1 - y_2| + C_z|m - l|h)}{\pi[V(mh) + V(lh)]}\right\} \tag{20}$$

is the spatial coherence funciton (Safak and Foutch 1987), in which $\varepsilon = \sqrt{1+r^2}/(1+r)$ and $r = C_y B/(C_z H)$, and where $H = S \times h$ is the height of the building and C_y, C_z are the exponential decay coefficients for y, z directions respectively.

The element in submatrix $S_{vv}(\omega)$ is the cross spectrum density of across-wind force of the l and mth floor. It is as follows :

$$S_{v_m v_l}(\omega) = \left(\frac{1}{2}\rho_A V_{10}^2\right)^2 C_L^2 B^2 h^2 \left(\frac{mh}{10}\right)^{2\alpha} \left(\frac{lh}{10}\right)^{2\alpha} J_{wz} \sqrt{S_w(mh, \omega)} \sqrt{S_w(1h, \omega)} \tag{21}$$

in which, C_L is the RMS lift coefficient;

$$J_{WZ} = \cos\left(\alpha_1 \frac{h|m-1|}{B}\right) \exp\left\{-\left[\frac{h|m-1|}{B\alpha_2}\right]^2\right\} \tag{22}$$

is the vertical coherence function of across wind force (Solari 1985), in which α_1 and α_2 are nondimensional coefficients related to the correlation length;

$$S_w(Z, \omega) = \frac{\dfrac{\beta_y(1 - 0.64\beta_y^2)\omega}{4\pi^2(0.964 - 0.353\beta_y)n_y^2}}{\left[1 - (1 - 0.64\beta_y^2)\dfrac{\omega^2}{4\pi^2 n_y^2}\right]^2 + 2.56\beta_y^2(1 - 0.64\beta_y^2)\dfrac{\omega^2}{4\pi^2 n_y^2}} \tag{23}$$

is the wake excitaiton spectral function proposed by Solari (1985), in which β_y is the bandwidth parameter and $n_y = S_y V(z) / B$ is the vortex shedding frequency, and where S_y is the Strouhal number.

The element in submatrix $S_{\theta\theta}(\omega)$ is the cross spectral density of torque of the l and mth floor, and it can be expressed as

$$S_{\theta_m\theta_l}(\omega) = S^{(1)}_{\theta_m\theta_l}(\omega) + S^{(2)}_{\theta_m\theta_l}(\omega) \tag{24}$$

in which,

$$S^{(1)}_{\theta_m\theta_1}(\omega) = \rho_A^2 V_{10}^2 h^2 \left(\frac{mh}{10}\right)^{\alpha}\left(\frac{lh}{10}\right)^{\alpha} S_u(\omega)$$
$$\left[(C_1^2 + C_2^2)\int_{-\frac{B}{2}}^{\frac{B}{2}}\int_{-\frac{B}{2}}^{\frac{B}{2}} y_1 y_2 \mathrm{coh}(y_1, y_2, z_1, z_2, \omega) dy_1 dy_2 \right.$$
$$\left. + (C_3^2 + C_4^2)\int_{-\frac{D}{2}}^{\frac{D}{2}}\int_{-\frac{D}{2}}^{\frac{D}{2}} x_1 x_2 \mathrm{coh}(x_1, x_2, z_1, z_2, \omega) dx_1 dx_2\right] \tag{25}$$

is the torque spectrum due to along-wind turbulence, where C_i is the averaged pressure coefficient on the ith vertical face, D is the building depth ;

$$\mathrm{coh}(x_1, x_2, z_1, z_2, \omega) = \exp\left\{-\frac{\delta\omega(C_x|x_1 - x_2| + C_z|m - l|h)}{\pi[V(mh) + V(lh)]}\right\} \tag{26}$$

is the spatial coherence function, in which $\delta = \sqrt{1 + f^2}/(1 + f)$ and $f = C_x D / (C_z H)$, and where C_x is the exponential decay coefficient for x direciton ;

$$S^{(2)}_{\theta_m\theta_l}(\omega) = d_{y_m} d_{y_l} S_{u_m u_l}(\omega) + d_{x_m} d_{x_l} S_{v_m v_l}(\omega) \tag{27}$$

is the torque spectrum due to eccentricity between mass and geometric center, in which, d_y and d_x are the coordinates of mass center in the Oxy coordinate system. $S_{u\theta}(\omega)$, $S_{\theta u}(\omega)$, $S_{v\theta}(\omega)$ and $S_{\theta v}(\omega)$ are the cross spectral submatrices between along-wind force, across-wind force and torque respectively.

The element in $S_{\theta u}(\omega)$ and $S_{u\theta}(\omega)$ is

$$S_{\theta_m u_l}(\omega) = S_{u_l\theta_m}(\omega) = d_{y_m} S_{u_m u_l}(\omega) \tag{28}$$

The element in $S_{v\theta}(\omega)$ and $S_{\theta v}(\omega)$ is

$$S_{v_l\theta_m}(\omega) = S_{\theta_m v_l}(\omega) = -d_{x_m} S_{v_m v_l}(\omega) \tag{29}$$

Consequently, the RMS displacements and velocities of liquid column of the ith TLCD in x- and y-direction are as follows :

$$\sigma_{x_i} = \left[\int_0^{\infty}\sum_{k=1}^{r}\sum_{j=1}^{r} H_{ik}(i\omega)H^*_{ij}(i\omega)S_{F^*_{kj}}(\omega)d\omega\right]^{1/2}, \qquad i = r + 1, r + n \tag{30}$$

$$\sigma_{y_i} = \left[\int_0^{\infty}\sum_{k=1}^{r}\sum_{j=1}^{r} H_{ik}(i\omega)H^*_{ij}(i\omega)S_{F^*_{kj}}(\omega)d\omega\right]^{1/2}, \qquad i = r + n + 1, r + n + m \tag{31}$$

$$\sigma_{\dot{x}_i} = \left[\int_0^{\infty}\omega^2\sum_{k=1}^{r}\sum_{j=1}^{r}H_{ik}(i\omega)H_{ij}^*(i\omega)S_{F_{kj}^*}(\omega)d\omega\right]^{1/2}, \qquad i = r+1, r+n \tag{32}$$

$$\sigma_{\dot{y}_i} = \left[\int_0^{\infty}\omega^2\sum_{k=1}^{r}\sum_{j=1}^{r}H_{ik}(i\omega)H_{ij}^*(i\omega)S_{F_{kj}^*}(\omega)d\omega\right]^{1/2}, \qquad i = r+n+1, r+n+m \tag{33}$$

When the contributions of higher modes and cross variances between two modes are neglected, the RMS acceleraiton of a point(x, y) located on the mth floor in x-and y-direction can be calculate by the first three modes with the following equations respectively :

$$\sigma_{\ddot{X}} = \left[\sum_{j=1}^{3}\sigma_j^2\varphi_{x_mj}^2 - 2(y-d_{y_m})\sum_{j=1}^{3}\sigma_j^2\varphi_{x_mj}\varphi_{\theta_mj} + (y-d_{y_m})^2\sum_{j=1}^{3}\sigma_j^2\varphi_{\theta_mj}^2\right]^{1/2} \tag{34}$$

$$\sigma_{\ddot{Y}} = \left[\sum_{j=1}^{3}\sigma_j^2\varphi_{y_mj}^2 + 2(x-d_{x_m})\sum_{j=1}^{3}\sigma_j^2\varphi_{y_mj}\varphi_{\theta_mj} + (x-d_{x_m})^2\sum_{j=1}^{3}\sigma_j^2\varphi_{\theta_mj}^2\right]^{1/2} \tag{35}$$

Therefore, the total acceleration of this point can be expressed as

$$a_m(x, y) = \mu_a(\sigma_{\ddot{X}}^2 + \sigma_{\ddot{Y}}^2)^{\frac{1}{2}} \tag{36}$$

in which, μ_a is peak factor, which can be evaluated according to reference Davenport (1964).

3. The relationship between reduction ratio of structural responses and parameters of the TLCD system

In order to obtain better control effects by using the TLCD system, a large number of calculations were conducted to investigate the relationship between the reduction ratio of structural responses and the parameters of the TLCD system. The parameters of the TLCD system include location, liquid mass, and natural frequency of TLCD, the ratio of the width of the tube to the length of liquid column and equivalent damping ratio of TLCD. The reduction ratio of structural responses can be expressed as: $\eta = 1-\{\sigma_a^c/\sigma_a\}$ in which, σ_a^c is the RMS acceleration of a corner point at the top of the building with the TLCD system, and σ_a is the RMS acceleration of the corner point at the top of the building without the TLCD system. In this paper, the corner point whose RMS acceleration is the maximum of the four corner points at the top of the building is adopted for analysis.

3.1. The location of the TLCD system and the reduction ratio

Generally speaking, when the TLCD system is located at the top of a building, its effects for controlling structural displacement and acceleration will increase (Qu 1991). Furthermore, the calculated results show that when the mass, stiffness and geometry center of a building coincide with each other, a better control effect can be obtained by distributing the tuned liquid column

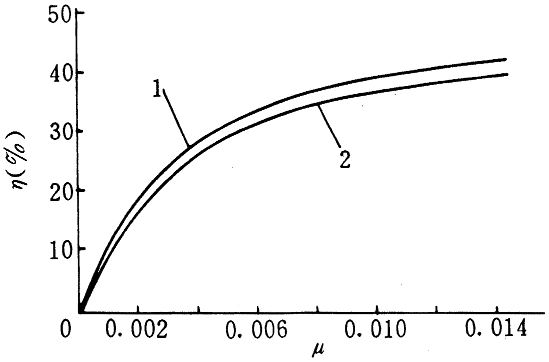

Fig. 3 Reduction ratio η versus mass ratio μ for building A. 1-TCLD near four sides, 2-TLCD at building center

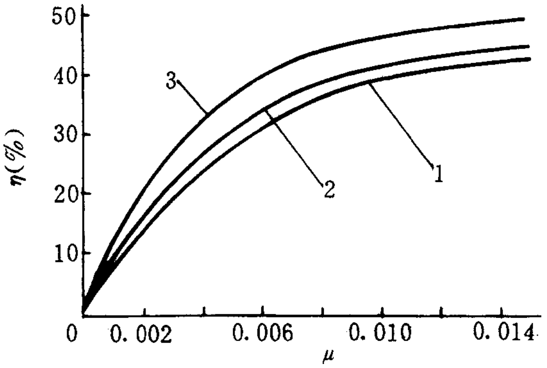

Fig. 4 Reduction ratio η versus mass ratio μ for building B. 1-TLCD at building center, 2-TLCD near four sides, 3-TLCD near side 1 and 4

dampers of the system near the peripheries of the building symmetrically.

For a structurally symmetrical rectangular tall building (building A), letting four TLCD of the system be parallel to x-direction near side 3 and 4, and the other four TLCD be parallel to y-direciton near side 1 and 2, the control effects of this TLCD system are better than those of the same TLCD placed at building center as shown in Fig. 3, in which mass ratio μ is the ratio of all liquid mass of the TLCD system to building mass. Because torsional responses of a structurally symmetrical building is not large, the advantage of the spatially placed TLCD system in suppressing 3-D structural responses is not remarkable in this case.

When the centers of mass, stiffness and geometry of a building are noncoincident, calculation results indicate that better control effects can be obtained by placing all TLCD far off the stiffness center; the farther the better. For example, if the stiffness center of a rectangular building is located in the first quadrant, a larger reduction ratio can be obtained by placing all of TLCD near side 1 and 4 of the building (Fig. 2). For a tall building (building B) whose geometry dimension B : D : H = 30 m : 40 m : 161 m and the coordinates of mass and stiffness center of every floors in Oxy coordinate system are (2, 0) and (6, 3) respectively, the reduction ratios versus mass ratio are shown in Fig. 4, in which, curve 1, 2 and 3 represent the reduction ratios for building B with a TLCD system composed of 4 TLCD located at building center, near four sides and near side 1 and 4 respectively.

Fig. 4 shows that there is no obvious difference between curve 1 and 2, but when all TLCD are located near the farther sides from the stiffness center of the building, the reduction ratio, curve 3, increases notably.

3.2. The equivalent damping ratio, the ratio B/L of TLCD and the reduction ratio

The mathematics model of a TLCD is based on the condition that the displacemment of liquid column should be limited in the vertical section of the tube, that is

$$\mu_L \sigma_L \le \frac{L-B}{2} \tag{37}$$

in which, μ_L is peak factor (usually $\mu_L = 2\sim4$); B is the width of the tube; σ_L is the RMS displacement of liquid column in the tube. For obtaining a greater control force, the ratio B/L should be as large as possible on the condition of Eq. (37). Meanwhile, the optimum of the equivalent damping ratio ζ_L of a TLCD should be selected on the condition of Eq. (37), hence the optimum of the damping coefficient ξ is also determined by tuning the orifice according to Eq. (13).

Since the equivalent damping ratio ζ_L is a function of the standard deviation of the liquid column velocities according to Eq. (13), the optimums of the equivalent damping ratio ζ_L as well as damping coefficient ξ should be determined for a given wind speed. This wind speed is so-called the design wind speed of a TLCD system. When the design wind speed of a TLCD system is relatively low and/or the mass ratio μ is relatively large, the oscillations of the liquid column in the tubes of TLCD are fairly small, and the optimums of ζ_L as well as ξ are not constrained by Eq. (37). In contrast, when the design wind speed of a TLCD system is relatively high and/or the mass ratio μ is relatively small, the optimums of ζ_L as well as ξ are constrained by Eq. (37). In the latter cases, both the ratio B/L and damping coefficient ξ should be properly tuned to obtain better control effects.

For building B, when mean wind speed at 10 m height $V_{10} = 20$ m/s and mass ratio $\mu = 0.01$, by placing all TLCD near the side 1 and 4, the reduction ratio η versus the equivalent damping ratio of 4 TLCD is shown in Fig. 5. Fig. 5 shows that the optimum of equivalent damping ratio of two TLCD in y-direction is 0.0579, and the optimum of the equivalent damping ratio of the other two TLCD in x-direction is 0.052. According to Eq. (13), the corresponding damping coefficients are 4.94 and 9.173 respectively.

After the damping coefficients of TLCD are selected, the reduction ratio of structural responses will slightly decrease as wind speed changes, In the above example, when damping coefficients of

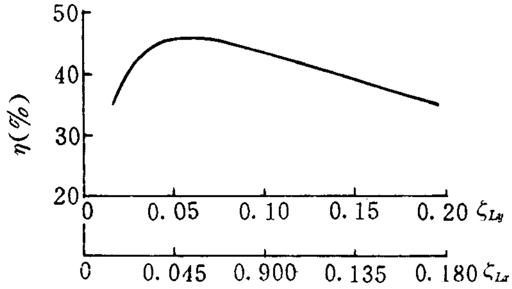

Fig. 5 Reduction ratio η versus equivalent damping ratio ζ_{Ly} and ζ_{Lx} for building B

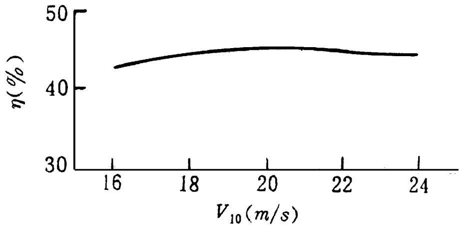

Fig. 6 Reduction Ratio η versus mean wind speed V_{10} for building B when damping coefficient ξ are given

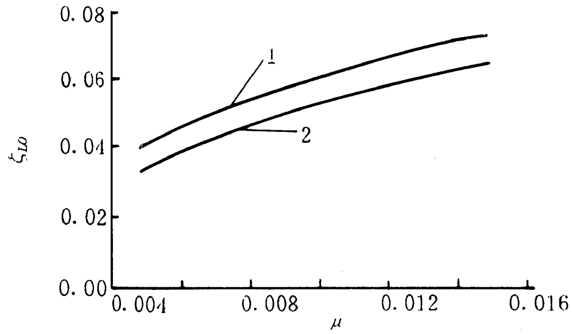

Fig. 7 Optimum of equivalent damping ratio ζ_{L0} versus mass ratio μ for building B, 1-$\zeta_{L_y\mathrm{O}}$, 2-$\zeta_{L_x\mathrm{O}}$

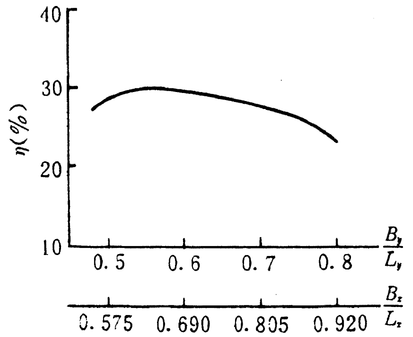

Fig. 8 Reduction ratio η versus the ratio $\{B_y / L_y\}$ and $\{B_x / L_x\}$ for building B on the condition of Eq. (37)

TLCD in y- and x-direction are 4.94 and 9.173 respectively, the reduction ratio η versus mean wind speed V_{10} is shown in Fig. 6. The curve in Fig. 6 indicates that the value of reduction ratio is not sensitive to the change of wind speed if Eq. (37) is satisfied. However, when the mean wind speed exceeds 25 m/s in this case, the damping coefficients of TLCD must be increased by tuning the orifices so as to satisfy Eq. (37).

Along with the increase of mass ratio, the optimums of the equivalent damping ratios of TLCD, ζ_{L0}, will gently increase. In Fig. 7, the curves show that the optimums of the equivalent damping ratios, ζ_{L_x0} and ζ_{L_y0}, vary with mass ratio μ for the asymmetrically placed TLCD system on building B. When wind speed is relatively high and/or the mass ratio μ is relatively small, the optimum of B/L of TLCD may be as low as 0.5~0.6 to obtain the maximum reduction ratio on the condition of Eq. (37). When $V_{10} = 25$ m/s and $\mu = 0.005$ for building B, the optimums of the equivalent damping ratios of TLCD are constrained by Eq. (37). In this case, by tuning both the ratio B/L and damping coefficient ξ of TLCD of the asymmetrically placed TLCD system, the maximum reduction ratio can be obtained when $B_y / L_y = 0.56$ and $B_x / L_x = 0.644$ as shown in Fig. 8.

3.3. The natural frequency of TLCD and the reduction ratio

In order to investigate the optimum natural frequencies of TLCD of the system, two numerical examples, building A with the symmetrical TLCD system and building B with the asymmetrical TLCD system are presented to illustrate the relationship between the reduction ratio η and frequency ratio $\{\omega_{L_x} / \omega_{x_1}\}$ and $\{\omega_{L_y} / \omega_{y_1}\}$.

The natural frequencies of TLCD in each direciton of the two horizontal principal axes of the buildings can be the same for all or be different from each other. The numerical examples show the reduction ratios reach the maximums when the natural frequencies of TLCD in each direction are the same and smaller than the lowest natural frequencies of the translational modes or translational dominant modes of the buildings in the corresponding direction.

According to Liang (1997), for mechanically uncoupled tall buildings whose lowest two natural frequencies are of translational modes, wind induced torsional responses of the corner points of the buildings are less than 20 percent of the total responses; for mechanically coupled tall buildings whose lowest two natural frequencies are of translational dominant modes, only less than 3 percent of wind induced translational responses of the corner points of the buildings is contributed by the mode shape coupling of torsional dominant mode, and in contrast, an important part, even the main part, of wind induced torsional responses of the corner points of the buildings is contributed by the mode shape coupling of translational dominant modes. Hence, it is understandable that the best control effects can be obtained by letting the natural frequencies of all TLCD be near the lowest natural frequencies of the translational modes or translational dominant modes of the tall buildings in each corresponding direction. Actually, the two numerical examples indicate : when the natural frequencies of partial TLCD of the TLCD system are equal to or near the lowest torsional natural frequency of building A and the lowest torsional dominant natural frequency of building B, the reduction ratios will obviously decrease as compared with letting the frequencies of all TLCD of the same mass ratio be equal to or near the lowest translational natural frequency of building A and the lowest translational dominant natural frequency of building B in each corresponding direction.

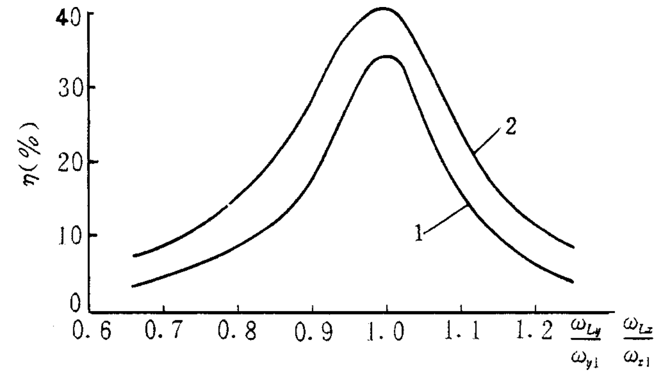

Fig. 9 Reduction ratio η versus frequency ratio $\{\omega_{L_y} / \omega_{y_1}\}$ and $\{\omega_{L_x} / \omega_{x_i}\}$ 1-building A, 2-building B

Fig. 9 shows when the natural frequencies of TLCD in each direction of the two horizontal principal axes of the buildings are the same, the curves of reduction ratio versus frequency ratio are single peak shapes, and the frequency ratios versus the maximums of reduction ratios are somewhat smaller then 1.0 in both directions.

4. An example

A 60-story tall building is taken as an example to show the application of the method and formulas presented above.

- Structural data : The geometry dimension of the building is 30 m : 30 m : 210 m; the height of every story $h = 3.5$ m; the coordinates of mass center and stiffness center in Oxy coordinate system for every floor are (1,1) and (4,4) respectively; the shear stiffness between two floors in x-and y-direction, k_x and k_y, equal 1.5×10^9 N/M and torsional stiffness between two floors $k_t = 4.9\times 10^{11}$ N-M/rad; the mass of every floor is 630,000 kg; the inertia moment of every floor to its mass center is 1.1×10^8 kg-m^2; the damping ratios of the first three modes are assumed, respectively, as $\zeta_1 = 0.01$, $\zeta_2 = 0.01$, $\zeta_3 = 0.108$; the natural frequencies of the first three modes n_1, n_2 and n_3 equal 0.1917, 0.2016 and 0.2901 Hz respectively according to calculated results; the mode shape values of the first three modes at the top floor are also obtained by calculation as follows : $\varphi_{x601} = 1$, $\varphi_{y601} = -0.9996$, $\varphi_{\theta 601} = 0.032$, $\varphi_{x602} = 0.9997$, $\varphi_{y602} = 1$, $\varphi_{\theta 602} = -7.1\times 10^{-6}$, $\varphi_{x603} = 2.8048$, $\varphi_{y603} = -2.8051$, $\varphi_{\theta 603} = -1$.
- Aerodynamic data : the Drag coefficient $C_D = 1.3$; the air density $\rho_A = 1.2$ kg/m^3; the exponent for the mean wind velocity profile power law $\alpha = 0.25$; the surface drag coefficient $Z_0 = 0.01$; the exponential decay coefficient C_x, C_y and C_z equal 6, 16 and 10 respectively (Simiu and Scanlan 1985); the RMS lift coefficient $C_L = 0.5$; the bandwidth parameter $\beta_y = 0.25$ and the Strouhal number $S_y = 0.1$ (Solari 1985); α_1 and α_2, the nondimensional coefficients in Eq. (22), are equal to 0.5 and 5 respectively; the averaged pressure coefficient on four vertical faces c_1, c_2, c_3 and c_4 equal 0.8, -0.5, -0.8 and -0.8 respectively.
- The design of the TLCD system : The design wind speed of the TLCD system $V_{10} = 18$ m/s,

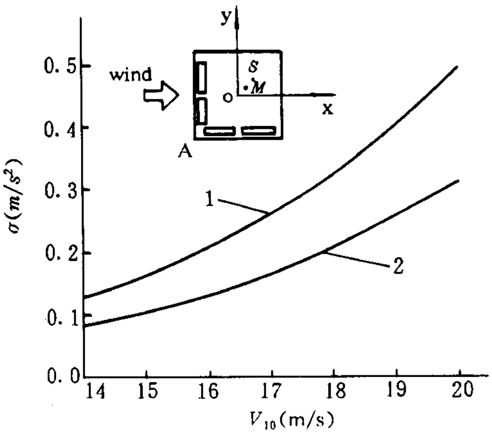

Fig. 10 RMS accelerations at point A versus mean wind speeds 1-σ_a, 2-σ_a^c

which is the extreme wind speed of the 6 year return period at the building's location; the mass ratio $\mu = 1/150$, hence the mass of water of the TLCD system = 252,000 kg; four TLCD of the system are placed as shown in Fig. 10; l_u of two TLCD in y direction $= l_v$ of two TLCD in x direction $= -15$ m, and $r_E = 16.29$ m; according to calculation, when $\omega_{Lx_1} = \omega_{Lx_2} = 0.98\omega_1$ and $\omega_{Ly_1} = \omega_{Ly_2} = 0.98\omega_2$, the reduction ratio of the building reaches its maximum, so let $\omega_{Lx_1} = \omega_{Lx_2} = 1.18$ rad/sec($L_{x_1} = L_{x_2} = 14.073$ m) and $\omega_{Ly_1} = \omega_{Ly_2} = 1.24$ rad/sec($L_{y_1} = L_{y_2} = 12.716$ m); for $V_{10} = 18$ m/s, the optimum of ζ_{Lx} and $\zeta_{Ly} = 0.04237$ and 0.04027 respectively, hence $\xi_{x_1} = \xi_{x_2} = 2.545$ and $\xi_{y_1} = \xi_{y_2} = 2.324$. When wind speed V_{10} exceeds 20 m/s, the Eq. (37) can not hold and ξ_x, ξ_y should be raised by tuning the orifices.

The RMS accelerations, at point A on the top floor of the building with and without the TLCD system, versus wind speeds are shown in Fig. 10. In Fig. 10, the reduction ratios are between 0.37~0.38 for all wind speeds.

5. Conclusions

The main conclusions from this investigation are as follows :

(1) The spatially placed TLCD system is very effective on suppressing wind induced 3-D responses, especially 3-D coupled responses, of tall buildings with a fairly small mass ratio. The advantage of this kind of TLCD system is that it can control torsional vibration as well as translational vibration of a building at the same time.

(2) The better control effects can be obtained by placing all TLCD near the sides far from the stiffness center of a tall building as compared with all TLCD at the building center, and a mechanically coupled tall building, as compared with all TLCD near four sides far from the geometry center of the tall building, or near the sides far from the mass center of the tall building.

(3) When the damping coefficients of TLCD are given, the control effects of the TLCD system will not obviously change along with the change of wind speed if the displacements of liquid columns are in the vertical sections of TLCD.

(4) When mass ratio is given and wind speed exceeds a certain threshold, reducing the ratio of the tube width to the liquid column length of TLCD is more beneficial than raising the damping coefficient of TLCD for obtaining a larger reduction ratio on the condition of Eq. (37).

(5) The optimum of the equivalent damping ratio of TLCD gently increases along with the increase of mass ratio, and the optimum of the frequency ratio is somewhat smaller than 1.0.

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