

## Development of computational software for flutter reliability analysis of long span bridges

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**Abstract.** The flutter reliability analysis of long span bridges requires use of a software tool that predicts the uncertainty in a flutter response due to uncertainties in the model formulation and input parameters. Existing flutter analysis numerical codes are not capable of dealing with stochastic uncertainty in the analysis of long span bridges. The goal of the present work is to develop a software tool (FREASB) to enable designers to efficiently and accurately conduct flutter reliability analysis of long span bridges. The FREASB interfaces an open-source Matlab toolbox for structural reliability analysis (FERUM) with a typical deterministic flutter analysis code. The paper presents a brief introduction to the generalized first-order reliability method implemented in FREASB and key steps involved in coupling it with a typical deterministic flutter analysis code. A numerical example concerning flutter reliability analysis of a long span suspension bridge with a main span of 1385 m is presented to demonstrate the application and effectiveness of the methodology and the software.

**Keywords:** computer software; structural reliability; flutter; long span bridges

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### 1. Introduction

Long span bridges are vulnerable to wind actions because of their flexible structure. Flutter is one of the critical issues to be addressed in the wind-resistant design of long span bridges. The flutter analysis of long span bridges has been the subject of considerable researches (Larsen 1997, Agar 1998, Chen and Cai 2003, Cai *et al.* 1999, Chen *et al.* 2001, Hua *et al.* 2007). These studies were based on the assumption of complete determinacy of analysis parameters. This is usually referred to as deterministic analysis. In reality, however, there are uncertainties in design variables. These uncertainties include geometric properties such as cross-sectional properties and dimensions, material mechanical properties such as modulus and strength, and load magnitude and distribution, etc. As a result the deterministic analysis cannot provide complete information regarding the flutter responses of long span bridges. Therefore, the flutter behavior of long span bridges should be more rationally studied under a probabilistic viewpoint.

In order to conduct probabilistic flutter analysis of long span bridges efficiently, there is a need for developing a computer-support tool to help designers at wind-resistant design stage. However, commercial finite element programs currently used in civil engineering cannot be readily used for the probabilistic flutter analysis as they lack some capabilities like the calculation of self-excited

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forces acting on the bridge deck, the prediction of critical flutter velocity, and determination of target configurations of long span bridges under dead loads. Many existing flutter analysis numerical codes are not capable of dealing with stochastic uncertainties involved in the analysis of long span bridges. The flutter reliability software appears as a consequence of these necessities.

While reliability analysis of structures subjected to various loads has been topics of extensive research (Wong 2005, Cheng and Xiao 2005, Pourzeynali1 and Datta 2005, Dey and Mahadevan 2000), comparatively less work is reported on the development of structural reliability software. A good state of the art report on the development of structural reliability software is given in a special issue of International Journal of Structural Safety (Schuëller 2006) and Haukaas and Der Kiureghian (Haukaas 2007). There has been actually practically no work reported on the development of flutter reliability software although in recent years some studies have been conducted on the flutter reliability analysis (Ostenfeldrosenthal *et al.* 1992, Pourzeynail and Datta 2002, Cheng *et al.* 2005).

Developing a flutter reliability software is not a trivial task. The need for conducting the probabilistic analysis may lead to additional immense programming efforts. Because of not being able to interact with existing flutter analysis software, probabilistic analysis capabilities of many available software systems such as NESSUS (Thacker 2006), COSSAN (Schueller and Pradlwarter 2005), FERUM (Der Kiureghian *et al.* 2006), STRUREL (Gollwitzer *et al.* 2006), VaP (Petschacher 2007) and Proban (Tvedt 2006) are limited. In this study, as the limit state for flutter reliability analysis of a long span bridge is an implicit function of random variables as evaluated with the deterministic flutter analysis method, a numerical approach is required to evaluate the reliability index. The response surface method proposed by Cheng *et al.* (2005) can be used for this purpose. However, their algorithm may be not able to interact with available probabilistic analysis software system (i.e., FERUM). The generalized first-order reliability method (FORM) (Melchers 2002) is another possibility. Ge *et al.* (2000) have applied the FORM to study the failure probability of bridges due to flutter. However, their study focused on flutter reliability analysis of bridge with explicit limit state functions. The FORM method is extended here to analyze bridge flutter reliability adopting implicit limit state functions. It should be mentioned that the FORM method encountered some difficulties (Lee and Kwak 2006) such as numerical difficulty in finding the most probable failure point (MPFP), errors involved in the nonlinear failure surface including the possibility of multiple design points, errors caused by non-normality of variables. On the other hand, it is computationally efficient (Elhewy 2006) and has the ability to identify one or more most probable point of failure quickly (Lin and Khalessi 2006).

The purposes of this paper are to develop a software tool to enable the designer to efficiently and accurately conduct flutter reliability analysis and to demonstrate the applicability and merits of using the FORM in the flutter reliability analysis.

The remainder of this paper is structured as follows. Section 2 describes the deterministic flutter analysis method employed in the development of flutter reliability software system. Section 3 outlines the flutter reliability analysis procedures including formulation of a flutter reliability problem and a brief description of the FORM. Section 4 discusses the issues and steps involved in developing the flutter reliability software. Section 5 demonstrates features of the methodology and the developed software by carrying out flutter reliability analysis of a long span suspension bridge. Finally, conclusions are drawn in Section 6.

## 2. Deterministic flutter analysis method

Since the reliability analysis is based on tracking the uncertainty propagating through the steps of deterministic analysis, the deterministic flutter analysis method is necessary in the implementation of the flutter reliability software system. The deterministic flutter analysis method proposed in (Zhang 2000) is employed here. Only a brief description for the deterministic flutter analysis method is given here. A more detailed description may be found in (Zhang 2000).

The equations of motion with respect to the static equilibrium position of a bridge caused by self-excited aerodynamic forces can be expressed as (Zhang 2000)

$$\begin{aligned}
 [M]\{\ddot{q}(x, y)\} + [D]\{\dot{q}(x, t)\} + [K]\{q(x, t)\} &= \{F_{se}\} \\
 \{F_{se}\} &= \frac{1}{2}\rho U^2 \left( [A_s]\{q(x, t)\} + \frac{1}{U}[A_d]\{\dot{q}(x, t)\} \right)
 \end{aligned} \tag{1}$$

where  $[M]$  is the mass matrix ;  $[D]$  is the structural damping matrix;  $[K]$  is the stiffness matrix;  $\{F_{se}\}$  represents the vector of the self-excited aerodynamic forces acting on the bridge;  $\ddot{q}(x, y)$ ,  $\dot{q}(x, t)$ ,  $q(x, y)$  are the vectors of acceleration, velocity and displacement, respectively;  $[A_s]$  and  $[A_d]$  are the aerodynamic stiffness matrix and aerodynamic damping matrix, respectively;  $\rho$  is the air density; and  $U$  is the mean wind speed.

By introducing the displacement transformation  $\{q(x, t)\} = [\phi]\{\xi(t)\}$ , where  $[\phi]$  is the mode shape matrix;  $\{\xi(t)\}$  is the generalized coordinate vector, and by assuming that the generalized coordinates have a damped harmonic form, Eq.(1) can be expressed as (Lau David 2000)

$$\left[ [M]^g \left( \frac{U}{B} \right)^2 P^2 + [D]^g \left( \frac{U}{B} \right) P + [K]^g - \frac{1}{2}\rho U^2 \left( [A_s]^g + \frac{1}{B}[A_d]^g P \right) \right] \{R\} \exp\left( \frac{U}{B} P t \right) = \{0\} \tag{2}$$

$$[M]^g = [\phi]^T [M] [\phi], \quad [D]^g = [\phi]^T [D] [\phi], \quad [K]^g = [\phi]^T [K] [\phi] \tag{3}$$

$$[A_s]^g = [\phi]^T [A_s] [\phi], \quad [A_d]^g = [\phi]^T [A_d] [\phi] \tag{4}$$

where  $[M]^g$ ,  $[D]^g$ ,  $[K]^g$  are the generalized mass, damping and stiffness matrices, respectively;  $\{R\}$  is the response amplitude vector;  $B$  is the bridge deck width;  $[A_s]^g$ ,  $[A_d]^g$  are the generalized aerodynamic stiffness matrix and aerodynamic damping matrix, respectively;  $P = K(\delta + i)$ ;  $K$  is the reduced frequency;  $\delta$  is the logarithmic decrement; and  $i = \sqrt{-1}$ .

To obtain a nontrivial solution for Eq. (2), the determinant of the expression in the square bracket must be zero (Lau David 2000). The PK-F method (Namini 1992) in conjunction with an iteration scheme is employed for the solution of the flutter problem of bridges.

## 3. Flutter reliability analysis

The flutter reliability analysis involves formulating the flutter reliability for a long span bridge and solving the resulting flutter reliability problem by the FORM. In the following sections, two issues are briefly discussed.

### 3.1. Flutter reliability problem formulation

The flutter reliability problem considered in this research is of estimation of the probability of extreme wind velocity,  $V_e$ , exceeding the critical flutter velocity,  $V_{cr}$ . The flutter reliability problem is formulated as the multiple integral of the joint probability density function  $f(X)$  over the failure domain

$$P_f = \int_{g \leq 0} f(X) dX \quad (5)$$

where  $p_f$  is the failure probability of interest;  $\{X\}$  is the vector of uncertainty parameters;  $g = V_{cr} - V_e$  is referred to as the limit state function. For a long span bridge, the critical flutter velocity has to be calculated by a deterministic flutter analysis. This brings another complex problem to the flutter reliability analysis because the limit state function  $g$  in Eq. (5) is not available as an explicit, closed-form function of input random variables.

### 3.2. Solution of the flutter reliability problem

A number of computational approaches for solving Eq. (5) can be found in the literature such as Monte Carlo simulation, response surface methods and the first-order reliability methods. However, our focus is on the application of the FORM since recent studies have shown that it is computationally efficient (Elhewy 2006) and has the ability to identify one or more most probable point of failure quickly (Lin and Khalessi 2006). In the FORM, the limit state  $g$  is linearized and nonnormal distributions are converted to equivalent normal distributions at the most probable point of failure, which can be found through the improved HLRF algorithm developed by Zhang and Der Kiureghian (1997) (the original HLRF algorithm as developed by Hasofer and Lind (1974) and extended by Rackwitz and Fiessler (1978) for nonnormal random variables). This procedure requires repeated computations of the limit-state function and its gradient. However, evaluation of derivatives of the implicit limit state function in Eq.(5) is not an easy task. For the sake of simplicity, the derivatives of the limit state function are obtained by integrating the finite difference method and deterministic flutter analysis method in this study. More detailed information concerning the FORM for reliability analysis is given in (Melchers 2002)

## 4. Development of flutter reliability software

Flutter reliability problems have arisen in wind-resistant design of long span bridges, which demand developments of new analysis tools. The deterministic flutter analysis method described previously has gained its popularity for solving deterministic flutter problems in the past, while the FORM is being widely employed in structural reliability analysis. Even though both methods are well-documented in the published literature, but their analyses and computational procedures are different. Combining both methods in single software normally creates programming difficulty, mainly from different backgrounds of programmers who have expertise in different fields. In addition, software maintenance including the implementation of new developments can also pose additional difficulties.

On the other hand, flutter analysis numerical codes have been developed for deterministic flutter analysis of long span bridges (Zhang 2000). Actually, several free structural reliability analysis software systems are available such as FERUM and OpenSees (Der Kiureghian *et al.* 2006). It is usually advantageous to interface the flutter analysis code with the structural reliability analysis software system, in order to develop flutter reliability software with minimal or no intrusive modifications. Thus an appropriate intervening interface is essential to facilitate the efficient transfer of data and information between various numerical codes.

Matlab is well suited for building the interface. It has the advantage of not having to compile an algorithm before executing it (scripting languages compile at running time). Also, it allows users to control the execution and inputs/outputs from a number of various numerical codes. Due to these advantages, Matlab has numerous applications in computational engineering community. Recently, Matlab has been applied by Der Kiureghian and Haukaas (2007) to develop structural reliability code termed as FERUM. This study proposes a procedure to combine FERUM with existing deterministic flutter analysis code, all under the Matlab computing environment.

FREASB is a prototype Matlab interface software developed for analyzing flutter reliability problems by integrating the deterministic flutter analysis method with the FORM. FREASB interfaces the FERUM with the deterministic flutter analysis code (*NFACSB*) developed by Zhang (2000). This software will automatically update the *NFACSB* input file, execute the *NFACSB*, and extract the necessary data from its output file, and deliver it to FERUM for performing the required reliability analysis. The *NFACSB* input file is updated through a series of dynamic data exchanges using the data obtained from the FERUM. An interface file named *user\_lsf.m* is provided to specify the limit-state function in Eq.(5). The critical flutter velocity value in the limit-state function is extracted from the *NFACSB* output file. A system command in Matlab is issued to execute the *NFACSB* so that the user can easily conduct the flutter reliability analysis without necessarily having to understand the *NFACSB* and access to the source code of *NFACSB*. The capacity of the software is very powerful, as it provides additional flexibility and functions. It should be noted that in addition to the FORM described earlier, the software incorporates other reliability methods such as the second-order reliability method (SORM), importance sampling (IS) and direct Monte Carlo Simulation (MCS). The detailed descriptions of the other methods can be found in the referenced publications (Der Kiureghian *et al.* 1987, Hohenbichler and Rackwitz 1988). In the subsequent study, emphasis is placed on using the FORM for solving the flutter reliability problem of long span bridges since it is computationally efficient and has the ability to quickly identify one or more most probable point of failure at the same time.

The program flow of the FREASB software is shown in Fig. 1. As an example, the architecture of the FORM based flutter reliability analysis method in the FREASB is presented in Fig. 2.

## 5. Numerical example

In this section, a numerical example that illustrates the application of the FREASB to flutter reliability problem is presented. The Jiang Yin Suspension Bridges, which is one of the longest suspension bridges in China, is chosen as the numerical example, for which the structural parameters were taken from the literature (Xiang 1987). Of course, other types of long span bridges (i.e., cable-stayed bridges) can be used for this purpose. In the FREASB, we have chosen the *NFACSB* as the deterministic flutter analysis code for the case study. However, the approach for

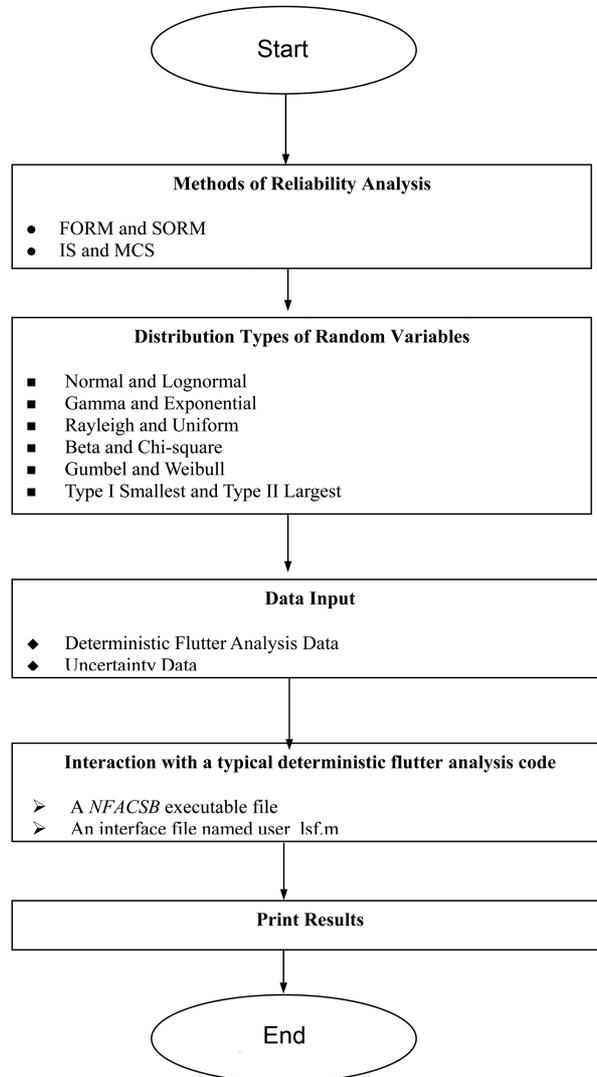


Fig. 1 Flow chart of the FREASB software

developing the FREASB is quite general and can be used in conjunction with any existing flutter analysis code of long span bridges.

### 5.1 Description of the Jiang Yin Bridge

The Jiang Yin Bridge span arrangements are (336 + 1385 + 309) m. The elevation view of the bridge is shown in Fig. 3. The deck cross section is an aerodynamically shaped closed box steel girder with 36.9 m wide and 3.0 m high (Fig. 4.). The distance between the two cables is 32.5 m; the hanger spacing is 16.0 m; section material and geometrical features of the main members are indicated in Table 1. For more details of the bridge, readers are referred to (Xiang 1987, Cheng *et al.* 2002).

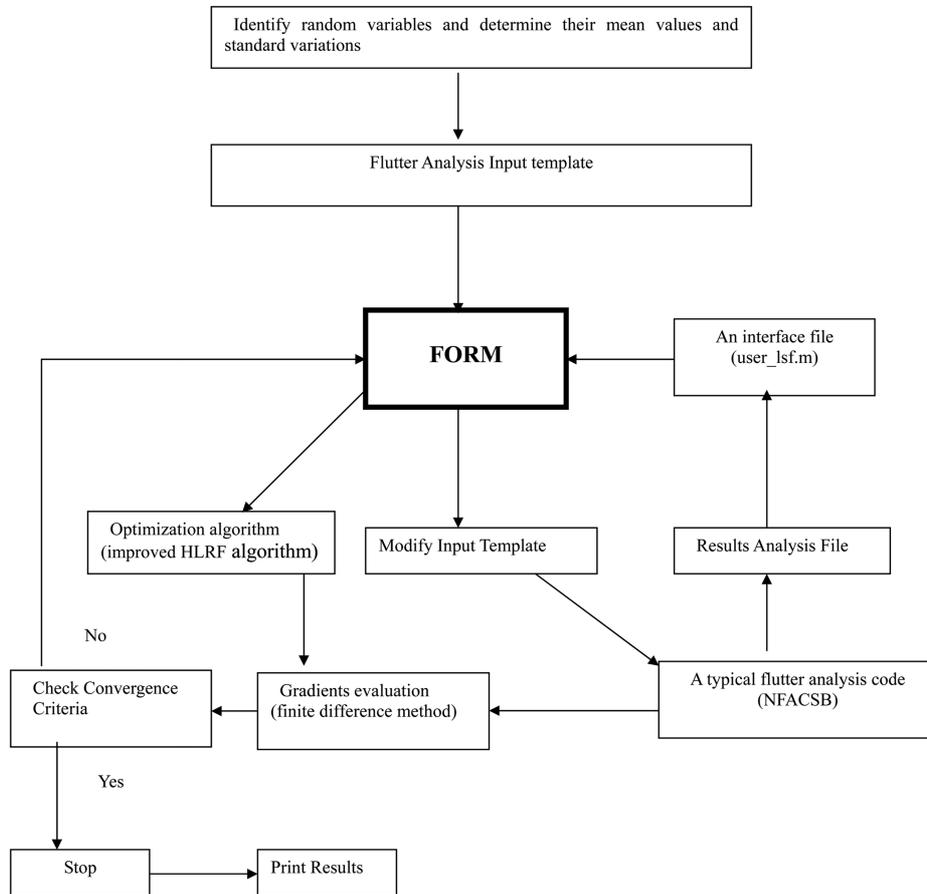


Fig. 2 Architecture of the FORM based flutter reliability analysis method in FREASB

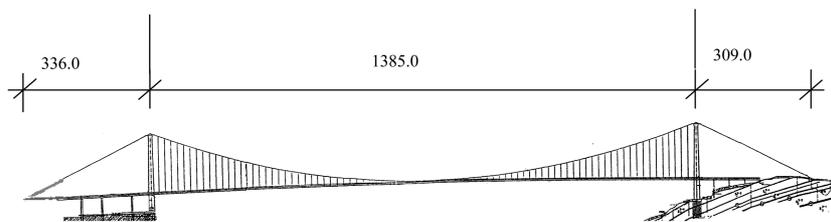


Fig. 3 Elevation of Jiang Yin Bridge (Unit: m)

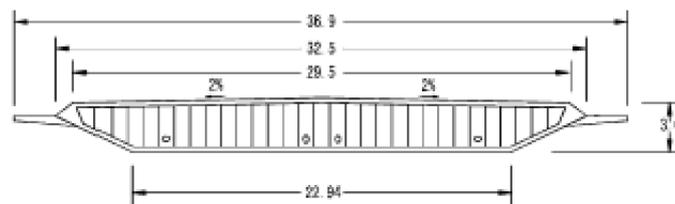


Fig. 4 Cross section of deck of Jiang Yin Bridge (Unit: m)

Table 1 Section geometrical and material feature of the main member

Substructures	$J_d$ (m <sup>4</sup> )	$I_2$ (m <sup>4</sup> )	$I_3$ (m <sup>4</sup> )	$E$ (MPa)	$\gamma$
Steel box girder	4.8218	93.3181	1.844	210000.0	0.3
Cable	-	-	-	200000.0	-
Hanger	-	-	-	140000.0	-

(Notes:  $E$  = modulus of elasticity;  $J_d$  = St. Venant constant;  $I_2$  = Out-of-plane moments of inertia;  $I_3$  = In-plane moments of inertia;  $\gamma$  = Poisson ratio )

### 5.2 Finite element modeling

A three-dimensional finite element model has been established for the Jiang Yin Bridge (Cheng *et al.* 2002). Three-dimensional beam elements were used to model the two bridge towers. The cables and suspenders were modeled by three-dimensional truss element accounting for geometric nonlinearity due to the cable sag. The bridge deck is represented by a single beam and the cross-section properties of the bridge deck are assigned to the beam as equivalent properties. In the bridge model, the girder was assumed to be pinned at the ends, i.e., only rotations were allowed. The main cables were assumed to be fixed at the pylon tops and the pylons were rigidly fixed to the piers. The structural damping ratio is assumed to be 0.005. The flutter derivatives were obtained from wind-tunnel experiments of the sectional model (Xiang 1987). For the sake of simplicity, only the flutter derivatives at the wind attack angle of 0 degree are considered in the following analyses.

### 5.3 Uncertainties in the analysis of the Jiang Yin Bridge

As is well known, the system uncertainties such as material properties, geometric parameters of the bridge may fluctuate in the vicinity of the nominal values, caused during the process of structural element manufacturing and erection of the bridge. Therefore, system parameters should be treated as random rather than deterministic variables. Besides, the structural damping ratio, flutter derivatives and extreme wind velocity at the bridge site are also random variables due to their natures and/or insufficient measured data, experimental error and wind characteristics. Table 1 shows the statistics of these random variables. As the objective of this study is to develop a software tool to enable designers to conduct flutter reliability analysis of long span bridges efficiently and accurately, all random parameters in the present analysis are based on arbitrary but typical values. On the other hand, since the determination of the interrelation of the random parameters is a difficult task, using the independence assumption can greatly simplify the reliability assessment. Therefore, all random parameters considered in the paper are treated as stochastically independent from each other.

### 5.4 Reliability evaluation and limit state function

Reliability is always estimated corresponding to a performance function or limit state function. In this study, the limit state function can be represented as (Cheng *et al.* 2005)

$$g(X) = X_{14} \cdot V_{cr}(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{13}) - X_{11} \cdot X_{12} \quad (6)$$

Table 2 Statistics of the random variables for Jing Yin Bridge

Random variables	Substructures	Distribution types	Mean value	Standard deviation	Sources		
Elastic modulus	X <sub>1</sub>	Girder	Normal	2.1×10 <sup>11</sup> N/m <sup>2</sup>	2.1×10 <sup>10</sup> N/m <sup>2</sup>	Assumed	
	X <sub>2</sub>	Cables	Normal	2.0×10 <sup>11</sup> N/m <sup>2</sup>	2.0×10 <sup>10</sup> N/m <sup>2</sup>		
	X <sub>3</sub>	Girder	Normal	1.100842 m <sup>2</sup>	0.1100842 m <sup>2</sup>		
Cross sectional areas	X <sub>4</sub>	Cables (main cables)	Normal	0.4825 m <sup>2</sup>	0.04825 m <sup>2</sup>		
	X <sub>5</sub>	Girder	Normal	1.844 m <sup>4</sup>	0.1844 m <sup>4</sup>		
Sectional moments of inertia	X <sub>6</sub>	Girder	Normal	93.3181 m <sup>4</sup>	9.3181 m <sup>4</sup>		
	X <sub>7</sub>	Girder	Normal	4.8218 m <sup>4</sup>	0.42818 m <sup>4</sup>		
	X <sub>8</sub>	Girder	Normal	16351.1 kg/m <sup>3</sup>	1635.11 kg/m <sup>3</sup>		
Mass density	X <sub>9</sub>	Cables	Normal	8236.9 kg/m <sup>3</sup>	823.69 kg/m <sup>3</sup>		
	X <sub>10</sub>	Girder	Normal	1.4257 × 10 <sup>6</sup> kg·m <sup>2</sup> /m	1.4257 × 10 <sup>5</sup> kg·m <sup>2</sup> /m		
Basic wind velocity	X <sub>11</sub>		Type I	11.86 m/s	2.85 m/s		Cheng <i>et al.</i> (2005)
				17.60 m/s	3.18 m/s		
Gust speed factor	X <sub>12</sub>		Normal	1.32	0.09		Pourzeynail and Datta (2002)
Modal damping ratio flutter derivatives parameter	X <sub>13</sub>		Lognormal	0.005	0.002		
			Lognormal	1.0	0.2		

(Note: X<sub>5</sub> is in-plane moment of inertia; X<sub>6</sub> is out-of-plane moment of inertia; X<sub>7</sub> is torsional moment of inertia)

The definitions of variables X<sub>i</sub> are given in Table 2. As described by Cheng *et al.* (2005), the flutter reliability of the bridge is dependent on the incident wind directionality. Two cases with different basic wind velocity X<sub>11</sub> are considered herein. The values of X<sub>11</sub> are: for Case I (considering wind directionality)- mean value ( $\mu = 11.86$ ) and standard deviation ( $\sigma = 2.85$ ) and for Case II (without considering wind directionality)-mean value ( $\mu = 17.60$ ) and standard deviation ( $\sigma = 3.18$ ).

### 5.5 Results and discussion

Using the FREASB, the flutter reliability is estimated for all the two cases using the FORM method described in Section 3.2. The results are tabulated in Table 2 and compared with the available reliability indices (Cheng *et al.* 2005).

The present reliability indices in Table 3 are found to agree well with the exact results. From Fig. 5, one can see that the FORM converges quickly, since with less than 10 numbers of iterations, the computed limit state function is less than the convergence value specified by the user (i.e.,  $1 \times 10^{-6}$ ). The results demonstrate that the FORM method is accurate and efficient in estimating the flutter reliability of long span bridges, broadening the application potential of the FORM method.

As mentioned earlier in Section 4, deterministic flutter analysis code can be readily linked to the open-source structural reliability analysis code (FERUM) through the user\_lsf.m file in the FREASB. This integration provides the ease in extending a modern flutter analysis code with

Table 3 Comparison of results for various cases

	Case I		Case II	
	FORM	Cheng <i>et al.</i> (2005)	FORM	Cheng <i>et al.</i> (2005)
Reliability index	4.327	4.371	3.595	3.633

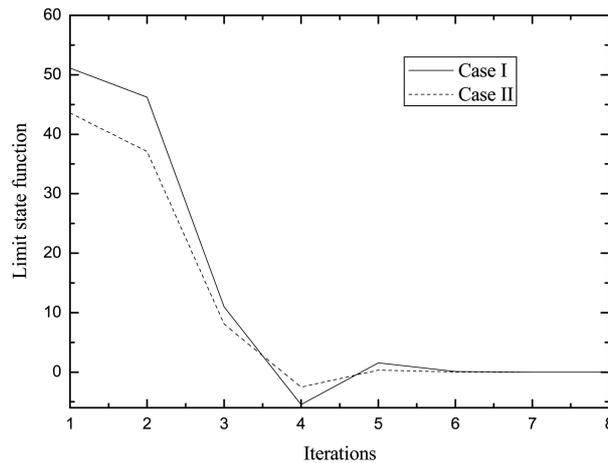


Fig. 5 Convergence histories of the FORM method for various cases

reliability analysis function so as to have the capability to conduct flutter reliability analysis and result in significant savings in terms of time and money. The presented hybrid approach can provide a tradeoff for the lengthy time spent creating and including flutter analysis code in the reliability software. Also, the approach removes the need to access and modify the source code of the deterministic flutter analysis code, which is often not available.

## 6. Conclusions

This study developed a software tool (FREASB) to enable designers to conduct flutter reliability analysis of long span bridges efficiently and accurately. The FREASB interfaces the FERUM with the NFACSB and has a number of attractive features: (1) users familiar with deterministic flutter analysis method and possessing a basic understanding of probability theory will be able to use the FREASB; (2) no access to the source code of the NFACSB is required to achieve a clear-cut separation in functionality and responsibility; and (3) future extensions of the software, for example to fatigue reliability problems of long span bridges due to the gustiness of wind action and safety problem for buffeting of long-span bridges, can be readily implemented.

In this paper strategies for interfacing the generalized first-order reliability method (FORM) with deterministic flutter reliability analysis numerical codes were presented and realized. A brief introduction to the FORM was outlined and the issues involved while using it in conjunction with deterministic flutter reliability analysis software were addressed. The usefulness and capabilities of such a coupling with the help of a numerical example were demonstrated. The numerical example also demonstrated that the FORM algorithm can be used to estimate the flutter reliability of long span bridges, broadening the application potential of the FORM method. The coupling presented

allows the user to link other reliability algorithms such as the second-order reliability method and simulation-based methods and ‘in-house’ stand-alone deterministic flutter analysis numerical codes to estimate flutter reliability of long span bridges.

Future development of the FREASB system is required to include exploitation of parallelization opportunities and a user-friendly graphical system for pre- and post-processing (AUI). These capabilities make the FREASB system a powerful tool for design and analysis of bridge structures.

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## Notation

*The following symbols are used in this paper:*

[M] = mass matrix;

[D] = structural damping matrix;

[K] = stiffness matrix;

$\{F_{se}\}$  = vector of the self-excited aerodynamic forces acting on the bridge;

$\ddot{q}(x, t), \dot{q}(x, t), q(x, t)$  = vectors of acceleration, velocity and displacement, respectively;

$[A_s]$  and  $[A_d]$  = aerodynamic stiffness matrix and aerodynamic damping matrix, respectively;

$\rho$  = air density;

U = mean wind speed;

$[M]^g, [D]^g, [K]^g$  = generalized mass, damping and stiffness matrices, respectively;

$\{R\}$  = response amplitude vector;

B = bridge deck width;

$[A_s]^g, [A_d]^g$  = generalized aerodynamic stiffness matrix and aerodynamic damping matrix, respectively;

$P-K(\delta + i)$ ;

K = reduced frequency;

$\delta$  = logarithmic decrement;

$i = \sqrt{-1}$ ;

$p_f$  = failure probability of interest;

$\{X\}$  = vector of uncertainty parameters; and

g = limit state function.

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