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Analysis of buckling response of functionally graded sandwich plates using a refined shear deformation theory

Z. Abdelhak[1,2], L. Hadji *[1,3], Z. Khelifa[1,3], T. Hassaine Daouadji[1,3] and E.A. Adda Bedia[1]

[1]Department of Civil Engineering, Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Algérie
[2]Centre Universitaire Ahmed Zabana, 48000 Relizane, Algérie
[3]Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algérie

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Abstract. In this paper, a refined shear deformation plate theory which eliminates the use of a shear correction factor was presented for FG sandwich plates composed of FG face sheets and an isotropic homogeneous core. The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the plate. The mechanical properties of the plate are assumed to vary continuously in the thickness direction by a simple power-law distribution in terms of the volume fractions of the constituents. Based on the present refined shear deformation plate theory, the governing equations of equilibrium are derived from the principle of virtual displacements. Numerical illustrations concern buckling behavior of FG sandwiches plates with Metal–Ceramic composition. Parametric studies are performed for varying ceramic volume fraction, volume fraction profiles, Boundary condition, and length to thickness ratios. The accuracy of the present solutions is verified by comparing the obtained results with the existing solutions.

Keywords: mechanical properties; functionally graded sandwich plate; buckling; shear deformation; volume fraction

1. Introduction

The conventional sandwich structures are generally fabricated form three homogeneous layers, two face sheets adhesively bonded to the core. However, the sudden change in material properties across the interface between different materials can result in large interlaminar stresses. To overcome these adverse effects, a new class of advanced inhomogeneous composite materials, that compose of two or more phases with different material properties and continuously varying composition distribution (using a simple functional law or an exponential law), has been developed which is referred to as functionally graded materials (FGMs). Such materials were introduced as to take advantage of the desired material properties of each constituent material without interface problems. The sandwich plate faces are typically made from a mixture of two materials. While the core of this sandwich plate is fully homogeneous material. Studies related to

∗Corresponding author, Dr., E-mail: had_laz@yahoo.fr

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FGM sandwich structures are few in numbers. Zenkour and Sobhy (2010) investigated the thermal buckling of various types of FGM sandwich plate using sinusoidal shear deformation plate theory (SPT). Zenkour (2005) studied the mechanical bending response, buckling and free vibration of simply supported FGM sandwich plate in that paper. Three-dimensional finite element simulations for analyzing low velocity impact behavior of sandwich panels with a functionally graded core were conducted by Etemadi *et al*. (2009). An exact thermoelasticity solution for a two-dimensional sandwich structures with functionally graded coating was presented by Shodja *et al*. (2007). Tounsi and his co-workers (Hadji *et al*, 2011, Bachir Bouiadjra *et al*. 2012, Fekrar *et al*. 2012, Tounsi et al. (2013), Klouche Djedid *et al*. 2014, Ait Yahia *et al*. 2015) developed new shear deformation plates theories involving only four unknown functions. Wang and Shen. (2011) analyzed non-linear free vibration, non-linear bending and postbuckling of sandwich plates with FGM face sheets resting on an elastic foundation of Pasternak type. Kiani and Eslami (2012) presented a simple approximate closed-form expression to predict the postbuckling response of sandwich plates with FGM face sheets, which were subjected to uniform temperature rise loading. Recently Ait Amar Meziane *et al*. (2014) proposed an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Zidi *et al.* (2014) analyzed the bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. Belabed *et al*. (2014) presented an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Hebali *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Bourada *et al.* (2015) analyzed the thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations. Mahi *et al.* (2015) studied the bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates using a new hyperbolic shear deformation theory. Hamidi *et al.* (2015) investigated a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Mahi *et al.* (2015) analyzed the bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates using a new hyperbolic shear deformation theory. Al-Basyouni *et al.* (2015) presented size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Zemri et al. (2015) studied the mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory. Bennoun *et al.* (2016) analyzed the vibration of functionally graded sandwich plates using a novel five variable refined plate theory. Ait Atmane *et al.* (2016) studied the effect of thickness stretching and porosity on mechanical response of functionally graded beams resting on elastic foundations.

In this paper, a refined shear deformation plate theory which eliminates the use of the shear correction factor is developed for FG sandwich plates composed of FG face sheets and an isotropic homogeneous core. Equations of motion and boundary conditions are derived from Hamilton’s principle. Analytical solutions for rectangular plates under various boundary conditions are obtained. Numerical examples are presented to verify the accuracy of the present theory in predicting the buckling responses of FG sandwich plates.

2. Theoretical formulations

Consider a sandwich plate composed of three layers as shown in Fig. 1. Two FG face sheets are made from a mixture of a metal and a ceramic, while a core is made of an isotropic homogeneous material. The material properties of FG face sheets are assumed to vary continuously through the plate thickness by a power law distribution as

$$P(z) = P_M + \left(P_C - P_m\right)^k V \tag{1}$$

where P represents the effective material property such as Young's modulus E, Poisson's ratio ν, and mass density ρ; subscripts c and m denote the ceramic and metal phases, respectively; and V is the volume fraction of the ceramic phase defined by

$$V^{(1)} = \left(\frac{z-h_0}{h_1-h_0}\right)^k \qquad z \in [h_0, h_1] \tag{2a}$$

$$V^{(2)} = 1 \qquad z \in [h_1, h_2] \tag{2b}$$

$$V^{(3)} = \left(\frac{z-h_3}{h_2-h_3}\right)^k \qquad z \in [h_2, h_3] \tag{2c}$$

where k is the power law index that governs the volume fraction gradation.

2.1 Kinematics and constitutive equations

The displacement field of the present refined shear deformation plate theory is given by

$$\begin{aligned} U(x,y,z) &= u(x,y) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ V(x,y,z) &= v(x,y) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ W(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{3a}$$

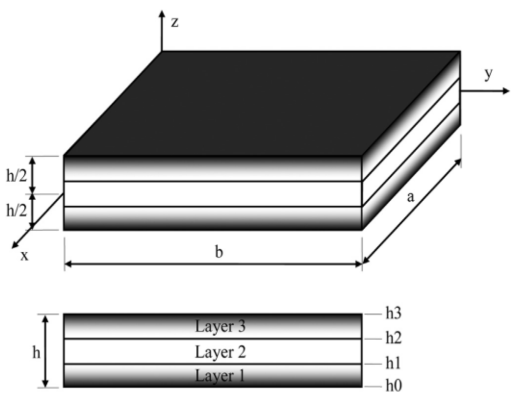

Fig. 1 Geometry and coordinates of FG sandwich plates

where

$$f(z) = z - ze^{-2(z/h)^2} \tag{3b}$$

The strains associated with the displacements in Eq. (3) are

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + zk_x^b + \mathrm{f}(\mathrm{z})\mathrm{k}_\mathrm{x}^\mathrm{s} \\
\varepsilon_y &= \varepsilon_y^0 + zk_x^b + \mathrm{f}(\mathrm{z})\mathrm{k}_\mathrm{y}^\mathrm{s} \\
\gamma_{xy} &= \gamma_{xy}^0 + \mathrm{z}k_{xy}^b + \mathrm{f}(\mathrm{z})\mathrm{k}_\mathrm{xy}^\mathrm{s} \\
\gamma_{yz} &= g\gamma_{yz}^s \\
\gamma_{xz} &= g\gamma_{xz}^s \\
\varepsilon_z &= 0
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
&\varepsilon_x^0 = \frac{\partial u}{\partial x}, k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, k_x^s = -\frac{\partial^2 w_s}{\partial x^2} \\
&\varepsilon_x^0 = \frac{\partial v}{\partial y}, k_y^b = -\frac{\partial^2 w_b}{\partial y^2}, k_y^s = -\frac{\partial^2 w_s}{\partial y^2} \\
&\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, k_{xy}^b = -2\frac{\partial^2 w_b}{\partial x \partial y}, k_{xy}^s = -2\frac{\partial^2 w_s}{\partial x \partial y} \\
&\gamma_{yz}^s = \frac{\partial w_s}{\partial y}, \gamma_{xz}^s = \frac{\partial w_s}{\partial x}, g = 1 - f'(z), f'(z) = \frac{df(z)}{dz}
\end{aligned} \tag{5}$$

For elastic and isotropic FGMs, the constitutive relations can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^{(n)} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}^{(n)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{(n)} \text{ and } \begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix}^{(n)} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix}^{(n)} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(n)} \tag{6}$$

where

$$\begin{aligned}
&C_{11}(z) = C_{22}(z) = \frac{E(z)}{1-\nu^2}, C_{12}(z) = \nu C_{11}(z) \\
&C_{44}(z) = C_{55}(z) = C_{66}(z) = \frac{E(z)}{2(1+\nu)},
\end{aligned} \tag{7}$$

2.2 Governing equations

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The principle of virtual work in the present case yields

$$\int_{-h/2}^{h/2}\int_{\Omega}\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right]d\Omega dz-\int_{\Omega}\overline{N}\delta w d\Omega=0, \tag{8}$$

with

$$\overline{N}=\left[N_x^0\frac{\partial^2(w_b+w_s)}{\partial x^2}+N_y^0\frac{\partial^2(w_b+w_s)}{\partial y^2}+2N_{xy}^0\frac{\partial^2(w_b+w_s)}{\partial x\partial y}\right] \tag{9}$$

Substituting Eqs. (5) and (6) into Eq. (9) and integrating through the thickness of the plate, Eq. (9) can be rewritten as:

$$\int_{\Omega}\begin{bmatrix}N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x^b\delta k_x^b+M_y^b\delta k_y^b+M_{xy}^b\delta k_{xy}^b+M_x^s\delta k_x^s\\ +M_y^s\delta k_y^s+M_{xy}^s\delta k_{xy}^s+S_{yz}^s\delta\gamma_{yz}^s+S_{xz}^s\delta\gamma_{xz}^s\end{bmatrix}d\Omega-\int_{\Omega}\overline{N}\delta w d\Omega=0, \tag{10}$$

where

$$\begin{aligned}
\left(N_x,N_y,N_{xy}\right)&=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\left(\sigma_x,\sigma_y,\tau_{xy}\right)dz\\
\left(M_x^b,M_y^b,M_{xy}^b\right)&=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\left(\sigma_x,\sigma_y,\tau_{xy}\right)z dz\\
\left(M_x^s,M_y^s,M_{xy}^s\right)&=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\left(\sigma_x,\sigma_y,\tau_{xy}\right)f dz\\
\left(S_{xy}^s,S_{yz}^s\right)&=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\left(\tau_{xy},\tau_{yz}\right)g(z)dz
\end{aligned} \tag{11}$$

where h_{n+1} and h_n are the top and bottom $z-$coordinates of the nth layer.

Substituting Eq. (6) into Eq. (11) and integrating through the thickness of the plate, the stress resultants are given as

$$\begin{Bmatrix}N\\ M^b\\ M^s\end{Bmatrix}=\begin{bmatrix}A & B & B^s\\ B & D & D^s\\ B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\ k^b\\ k^s\end{Bmatrix},\begin{Bmatrix}S_{yz}^s\\ S_{xz}^s\end{Bmatrix}=\begin{bmatrix}A_{44}^s & 0\\ 0 & A_{55}^s\end{bmatrix}\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{12}$$

in which

$$N=\left\{N_x,N_y,N_{xy}\right\}^t,M^b=\left\{M_x^{\ b},M_y^{\ b},M_{xy}^{\ b}\right\}^t,M^s=\left\{M_x^s,M_y^s,M_{xy}^s\right\}^t \tag{13a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t \tag{13b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{13c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{13d}$$

and stiffness components and inertias are given as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \sum_{n=1}^{3} \int_{h}^{hn+1} Q_{11}^{(n)} \left(1, z, z?\ f(z), zf(z), f?z)\right) \begin{Bmatrix} 1 \\ v^{(n)} \\ \dfrac{1-v^{(n)}}{2} \end{Bmatrix} dz \tag{14a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), Q_{11}^{(n)} = \frac{E(z)}{1-v^2} \tag{14b}$$

$$A_{44}^s = A_{55}^s = \sum_{n=1}^{3} \int_{h_n}^{h_{n+1}} \frac{E(z)}{2(1+v)} \left[g(z)\right]^2 dz \tag{14c}$$

The governing equations of equilibrium can be derived from Eq. (16) by integrating the displacement gradients by parts and setting the coefficients zero δu_0, δv_0, δw_b, and δw_s separately. Thus one can obtain the equilibrium equations associated with the present refined shear deformation plate theory

$$\begin{aligned} \delta u &= \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ \delta v &= \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ \delta w_b &= \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy\,xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + \overline{N} = 0 \\ \delta w_b &= \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy\,xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + \overline{N} = 0 \end{aligned} \tag{15}$$

Eq. (15) can be expressed in terms of displacements $\left(u_s, v_s, w_b, w_s\right)$ by substituting for the stress resultants from Eq. (12). For FG sandwich plates, the equilibrium Eq. (15) take the form

$$A_{11}\frac{\partial^2 u}{\partial x^2}+A_{66}\frac{\partial^2 u}{\partial y^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 v}{\partial x\partial y}-B_{11}\frac{\partial^3 w_b}{\partial x^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_b}{\partial x\partial y^2}$$
$$-B_{11}^s\frac{\partial^3 w_s}{\partial x^3}-\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x\partial y^2}=0 \tag{16a}$$

$$\left(A_{12}+A_{66}\right)\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 v}{\partial x^2}+A_{22}\frac{\partial^2 v}{\partial y^2}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_b}{\partial x^2\partial y}-B_{22}\frac{\partial^3 w_b}{\partial y^3}$$
$$-B_{22}^s\frac{\partial^3 w_s}{\partial y^3}-\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x^2\partial y}=0 \tag{16b}$$

$$B_{11}\frac{\partial^3 u}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u}{\partial x\partial y^2}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v}{\partial x^2\partial y}+B_{22}\frac{\partial^3 v}{\partial y^3}-D_{11}\frac{\partial^4 w_b}{\partial x^4}$$
$$-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_b}{\partial y^4}-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2} \tag{16c}$$
$$-D_{22}^s\frac{\partial^4 w_s}{\partial y^4}+\overline{N}=0$$

$$B_{11}^s\frac{\partial^3 u}{\partial x^3}+(B_{12}^s+2B_{66}^s)\frac{\partial^3 u}{\partial x\partial y^2}+(B_{12}^s+B_{66}^s)\frac{\partial^3 v}{\partial x^2\partial y}+B_{22}^s\frac{\partial^3 v}{\partial y^3}-D_{11}^s\frac{\partial^4 wb}{\partial x^4}$$
$$-2(D_{12}^s+2D_{66}^s)\frac{\partial^4 wb}{\partial x^2\partial y^2}-D_{22}^s\frac{\partial^4 wb}{\partial y^4}-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2(H_{12}^s+2H_{66}^s)\frac{\partial^4 w_s}{\partial x^2\partial y^2} \tag{16d}$$
$$-H_{22}^s\frac{\partial^4 w_s}{\partial x^4}+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+A_{44}^s\frac{\partial^2 w_s}{\partial y^2}+\overline{N}=0$$

3. Analytical solution

Consider a rectangular plate with length a and width b under in-plane forces in two directions $\left(N_x^0=\gamma_1 N_{cr}, N_y^0=\gamma_2 N_{cr}, N_{xy}^0=0\right)$. The analytical solution of Eq. (16) can be obtained for rectangular plates under various boundary conditions by using the following expansions of generalized displacements

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}X'_m(x)Y_n(y) \\ V_{mn}X_m(x)Y'_n(y) \\ W_{bmn}X_m(x)Y_n(y) \\ W_{smn}X_m(x)Y_n(y) \end{Bmatrix} \tag{17}$$

where U_{mn}, V_{mn}, W_{bmn} and W_{smn} are coefficients. The functions $X_m(x)$ and $Y_n(y)$ given in Table 1 are suggested by Sobhy (2013) to satisfy various boundary conditions

Clamped edge

$$u = v = w_b = \frac{\partial w_b}{\partial x} = \frac{\partial w_b}{\partial y} = w_s = \frac{\partial w_s}{\partial x} = \frac{\partial w_s}{\partial y} = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad y = 0,\ b \tag{18a}$$

Simply supported edge

$$v = w_b = \frac{\partial w_b}{\partial y} = w_s = \frac{\partial w_s}{\partial y} = N_x = M_x^b = M_x^s = 0 \quad \text{at} \quad x = 0,\quad a \tag{19a}$$

$$u = w_b = \frac{\partial w_b}{\partial x} = w_s = \frac{\partial w_s}{\partial x} = N_y = M_y^b = M_y^s = 0 \quad \text{at} \quad y = 0,\quad b \tag{19a}$$

with $\lambda = m\pi / a,\quad \mu = n\pi / b$.

Substituting Eq. (17) into Eq. (16), the analytical solutions can be obtained from

$$\left\{\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k & k \\ 0 & 0 & k & k \end{bmatrix}\right\} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{20}$$

where

$$\begin{aligned}
S_{11} &= \int_0^a\int_0^b \left(A_{11} X_m''' Y_n + A_{66} X_m' Y_n''\right) X_m' Y_n \, dxdy \\
S_{12} &= \int_0^a\int_0^b \left(A_{11} + A_{66}\right) X_m' Y_n'' X_m' Y_n \, dxdy \\
S_{13} &= -\int_0^a\int_0^b \left[B_{11} X_m'' Y_n + \left(B_{12} + 2B_{66}\right) X_m' Y_n''\right] X_m' Y_n \, dxdy \\
S_{14} &= -\int_0^a\int_0^b \left[B_{11}^s X_m''' Y_n + \left(B_{12}^s + 2B_{66}^s\right) X_m' Y_n''\right] X_m' Y_n \, dxdy
\end{aligned} \tag{20a}$$

$$\begin{aligned}
S_{21} &= \int_0^a\int_0^b \left(A_{11} + A_{66}\right) X_n'' Y_m' X_m Y_n' \, dxdy \\
S_{22} &= \int_0^a\int_0^b \left(A_{22} X_m Y_n''' + A_{66} X_m'' Y_n'\right) X_m Y_n' \, dxdy
\end{aligned} \tag{20b}$$

$$S_{23} = -\int_0^a\int_0^b \left[B_{22}X_mY_n^{'''} + \left(B_{12}+2B_{66}\right)X_m^{''}Y_n^{'}\right]X_mY_n^{'}dxdy$$

$$S_{24} = -\int_0^a\int_0^b \left[B_{11}^{s}X_mY_n^{'''} + \left(B_{12}^{s}+2B_{66}^{s}\right)X_m^{''}Y_n^{'}\right]X_mY_n^{'}dxdy \tag{20c}$$

$$S_{31} = \int_0^a\int_0^b \left[B_{11}X_m^{''''}Y_n + \left(B_{12}+2B_{66}\right)X_m^{''}Y_n^{''}\right]X_mY_ndxdy$$

$$S_{32} = \int_0^a\int_0^b \left[B_{22}X_mY_n^{''''} + \left(B_{12}+2B_{66}\right)X_m^{''}Y_n^{''}\right]X_mY_ndxdy$$

$$S_{33} = \int_0^a\int_0^b -\left[B_{22}X_m^{''''}Y_n + 2\left(D_{12}+2D_{66}\right)X_m^{''}Y_n^{''} + D_{22}X_mY_n^{''''}\right]X_mY_ndxdy \tag{20d}$$

$$S_{34} = \int_0^a\int_0^b -\left[D_{11}^{S}X_m^{''''}Y_n + 2\left(D_{12}^{s}+2D_{66}^{s}\right)X_m^{''}Y_n^{''} + D_{22}^{s}X_mY_n^{''''}\right]X_mY_ndxdy$$

$$S_{41} = \int_0^a\int_0^b \left[B_{11}^{s}X_m^{''''}Y_n + \left(B_{12}^{s}+2B_{66}^{s}\right)X_m^{''}Y_n^{''}\right]X_mY_ndxdy$$

$$S_{42} = \int_0^a\int_0^b \left[B_{22}^{s}X_mY_n^{''''} + \left(B_{12}^{s}+2B_{66}^{s}\right)X_m^{''}Y_n^{''}\right]X_mY_ndxdy \tag{20e}$$

$$S_{43} = \int_0^a\int_0^b -\left[D_{11}^{S}X_m^{''''}Y_n + 2\left(D_{12}^{s}+2D_{66}^{s}\right)X_m^{''}Y_n^{''} + D_{22}^{s}X_mY_n^{''''}\right]X_mY_ndxdy$$

Table 1 The admissible functions $X_m(x)$ and $Y_n(y)$

Boundary conditions					The fonctions $X_m(x)$ and $Y_n(y)$	
Notation	$x=0$	$y=0$	$x=a$	$y=b$	$X_m(x)$	$Y_n(y)$
SSSS	S	S	S	S	$\sin(\lambda x)$	$\sin(\mu x)$
CSCS	C	S	C	S	$\sin^2(\lambda x)$	$\sin(\mu x)$
CCCC	C	C	C	C	$\sin^2(\lambda x)$	$\sin^2(\mu x)$
FCFC	F	C	F	C	$\cos^2(\lambda x)\left[\sin^2(\lambda x)+1\right]$	$\sin^2(\mu x)$

$$S_{44} = \int_0^a \int_0^b -\left(\left[H_{11}^S X_m'''' Y_n + 2\left(H_{12}^s + 2H_{66}^s\right) X_m'' Y_n'' + H_{22}^s X_m Y_n''''\right] + A_{55}^s X_m'' Y_n + A_{44}^s X_m Y_n''\right) X_m Y_n dxdy$$
$$k = N_{cr} \int_0^a \int_0^b \left(\gamma_1 X_m'' Y_n + \gamma_2 X_m Y_n''\right) X_m Y_n dxdy \tag{20f}$$

4. Numerical results and discussion

In this section, a number of numerical examples is presented and discussed to verify the accuracy of the present theory and investigate the effects of the power law index, thickness ratio of layers, i.e., scheme, and transverse shear deformation on critical buckling load of FG sandwich plates.

A simply supported Al/Al_2O_3 sandwich plate composed of aluminum face sheets (as metal) and an alumina core (as ceramic) is considered. Young's modulus, Poisson's ratio and density of aluminum are $E_m = 70GPa$, $\nu_m = 0.3$, respectively, and those of alumina are $E_C = 380GPa$, $\nu_C = 0.3$. Five different schemes of sandwich plate are considered. A four-letter notation as shown in Table 1 is used to describe the boundary conditions of the plate. The ratio of the thickness of each layer from bottom to top is denoted by the combination of three numbers, i.e., (1-0-1), (2-1-2) and so on. The following dimensionless form is used

$$\overline{N} = \frac{Na^2}{100h^3E_0}, \; E_0 = 1GPa \tag{21}$$

The critical buckling loads of the system are calculated by the stability Eq. (20) as an eigenvalue problem. The critical buckling loads of FG sandwich plates are presented here to estimate the accuracy of the present refined shear deformable plate theory. A moderately thick square plate with the thickness ratio equal to 10 and the power law index varied from 0 to 10 is analyzed. Dimensionless critical buckling loads $\overline{N}$ of square plates under uniaxial and biaxial compressions are presented in Tables 2 and 3, respectively. The obtained results are compared with those generated by El Meiche *et al*. (2011) based on the HSDT and Tai *et al*. (2014) based on the NFSDPT. An excellent agreement between the results is obtained for all schemes and values of power law index. Thus, it can be stated that the present model is not only accurate but also simple in predicting the critical buckling load of FG sandwich plates.

The effects of the power law index k on critical buckling load of FG sandwich square plates is illustrated in Fig. 2. The thickness ratio of the plate is taken equal to 10. It can be seen that increasing the power law index k result in a reduction of buckling load. This is due to the fact that higher power law index k corresponds to lower volume fraction of the ceramic phase V. In other word, increasing the power law index will reduce the stiffness of the plate due to high portion of metal in comparison with the ceramic part, and consequently, lead to a reduction of both buckling load.

Table 2 Dimensionless buckling load $\overline{N}$ of square plates under uniaxial compression $(\gamma_1 = -1, \gamma_1 = 0, a/h = 10)$

k	Theory	Scheme				
		1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
0	HSDT (El Meiche *et al.* 2011)	13.0055	13.0055	13.0055	13.0055	13.0055
	NFSDT (Tai 2014)	13.0045	13.0045	13.0045	13.0045	13.0045
	Present	13.0093	13.0093	13.0093	13.0093	13.0093
0.5	HSDT (El Meiche *et al.* 2011)	7.3638	7.9405	8.4365	8.8103	9.2176
	NFSDT (Tai 2014)	7.3634	7.9403	8.4361	8.8095	9.2162
	Present	7.3677	7.9438	8.4386	8.8253	9.2175
1	HSDT (El Meiche *et al.* 2011)	5.1663	5.8394	6.4645	6.9495	7.5072
	NFSDT (Tai 2014)	5.1648	5.8387	6.4641	6.9485	7.5056
	Present	5.1702	5.8427	6.4665	6.9809	7.5066
5	HSDT (El Meiche *et al.* 2011)	2.6568	3.0414	3.5787	4.1116	4.7346
	NFSDT (Tai 2014)	2.6415	3.0282	3.5710	4.1024	4.7305
	Present	2.6621	3.0456	3.5818	4.1856	4.7352
10	HSDT (El Meiche *et al.* 2011)	2.4857	2.7450	3.1937	3.7069	4.2796
	NFSDT (Tai 2014)	2.4666	2.7223	3.1795	3.6901	4.2728
	Present	2.4916	2.7498	3.1973	3.78793	4.2808

Table 3 Dimensionless buckling load $\overline{N}$ of square plates under biaxial compression $(\gamma_1 = -1, \gamma_2 = -1, a/h = 10)$

k	Theory	Scheme				
		1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
0	HSDT (El Meiche *et al.* 2011)	6.5028	6.5028	6.5028	6.5028	6.5028
	NFSDT (Tai 2014)	6.5022	6.5022	6.5022	6.5022	6.5022
	Present	6.5046	6.5046	6.5046	6.5046	6.5046
0.5	HSDT (El Meiche *et al.* 2011)	3.6819	3.9702	4.2182	4.4051	4.6088
	NFSDT (Tai 2014)	3.6817	3.9702	4.2181	4.4047	4.6081
	Present	3.6839	3.9719	4.2193	4.4126	4.6088
1	HSDT (El Meiche *et al.* 2011)	2.5832	2.9197	3.2323	3.4748	3.7536
	NFSDT (Tai 2014)	2.5824	2.9193	3.2320	3.4742	3.7528
	Present	2.5851	2.9214	3.2332	3.4904	3.7533
5	HSDT (El Meiche *et al.* 2011)	1.3284	1.5207	1.7894	2.0558	2.3673
	NFSDT (Tai 2014)	1.3208	1.5141	1.7855	2.0512	2.3652
	Present	1.3310	1.5228	1.7909	2.0928	2.3676
10	HSDT (El Meiche *et al.* 2011)	1.2429	1.3725	1.5969	1.8534	2.1398
	NFSDT (Tai 2014)	1.2333	1.3612	1.5897	1.8450	2.1364
	Present	1.2458	1.3749	1.5986	1.8939	2.1404

In order to investigate the effect of shear deformation on buckling load of FG sandwich plates, Fig. 3 display the variation of critical buckling load, with respect to thickness ratio a/h. The power law index is taken equal to 1. The dimensionless buckling load is obtained using the present theory and CPT. Since the CPT neglects the shear deformation, it overestimates buckling load (see

Fig. 3). The difference between the present theory and CPT is significant for thick to moderately thick FG sandwich plates, but it is negligible for thin plates with $a/h \succ 20$. This means that the inclusion of shear deformation results in an reduction of buckling load, and the effect of shear deformation is considerable for thick plates, but negligible for thin plates.

The effect of boundary conditions on buckling load is shown in Fig. 4 and Table 4. It is observed that the hardest and softest plates correspond to the FCFC and SSSS ones, respectively.

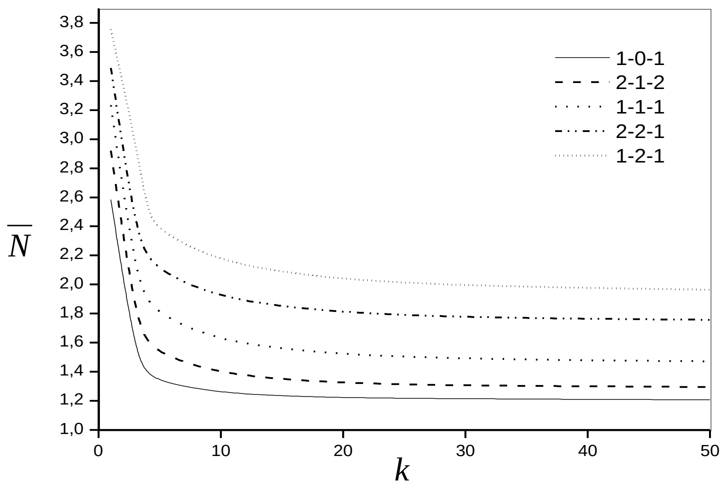

Fig. 2 Effect of power law index k on dimensionless critical buckling load $\overline{N}$ of square plates under biaxial compression $(\gamma_1 = \gamma_2 = -1, a = 10h)$

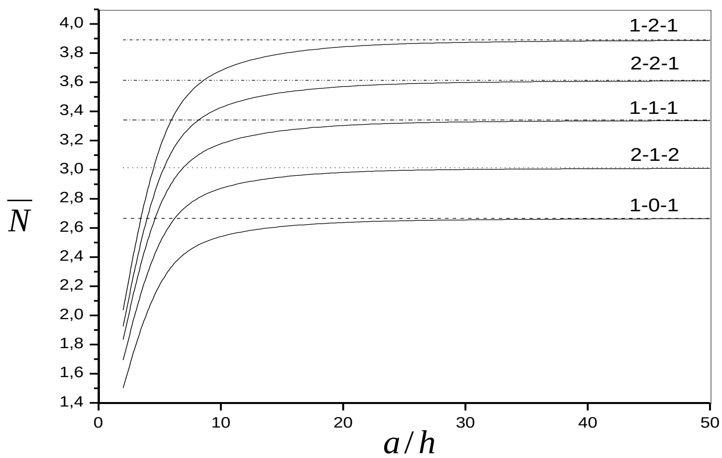

Fig. 3 Effect of shear deformation on dimensionless critical buckling load $\overline{N}$ of square plates under biaxial compression $(\gamma_1 = \gamma_2 = -1, k = 1)$

Table 4 Dimensionless buckling load N of square plates ($\gamma_1 = \gamma_2 = -1, a/h = 10$)

Boundary conditions	k	Scheme				
		1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
SSSS	0	6.5046	6.5046	6.5046	6.5046	6.5046
	0.5	3.6839	3.9719	4.2193	4.4126	4.6088
	1	2.5851	2.9214	3.2332	3.4904	3.7533
	2	1.7797	2.0833	2.4052	2.6995	2.9935
	5	1.3310	1.5228	1.7909	2.0928	2.3676
	10	1.2458	1.3749	1.5986	1.8939	2.1404
CSCS	0	11.9569	11.9569	11.9569	11.9569	11.9569
	0.5	6.8664	7.4006	7.8535	8.2187	8.5600
	1	4.8486	5.4784	6.0548	6.5607	7.0068
	2	3.3539	3.9294	4.5298	5.1349	5.6160
	5	2.5085	2.8840	3.3889	4.0383	4.4621
	10	2.3367	2.6054	3.0297	3.6764	4.0412
CCCC	0	15.9404	15.9404	15.9404	15.9404	15.9404
	0.5	9.2481	9.9649	10.5667	11.0326	11.4988
	1	6.5612	7.4124	8.1837	8.8220	9.4482
	2	4.5549	5.3400	6.1489	6.9016	7.6011
	5	3.4068	3.9315	4.6170	5.4105	6.0602
	10	3.1619	3.5533	4.1323	4.9161	5.4961
FCFC	0	18.6306	18.6306	18.6306	18.6306	18.6306
	0.5	10.8843	11.7258	12.4273	12.9730	13.5088
	1	7.7469	8.7513	9.6548	10.4128	11.1287
	2	5.3917	6.3238	7.2760	8.1828	8.9761
	5	4.0326	4.6659	5.4771	6.4469	7.1736
	10	3.7335	4.2184	4.9061	5.8695	6.5120

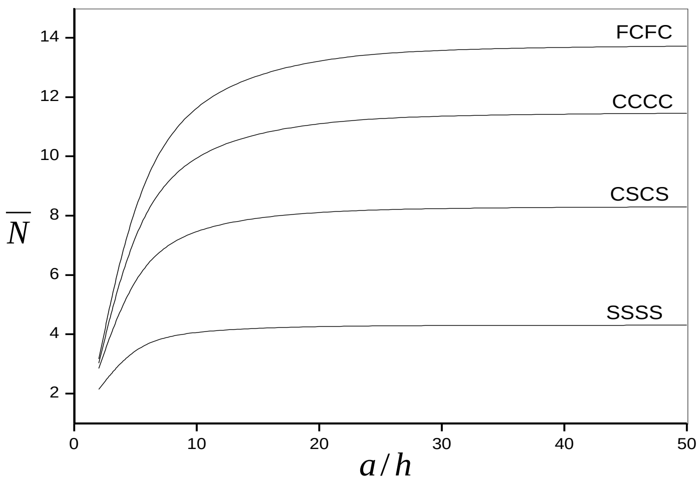

Fig. 4 Effect of boundary conditions on dimensionless critical buckling load $\overline{N}$ of (1-3-1) FG sandwich square plates under biaxial compression (k=1).

5. Conclusions

In the present study, a refined shear deformation plate theory which eliminates the use of a shear correction factor was presented for FG sandwich plates composed of FG face sheets and an isotropic homogeneous core. Governing equations and boundary conditions are derived from principle of virtual displacements. Analytical solutions for buckling analysis of simply supported plates are presented. Numerical examples show that the proposed theory gives solutions which are almost identical with those obtained using other shear deformation theories.

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