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Reliability based analysis of torsional divergence of long span suspension bridges

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Abstract. A systematic reliability evaluation approach for torsional divergence analysis of long span suspension bridges is proposed, consisting of the first order reliability method and a simplified torsional divergence analysis method. The proposed method was implemented in the deterministic torsional divergence analysis program SIMTDB through a new strategy involving interfacing the proposed method with SIMTDB via a freely available MATLAB software tool (FERUM). A numerical example involving a detailed computational model of a long span suspension bridge with a main span of 888 m is presented to demonstrate the applicability and merits of the proposed method and the associated software strategy. Finally, the most influential random variables on the reliability of long span suspension bridges against torsional divergence failure are identified by a sensitivity analysis.

Keywords: torsional divergence; wind effect; structural reliability; suspension bridges; long span structures; displacement-dependent wind loads.

1. Introduction

When the deformed shape of the bridge deck of a long span suspension bridge produces an increase in the value of three components of displacement dependent wind loads distributed along the bridge deck, the bridge will occur a loss of overall stability due to the action of the three components of displacement dependent wind loads. This paper considers specifically the aerostatic instability of suspension bridges, whose importance has been stressed by different authors (Cheng, *et al*. 2002, Boonyapinyo, *et al*. 2006).

The aerostatic instability of suspension bridges in many cases appears to be dominated by torsional modes (Cheng, *et al.* 2003). Several investigations concerning torsional divergence instability of long-span suspension bridges (e.g., (Cheng, *et al*. 2002, Simiu and Scanlan 1978, Xiang 1996)) have been performed. There are two basic approaches to this torsional divergence problem: (1) linear methods, e.g., Simiu and Scanlan (1978) and Xiang (1996) or (2) nonlinear methods, e.g., Cheng, *et al*. (2002). The linear methods are relatively more popular because of simplicity. However, these methods were all based

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on assumptions of a linearized derivative of pitch moment as well as linear structural stiffness matrix; the nonlinear effects arising from a bridge structure and the three components of wind load were not considered. Therefore, the critical wind velocity causing the torsional divergence instability cannot be accurately calculated, the mode of instability as well as the coupling effect cannot be considered, and the wind velocity-deformation path of the bridge from applied wind velocity to divergence cannot be traced. While the nonlinear methods were found to provide an accurate estimate of critical wind velocity for solving the torsional divergence problem, the use of nonlinear methods is usually computationally too intensive for large scale structures, and data input is time-consuming for long span suspension bridges.

A simplified nonlinear torsional divergence analysis procedure for suspension bridges has been developed (Cheng, *et al*. 2003). The procedure, which was based on the deflection theory of suspension bridges and involved the series method, took into account the geometric nonlinearity of a structure and the associated three components of displacement-dependent wind loads. A numerical example involving a detailed computational model of a long span suspension bridge with a main span of 888 m was presented in Ref. (Cheng, *et al*. 2003) to demonstrate the simplicity and accuracy of the new procedure. It was revealed that the procedure is simple to use and has a high convergence rate, thus significantly reducing computation time. The advantage of the procedure becomes more remarkable for reliability analysis since hundreds, thousands even millions of similar torsional divergence analyses may be required for reliability assessment of a long span suspension bridge.

The above-mentioned torsional divergence analysis methods of suspension bridges were based on the assumption of complete determinacy of structural parameters. However, these deterministic torsional divergence analysis methods fail to consider the uncertainty in the variables, and thus cannot be used for reliability analysis. If the basic variables are uncertain, every critical wind velocity computed by the deterministic torsional divergence analysis is also uncertain. The currently available reliability methods can still be used if the uncertainty in the critical wind velocity can be tracked in terms of variations of the basic variables at every step of the deterministic torsional divergence analysis. To capture the desirable features of these two approaches, they needed to be combined, leading to the concept of the systematic reliability method (SRM).

In recent years, a number of SRM algorithms have been proposed to calculate the reliability of complex structures (Cheng and Xiao 2005, Cheng, *et al*. 2008, Lee 2000, Frangopol and Imai 2004, Imai and Frangopol 2002, Pourzeynali and Datta 2002). However, to the knowledge of the authors, there has not been an attempt to solve torsional divergence problem of suspension bridges in the context of SRM. Therefore, this research attempts to fill the gap by: (1) developing a SRM based reliability analysis method for analysis of the torsional divergence of suspension bridges; (2) using the proposed SRM method to evaluate the reliability of an existing long span suspension bridge against torsional divergence instability, and (3) conducting a sensitivity analysis to ascertain the effect of parameter uncertainty on the computational results.

2. Three components of wind loads

Before proceeding to the development of the proposed SRM method, it is necessary to briefly outline the modeling of the underlying three components of wind loads (drag force, lift force and pitch moment) for the sake of future reference. A section of bridge deck in a smooth flow is considered herein, as shown in Fig. 1. Assuming that under action of mean wind speed V with an angle of incidence α_0, torsional displacement of the deck is θ. Then the effective wind angle of attack is $\alpha = \alpha_0 + \theta$. The components of wind forces per unit span acting on the deformed deck can be written in wind axes as

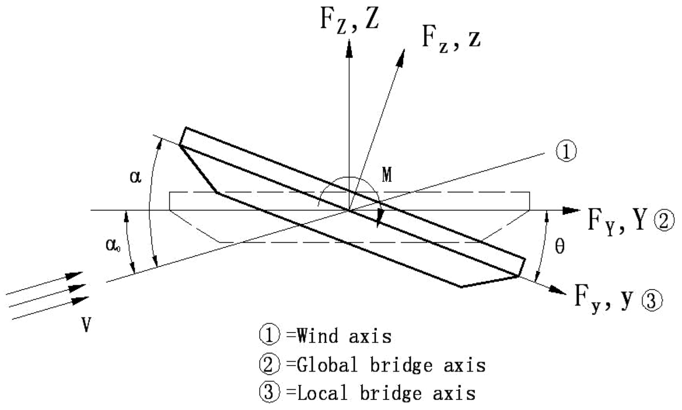

Fig. 1 Motion of Bridge deck and three components of wind loads in different axes

$$\text{Drag force: } F_y(\alpha) = \frac{1}{2}\rho V^2 C_y(\alpha) D \tag{1a}$$

$$\text{Lift force: } F_z(\alpha) = \frac{1}{2}\rho V^2 C_z(\alpha) B \tag{1b}$$

$$\text{Pitch moment: } M(\alpha) = \frac{1}{2}\rho V^2 C_M(\alpha) B^2 \tag{1c}$$

where $C_y(\alpha)$, $C_z(\alpha)$ and $C_M(\alpha)$ = coefficients of drag force, lift force, and pitch moment in local bridge axes, respectively; B = deck width; D = vertical projected area.

The wind forces in Eq. (1) are functions of the torsional displacement of the structure. They vary as the girder displaces. Therefore, the three components of wind loads are displacement dependent. Theoretical coefficient curves of the three components of wind loads fitted to available experimental points are generally nonlinear. In this paper, the nonlinear curves are approximately represented by linear curves between two experimental points.

3. Deterministic torsional divergence analysis method

Since the deterministic torsional divergence analysis will be used iteratively, the efficiency of the deterministic torsional divergence analysis method plays a very important role in the successful implementation of the proposed SRM method, particularly in the nonlinear aerostatic stability analysis of suspension bridges. It has been reported in the Ref. (Cheng, *et al.* 2003) that the above-mentioned simplified method is reasonable for the deterministic torsional divergence analysis of suspension bridges. In the simplified method, by introducing the series method, the analysis of the nonlinear torsional divergence of suspension bridges is greatly simplified, thus significantly reducing computational efforts. Different sources of nonlinear, especially due to the presence of displacement-dependent wind loads, can be incorporated without losing simplicity. It is very accurate and efficient in analyzing the torsional divergence of suspension bridges. Although the details of the simplified method cannot be given here due to lack of space, they can be found in the Ref. (Cheng, *et al.* 2003).

4. Proposed SRM method for torsional divergence analysis

In general, simulation or the first order reliability methods (FORM) can be used for structural reliability evaluation. Simulation techniques may not be desirable for the class of problems considered here, because of cost and inherent inefficiency of such approaches. In the context of the FORM, the limit state function of a structure needs to be defined in an explicit form. However, the limit state function for the class of problems under consideration is implicit in nature. The response surface method (RSM) can be used to approximately generate the implicit limit state function; however, it may need to be modified before it can be applied. Also, the difficulties in the RSM caused by the non-smoothness of the response surface, inappropriate evaluation range that is controlled by the parameter, h_i, and unrealistic design points are difficult to overcome (Wang, *et al.* 2005). Considering the advantages and disadvantages of currently available methods, an efficient SRM method for the torsional divergence analysis of suspension bridges is proposed in this paper through combination of the advantages of the simplified method described in the previous sections and the FORM. In the proposed SRM method, the FORM is used to estimate the reliability of structures. In this context, the limit state g is linearized and nonnormal distributions are converted to equivalent normal distributions at the most probable point of failure, which can be found through the improved Hasofer-Lind and Rackwitz-Fiessler (HLRF) algorithm developed by Zhang and Der Kiureghian (1997). This procedure requires repeated computations of the limit-state function and its gradient. For the torsional divergence problem under consideration, the critical wind speed for a suspension bridge is an implicit function of the random variables. In other words, a closed form solution of the critical wind speed of a suspension bridge is not available due to its complexity nature. Under such a condition, evaluation of the derivatives of the implicit limit state function is not an easy task. For the sake of simplicity, the derivatives of the limit state function are obtained by integrating the finite difference method and the simplified method described in the previous sections. More detailed information concerning the FORM for reliability analysis is given in (Melchers 2002).

5. Sensitivity analysis

An important step in the structural reliability analyses is sensitivity analysis of reliability indices. This helps to identify important parameters. On the other hand, sensitivity analysis is also useful in reducing the size of problems with a large numbers of random variables. This is because that, in general, only a few variables have a significant effect on the structural reliability (Haldar and Mahadevan 2000). In this section, sensitivity analysis is used to identify the influences of the random variables on the reliability of the torsional divergence of suspension bridges. Since the random variables considered in this study are independent from each other, there is one-to-one mapping between the standard normal random variables Y and the original random variables X. In that case, an importance ordering of the elements of Y implies the same importance ordering of the elements of X. To this end, the relative importance of the random variables in the reliability analysis can be measured by means of the vector α^* defined as (Sudret and Kiureghian 2000):

$$\alpha^* = \frac{Y^*}{\|Y^*\|} \tag{24}$$

where Y^* denotes the coordinates of the design point in the standard normal space.

6. Computer implementation

The SRM method needed to evaluate the reliability of suspension bridges against torsional divergence instability is now available. However, the implementation of the SRM-based reliability analysis needs to be done in a computer environment. This requires the development of a computer-support tool for helping designers at wind-resistant design stage. The present paper is regarded as a contribution to this work.

Over the past three decades, a significant amount of effort has gone into developing commercial finite element analysis software. Many such software tools are available, among which the most noted ones are ANSYS, ADINA and SAP. However, these commercial finite element analysis programs cannot be readily used for the torsional divergence analysis of suspension bridges as they lack some capabilities like the calculation of displacement-dependent wind loads, the prediction of critical wind speed, and determination of target configurations of suspension bridges under dead loads. The torsional divergence analysis software appears as a consequence of these necessities.

A computer program denoted hereafter as SIMTDB is specially developed to realize this purpose. The program was developed by Cheng, *et al*. (2003) and adopts a simplified torsional divergence analysis method described previously. However, the program is not capable of dealing with stochastic uncertainties involved in the analysis of long span suspension bridges. The need for conducting the probabilistic analysis may lead to additional immense programming efforts. Because of not being able to interact with the SIMTDB program, probabilistic analysis capabilities of many available software systems such as NESSUS (Thacker, *et al.* 2006), COSSAN (Schueller and Pradlwarter 1996-2005) and FERUM (Der Kiureghian, *et al*. 2006) are limited. In view of this situation, it is usually advantageous to interface the torsional divergence analysis code with the structural reliability analysis software, in order to develop reliability based torsional divergence analysis software with minimal or no intrusive modifications.

As mentioned above, many structural reliability analysis software systems are available. FERUM was selected as the probabilistic analysis platform in this study due to its open-source philosophy and its simplicity. However, other structural reliability analysis software systems could be easily adopted. A brief outline of how the FERUM can be coupled with the SIMTDB program is presented in the next part.

Fig. 2 outlines an overview of the steps involved in the coupled software framework. It shows various levels of interactions between the two computational tools. The efficient transfer of data and information between the two computational tools is facilitated via an intervening interface. Here the intervening interface is the scripting language MATLAB. An interface file named *gfun.m* is provided to specify the limit-state function. The critical wind speed value in the limit-state function is extracted from the SIMTDB output file. A system command in Matlab is issued to execute the SIMTDB so that the user can easily conduct the deterministic torsional divergence analysis using the simplified method described previously. The SIMTDB input file is updated through a series of dynamic data exchanges using the data obtained from the FERUM. A SIMTDB input file is generated containing bridge model data and solution information for the deterministic torsional divergence analysis of suspension bridges. Thus if any of given parameters is changed, the new bridge model will be regenerated accordingly. The input file is fed into the SIMTDB to compute the critical wind speed which is eventually used by the FERUM to compute the defined limit-state function.

7. Case study

7.1. Long span suspension bridge

The long span suspension bridge selected for the case study is the Hu Men suspension bridge

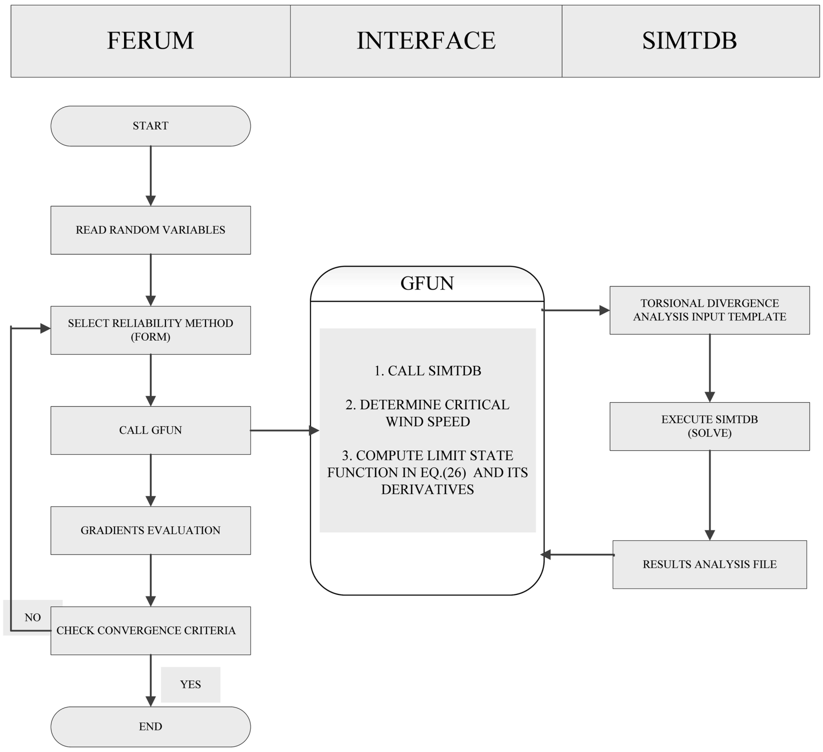

Fig. 2 Flowchart of the coupled software

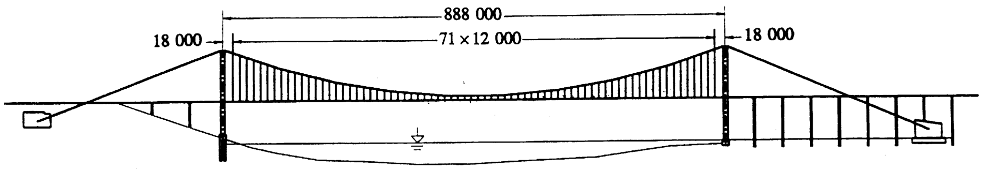

Fig. 3 General configuration of Hu Men Bridge (Unit: mm)

located in Guang-Dong Province, China. The general configurations of the bridge shown in Fig. 3 are summarized as follows: a suspension bridge of main span 888m; portal frame shape towers with 150m height; closed-box deck with 3.012 m depth and 35.6 m width; width between center lines of cables is 33 m; the spacing between two hangers is 12.0m. A detailed description of the bridge may be obtained from (Xiang 1994).

7.2. Deterministic structural data and parameters

The deterministic structural data of the bridge is listed in Table 1. Complete structure data is given by Xiang (Xiang 1994). The static aerodynamic coefficients for the bridge (Xiang 1994) are shown in Fig. 4. The three components of the displacement-dependent wind loads were only considered for the bridge deck while for the cables only the initial drag force was taken into account. Drag force

Table 1 Statistics of the random variables for Hu Men Bridge

Variable	μ	σ	Dimension	Distribution	Sources
E_c	2.0×10^{11}	2.0×10^{10}	N/m^2	Normal	
A_c	1.229	0.0615	m^2	Lognormal	
GI_d	4.12×10^{11}	4.12×10^{10}	$N\cdot m^2$	Lognormal	Assumed
EI	4.158×10^{11}	4.158×10^{10}	$N\cdot m^2$	Lognormal	
EI_y	261.219×10^{11}	261.219×10^{10}	$N\cdot m^2$	Lognormal	
V_{10}	17.86	3.18	m/s	Type I	Ref. (Cheng, *et al*. 2005)
g_s	1.32	0.09	-	Normal	
φ	1.0	0.2	-	Lognormal	Assumed

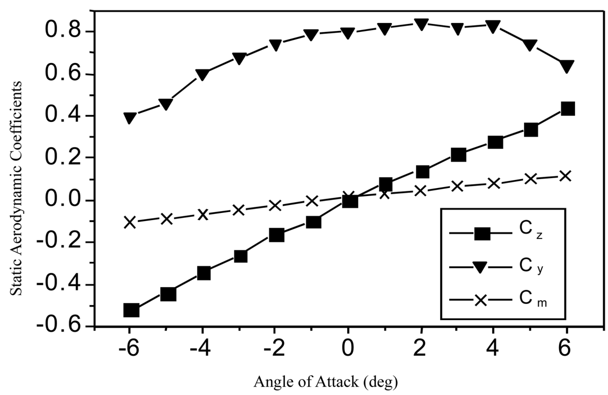

Fig. 4 Static aerodynamic coefficients as function of angle of attack

acting on the towers was not included in the analysis.

The following parameters are used in the analysis: (1) the angle of incidence $\alpha_0 = 0$; (2) $\mu = 18.68$, $\frac{r}{b} = 0.65$, $C'_{M0} = 1.07$, these datum can be obtained from (Cheng and Xiao 2005); (3) From Fig. 5, it can be seen that the coefficient curves of lift force and pitch moment of wind loads are almost linear. Therefore, linear curve fitting is used. The coefficients (e_1, e_2, c_1, c_2) are given by $e_1 = 0.00877$, $e_2 = 0.01838$, $c_1 = -0.02462$, $c_2 = 0.0789$. However, because coefficient curve of drag force of wind loads is nonlinear, the nonlinear curve is piecewise linearized. The coefficients (d_1, d_2) are given by

For $0 \le \theta \le 1$, $d_1 = 0.81993$; $d_2 = 0.02075$
For $1 < \theta \le 2$, $d_1 = 0.83759$; $d_2 = 0.00309$
For $2 < \theta \le 3$, $d_1 = 0.87531$; $d_2 = -0.01577$
For $3 < \theta \le 4$, $d_1 = 0.9354$; $d_2 = -0.0358$
For $4 < \theta \le \infty$, $d_1 = 1.0202$; $d_2 = -0.057$
$r = 1,3,5,7,9$.

7.3. Uncertainties in the analysis of the bridge

Referring to Eqs. (2), (8) and (17), several design parameters are sources of uncertainties influencing the torsional divergence behavior of the bridge. The major uncertain parameter in the stiffened girder is the elastic modulus, E, while the uncertain parameters in the main cables are the elastic modulus of main cables, E_c, and the cross section area, A_c. Besides the above parameters, the three components of the displacement-dependent wind loads (M, F_z and F_y) could be another source of uncertainty in the torsional divergence analysis of the bridge.

The variability of the elastic modulus of a stiffened girder can be represented by the statistical mean and variance of the elastic modulus of the girder material. Instead of using two separate parameters, E and I, the vertical stiffness of the girder (EI) is often used in practice and considered as one design parameter. As I only depends on the girder cross-section geometry, the variability of E contributes the greater part of the variability of the vertical stiffness of the girder. In practice, it is common to use a single random variable EI to represent the variability of the vertical stiffness along the length of a girder. Similar to the case of vertical stiffness of the girder, both the torsional stiffness (GI_d) and lateral stiffness (EI_y) are considered as a single random variable, respectively.

In contrast to the elastic modulus of the girder, the variability in the three components of the displacement-dependent wind loads is more complex. A considerable amount of the uncertainty in the three components of the displacement-dependent wind loads come from the uncertainties in the static aerodynamic coefficients and the extreme wind velocity at the bridge site. Deterministic prediction of both static aerodynamic coefficients and extreme wind velocity at the bridge site, based on the experimental results or single observation data, is likely to overestimate the displacement-dependent wind loads. In practice, the static aerodynamic coefficients and the extreme wind velocity at the bridge site may be often varied around the nominal values due to their natures and/or experimental error and boundary layer wind characteristics. Therefore, deterministic values do not account for the scatter in the static aerodynamic coefficients and the extreme wind velocity at the bridge site.

There is no guideline on how to consider the uncertainties in the static aerodynamic coefficients. For the sake of simplicity, a parameter φ is introduced to incorporate the uncertainties arising from insufficient knowledge of the static aerodynamic coefficients. The uncertainties in the static aerodynamic coefficients are actually considered indirectly in this study. Note that the extreme wind velocity at the bridge site can be expressed as $V_e = g_s \cdot V_{10}$, where g_s is the gust speed factor and V_{10} is the basic wind speed. The uncertainty in the extreme wind velocity at the bridge site is successfully considered in this study by considering both gust speed factor and basic wind speed to be random variables. Table 1 shows the statistics of these random variables. As the objective of this study is to propose an efficient method for the reliability based torsional divergence analysis of suspension bridges, all random parameters in the analysis are based on arbitrary but typical values. On the other hand, since the determination of the interrelation of the random parameters is a difficult task, using the independence assumption can greatly simplify the reliability assessment. Therefore, all random parameters in the paper are treated as stochastically independent from each other.

7.4. Limit state function

Structural reliability assessment traditionally considers a limit state to define a failure event. A limit state is a state of the structure including its load at which the structure is just on the point of not satisfying the requirement (Ditlevsen and Madsen 1996). Failure is said to occur when the following condition is satisfied,

$$S = G(X) \leq 0 \tag{25}$$

where S is the safety margin, X means the vector of the random variables discussed above and $G(X)$ is the so-called limit state function which separates the acceptable region from that which is characterized as failure.

The failure mode considered for the present analysis is the torsional divergence failure of the bridge. The bridge is assumed to have failed when the extreme wind speed, V_e reaches or exceeds corresponding critical wind speed, V_{cr} estimated from the deterministic torsional divergence analysis presented in Section 3. Therefore, the limit state function of a long span suspension bridge against the torsional divergence failure is defined as

$$G(X) = \varphi \cdot V_{cr}(X) - g_s \cdot V_{10} \tag{26}$$

7.5. Verification of the proposed method

The success of the proposed method depends on the accuracy and efficiency of the deterministic method used. The basic deterministic method used in this study was discussed briefly in the previous section. The verification of the accuracy and efficiency of the deterministic method is referred to Ref. (Cheng, *et al*. 2003). The accuracy of the proposed method is established using direct Monte Carlo simulations. Using direct Monte Carlo simulation, it is found that the probability of failure of the bridge is very close to zero, making the verification of the proposed method difficult. For verification purposes, the critical wind speed, V_{cr} mentioned earlier is multiplied by a reduction factor equal to 0.3. This is done to reduce the probability of failure, P_f, so that the analysis can be performed using direct Monte Carlo simulation. The limit state function in Eq. (26) can then be roughly represented by the following function

$$G(X) = 0.3\varphi \cdot V_{cr}(X) - g_s \cdot V_{10} \tag{27}$$

Applying the direct Monte Carlo simulation with 10,000 samples, the probability of failure of the bridge against torsional divergence instability is found to be 0.0445. Using the proposed method, the probability of failure of the bridge against torsional divergence instability is 0.0406. Comparing the results of the proposed method with those of the direct Monte Carlo simulation, it is noticed that the proposed method provides reasonably accurate results, which demonstrate that the proposed method can be used to estimate the reliability of long span suspension bridges against torsional divergence instability. The results also demonstrate the usefulness and capabilities of the present coupling strategy involving interfacing the proposed method with deterministic torsional divergence analysis codes (i.e. SIMTDB) via a freely available MATLAB software tool (FERUM). The coupling presented removes the immense effort required in coding ones own reliability analysis codes by utilizing already existing reliability analysis software.

7.6. Reliability analysis results

As described in the previous sections, much of the past research has been limited to the deterministic torsional divergence analysis. Also during the last two decades, considerable progress has been made in the field of structural reliability (Cheng and Xiao 2005, Cheng, *et al*. 2008, Lee 2000, Frangopol, *et al*. 2004, Imai and Frangopol 2002, Pourzeynali and Datta 2002, Wong, *et al*.

2005, Zhang and Der Kiureghian 1997, Melchers 2002). However, most of the recent research has concentrated on developing methods to identify the reliability of structural systems. Seldom have the effects of the geometric nonlinearity of a structure and the three components of displacement-dependent wind loads been considered in the reliability analysis of a suspension bridge against torsional divergence instability. Ignoring the 0.3 reduction factor used previously, the probability of failure of the bridge due to torsional divergence instability is evaluated using the proposed method so as to show the importance of the consideration of the geometric nonlinearity of the structure and the three components of displacement-dependent wind loads in the calculation of the probability of failure of the bridge. For this purpose, the linear method proposed by Xiang (1996) is coupled with the FORM described previously. The coupled algorithm will be called "the linear method based SRM" in order to distinguish it from the proposed method. The role of linear method in the linear method based SRM is similar to the simplified method used in proposed method. The difference between the linear method based SRM and the proposed method is that the nonlinear effects arising from a bridge structure and the three components of wind load are not considered in the linear method based SRM, while the geometric nonlinearity of a structure and the three components of displacement-dependent wind loads are taken into account in the proposed method. From the sensitivity analysis discussed next, the resistance-related parameters were found to have very low sensitivity indexes compared to the others. For simplicity, only φ, g_s and V_{10} in Table 1 are considered to be random variables in this section. The probabilities of failure obtained by the linear method based SRM and the proposed method are listed in Table 2. Several observations can be made from Table 2.The probabilities of failure obtained by the linear method based SRM and the proposed method are found to be very different, indicating that the effect of the geometric nonlinearity of a structure and the three components of displacement-dependent wind loads may be an important factor in evaluating the torsional divergence failure of a suspension bridge. Also, the probability of failure obtained by the proposed method is much higher than that by the linear method based SRM. This implies that the linear method based SRM underestimates the probability of failure of the suspension bridge against torsional divergence failure and gives unsafe results. Therefore, the effect of the geometric nonlinearity of a structure and the three components of displacement-dependent wind loads should be considered in the reliability analysis of a suspension bridge against torsional divergence failure if more reliable results are desired.

7.7. Sensitivity analysis results

The results of the sensitivity analysis are presented in Fig. 5. The results show that the reliability index of the bridge is highly influenced by the static aerodynamic coefficient parameter, basic wind speed and gust speed factor. This means that accurate determination of such parameters is very important in obtaining reliable probabilistic results. On the other hand, the displacement dependent wind loads-related variables (the static aerodynamic coefficient parameter and basic wind speed) are the most sensitive variables for the reliability analysis of the bridge. Therefore, the effects of uncertainty in the displacement dependent wind loads are very significant.

Table 2 Comparison of results for various methods

Method	Linear method based SRM	Proposed method
Probability of failure	3.760×10^{-9}	1.713×10^{-8}

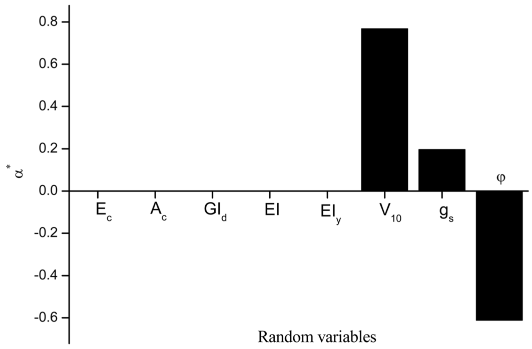

Fig. 5 The results of the sensitivity analysis

8. Conclusions

1) The proposed method intelligently integrates the first order reliability method and a simplified torsional divergence analysis method. The proposed method is verified using Monte Carlo simulation. As expected, it estimates the probability of failure very accurately, and is very efficient in comparison with simulation.

2) The proposed method was implemented in the deterministic torsional divergence analysis program SIMTDB through a new strategy involving interfacing the proposed method with SIMTDB via a freely available MATLAB software tool (FERUM). The new strategy has a number of attractive features: (1) an existing deterministic torsional divergence analysis program (i.e., SIMTDB) can be easily extended with sensitivity and reliability analysis capabilities. This allows us to characterize material, load and geometry parameters as random variables in order to investigate the propagation of uncertainties, estimate failure probabilities and identify the most important parameters; (2) direct access to the extensive statistical distribution functions and reliability method library of FERUM becomes possible; and (3) future developments of the strategy, for example to other aerostatic stability problems and system reliability problems, can be readily implemented.

3) The reliability of an existing long span suspension bridge against torsional divergence failure was evaluated. Based on the results, the following observations are made: (1) the effect of the geometric nonlinearity of the structure and the three components of the displacement-dependent wind loads is very significant; (2) the failure probability of the bridge under the torsional divergence failure mode is sensitive to the static aerodynamic coefficient parameter, basic wind speed and gust speed factor. However, the failure probability of the bridge is insensitive to the resistance-related parameters; and (3) the displacement dependent wind loads-related variables (the static aerodynamic coefficient parameter and basic wind speed) are the most sensitive variables for the reliability analysis of the bridge.

4) The torsional divergence problem explored here were relatively simple in a sense that only eight random variables were considered. This is not a limitation of the methodology, but rather of the computational costs. This problem required considerable computational effort, and it is recommended that future work in this area be directed toward reducing these costs. Such a reduction may also allow the inclusion of more accurate torsional divergence analysis methods such as nonlinear finite element method as well as the potential for system reliability analysis.

Acknowledgments

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