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Bending analysis of FGM plates using a sinusoidal shear deformation theory

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Abstract. The response of functionally graded ceramic-metal plates is investigated using theoretical formulation, Navier's solutions, and a new displacement based on the high-order shear deformation theory are presented for static analysis of functionally graded plates. The theory accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The plates are assumed to have isotropic, two-constituent material distribution through the thickness, and the modulus of elasticity of the plate is assumed to vary according to a power-law distribution in terms of the volume fractions of the constituents. Numerical results of the new refined plate theory are presented to show the effect of the material distribution on the deflections, stresses and fundamental frequencies. It can be concluded that the proposed theory is accurate and simple in solving the static and free vibration behavior of functionally graded plates.

Keywords: theoretical formulation; Navier's solutions; FGM plate; static

1. Introduction

Functionally graded materials (FGMs) are a class of composites that have a continuous variation of aterial properties from one surface to another. These materials can be fabricated by varying the percentage content of two or more materials such that the new materials have the desired property gradation in spatial directions. The gradation in the properties of the materials reduces thermal stresses, residual stresses and stress concentration factors found in laminated composites. FGMs have gained widespread applicability as thermal barrier structures, wear- and corrosion-resistant coatings other than joining dissimilar materials. They are usually made from a mixture of ceramics and metals to attain the significant requirement of material properties.

Due to the increased relevance of the FGMs structural components in the design of engineering structures, many studies have been reported on the static, and vibration analyses of functionally graded (FG) plates. Fekrar, El Meiche *et al.* (2012) analyzed the buckling response of FG hybrid composite plates using a new four variable refined plate theory. Bouremana, Benrahou *et al*. (2013)

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proposed a novel first shear deformation beam theory based on neutral surface position for FG beams. Mantari and Guedes Soares (2012) studied the bending analysis of thick exponentially graded plates using a new trigonometric higher order shear deformation theory. Tai and Kim (2013a) used a simple quasi-3D sinusoidal shear deformation theory for functionally graded plates. Tai and Kim (2013b) proposed a simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded plates. Zhang (2013) studied the modeling and analysis of FGM rectangular plates based on physical neutral surface and high order shear deformation theory. Prakash, Singha *et al.* (2009) studied the Influence of neutral surface position on the nonlinear stability behavior of functionally graded plates. Belabed, Houari *et al*. (2014) used an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Tai, Nguyen *et al.* (2014) studied the analysis of functionally graded sandwich plates using a new first-order shear deformation theory. Chen, Yang *et al*. (2016) studied the free and forced vibrations of shear deformable functionally graded porous beams. Sina, Navazi *et al*. (2009) investgated an analytical method for free vibration analysis of functionally graded beams. Xiang and Yang (2008) analyzed the free and forced vibration of a laminated FGM Timoshenko beam of variable thickness under heat conduction. Kocatürk and Akbaş (2013) studied the thermal post-buckling analysis of functionally graded beams with temperature-dependent physical properties. Al-Basyouni, Tounsi *et al.* (2015) investigated size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Tai *et al.* (2012) studied the bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories. Bourada, Kaci *et al.* (2015) used a new simple shear and normal deformations theory for functionally graded beams. Hebali, Tounsi *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Ait Yahia, Ait Atmane *et al.* (2015) analyzed the wave propagation in functionally graded plates with porosities. Bennoun, Houari *et al.* (2016) analyzed the vibration of functionally graded sandwich plates using a novel five variable refined plate theory. Ait Amar Meziane, Abdelaziz *et al.* (2014) proposed an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Mahi, Adda Bedia *et al.* (2015) studied the bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates using a new hyperbolic shear deformation theory. Bousahla, Houari *et al*. (2014) investigated a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Bellifa *et al.* (2016) studied the bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position. Bounouara, Benrahou *et al.* (2014) studied the free vibration of functionally graded nanoscale plates resting on elastic foundation using a nonlocal zeroth-order shear deformation theory. Belkorissat, Houari *et al.* 2015) developed new shear deformation plates theories involving only four unknown functions. Larbi Chaht *et al.* (2015) studied the bending and buckling of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Ahouel, Houari *et al.* (2016) investigated a size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Zemri, Houari *et al*. (2015) proposed an assessment of a refined nonlocal shear deformation theory beam theory for a mechanical response of functionally graded nanoscale beam. Nedri, El Meiche *et al.* (2014) developed new shear deformation plate theorie involving only four unknown functions for free vibration analysis of laminated composite plates resting on elastic foundations. Tounsi, Houari *et* *al.* (2013) use a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Zidi, Tounsi *et al*. (2014) study hygro-thermo-mechanical loading for the bending of FGM plates using a four variable refined plate theory. Bouderba, Houari *et al.* (2013) studied the thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations. Bouderba, Houari *et al.* (2016) studied the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Attia, Tounsi *et al.* (2015) developed the free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories.

Bakora and Tounsi (2015) investigated the thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations. Boukhari, Ait Atmane *et al*. (2016) used an efficient shear deformation theory for wave propagation of functionally graded material plates.

The purpose of this study is to develop a shear deformation plate theory for FG plates which is simple to use. The number of independent unknowns of present theory is four, as against five in other shear deformation theories. The material properties of plate are assumed to vary according to power law distribution of the volume fraction of the constituents whereas Poisson's ratio is constant. The accuracy and convergence of the present method are demonstrated through numerical results. A detailed parametric study is carried out to highlight the influences of aspect and thickness ratios, material property graded indexes on the static of the FG plate.

2. Theoretical formulation

Consider a rectangular FGM plate having the thickness *h*, length a, and width *b*. A Cartesian coordinate system (x, y, z) is used to label the material point of the plate in the unstressed reference configuration, as depicted in Fig. 1. It is assumed that the material is isotropic and grading is assumed to be only through the thickness. The *xy* plane is taken to be the undeformed mid plane of the plate with the *z* axis positive upward from the mid plane.

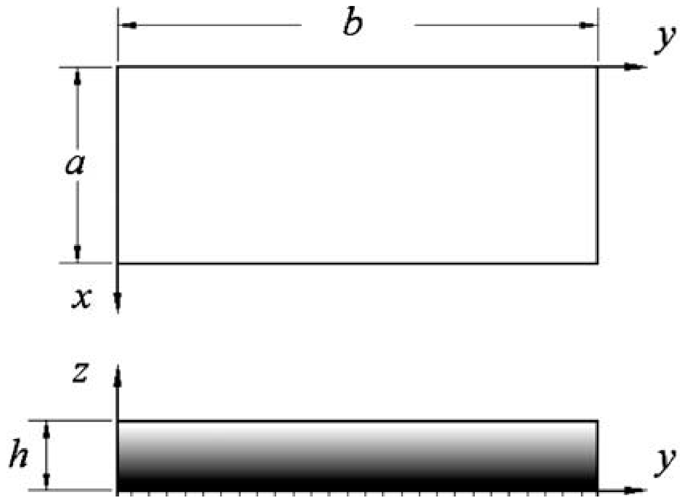

Fig. 1 Geometry of rectangular FG plate and coordinates

2.1 Basic assumptions

The assumptions of the present theory are as follows:

The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

The transverse displacement W includes two components of bending w_b , and shear w_s. These components are functions of coordinates x, y, and time t only.

$$W(x,y,z) = w_b(x,y) + w_s(x,y) \tag{1}$$

The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

The displacements U in x-direction and V in y-direction consist of extension, bending, and shear components

$$U = u + u_b + u_s, \quad V = v + v_b + v_s \tag{2}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b = -z\frac{\partial w_b}{\partial x} \qquad v_b = -z\frac{\partial w_b}{\partial y} \tag{3a}$$

The shear components $\boldsymbol{u_s}$ and $\boldsymbol{v_s}$ give rise, in conjunction with $\boldsymbol{w_s}$, to the parabolic variations of shear strains $\boldsymbol{\gamma_{xz}}$, $\boldsymbol{\gamma_{yz}}$ and hence to shear stresses $\boldsymbol{\tau_{xz}}$, $\boldsymbol{\tau_{yz}}$ through the thickness of the plate in such a way that shear stresses $\boldsymbol{\tau_{xz}}$, $\boldsymbol{\tau_{yz}}$ are zero at the top and bottom faces of the plate. Consequently, the expression for $\boldsymbol{u_s}$ and $\boldsymbol{v_s}$ can be given as

$$u_s = -f(z)\frac{\partial w_s}{\partial x} \qquad v_s = -f(z)\frac{\partial w_s}{\partial y} \tag{3b}$$

2.2 Displacement fields and strains

Based on the assumptions made in the preceding section, the displacement field can be obtained

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\dfrac{\partial w_b}{\partial x} - f(z)\dfrac{\partial w_s}{\partial x} \\ v(x,y,z) = v_0(x,y) - z\dfrac{\partial w_b}{\partial y} - f(z)\dfrac{\partial w_s}{\partial y} \\ w(x,y,z) = w_b(x,y) + w_s(x,y) \end{cases} \tag{4}$$

where $\boldsymbol{u_0}$ and $\boldsymbol{v_0}$ are the mid-plane displacements of the plate in the $\boldsymbol{x}$ and $\boldsymbol{y}$ direction, respectively; $\boldsymbol{w_b}$ and $\boldsymbol{w_s}$ are the bending and shear components of transverse displacement, respectively.

The kinematic relations can be obtained as follows

$$\begin{cases} \varepsilon_x = \varepsilon_x^0 + z\, k_x^b + f(z)\, k_x^s \\ \varepsilon_y = \varepsilon_y^0 + z\, k_y^b + f(z)\, k_y^s \\ \gamma_{xy} = \gamma_{xy}^0 + z\, k_{xy}^b + f(z)\, k_{xy}^s \\ \gamma_{yz} = g(z)\, \gamma_{yz}^s \\ \gamma_{xz} = g(z)\, \gamma_{xz}^s \\ \varepsilon_z = 0 \end{cases} \tag{5}$$

Where

$$\begin{cases} \varepsilon_x^0 = \dfrac{\partial u_0}{\partial x}, k_x^b = -\dfrac{\partial^2 w_b}{\partial x^2},\ k_x^s = -\dfrac{\partial^2 w_s}{\partial x^2} \\ \varepsilon_y^0 = \dfrac{\partial v_0}{\partial y}, k_y^b = -\dfrac{\partial^2 w_b}{\partial y^2}, k_y^s = -\dfrac{\partial^2 w_s}{\partial y^2} \\ \gamma_{xy}^0 = \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}, k_{xy}^b = -2\dfrac{\partial^2 w_b}{\partial x \partial y}, k_{xy}^s = -2\dfrac{\partial^2 w_s}{\partial x \partial y} \\ \gamma_{yz}^s = \dfrac{\partial w_s}{\partial y}, \gamma_{xz}^s = \dfrac{\partial w_s}{\partial x} \\ g(z) = 1 - f'(z) \ \ and \ \ f'(z) = \dfrac{df(z)}{dz} \end{cases} \tag{6}$$

while $\boldsymbol{f}(\boldsymbol{z})$ represents shape functions determining the distribution of the transverse shear strains and stresses along the thickness and is given as

$$f(z) = z - \frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right) \tag{7}$$

2.3 Constitutive relations

The material properties of FG plate are assumed to vary continuously through the thickness of the plate in accordance with a power law distribution as

$$P(z) = (P_t - P_b)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_b \tag{8}$$

Where P denotes a generic material property like modulus, P_t and P_b denotes the property of the top and bottom faces of the plate respectively, and k is a parameter that dictates material variation profile through the thickness. Here, it is assumed that modules E and G vary according to the Eq. (8) and v is assumed to be a constant. The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$
$$\begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{9}$$

Where

$$\begin{cases} Q_{11} = Q_{22} = \dfrac{E(z)}{1-\nu^2} \\ Q_{12} = \dfrac{\nu E(z)}{1-\nu^2} \\ Q_{44} = Q_{55} = Q_{66} = \dfrac{E(z)}{2(1+\nu)} \end{cases} \tag{10}$$

2.4 Governing equations

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The principle of virtual work in the present case yields

$$\int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_{\Omega} (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}).d\Omega.dz - \int_{\Omega} q.\delta w.d\Omega = 0 \tag{11a}$$

Where Ω is the top surface.

Substituting Eqs. (5) and (9) into Eq. (11(a)) and integrating through the thickness of the plate, Eq. (11(a)) can be rewritten as

$$\int_{\Omega} \begin{pmatrix} N_x.\delta\varepsilon_x^0 + N_y.\delta\varepsilon_y^0 + N_{xy}.\delta\gamma_{xy}^0 + M_x^b.\delta k_x^b + M_y^b.\delta k_y^b \\ +M_{xy}^b.\delta k_{xy}^b + M_x^s.\delta k_x^s + M_y^s.\delta k_y^s + M_{xy}^s.\delta k_{xy}^s \\ +S_{yz}^s.\delta\gamma_{yz}^s + S_{xz}^s.\delta\gamma_{xz}^s - q(\delta w_b + \delta w_s) \end{pmatrix} d\Omega = 0 \tag{12}$$

Where

$$\begin{Bmatrix} N_x, & N_y, & N_{xy}, \\ M_x^b, & M_y^b, & M_{xy}^b, \\ M_x^s, & M_y^s, & M_{xy}^s, \end{Bmatrix} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{13a}$$

$$(S_{xz}^{s}, S_{yz}^{s}) = \int_{-\frac{h}{2}}^{+\frac{h}{2}} (\tau_{xz}, \tau_{yz}) g(z) dz \tag{13b}$$

The governing equations of equilibrium can be derived from Eq. (12) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0, δw_b, and δw_s zero separately. Thus, one can obtain the equilibrium equations associated with the present shear deformation theory

$$\begin{cases} \delta u : \dfrac{\partial N_x}{\partial x} + \dfrac{\partial N_{xy}}{\partial y} = 0 \\ \delta v : \dfrac{\partial N_{xy}}{\partial x} + \dfrac{\partial N_y}{\partial y} = 0 \\ \delta w_b : \dfrac{\partial^2 M_x^b}{\partial x^2} + 2\dfrac{\partial^2 M_{xy}^b}{\partial x \partial y} + \dfrac{\partial^2 M_y^b}{\partial y^2} + q = 0 \\ \delta w_s : \dfrac{\partial^2 M_x^s}{\partial x^2} + 2\dfrac{\partial^2 M_{xy}^s}{\partial x \partial y} + \dfrac{\partial^2 M_y^s}{\partial y^2} + \dfrac{\partial S_{xz}^s}{\partial x} + \dfrac{\partial S_{yz}^s}{\partial y} + q = 0 \end{cases} \tag{14}$$

Using Eq. (9) in Eq. (13), the stress resultants of a sandwich plate made up of three layers can be related to the total strains by

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ A & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \; S = A^s \gamma \tag{15}$$

Where

$$N = \left\{ N_x, N_y, N_{xy} \right\}^t, M^b = \left\{ M_x^b, M_y^b, M_{xy}^b \right\}^t, M^s = \left\{ M_x^s, M_y^s, M_{xy}^s \right\}^t \tag{16a}$$

$$\varepsilon = \left\{ \varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0 \right\}^t, \; k^b = \left\{ k_x^b, k_y^b, k_{xy}^b \right\}^t, \; k^s = \left\{ k_x^s, k_y^s, k_{xy}^s \right\}^t \tag{16b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \; B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{16c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{16d}$$

$$S=\left\{S_{xz}^{z},S_{yz}^{s}\right\}^{t},\ \gamma=\left\{\gamma_{xz},\gamma_{yz}\right\}^{t},\ A^{s}=\begin{bmatrix}A_{44}^{s} & 0\\ 0 & A_{55}^{s}\end{bmatrix} \tag{16e}$$

The stiffness coefficients A_{ij} and B_{ij} , etc., are defined as

$$\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^{s} & D_{11}^{s} & H_{11}^{s}\\ A_{12} & B_{12} & D_{12} & B_{12}^{s} & D_{12}^{s} & H_{12}^{s}\\ A_{66} & B_{66} & D_{66} & B_{66}^{s} & D_{66}^{s} & H_{66}^{s}\end{Bmatrix}= \int_{-\frac{h}{2}}^{+\frac{h}{2}} Q_{11}(1,z,z^{2},f(z),zf(z),f^{2}(z))\begin{Bmatrix}1\\ \nu\\ \dfrac{1-\nu}{2}\end{Bmatrix}dz \tag{17a}$$

$$(A_{22},B_{22},D_{22},B_{22}^{s},D_{22}^{s},H_{22}^{s})=(A_{11},B_{11},D_{11},B_{11}^{s},D_{11}^{s},H_{11}^{s}) \tag{17b}$$

$$Q_{11}=\frac{E(z)}{1-\mu^{2}},\ A_{44}^{s}=A_{55}^{s}=\int_{-\frac{h}{2}}^{+\frac{h}{2}}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^{2}dz \tag{17c}$$

Substituting from Eq. (15) into Eq. (14), we obtain the following equation

$$A_{11}d_{11}u_{0}+A_{66}d_{22}u_{0}+\left(A_{12}+A_{66}\right)d_{12}v_{0}-B_{11}d_{111}w_{b}-\left(B_{12}+2B_{66}\right)d_{122}w_{b}$$
$$-\left(B_{12}^{S}+2B_{66}^{S}\right)d_{122}w_{S}-B_{11}^{S}d_{111}w_{S}=0 \tag{18a}$$

$$A_{22}d_{22}v_{0}+A_{66}d_{11}v_{0}+\left(A_{12}+A_{66}\right)d_{12}u_{0}-B_{22}d_{222}w_{b}-\left(B_{12}+2B_{66}\right)d_{112}w_{b}$$
$$-\left(B_{12}^{S}+2B_{66}^{S}\right)d_{112}w_{S}-B_{11}^{S}d_{222}w_{S}=0 \tag{18b}$$

$$B_{11}d_{111}u_{0}+\left(B_{12}+2B_{66}\right)d_{122}u_{0}+\left(B_{12}+2B_{66}\right)d_{112}v_{0}+B_{22}d_{222}v_{0}-D_{11}d_{1111}w_{b}$$
$$-2\left(D_{12}+2D_{66}\right)d_{1122}w_{b}-D_{22}d_{2222}w_{b}-D_{11}^{S}d_{1111}w_{S}-2\left(D_{12}^{S}+2D_{66}^{S}\right)d_{1122}w_{S}$$
$$-D_{22}^{S}d_{2222}w_{S}=0 \tag{18c}$$

$$B_{11}^{S}d_{111}u_{0}+\left(B_{12}^{S}+2B_{66}^{S}\right)d_{122}u_{0}+\left(B_{12}^{S}+2B_{66}^{S}\right)d_{112}v_{0}+B_{22}^{S}d_{222}v_{0}-D_{11}^{S}d_{1111}w_{b} \tag{18d}$$

$$-2\left(D_{12}^{S}+2D_{66}^{S}\right)d_{1122}w_b - D_{22}^{S}d_{2222}w_b - H_{11}^{S}d_{1111}w_S - 2\left(H_{12}^{S}+2H_{66}^{S}\right)d_{1122}w_S$$

$$-H_{22}^{S}d_{2222}w_S + A_{55}^{S}d_{11}w_S + A_{44}^{S}d_{22}w_S = 0$$

where d_{ij} , d_{ijl} , and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i \partial x_j},\ d_{ijl}=\frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\ d_{ijlm}=\frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},\ \left(i,j,l,m=1,2\right) \tag{19}$$

2.4 Closed-form solution for simply supported plates

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eqs. (18(a)-18(d)) for a simply supported FG plate. The following boundary conditions are imposed at the side edges

$$v_0 = w_b = w_s = \frac{\partial w_s}{\partial y} = N_x = M_x^b = M_x^s = 0 \quad \text{at} \quad x=-a/2,\ a/2 \tag{20a}$$

$$u_0 = w_b = w_s = \frac{\partial w_s}{\partial y} = N_y = M_y^b = M_y^s = 0 \quad \text{at} \quad y=-b/2,\ b/2 \tag{20b}$$

To solve this problem, Navier assumed that the transverse mechanical load, q in the form of a double trigonometric series as

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin\left(\lambda x\right)\sin\left(\mu y\right) \tag{21}$$

The coefficients q_{mn} for the case of uniformly distributed load (UL) are defined as follows

$$q_{mn}=\frac{16 q_0}{mn\pi^2} \qquad \text{for m,n odd} \tag{21a}$$

$$q_{mn}=0 \qquad \text{for m,n even} \tag{21b}$$

For the case of a sinusoidally distributed load, we have

$$m=n=1 \ \text{ and } \ q_{11}=q_0 \tag{21c}$$

Where $\lambda = m\pi / a$, $\mu = n\pi / b$ and q_0 represents the intensity of the load at the plate center.

Following the Navier solution procedure, we assume the following solution form for $\boldsymbol{u_0}$, $\boldsymbol{v_0}$, $\boldsymbol{w_b}$ and $\boldsymbol{w_b}$ that satisfies the boundary conditions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\lambda x)\sin(\mu y) \\ V_{mn}\sin(\lambda x)\cos(\mu y) \\ W_{bmn}\sin(\lambda x)\sin(\mu y) \\ W_{smn}\sin(\lambda x)\sin(\mu y) \end{Bmatrix} \tag{22}$$

U_{mn}, V_{mn}, W_{bmn}, and W_{smn} are arbitrary parameters to be determined.
Eqs. (21) and (22) reduce the governing Eqs. (18) to the following form

$$[C]\{\Delta\} = \{P\} \tag{23a}$$

Where
$\{\Delta\} = \{U, V, W_b, W_s\}^t$, $[C]$ and $[G]$ refers to the flexural stiffness.

$$[C] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \tag{24}$$

in which

$$\begin{aligned}
a_{11} &= A_{11}\lambda^2 + A_{66}\mu^2 \\
a_{12} &= \lambda\,\mu\left(A_{12} + A_{66}\right) \\
a_{13} &= -\lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2] \\
a_{14} &= -\lambda\,[B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\,\mu^2] \\
a_{22} &= A_{66}\lambda^2 + A_{22}\mu^2 \\
a_{23} &= -\mu\,[(B_{12} + 2B_{66})\,\lambda^2 + B_{22}\mu^2] \\
a_{24} &= -\mu\,[(B_{12}^s + 2B_{66}^s)\,\lambda^2 + B_{22}^s\mu^2] \\
a_{33} &= D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 \\
a_{34} &= D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\,\mu^2 + D_{22}^s\,\mu^4 \\
a_{44} &= H_{11}^s\lambda^4 + 2(H_{12}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2
\end{aligned} \tag{25}$$

3. Results and discussion

In numerical analysis, static analysis of simply supported FG Plates is evaluated. The FG plate is taken to be made of aluminum and alumina with the following material properties

Ceramic (P_C : Alumina, Al_2O_3): $E_c = 380$ GPa; $\nu = 0.3$;

Metal (P_M : Aluminium, Al): $E_m = 70$ GPa; $\nu = 0.3$;

And their properties change through the thickness of the plate according to power-law. The bottom surfaces of the FG plate are aluminum rich, whereas the top surfaces of the FG plate are alumina rich.

For convenience, the following dimensionless form is used

$$\bar{w} = 10\frac{E_C h^3}{q_0 a^4} w\left(\frac{a}{2},\frac{b}{2}\right), \quad \bar{u} = 100\frac{E_C h^3}{q_0 a^4} u\left(0,\frac{b}{2},\frac{-h}{4}\right), \quad \bar{v} = 100\frac{E_C h^3}{q_0 a^4} v\left(\frac{a}{2},0,\frac{-h}{6}\right)$$

$$\overline{\sigma_x} = \frac{h}{hq_0}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right), \quad \overline{\sigma_y} = \frac{h}{hq_0}\sigma_y\left(\frac{a}{2},\frac{b}{2},\frac{h}{3}\right), \quad \overline{\tau_{xy}} = \frac{h}{hq_0}\tau_{xy}\left(0,0,-\frac{h}{3}\right)$$

$$\overline{\tau_{xz}} = \frac{h}{hq_0}\tau_{xz}\left(0,\frac{b}{2},0\right), \quad \overline{\tau_{yz}} = \frac{h}{hq_0}\tau_{yz}\left(\frac{a}{2},0,\frac{h}{6}\right).$$

The effect of volume fraction exponent on the dimensionless stresses and displacements of a FGM square plate subjected to uniform and sinusoidal distributed loads is shown in Tables 1 and 2, respectively. As it can be seen, the dimensionless stresses and displacements results for sinusoidal load distribution always less than uniform distribution load. As the plate becomes more and more metallic, the difference increases for deflection $\bar{w}$ and in-plane longitudinal stress $\overline{\sigma_x}$ while it decreases for in-plane normal stress $\overline{\sigma_y}$. It is important to observe that the stresses for a fully ceramic plate are the same as that for a fully metal plate. This is because the plate for these two cases is fully homogeneous and the stresses do not depend on the modulus of elasticity. As seen from this tables, the results of the present theory are close to those obtained by the SSDT of Zenkour (2006).

The effect of aspect and side-to thickness ratios on the center deflection of the FGM plate under uniform distributed load are plotted in Figs. 2 and 3, respectively. As in can be seen, in the case of ceramic plate the deflection is less than of metallic plate.

Table 1 Comparison of nondimensional deflection and stresses of square plate under uniformly distributed load (m, n = 100 term series, a = 10h)

k	Method	$\bar{w}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\sigma_{yz}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{xy}}$
Ceramic	SSDT	0.4665	2.8932	1.9103	0.4429	0.5114	1.2850
	Present	0.4665	2.8932	1.9103	0.4429	0.5114	1.2850
1	SSDT	0.9287	4.4745	2.1962	0.5446	0.5114	1.1143
	Present	0.9287	4.4499	2.1647	0.5379	0.5375	1.1097
2	SSDT	1.1940	5.2296	2.0338	0.5734	0.4700	0.9907
	Present	1.1936	5.1737	1.9607	0.5998	0.4385	0.9867
3	SSDT	1.3200	5.6108	1.8593	0.5629	0.4367	1.0047
	Present	1.3268	5.6876	2.0435	0.5754	0.4360	0.9908
metal	SSDT	2.5327	2.8932	1.9103	0.4429	0.5114	1.2850
	Present	2.5327	2.8932	1.9103	0.4429	0.5114	1.2850

Table 2 Comparison of nondimensional deflection and stresses of square plate under sinusoidally distributed load (a=10h)

k	Method	$\overline{w}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\sigma_{yz}}$	$\overline{\sigma_{xz}}$	$\overline{\sigma_{xy}}$
Ceramic	SSDT	0.2960	1.9955	1.3121	0.2132	0.2462	0.7065
	Present	0.2960	1.9955	1.3121	0.2132	0.2462	0.7065
1	SSDT	0.5889	3.0870	1.4894	0.2622	0.2462	0.6110
	Present	0.5889	3.1212	1.4831	0.2610	0.2458	0.6107
2	SSDT	0.7572	3.6094	1.3954	0.2763	0.2265	0.5441
	Present	0.7573	3.6720	1.3854	0.2760	0.2254	0.5434
3	SSDT	0.8372	3.8742	1.2748	0.2715	0.2107	0.5525
	Present	0.8376	3.9571	1.2632	0.2708	0.2101	0.5520
metal	SSDT	1.6071	1.9955	1.3121	0.2132	0.2462	0.7065
	Present	1.6071	1.9955	1.3121	0.2132	0.2462	0.7065

The variation of the shear stresses τ_{xy} and τ_{yz} of the simply supported FGM plate under uniform load are shown in Figs. 4 and 5, respectively. The gradient index is taken as *k=2*, and the thickness ratio a/h=10. It to be noted, that the stresses are tensile at the top surface and compressive at the bottom surface.

Figs. 6 and 7 shows the variation of transversal shear stress τ_{xz} and in-plane longitudinal stress σ_{xx} of the FGM plate respectively. It's clear that the distributions transversal shear stress τ_{xz} are not parabolic.

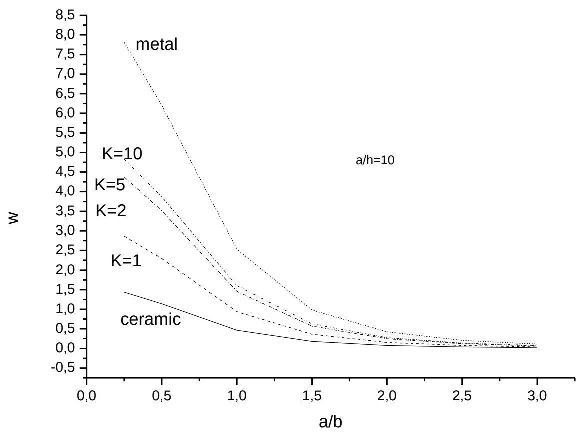

Fig. 2 Dimensionless center deflection as function of the aspect ratio (a/b) of an FGM

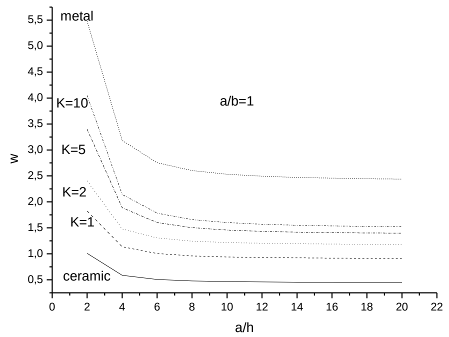

Fig. 3 Dimensionless center deflection as a function of the side-to-thickness ratio (a/h) of an FGM square plate

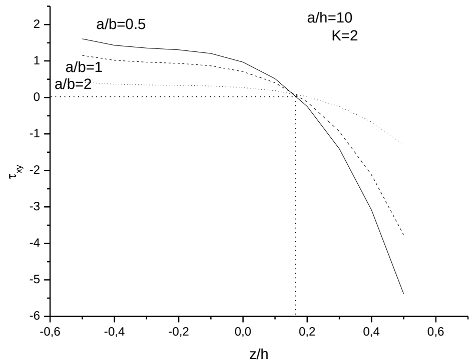

Fig. 4 Variation of longitudinal tangential stress (τ_{xy}) through-the thickness of an FGM plate for different values of the aspect ratio

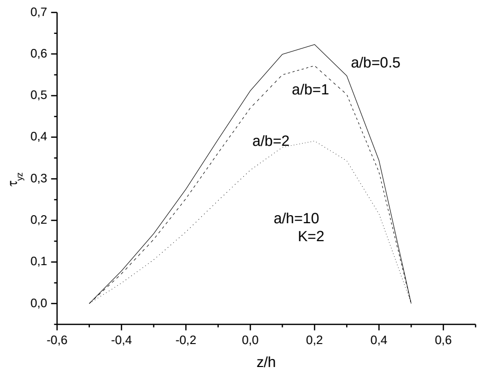

Fig. 5 Variation of transversal shear stress (τ_{yz}) through-the thickness of an FGM plate for different values of the aspect ratio

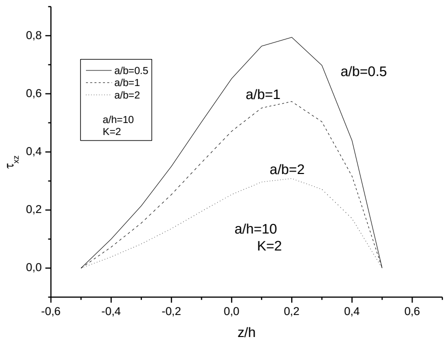

Fig. 6 Variation of transversal shear stress (τ_{xz}) through-the thickness of an FGM plate for different values of the aspect ratio

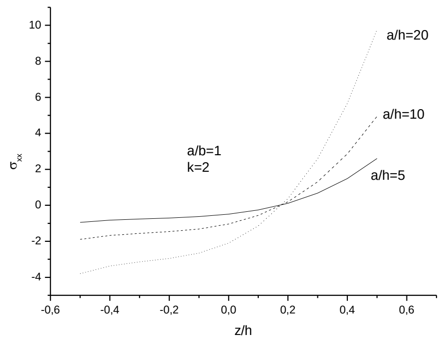

Fig. 7 Variation of in-plane longitudinal stress (σ_{xx}) through-the thickness of an FGM plate for different values of the side -to-thickness ratio

4. Conclusions

In this work, a refined plate theory based on the high order shear deformation theory is successfully developed for bending of a simply supported FG plates. The theory accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. Accuracy and convergence of the present refined plate theories was verified by comparing the results obtained with those reported in the literature for the FG plate. Parametric studies for varying of the power low index, the aspect and side-to-thickness ratio are discussed and demonstrated through illustrative numerical examples.

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