

Effects of frequency ratio on bridge aerodynamics determined by free-decay sectional model tests

X.R. Qin^{1,2*}, K.C.S. Kwok^{1,3}, C.H. Fok¹ and P.A. Hitchcock¹

¹CLP Power Wind/Wave Tunnel Facility, HKUST, H.K. S.A.R. P.R. China

²Institute of Mechanical Design and Its Theory, School of Mechanical Engineering, Tongji University, Shanghai, P.R. China

³School of Engineering, University of Western Sydney, Australia

(Received March 14, 2008, Accepted May 12, 2009)

Abstract. A series of wind tunnel free-decay sectional model dynamic tests were conducted to examine the effects of torsional-to-vertical natural frequency ratio of 2DOF bridge dynamic systems on the aerodynamic and dynamic properties of bridge decks. The natural frequency ratios tested were around 2.2:1 and 1.2:1 respectively, with the fundamental vertical natural frequency of the system held constant for all the tests. Three 2.9 m long twin-deck bridge sectional models, with a zero, 16% (intermediate gap) and 35% (large gap) gap-to-width ratio, respectively, were tested to determine whether the effects of frequency ratio are dependent on bridge deck cross-section shapes. The results of wind tunnel tests suggest that for the model with a zero gap-width, a model to approximate a thin flat plate, the flutter derivatives, and consequently the aerodynamic forces, are relatively independent of the torsional-to-vertical frequency ratio for a relatively large range of reduced wind velocities, while for the models with an intermediate gap-width (around 16%) and a large gap-width (around 35%), some of the flutter derivatives, and therefore the aerodynamic forces, are evidently dependent on the frequency ratio for most of the tested reduced velocities. A comparison of the modal damping ratios also suggests that the torsional damping ratio is much more sensitive to the frequency ratio, especially for the two models with nonzero gap (16% and 35% gap-width). The test results clearly show that the effects of the frequency ratio on the flutter derivatives and the aerodynamic forces were dependent on the aerodynamic cross-section shape of the bridge deck.

Keywords: wind-induced vibration; flutter derivative; system identification; frequency ratio; wind tunnel test.

1. Introduction

Flutter derivatives are aerodynamic parameters commonly used to define the flutter properties of bridge decks, and can be identified by wind tunnel sectional model vibration tests, chiefly classified into forced-vibration test and free-vibration test. For the forced vibration test, the vibration is induced by harmonic excitation produced by custom-designed test device (e.g. Diana, *et al.* 2004), while for the free-vibration test, the vibration is induced by initial conditions and no complex

* Corresponding Author, E-mail: xianrongqin@hotmail.com

excitation device is required. Therefore, compared with the forced vibration test, the free-vibration test is more frequently utilized, even if it requires complicated system identification algorithms, demands more software work, and often encounters big difficulties.

One of the biggest difficulties of determining flutter derivatives of bridge decks by the free-vibration methodology is that the response, especially the vertical one, decays so quickly that inadequate free-decay response can be recorded for the identification process. In some studies (e.g. Gu, *et al.* 2001), applying a smaller frequency ratio, short for "the natural torsional-to-vertical frequency ratio of the 2DOF model-rig bridge deck system" in this paper, was regarded as one of the remedies for this problem, since it was often assumed that, at certain reduced velocities and for some sectional shapes, flutter derivatives are not sensitive to structural dynamic parameters of the bridge dynamic system, including the frequency ratio. The objective of this paper is to examine whether such a remedy is applicable for decks with different cross-section shapes, since the model tested in the aforementioned study (Gu, *et al.* 2001) was the sectional model of a particular bridge: Jiangyin Bridge, and there is reported result that for some cross-sectional shapes (e.g. a rectangular model with a width-to-depth ratio of 10, according to Matsumoto, *et al.* 1993), vibration mode, i.e. SDOF (single-degree-of-freedom) or 2DOF (two-degrees-of-freedom) vibrations, shows a remarkable influence on flutter derivatives.

Three twin-deck bridge sectional models, with a zero, 16% and 35% gap-to-width ratio, respectively, were tested and for each model, the frequency ratio was varied to be around 2.2:1 and 1.2:1, with the fundamental vertical natural frequency held constant at 1.72 Hz for all the tests. The Eigensystem Realization Algorithm (ERA, Juang and Pappa 1985) was utilized to simultaneously determine the eight flutter derivatives of the models at different wind speeds, as well as the corresponding modal parameters.

2. Basic formulations

At a specific wind speed, the motion of a bridge deck with two-degrees-of-freedom (2DOF), i.e. vertical bending h (m) and torsion α (rad), in smooth flow can be modelled as:

$$\mathbf{M}\{\ddot{\mathbf{y}}(t)\} + \mathbf{C}^0\{\dot{\mathbf{y}}(t)\} + \mathbf{K}^0\{\mathbf{y}(t)\} = \{\mathbf{f}_{se}(t)\} \quad (1)$$

in which: \mathbf{M} , \mathbf{C}^0 and \mathbf{K}^0 are the structural mass, damping and stiffness matrices of the 2DOF dynamic bridge system per unit span; $\{\mathbf{y}\} = [h \ \alpha]^T$ is the response vector; and $\{\mathbf{f}_{se}\} = [L_{se} \ M_{se}]^T$ is the aerodynamic force vector, where L_{se} and M_{se} are the self-excited lift force (N/m) and pitching moment (N·m/m) respectively, per unit span of bridge, as given by Sarkar (1994):

$$\begin{aligned} L_{se}(t) &= \rho U^2 B \left[K_h H_1^*(K_h) \frac{\dot{h}(t)}{U} + K_\alpha H_2^*(K_\alpha) \frac{B \dot{\alpha}(t)}{U} + K_\alpha^2 H_3^*(K_\alpha) \alpha(t) + K_h^2 H_4^*(K_h) \frac{h(t)}{B} \right] \\ M_{se}(t) &= \rho U^2 B^2 \left[K_h A_1^*(K_h) \frac{\dot{h}(t)}{U} + K_\alpha A_2^*(K_\alpha) \frac{B \dot{\alpha}(t)}{U} + K_\alpha^2 A_3^*(K_\alpha) \alpha(t) + K_h^2 A_4^*(K_h) \frac{h(t)}{B} \right] \end{aligned} \quad (2)$$

in which ρ is air density (kg/m^3); U is mean wind speed (m/s); B is bridge deck width (m); K_h and K_α are the dimensionless reduced frequencies for vertical bending and torsion respectively, and are

defined as $K_h = \omega_h B/U$ and $K_\alpha = \omega_\alpha B/U$; where ω_h and ω_α are the circular vibration frequencies (rad/s) at wind speed U , and $\omega_h = 2\pi f_h$, $\omega_\alpha = 2\pi f_\alpha$, in which f_h and f_α are the vibration frequencies (Hz); H_i^* and A_i^* ($i = 1,2,3,4$) are the dimensionless flutter derivatives. As shown in Eq. (2), H_1^* , H_4^* , A_2^* and A_3^* are only associated with motion of the model in one direction, and are therefore named “direct flutter derivatives”, whereas H_2^* , H_3^* , A_1^* and A_4^* are named “cross flutter derivatives”.

The natural frequencies and damping ratios, the modal frequencies and damping ratios at a specific wind speed, as well as the corresponding flutter derivatives, can be determined by system identification techniques, e.g. Sarkar (1994), Qin and Gu (2004) and Qin, *et al.* (2007), from the measured dynamic responses of the bridge system at zero and non-zero wind speeds.

It is also possible for Eq. (1) to be expressed by the following state equations:

$$\begin{aligned} \{\dot{x}(t)\} &= \mathbf{A}\{x(t)\} \\ \{y(t)\} &= \mathbf{C}\{x(t)\} \end{aligned} \tag{3}$$

in which $\{x\} = [y \ \dot{y}]^T$ is the state vector; \mathbf{A} and \mathbf{C} are the state and output matrices, respectively. The eigen-values (λ_r) of the state matrix are named the poles of the system, and associated with the modal parameters by Eq. (4):

$$\lambda_r = -\xi_r \omega_r + j\omega_r \sqrt{1 - \xi_r^2} \tag{4}$$

in which ξ_r and ω_r are the modal damping ratio and modal circular frequency (rad/s) of the r^{th} mode. The mode shape, another important modal parameter, is directly related to the eigen-vector of the state matrix. From this point of view, changing the torsional-to-vertical frequency ratio will subsequently change the eigen-structure of the system, which will affect the dynamic properties of a general dynamic system and it will also be possible to influence the interaction between a bridge deck dynamic system and the natural wind.

In the frequency-domain, the responses of the 2DOF linear time-invariant dynamic bridge system to an excitation $\{F(\omega)\}$ can be defined by Eq. (5).

$$\{Y(\omega)\} = \mathbf{G}(\omega)\{F(\omega)\} \tag{5}$$

in which, $\mathbf{G}(\omega)$ is the Frequency Response Function (FRF) matrix, modelling the transfer properties of the system and can be expressed as:

$$\mathbf{G}(\omega) = [-\omega^2 \mathbf{M} + j\omega \mathbf{C}^U + \mathbf{K}^U]^{-1} \tag{6}$$

in which \mathbf{C}^U and \mathbf{K}^U are the damping and stiffness matrices at a specific wind speed U , i.e. \mathbf{C}^0 and \mathbf{K}^0 aerodynamically modified by the self-excited forces. FRF can provide insight to the coupling effects, as illustrated in Fig. 1, and generally, the amplitude of a FRF will show a peak at each modal frequency, which means that the excitation will be dramatically magnified at those frequencies. Therefore, changing the torsional-to-vertical frequency ratio will change the transfer property of the bridge dynamic system, and subsequently affect the energy distribution of the responses, which is likely to affect the self-excited forces induced by the motion of the dynamic system.

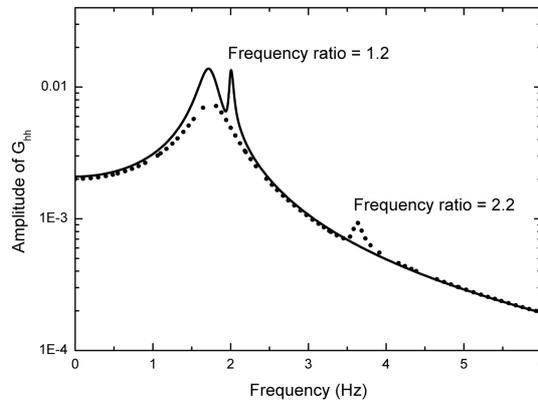


Fig. 1 Typical Frequency Response Function (FRF) of the 2DOF model-rig bridge dynamic system

3. Wind tunnel tests

3.1. Model, tests and parameter identification procedure

To examine the effects of torsional-to-vertical natural frequency ratio on the modal damping parameters, and subsequently the flutter derivatives of the 2DOF bridge deck system, a series of wind tunnel sectional model dynamic tests were conducted in the high-speed section (2 m high and 3 m wide) of the CLP Power Wind/Wave Facility at The Hong Kong University of Science and Technology.

During the wind tunnel tests, the tested model was elastically supported inside the wind tunnel by custom-designed support rigs and springs, as shown in Fig. 2, to reproduce a bridge dynamic system with two degrees-of-freedom (2DOF model-rig bridge dynamic system), i.e. vertical and torsional vibrations. For the model-rig bridge dynamic system, the elastic deformation of the bridge deck model was neglected, or in other words, the model was assumed to be a rigid-body, and only

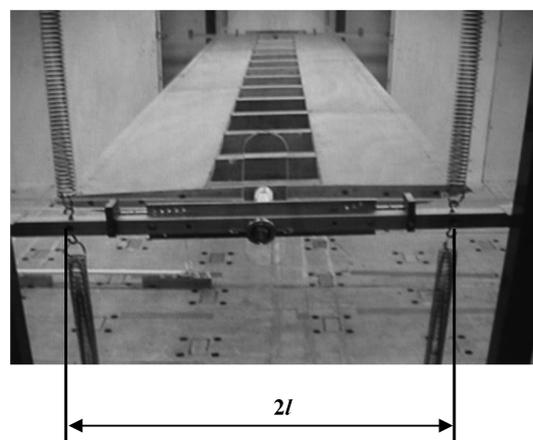


Fig. 2 Bridge sectional model suspended by a custom-designed rig-spring mechanism

provided the bridge dynamic system with mass.

Three 2.9 m long bridge sectional models, with cross section shapes shown in Fig. 3, were tested, the first one is a model to simulate the cross-section of a twin-deck bridge, with a gap-width (b) to total chord (B) ratio, defined in Fig. 3, of about 16%, the second one is a model of a twin-deck bridge with a gap-width to total chord ratio of about 35%, and the third one is a model with a zero gap-width to approximate a thin flat plate. All the three bridge sectional models have identical chord shapes and dimensions (0.244 m wide for a single deck and 0.044 m deep at model scale), and a similar fundamental natural frequency of the deck of around 10 Hz, which is considered sufficiently high to meet the above-mentioned rigid-body assumption.

For each test model, two cases were tested with the frequency ratio of the model-rig system adjusted to be around 2.2:1 and 1.2:1, respectively, with the mass, mass moment of inertia and the fundamental vertical natural frequency of the system held constant, as summarized in Table 1. The two different frequency ratios of the model-rig system were obtained by adjusting the space between the springs, i.e. the dimension $2l$ shown in Fig. 2, to adjust the torsional natural frequency of the system and the entire test rig was carefully calibrated before each test.

The models were tested at zero degree angle of wind incidence and in a nominally smooth flow. Model tests were conducted for the 2.2:1 frequency ratio configuration at model scale wind speeds ranging from 1 m/s up to the equivalent of a typical design wind speed for the prototype scale twin-deck bridge (10 m/s at model scale) at 1 m/s intervals. For the 1.2:1 frequency ratio, the tests were stopped when significant flutter behavior was observed, which generally occurred at wind speeds lower than the aforementioned design wind speed.

To excite the model, a strong string was attached to the downstream edge of the bridge model to give it an initial displacement (approximately 0.04 m for vertical displacement for all the tests). Four laser sensors, s_1 , s_2 , s_3 and s_4 , as shown in Fig. 4, were set up at both ends of the bridge model to record the displacement responses which were subsequently used to extract the corresponding vertical and torsional free decay responses of the model. The Eigensystem Realization Algorithm

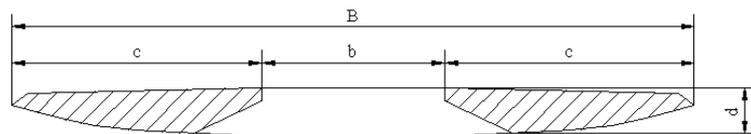


Fig. 3 Cross section shape of the bridge sectional models

Table 1 Parameters of the six test cases (three models and two frequency ratios for each model)

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Gap-width (b/B in Figs. 3 and 4) of the model	0	0	16%	16%	35%	35%
Mass (kg/m) of model-rig system	3.95	3.95	3.95	3.95	3.95	3.95
Mass moment of inertia ($\text{kg}\cdot\text{m}^2/\text{m}$) of model-rig system	0.195	0.195	0.195	0.195	0.195	0.195
Moment coefficient of the model	-0.004	-0.004	0.090	0.090	0.117	0.117
Vertical frequency (Hz)	1.72	1.72	1.72	1.72	1.72	1.72
Torsion-bending frequency ratio	1.2	2.2	1.2	2.2	1.2	2.2

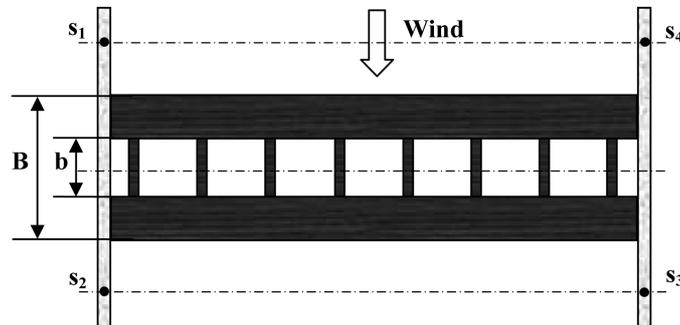


Fig. 4 Top view of model-rig system and the layout of laser sensors (s_1 , s_2 , s_3 and s_4)

(ERA, Juang and Pappa 1985) was utilized to simultaneously determine the eight flutter derivatives of the models at different wind speeds, as well as the corresponding modal parameters. The wind tunnel test and system identification methodologies have been verified and benchmarked (Qin, *et al.* 2007) by comparing the flutter derivatives of the current study and the previously published results, and verified further by comparing the measured and predicted free-decay responses.

3.2. Flutter derivatives of configurations with different frequency ratios

Figs. 5 to 7 present the comparison of the flutter derivatives for the 2DOF model-rig bridge dynamic system with different frequency ratios.

For the test case of 1.2 frequency ratio for the model with 35% gap-width (Case 5 in Table 1), the vertical free-decay response almost vanished when the test wind speed was greater than 8 m/s. Consequently the identified vertical damping ratio, and subsequently the identified flutter derivatives, contains considerable and unacceptable random uncertainties, and hence the corresponding results were discarded and not presented in Fig. 7.

The flutter derivatives were normalized by the actual vibration frequencies at a specific wind speed. Those associated with vertical motion (H_1^* , A_1^* , H_4^* and A_4^*) were normalized by the vertical frequency, while the flutter derivatives associated with torsion (H_2^* , A_2^* , H_3^* and A_3^*) were normalized by the torsional frequency.

Therefore, for the 2.2:1 frequency ratio configurations, the maximum reduced velocity relates to H_2^* , A_2^* , H_3^* and A_3^* is much smaller than that of the corresponding 1.2:1 frequency ratio configuration. For the zero gap width configuration, the total width of the bridge is much smaller than that of the nonzero gap width configurations, hence, flutter of that model configuration occurred at a much lower wind speed and the U/fB , where B is the total width of the bridge deck and is different for different test models, of 1.2:1 frequency ratio for that configuration is only up to around 6 in Fig. 5. Since H_4^* and A_4^* are generally regarded as less important, their comparison is not presented in this paper.

It is evident from Figs. 5 to 7 that the effects of the torsional-to-vertical frequency ratio of the 2DOF model-rig dynamic bridge system depend on the cross-sectional shape of bridge decks.

Fig. 5 suggests that for the model with a zero gap-width, a model to simulate a thin flat plate, all the six flutter derivatives, and subsequently the self-excited lift force and pitching moment, are not sensitive to the frequency ratio.

For the model with a 35% gap-width, at different frequency ratios, the two direct flutter

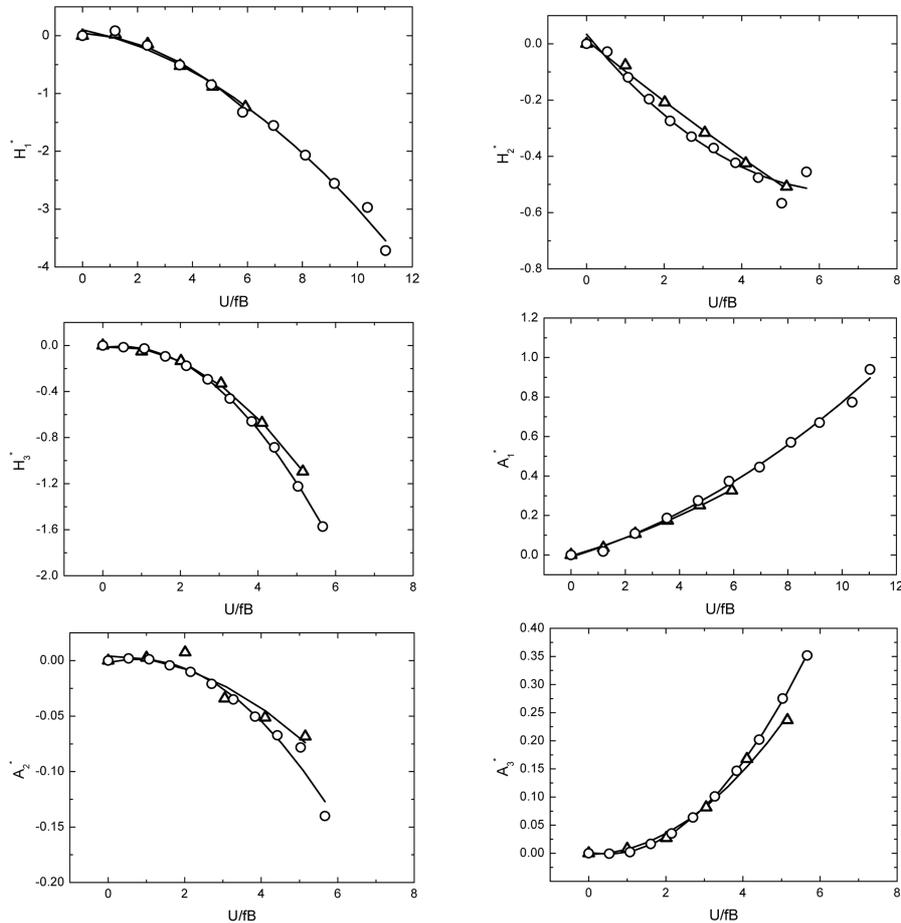


Fig. 5 Flutter derivatives of model with a zero gap-width
 Δ : frequency ratio =1.2 \circ : frequency ratio =2.2

derivatives (A_2^* and A_3^*) associated with the torsion of the model are quite similar for a large range of reduced velocity. However, for the model with a 16% gap-width ratio, when the frequency ratio is changed from 2.2 to 1.2, the two flutter derivatives are quite different for the majority of the test reduced velocities. It was also noticed that for both models with nonzero gap-width, the two cross flutter derivatives (H_2^* and A_1^*) are influenced by the frequency ratio. As the aerodynamic interference between the upstream and downstream chords is different for different gap-widths, and such interference is different in different directions, it is expected that the effects of frequency ratio on each flutter derivative of the models with 16% and 35% gap-width are also different. Anyway, it could be concluded from the above observations and Eq. (2) that the self-excited lift force and pitching moment of such models with 16% and 35% gap-widths will be affected by the frequency ratio.

Since the coupling effect between the vertical bending and torsion vibrations will be lowered with the increase of torsional-to-vertical frequency ratio, it can be deduced that when the frequency ratio approaches infinity, the coupling between the vibrations will diminish and the 2DOF vibration will “decompose to” two SDOF vibrations. It is noteworthy that the effects of frequency ratio on the

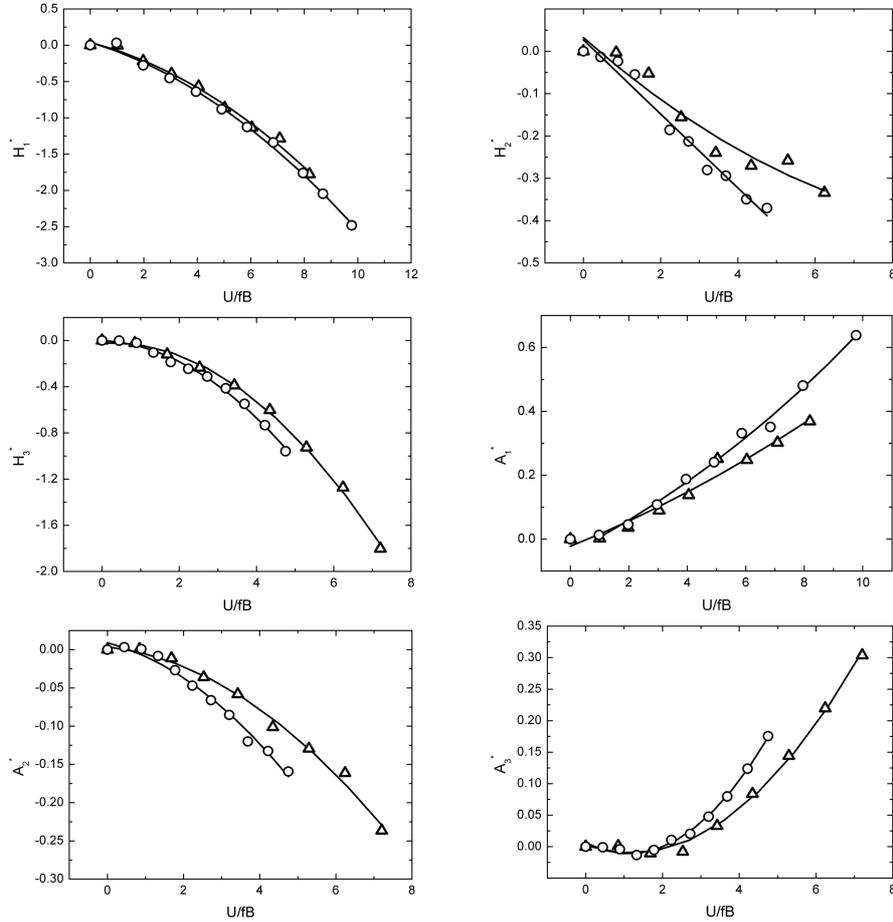


Fig. 6 Flutter derivatives of model with 16% gap-width
 Δ : frequency ratio =1.2 o: frequency ratio =2.2

aerodynamic forces and the flutter derivatives are dependent on cross-section shape (in the current tests different gap widths) and this is in general agreement with the conclusion of Matsumoto, *et al.* (1993), who conducted wind tunnel forced vibration tests and found that for some models (e.g. 2D rectangular model with a 20 width-to-depth ratio) the aerodynamic forces and the flutter derivatives are independent on the vibration mode, i.e. SDOF vibration or 2DOF coupled vibration. However, it was found that for other models (e.g. 2D rectangular model with a 10 width-to-depth ratio), such a conclusion does not hold, and each flutter derivative is strongly influenced by the DOF of the motion.

Another possible cause for such a dependence of the effects of frequency ratio on cross-sectional shape is the shift of the equilibrium position in non-zero wind speeds, as shown in Fig. 8, which will result in an “additional” angle of wind incidence (θ in Fig. 8), and such a shift is dependent on the aerodynamic shape of the model. Since the flutter derivatives of bridge decks are functions of the angle of wind incidence (Curami and Zasso 1993, Diana, *et al.* 2004), the effect of the “additional” angle of wind incidence is a likely cause of the differences between flutter derivatives for configurations with different frequency ratios. However, for the current tests, the additional

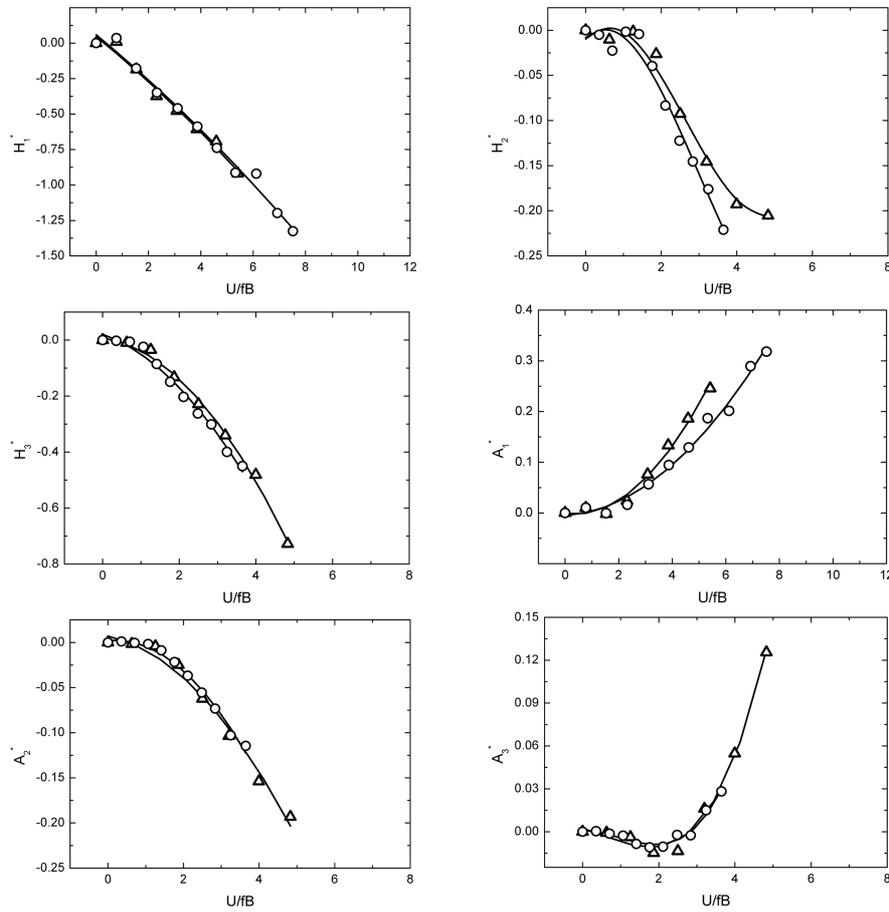


Fig. 7 Flutter derivatives of model with 35% gap-width
 Δ : frequency ratio =1.2 \circ : frequency ratio =2.2

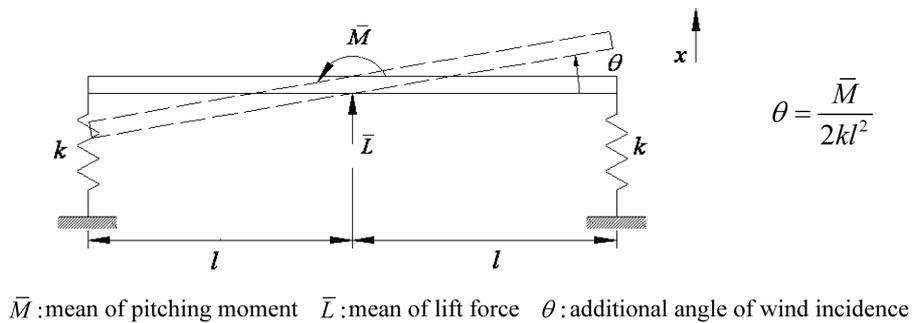


Fig. 8 Equilibrium position of the model-rig system at zero (solid line) and non-zero (dashed line) wind speeds

angle of wind incidence is not the dominant cause because it is always less than 0.5 degree, which is small compared with the targeted angle of wind incidence tested.

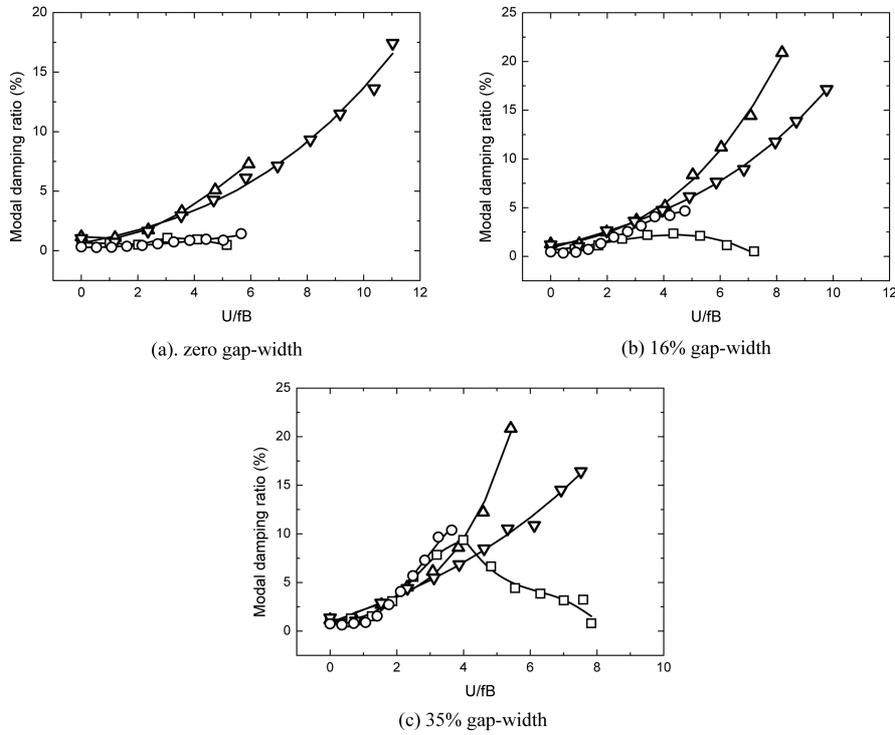


Fig. 9 Modal damping ratios of configurations with different gap-widths
 Δ : vertical mode (frequency ratio =1.2) \square : torsional mode (frequency ratio =1.2)
 ∇ : vertical mode (frequency ratio =2.2) o : torsional mode (frequency ratio =2.2)

3.3. Modal damping ratios of configurations with different frequency ratios

As shown in Eq. (2), flutter derivatives define the contribution of various responses (displacement or velocity) to the self-excited forces, which are defined in the coupled physical spaces. Fig. 9 presents the comparison of modal damping ratios, which are defined in the un-coupled modal spaces and offer a different perspective to examine the problem. Evidently, no remarkable effect was found for the modal damping ratios of the model with zero gap-width. In contrast, the modal damping ratios, especially the torsional one, of the models with 16% and 35% gap-width are much more sensitive to the frequency ratio.

4. Conclusions

This paper presented a framework to investigate the effects of frequency ratio on aerodynamic and dynamic properties of bridge decks. A series of wind tunnel dynamic sectional model tests were conducted to investigate the effects of torsional-to-vertical natural frequency ratio of the 2DOF model-rig bridge dynamic system on the aerodynamic and dynamic properties of bridge decks.

The effects of the frequency ratio on the flutter derivatives were found to be dependent on the gap-width of twin-deck bridges and therefore on the aerodynamic cross-section shape of the bridge

deck. For the model with zero gap-width, a model to approximate a thin flat plate, all the six flutter derivatives are not sensitive to frequency ratio, while for the models with nonzero gap-width (16% and 35% gap-width), some of flutter derivatives and therefore the self-excited lift force and pitching moment, are influenced by frequency ratio.

For wind tunnel free vibration test methodology, applying a smaller torsional-to-vertical natural frequency ratio is commonly adopted as a model test strategy to overcome the lack of free-decay response at high test wind speeds. It is suggested that parametric studies be conducted first to determine whether the aerodynamic properties of the subject model are sensitive to the frequency ratio. Otherwise erroneous results are likely to be generated from the test program.

Acknowledgements

This research was supported by the HKUST Targets of Opportunities Fund (Project: TOOF03/04.EG01), Research Grants Council of Hong Kong (Project: CA02/03.EG03), Natural Science Foundation of China (Project: 50308022), and National Key Technology R&D Program of China (Project: 2007BAF10B05). The authors also would like to gratefully acknowledge the contributions of the Highways Department of the Hong Kong SAR.

References

- Brownjohn, J.M.W. and Bogunovic Jakobsen, J. (2001), "Strategies for aeroelastic parameter identification from bridge deck free vibration data", *J. Wind Eng. Ind. Aerod.*, **89**, 1113-1136.
- Chen, X.Z., Matsumoto, M. and Kareem, A. (2000), "Aerodynamic coupling effects on flutter and buffeting of bridges", *J. Eng. Mech.* ASCE, **126**(1), 17-26.
- Curami, A. and Zasso, A. (1993), "Extensive identification of bridge deck aeroelastic coefficients: average angle of attack, Reynolds number and other parameter effects", *Proc. of the 3rd Asia-Pacific Symposium on Wind Engineering*, Hong Kong, pp. 143-148.
- Diana, G., Resta, F., Zasso, F., Belloli, M. and Rocchi, D. (2004), "Forced motion and free motion aeroelastic tests on a new concept dynamometric section model of the Messina suspension bridge", *J. Wind Eng. Ind. Aerod.*, **92**, 441-462.
- Gu, M., Zhang, R.X. and Xiang, H.F. (2000), "Identification of flutter derivatives of bridge decks", *J. Wind Eng. Ind. Aerod.*, **84**, 151-162.
- Gu, M., Zhang, R.X. and Xiang, H.F. (2001), "Parametric study of flutter derivatives of bridge decks", *Eng. Struct.*, **23**, 1607-1613.
- Juang, J.N. and Pappa, R.S. (1985), "An eigensystem realization algorithm for modal parameter identification and model reduction", *J. Guid. Control Dynam.*, **8**(5), 620-627.
- Matsumoto, M., Shiraishi, N., Shirato, H., Shigetaka, K. and Niihara, Y. (1993), "Aerodynamic derivatives of coupled/hybrid flutter of fundamental structural sections", *J. Wind Eng. Ind. Aerod.*, **49**, 575-584.
- Matsumoto, M., Shijo, R., Eguchi, A., Hikida, T., Tamaki, H. and Mizuno, K. (2004), "On the flutter characteristics of the separated two box girders", *Wind Struct.*, **7**(4), 281-291.
- Matsumoto, M., Shirato, H., Mizuno, K., Shijo, R. and Hikida, T. (2008), "Flutter characteristics of H-shaped cylinders with various side-ratios and comparisons with characteristics of rectangular cylinders", *J. Wind Eng. Ind. Aerod.*, **96**, 963-970.
- Noda, M., Utsunomiya, H., Nagao, F., Kanda, M. and Shiraishi, N. (2003), "Effects of oscillation amplitude on aerodynamic derivatives", *J. Wind Eng. Ind. Aerod.*, **91**, 101-111.
- Qin, X.R., Kwok, K.C.S., Fok, C.H., Hitchcock, P.A. and Xu, Y.L. (2007), "Wind-induced self-excited vibrations of a twin-deck bridge and the effects of gap-width", *Wind Struct.*, **10**(5), 463-479.

- Qin, X.R. and Gu, M. (2004), "Determination of flutter derivatives by stochastic subspace identification technique", *Wind Struct.*, **7**(3), 173-186.
- Qin, X.R., Kwok, K.C.S., Fok, C.H. and Hitchcock, P.A. (2004), "Wind-induced vibrations of a twin-deck bridge model", *Proc. of the 6th Asia-Pacific Conf. on Wind Engineering*, Korea, pp. 340-348.
- Sarkar, P.P. (1994), "Identification of aeroelastic parameter of flexible bridges", *J. Eng. Mech. ASCE*, **120**(8), 1718-1741.
- Sato, H., Kusuhara, H., Ogi, K. and Matsufuji, H. (2000), "Aerodynamic characteristics of super long-span bridges with slotted box girder", *J. Wind Eng. Ind. Aerod.*, **8**(8), 297-306.
- Scanlan, R.H. (1978), "The action of flexible bridges under wind, I: flutter theory", *J. Sound Vib.*, **60**(2), 187-199.
- Scanlan, R.H. and Tomko, J.J. (1971), "Airfoil and bridge deck flutter derivatives", *J. Eng. Mech. ASCE*, **97**(6), 1717-1737.
- Singh, L., Jones, N.P., Scanlan, R.H. and Lorendeaux, O. (1996), "Identification of lateral flutter derivatives of bridge decks", *J. Wind Eng. Ind. Aerod.*, **60**, 81-89.
- Zhang, X., Mark, J. and Brownjohn, W. (2004), "Effect of relative amplitude on bridge deck flutter", *J. Wind Eng. Ind. Aerod.*, **91**, 101-111.

JH