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# Mathematical modeling of actively controlled piezo smart structures: a review

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**Abstract.** This is a review paper on mathematical modeling of actively controlled piezo smart structures. Paper has four sections to discuss the techniques to: (i) write the equations of motion (ii) implement sensor-actuator design (iii) model real life environmental effects and, (iv) control structural vibrations. In section (i), methods of writing equations of motion using equilibrium relations, Hamilton's principle, finite element technique and modal testing are discussed. In section (ii), self-sensing actuators, extension-bending actuators, shear actuators and modal sensors/actuators are discussed. In section (iii), modeling of thermal, hygro and other non-linear effects is discussed. Finally in section (iv), various vibration control techniques and useful software are mentioned. This review has two objectives: (i) practicing engineers can pick the most suitable philosophy for their end application and, (ii) researchers can come to know how the field has evolved, how it can be extended to real life structures and what the potential gaps in the literature are.

**Keywords:** smart structure; piezoelectric materials; mathematical modeling; active vibration control; review.

## 1. Introduction

Since time immemorial, man has been controlling structural vibrations by modifying mass, stiffness and damping of the structure. This may increase overall mass of the structure and is found to be unsuitable for controlling low frequency vibrations (Fahy and Walker 1988, Brennan and Ferguson 2004). This method does not suit applications where less weight is desired and low frequency vibrations are encountered. For such applications, smart structures are being developed which are light weight and attenuate low frequency vibrations (Fuller *et al*. 1997). A structure in which external source of energy is used to control structural vibrations is called a 'smart structure' and the technique is called 'active vibration control (AVC)'. A smart structure essentially consists of sensors to capture the dynamics of the structure, a processor to manipulate the sensor signal, actuators to obey the order of processor and a source of energy to actuate the actuators (Fig. 1).

This field has recently gained lot of interest due to three main reasons: (i) increased interest of man in space exploration, nano-positioning, micro-sensing etc. (Moheimani and Fleming 2006) (ii) advent of fast processors, real time operating systems etc. (Piersol and Paez 2010) and (iii) development of stable and high performance sensors and actuators (Rao and Sunar 1994, Barlas and

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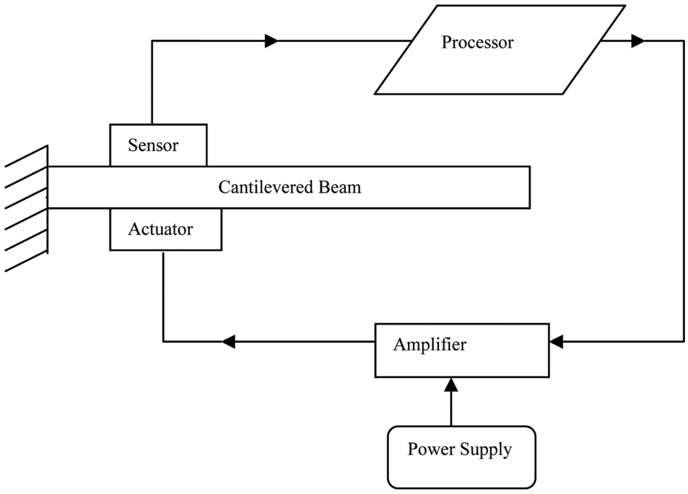


Fig. 1 A schematic of a smart structure

van Kuik 2010). Piezoelectric sensors/actuators are being used extensively for AVC because a piezoelectric material has excellent electromechanical properties: fast response, easy fabrication, design flexibility, low weight, low cost, large operating bandwidth, low power consumption, generation of no magnetic field while converting electrical energy into mechanical energy etc. Piezoelectric materials generate strains when an electric signal is applied on them and vice versa. So, they can be used as sensors and actuators for structural vibrations (Fabunni 1980). This effect occurs naturally in quartz but can be induced in other materials such as specially formulated ceramics consisting mainly of lead, zirconium and titanium (PZT). Piezoelectric materials can be used as sensors and actuators in the form of distributed layers (Bailey and Hubbard 1985, Hanagud *et al.* 1985, Balamurugan and Narayanan 2001, Tzou and Hollkamp 1994), surface bonded patches (Chandershekhara and Tanneti 1995, Crawley and de Luis 1987, Vautier and Moheimani 2005), embedded patches (Elsoufi *et al.* 2007, Bronowicki *et al*. 1996, Raja *et al.* 2002), cylindrical stacks (Li *et al.* 2008), piezo paint sensor (Lahtinen *et al.* 2007), screen printed piezoelectric layer (Glynne-Jones *et al.* 2001), active fibre composites (Raja *et al.* 2004), functionally graded piezo material (Yang and Xiang 2007) etc. Surface mounted or embedded piezo patches can control a structure better than distributed one because the influence of each patch on the structure response can be individually controlled (Tzou and Fu 1994).

As of now, AVC technique is being applied on very simple two dimensional structures and rarely on real life structures. It is very difficult to model a real life structure because of (i) complex three dimensional shape (ii) complex and large boundary conditions and (iii) ever evolving nature of real life structures. Also up till now, electromechanical interaction of sensors and actuators in real life environment has not been well understood. Lastly, this field demands small control algorithms which (i) can be completed in stipulated time (ii) are robust to parametric uncertainties and (iii) are adaptive to different environmental changes. When active vibration control is to be performed on a structure one has to: (i) understand the sensor-actuator mechanics (ii) select and place sensors & actuators suitably on the host structure (iii) model the whole dynamic system and, (iv) apply a suitable control law.

The field of AVC started in 1980's. In last three decades, lots of techniques have been used to model actively controlled piezo smart structures. In near future, these techniques can be technically harnessed and commercialized. Presentation of such techniques in a unified way would be of immense use to the scientists and practicing engineers working in this field. Researchers have reviewed work on: role of piezoelectric materials in AVC (Kandagal and Venkatraman 2006), essential aspects involved in the design of AVC (Alkhatib and Golnaraghi 2003), kinematics of 1-dimensional smart structure (Alzahrani and Alghamdi 2003), finite element modeling (Benjeddou 2000), modal sensing and actuation techniques (Fripp and Atalla 2001), piezoelectric actuation techniques (Niezrecki *et al.* 2001), active constrained layer damping (Park and Baz 1999), aeronautical applications of smart structures (Loewy 1997), optimal placement of piezoelectric sensors and actuators on a smart structure (Gupta *et al.* 2010) etc. However to the authors' best knowledge, a unified review presenting the techniques to: (i) write equations of motion (ii) implement sensor-actuator design (iii) model real life environmental effects and (iv) control structural vibrations, is absent in the literature. So, the authors are motivated to write this review article on this relatively new area of research.

Aim of this review article is to provide practicing engineers and scientists, a well knitted knowledge bank of various techniques used by researchers to mathematically model a smart structure instrumented with piezoelectric sensors and actuators. It can help in extending AVC technique to real life structures in near future. State-of-the-art for each technique is discussed so that: (i) practicing engineers can pick the most suitable technique for their end application (ii) researchers can come to know how the field has progressed and, (iii) the real and actual potential hindrances in the development of this field are identified. To make this review article a unified one, various control techniques and software used by researchers in AVC are also mentioned in a separately dedicated section.

A smart structure is intended to work in real-life environment where thermal, hygro, non-linear, fatigue and ageing effects etc. are present and need to be considered while modeling sensor-actuator mechanics. So in order to build an accurate mathematical model of piezo smart structure for AVC, one has to: (i) write equations of motion (ii) implement appropriate sensor-actuator design and, (iii) incorporate environmental effects in sensor-actuator equations. In this way, complete dynamics of piezo smart structure can be captured before implementing control algorithm. Keeping in view these important aspects of mathematical modeling, separate sections are dedicated to discuss the techniques to: (i) write equations of motion (ii) implement sensor-actuator design and (iii) model real life environmental effects (thermal, hygro, non-linearities etc), in this review article.

### *1.1 Techniques to write equations of motion of smart structure*

A smart structure is prepared by bonding piezoelectric patches over the host structure by employing finite bonding layer of glue. It is essential to understand the electromechanical interaction between the piezoelectric patch and the host structure. A simple model of two dimensional structure instrumented with piezoelectric patches can be developed by considering electromechanical coupling of piezoelectric patches but without considering mass and stiffness of piezoelectric patches (Lee *et al.* 1991, Dimitriadis *et al.* 1991). When a piezoelectric patch is instrumented on a structure, then the dynamics of the structure changes due to considerable mass and stiffness of the piezoelectric patch. Static and dynamic analytical models of surface bonded as well as embedded piezoelectric actuators can be obtained by considering the stiffness but ignoring the mass of piezoelectric patch (Crawley and de Luis 1987). A refined model can be developed by taking into consideration mass as well as stiffness of piezoelectric patch (Baz and Poh 1988). If mass and stiffness of piezoelectric patch is ignored then fundamental natural frequency of a beam structure would be wrong by as much as 18% even for open circuit conditions (Yang and Lee 1994a). Natural frequencies shift by as much as 14% if a smart structure is in a closed circuit. This happens due to induction of so called 'generalized stiffness' which is a function of structural mode shapes, piezoelectric patch location and piezoelectric constants (Yang and Lee 1994b, Yang and Jeng 1996). Inertial effect of the piezoelectric patch can be modelled by a grid of lumped masses and stiffness effect by arrays of springs oriented in direction parallel to edges of piezoelectric patch (Aoki *et al.* 2008). Models of two dimensional smart plate structures considering mass and stiffness of piezoelectric patches predict accurate dynamics of the smart structure (Chandrashekhara and Agarwal 1993, Sadri *et al.* 1999). Therefore, inclusion of inertia and stiffness effects of piezoelectric patches in mathematical modeling is important to capture the structural dynamics. Equilibrium relations, Hamilton's principle, finite element model, modal testing etc. are used to find the equations of motion (1) of the smart structure

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \tag{1}$$

Let us discuss these ways one by one.

### *1.1.1 Equilibrium relations*

Equilibrium relations Eqs. (2)-(4) can be solved to obtain strains and stresses in the piezoelectric substrate and bonding layer (Reddy 2005).

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{yx}}{\partial y} + \frac{\partial\sigma_{zx}}{\partial z} + f_x = \rho\ddot{u} \tag{2}$$

$$\frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} + \frac{\partial\sigma_{yz}}{\partial z} + f_y = \rho\ddot{v} \tag{3}$$

$$\frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_{zz}}{\partial z} + f_z = \rho\ddot{w} \tag{4}$$

These strains and stresses involve hyperbolic trigonometric functions and depend upon strains at the boundaries of piezoelectric actuator and on voltage applied on the piezoelectric actuator. For a perfectly bonded actuator, bonding layer is assumed to be infinitely stiff. Under this idealized assumption, strains do not involve trigonometric hyperbolic functions and the strains are transferred between the piezoelectric and structures at concentrated points at the ends of piezoelectric. These strains Eq. (5) and moments Eq. (6) are obtained as a sum of two factors: first depending upon strains in the substrate at the boundaries of piezoelectric patch and the second upon the voltage applied across the piezoelectric actuator (Crawley and de Luis 1987).

$$\varepsilon_s = \varepsilon_p = \frac{\Psi}{\Psi+\Omega}\left[\frac{\varepsilon_s^{s+}+\varepsilon_s^{s-}}{2} + \frac{\varepsilon_s^{s+}-\varepsilon_s^{s-}}{2}\bar{x}\right] + \frac{\Omega}{\Psi+\Omega}\left(\frac{d_{31}V}{t_p}\right) \tag{5}$$

$$M = R_s t_s^2 b_s\left[\frac{1}{\Psi+\Omega}\left(\frac{\varepsilon_s^{s+}+\varepsilon_s^{s-}}{2} + \frac{\varepsilon_s^{s+}-\varepsilon_s^{s-}}{2}\bar{x}\right) - \frac{1}{\Psi+\Omega}\left(\frac{d_{31}V}{t_p}\right)\right] \tag{6}$$

where $\Psi = \dfrac{Y_s t_s}{Y_p t_p}$, $\Omega$ is a constant depending upon the assumed beam strain distributions ratio of the beam and $\bar{x}$ represents the piezoelectric location. Similarly, equilibrium equations can be solved to obtain strains as well as moments in case of embedded actuators in bending as well as shear mode.

Equilibrium equations can be solved by assuming a solution and finding unknowns of the assumed solution. A 'stress function $f_1(x, z)$' and an 'electric displacement function $f_2(x, z)$' can be introduced such that equilibrium relations are satisfied. These functions vary quadratically through the length and their constants can be obtained by using strain compatibility Eq. (7) and electric compatibility Eq. (8).

$$\frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = 0 \tag{7}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0 \tag{8}$$

In this way stress, strain and electric displacement variations through the thickness and length of a piezoelectric patch can be found (Huang *et al.* 2007). Resultant force acting on a piezoelectric patch can be obtained by integrating stress defined by constitutive equations Eqs. (9) and (10).

$$\{\sigma\} = [c]\{\varepsilon\} - [e]^T\{E\} \tag{9}$$

$$\{D\} = [e]\{\varepsilon\} + [\in]\{E\} \tag{10}$$

Using this resultant force, equations of motion can be obtained (Geradin and Rixen 1993). Equilibrium equations can be integrated to get equations of motion involving forces and moments. These forces and moments can be obtained by integrating constitutive equations of piezoelectric as well as of the host structure (Kargarnowin *et al.* 2007).

### *1.1.2 Hamilton's principle*

Principle of virtual work or Hamilton's principle Eq. (11) is a powerful tool to model dynamic systems (Petyt 1990).

$$\delta \int_{t_1}^{t_2} L dt = 0 \tag{11}$$

where $L = T - U + W$. User of this tool has to visualize the dynamic system and then write expression for total kinetic energy ($T$), total potential energy ($U$) and total work done ($W$) at a particular instant of time. Assumed shape functions Eqs. (12)-(14) for flexural and longitudinal motion can be used in kinetic and potential energy expressions (Sadri *et al.* 1999).

$$u(x, y, z) = \{\psi(x, y)\}^T\{h(t)\} \tag{12}$$

$$v(x, y, z) = \{\zeta(x, y)\}^T\{f(t)\} \tag{13}$$

$$w(x, y, z) = \{\phi(x, y)\}^T\{g(t)\} \tag{14}$$

These expressions when substituted in Hamilton's principle give equations of motion of the dynamic system. Typically, a piezoelectric patch may be acted upon by body force, surface traction, point force and surface charge. These all quantities would do some work on the piezoelectric patch. Their work contributions can be added along with total kinetic energy and potential energy of the piezoelectric patch and finally substituted into Hamilton's principle to obtain dynamic equations of motion. Application of variations *w.r.t.* displacement variable gives equation of motion of smart structure and *w.r.t.* electric potential gives equation of piezoelectric sensor and actuator (Lin and Huang 1999, Huang and Sun 2001, Mirzaeifar *et al.* 2008). If the structure is rotating about an axis, then kinetic energy term '$T$' of the Lagrangian functional would also have rotational kinetic energy term of the whole structure (Akella *et al.* 1994). Stresses due to rotations contribute to potential energy term '$U$' and angular velocity will contribute to the kinetic energy term '$T$'. The equation of motion can be then derived using Lagrange's Eq. (15).

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \left(\frac{\partial T}{\partial q}\right) + \frac{\partial U}{\partial q} = Q \tag{15}$$

The equation of motion (16) so obtained has two extra stiffness matrices: geometric matrix ($K_G$) and rotational matrix ($K_R$). Both of these matrices are proportional to the square of the angular velocity (Dhuri and Sheshu 2007).

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K_E + K_G + K_R]\{x\} = \{F\} \tag{16}$$

In case of thermo-electro-mechanical loading, effect of initial stress caused by thermoelectric loading would be considered in potential energy term Eq. (17) of the Lagrangian (Yang and Xiang 2007, Oh *et al.* 2007).

$$U = b\int_0^l \int_{-t/2}^{t/2} \left(\sigma_x \varepsilon_x + \frac{\pi^2}{12}\tau_{xz}\gamma_{xz} + \chi - D_x E_x - D_z E_z\right) dz dx \tag{17}$$

where $\chi = \frac{1}{2}\sigma_x^0\left(\frac{\partial w}{\partial x}\right)^2$ shows the effect of initial stress '$\sigma_x^0$' that is caused by thermoelectric loading before dynamic deformation takes place.

### *1.1.3 Finite element technique*

Finite element technique is most widely used tool for the design and analysis of smart structures. In the equations of motion of a smart structure, there are two variables namely: displacement and electric potential. A finite element formulation can be constructed and these two variables can be interpolated in terms of nodal values of the element. A three dimensional tetrahedral element (Fig. 2(a)) is the most basic geometric form in modeling arbitrary continuum and can be used to model an arbitrary shaped piezoelectric patch (Allik and Hughes 1970). When such thick solid elements are used to model very thin continua, they produce models which are stiffer than actual ones. To overcome this problem, hexahedral solid elements (Fig. 2(b)) with three additional internal degrees of freedom can be used (Tzou and Tseng 1991, Ha *et al.* 1992). Such 3D elements with nodal electric potential dofs when used to model the host structure, increase size of the problem and computational efforts. Therefore, two dimensional 4-noded quadrilateral element (Fig. 2(c)) based upon 'Classical Plate Theory' with 12-mechanical and 1-electric degrees of freedom per element can be used to model a smart structure. These elements overcome excessive stiffness problem and reduce problem size as well as computational efforts (Hwang and Park 1993). However, to capture the dynamics well, three dimensional piezoelectric finite elements are preferred over two dimensional ones (Manna *et al.* 2009). But when these elements are applied to thin plate and shell structures,

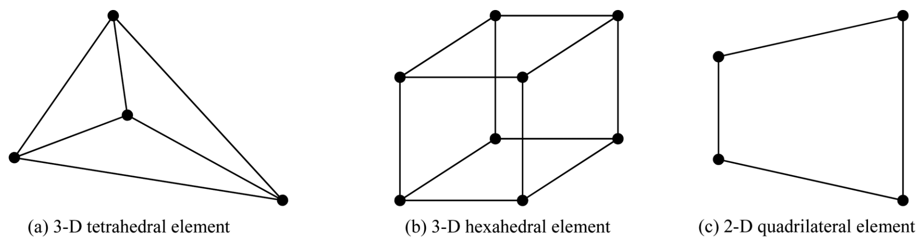


Fig. 2 Finite element shapes to model a smart structure

displacements for a given mesh are much smaller than numerically predicted and stresses are largely inaccurate. This happens due to locking of mesh. When structure is subjected to pure bending then 3D elements applied to thin structures give false shear stress and hence false shear strains in the thickness direction. These false shear strains absorb energy and some of the energy that should go into bending is lost. Therefore, such elements become too stiff in bending and the resulting deformation is smaller than actually it should be. In finite element literature, this phenomenon is known as shear locking (Bathe 1996). When curved structures bend, the vertical edges of element are oblique and can activate thickness/normal strains in thickness direction which lead to trapezoidal locking (MacNeal 1987) due to false thickness strains (Sze 2000). Thickness locking is introduced when false plane strain conditions ($\varepsilon_z = 0$) are predicted instead of expected plane-stress conditions ($\sigma_z = 0$) if the element is loaded by bending moment.

Various techniques have been developed to overcome locking effects. If we use (i) 20-noded 3-dimensional brick elements to model piezoelectric patch and the neighbourhood (Fig. 3(a)) (ii) 9-noded 2-dimensional shell elements to model rest of the host structure (Fig. 3(c)) (iii) 13-noded flat transition elements (Fig. 3(b)) to connect solid & shell elements and (iv) excessive aspect ratios of solid & transition elements are avoided then, computational efforts as well as locking effects are minimized (Kim *et al.* 1997). Also, dynamics of smart structure is well captured (Kim *et al.* 1996). In case of active constrained layer damping (ACLD), 3-dimensional solid elements to model ACLD area, 2-dimensional shell elements to model host structure and transition elements to connect solid and shell elements is a wise element selection (Varadan *et al*. 1996). Piezoelectric finite element models with independently assumed displacement and electric potential are called irreducible finite element models because the field variables cannot be further reduced. These models are too stiff, susceptible to mesh distortion and locking phenomena. Such models can also be improved if we use mixed/hybrid assumptions.

Hybrid tetrahedral finite element with assumed displacement, electric potential and electric displacement can be derived using 'hybrid variational principle' (Ghandi and Hagood 1997). However, 8-noded hexahedral hybrid piezoelectric finite element with assumed stress, assumed electric displacements

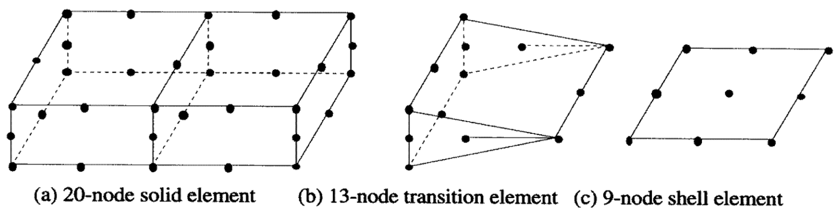


Fig. 3 A combination of elements to model a smart structure with piezoelectric patches

and both, is more accurate as well as less sensitive to element distortion and aspect ratio. This hybrid piezoelectric finite element is derived using 'general hybrid variational principle' which assumes displacement, stress, strain, electric potential, electric field and electric displacement as independent field variables and the functional is given by

$$\Pi = \int_{\Omega}\left[H - \begin{Bmatrix}\sigma \\ D\end{Bmatrix}\left(\begin{Bmatrix}\varepsilon \\ -E\end{Bmatrix} - \begin{Bmatrix}\nabla_m u \\ \nabla_p \Phi\end{Bmatrix}\right) - f^T u\right] dv$$

$$- \int_{S_T} (\bar{t})^T u ds - \int_{S_e} \Phi \bar{d} ds - \int_{S_u} (n_m \sigma)^T (u - \bar{u}) ds + \int_{S_\Phi} (n_e D)^T (\Phi - \bar{\Phi}) ds \tag{18}$$

where $H = \frac{1}{2}\begin{Bmatrix}\varepsilon \\ -E\end{Bmatrix}^T \begin{bmatrix} c & e^T \\ e & -\in \end{bmatrix}\begin{Bmatrix}\varepsilon \\ -E\end{Bmatrix}$ is electric enthalpy, $\nabla_m$ is differential operator in stress equilibrium equations (-), $\nabla_e$ is differential operator in charge conservation condition , $n_m$ & $n_e$ are matrices containing unit outward normal vectors of the respective boundaries and $\bar{t}$, $\bar{u}$ & $d$ are prescribed traction, displacement & electric displacement respectively (Sze and Pan 1999). ANS method (Sze and Yao 2000a, Bathe and Dvorkin 1986) removes shear locking by interpolating natural transverse shear strains from their samples along the element edges. These sample strains are common to the elements sharing the same edge. In this way, the number of independent shear strains is reduced. ANS method can also be used to remove trapezoidal locking by interpolating natural thickness strains at the midpoints of element corners (Stolarski 1991). Hybrid stress formulations along with ANS method are used to construct HS-ANS solid shell element which can remove shear locking and trapezoidal locking. Thickness locking can be removed by modifying the generalized stiffness of the laminate host (Sze and Yao 2000b). A more accurate and computationally efficient novel RHEAS element can be used to overcome shear locking and thickness locking (Zheng *et al.* 2004). It can be derived by incorporating enhanced assumed strain (EAS) modes (Simo and Armero 1992) and orthogonal interpolation modes (Chen and Cheung 1992) in the modified variational principle functional (19)

$$\Pi = \frac{1}{2}\begin{Bmatrix}\varepsilon \\ \nabla_e \Phi\end{Bmatrix}^T \begin{bmatrix} c & e^T \\ e & -\in \end{bmatrix}\begin{Bmatrix}\varepsilon \\ \nabla_e \Phi\end{Bmatrix} - (\sigma^T + \beta)(\varepsilon - \nabla_m u_q) + \sigma^T(\nabla_m u_\lambda) + W \tag{19}$$

where $\beta$ is a constant and $u_q$ & $u_\lambda$ are conventional displacement field & non-conforming displacement field respectively.

For actively controlling a smart structure, size of the mathematical model should be as small as possible. Large sized mathematical models can be reduced by employing modal truncation (Sharma *et al.* 2005), Guyan reduction (Guyan 1965) etc. Another option available for obtaining a small size model is to use spectral finite element. In spectral finite element, displacement function Eq. (20) is assumed to be product of spatial variable and a harmonic function in time.

$$u = U(x)e^{i\omega t} \tag{20}$$

Substitution of these displacement functions into equations of motion of smart structure yields spatial functions Eq. (21) as summation of harmonic functions in space.

$$U(x) = a_1 e^{ik_1 x} + a_2 e^{-ik_1 x} + a_3 e^{ik_2 x} + a_4 e^{-ik_2 x} \tag{21}$$

where $k_1$, $k_2$ are wave numbers of the system and $a_i$ depend upon structural properties. Finite element thus developed exactly captures the physics of the wave propagation and also, very less number of elements are required to model the dynamics of the structure (Baz 2000).

Thin structures can be analyzed using 'Classical Beam/Plate Theory' wherein shear deformations are ignored. It is adequate for most of the structures where thickness is small, by two orders of magnitude as compared to inplane dimensions (Ochoa and Reddy 1992). However, application of this theory on thicker beams/plates gives wrong results. Higher order displacement field through the thickness is essential to analyze thick beam/plate undergoing shear deformation. So, higher order (3$^{rd}$ order) displacement function Eq. (22) using Reddy's higher order shear deformation theory (Mitchell and Reddy 1995) can be assumed as

$$u(x, z) = u_0(x) + g_1(z)\frac{\partial w_0(x)}{\partial x} + g_2(z)\,\psi(x) \tag{22}$$

$$w(x, z) = w_0(x) \tag{23}$$

where $g_1(z) = \frac{4z^3}{3h_e^2}$, $g_2(z) = z - \frac{4z^3}{3h_e^2}$, $u_0$ & $w_0$ are the displacements of a point in the mid-plane, $\psi$ represents the shear effect and '$h_e$' is the thickness of the finite element. According to Gauss's law Eq. (24) (Griffiths 1999), divergence of electric displacement for a piezoelectric material is zero i.e., electric displacement should be constant throughout the thickness

$$\vec{\nabla} \cdot \vec{D} = 0 \tag{24}$$

since there is no free charge inside a piezoelectric material. Electric displacement is a function of strains and electric field. For a thick piezoelectric material undergoing shear deformation, strains through the thickness are non-uniform (Eqs. (22) and (23)). So, electric field through the thickness cannot be assumed constant because such assumption would make electric displacement variable through the thickness and Gauss's law would be violated. It is essential for thick structures, to take higher order through the thickness variation of electric field Eq. (25).

$$\phi_i(x, \tilde{z}_i) = f_{1i}(\tilde{z}_i)E_{ti}(x) + f_{2i}(\tilde{z}_i)E_{bi}(x) + f_{3i}(\tilde{z}_i)V_i(x) \tag{25}$$

where $f_{1i}$, $f_{2i}$ & $f_{3i}$ are third order functions in terms of $\tilde{z}_i\left(= \frac{z - z_{0i}}{h_i} + \frac{1}{2}\right)$ and $h_i$. $z_{oi}$ and $h_i$ are mid-plane position and thickness of the $i^{th}$ layer (Wang *et al.* 2007). Piezoelectrically actuated forces can be estimated efficiently if in-plane strains through the thickness of smart structure are modelled properly. In-plane strains through the thickness can be modelled by layer-wise displacement theory (Robbins and Reddy 1991). In layer-wise displacement theory, the structure is assumed to be made up of finite layers. Displacement fields *u*, *v* & *w* (Eqs. (26)-(28)) at any point in the volume of the structure is linear combination of values of displacement at the nodes of the finite layers.

$$u(x, y, z, t) = \sum_{j=1}^{N} U^J(x, y, t)\Phi^J(z) \tag{26}$$

$$v(x, y, z, t) = \sum_{j=1}^{N} V^J(x, y, t)\Phi^J(z) \tag{27}$$

$$w(x, y, z, t) = W(x, y, t) \tag{28}$$

where $U^J$ & $V^J$ are undetermined coefficients, $\Phi^J$ is the Lagrangian interpolation function through the thickness and $N$ is the number of degrees of freedom for the in-plane displacement along the thickness. So to model a composite beam, a two node beam element can be taken with four mechanical and 3 m degrees of freedom per node where '*m*' is the number of piezoelectric layers in an element (Han and Lee 1998, Han *et al*. 1999). Developments in piezoelectric finite elements are well tabulated showing element shape, assumptions made, beam/plate theory used etc. (Benjeddou 2000).

#### *1.1.4 Modal testing*

Many times, mathematical model of the smart structure is required in state space form Eq. (29) to apply

$$\{\dot{s}\} = [A]\{s\} + Bu \tag{29}$$

suitable control law. State space model of the system consists of natural frequencies, damping ratios, normalized mode shapes and modal actuation force per unit actuator voltage. All these constants can be obtained by manipulating frequency response functions (FRF) of the smart structure i.e., by performing modal test on the smart structure. If '*x*' is a vector of surface displacements in physical coordinates then for an applied force '*f*' at a spatial position '*l*' and a response measurement at spatial position '*k*', the FRF equation in physical coordinates is

$$\frac{x_k}{f_i} = \sum_{i=1}^{n}\left[\frac{\Phi_{ki}\Phi_{li}}{a_i(s-\lambda_i)} + \frac{\Phi_{ki}^*\Phi_{li}^*}{a_i^*(s-\lambda_i^*)}\right] = \sum_{i=1}^{n}\left[\frac{A_i^{k,l}}{(s-\lambda_i)} + \frac{A_i^{k,l*}}{(s-\lambda_i^*)}\right] \tag{30}$$

where '*' refers to the complex conjugate, $n$ is the number of modes and pole residue $A_l^{k,\,l} = \dfrac{\Phi_{ki}}{\Phi_{li}a_i}$. Natural frequency and damping ratios can be obtained by curve fitting the actual transfer function. Normalized mode shapes and modal actuation forces can be obtained by comparison of numerators of theoretical and experimental transfer functions. FRF equation of a structure gives the response measurement at a spatial position *w.r.t.* applied force at another spatial position. It is possible to convert the FRF so that voltage sensed by a piezoelectric sensor is available in response to voltage applied on a piezoelectric actuator. This way, pole residue modal model becomes compatible with broadly available modal analysis algorithms. Existing curve-fit software can be used to estimate system poles, residues and electromechanical coupling matrix (William *et al.* 1994).

## 2. Techniques to implement sensor-actuator design

Piezoelectric materials become electrically charged when subjected to mechanical strains and vice versa. These effects are known as direct and inverse piezoelectric effects. These effects can be written in the form of two constitutive equations (Cady 1964).

$$\text{Direct Piezoelectric Effect: } \{D\} = [e]\{\varepsilon\} + [\in]\{E\} \tag{31}$$

$$\text{Inverse Piezoelectric Effect: } \{\sigma\} = [c]\{\varepsilon\} - [e]^T\{\mathrm{E}\} \tag{32}$$

Eq. (31) associated with direct piezoelectric effect serves as the principle for piezoelectric sensor and Eq. (32) associated with inverse piezoelectric effect serves as the principle for piezoelectric actuator. In actively controlled piezo smart structures, piezoelectric materials can be used as: self-sensing actuators, extension-bending actuators, shear actuators and modal sensors/actuators. In the following sub-sections, state-of-the-art to implement these designs is discussed.

### *2.1 Self-sensing actuators*

A single piezoelectric patch can exhibit direct as well as inverse piezoelectric effect simultaneously. Hence, a single piezoelectric patch can be used as sensor/actuator simultaneously. Sensor/actuator which can be used to actuate/sense as well is called concurrent sensor/actuator. Self-sensing/concurrent-sensing concept ensures perfect sensor-actuator collocation which leads to better control stability, less weight and low cost of the smart structure (Dosch *et al.* 1992, Anderson *et al.* 1992).

In self-sensing piezoelectric actuators, main challenge is to separate sensor signal from the control signal. It can be done using a bridge circuit. Piezoelectric is introduced in bridge circuit in the form of a voltage source due to control signal and a pure capacitor ($C_{piezo}$) in series. The circuit is divided into two parts using principal of superposition: one with control signal and second with control plus sensing signal (Fig. 4). Bridge balance condition is achieved by introducing a matching capacitance ($C_{Matched}$). When perfectly balanced, this circuit acts as a simplified voltage divider which gives the difference between two parts leaving only sensor signal as out put. Capacitance '$C_1$' has been introduced in the circuit to represent signal conditioning device (Fig. 4) (Simmers *et al.* 2004). The obtained signal is then manipulated, amplified and fed back to actuator with suitable gains so as to induce control action. Bridge circuit should extract accurate mechanical strain rate signal i.e., sensor signal (Yang and Chiu 1994). Performance of self-sensing scheme also depends upon electromechanical coupling effects between piezoelectric material and the structural vibrations (Yang and Jeng 1996).

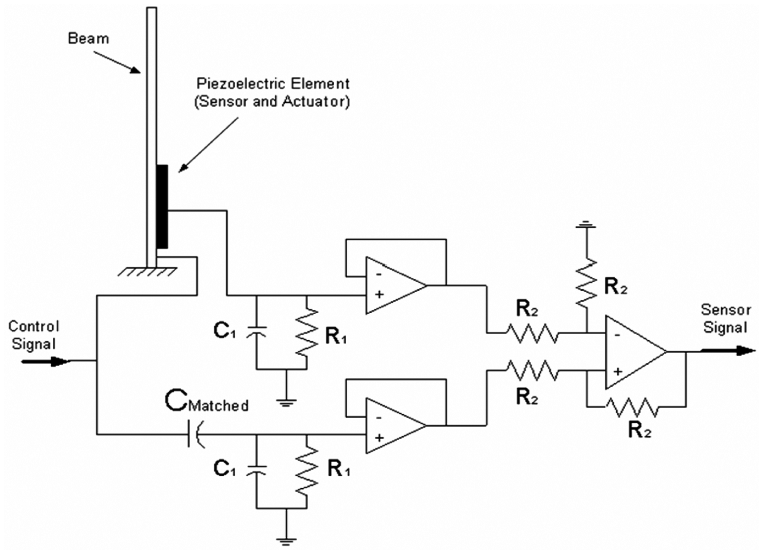


Fig. 4 Self-sensing actuator with a bridge circuit

Due to thermal dependence of piezoelectric capacitance, a perfect match of time constant cannot be maintained under varying thermal conditions. Hence as the temperature changes, capacitance of piezoelectric patch changes and match condition is lost. In such a situation, self-sensing cannot be done (Zhang *et al*. 2004). Match condition can be achieved if a set of capacitors is used such that a particular capacitor enters into the bridge circuit at a particular range of temperature. Such bridge circuit design enhances the stable operating thermal range by 95ºC (from 70ºC to 165ºC) (Simmers *et al.* 2007).

### *2.2 Extension-bending and shear actuators*

A piezoelectric material subjected to an external electric field extends or contracts depending on the direction of the applied field and the internal polarization (Cady 1964). Mostly for structural vibration control, electric field is applied parallel to the poling direction of the piezoelectric actuator (Fig. 5(a)) (Crawley and de Luis 1987, Baz and Poh 1988). It produces extension in the piezoelectric actuator and thereby bending of host structure. Such an actuator is called extension-bending actuator. It is also possible to excite the structure by applying electric field perpendicular to the poling direction of the piezoelectric actuator (Fig. 5(b)) (Sun and Zhang 1995, Zhang and Sun 1996). It shears the piezoelectric patch causing shear actuation and thereby bending the host structure. Shear actuation is more efficient then the extension actuation for stiff structures and thick piezoelectric actuators. Extension-bending actuators produce boundary concentrated forces and moments in the structure whereas sandwich shear actuators induce distributed moments in the structure. Therefore, less debonding problems occur with shear actuation. Extension actuators are effective when they are long and placed near the clamped end of beam whereas shear actuators are effective even for very short lengths and over a long range of position (Benjeddou *et al*. 2000). Shear actuation is more efficient in actively controlling the vibration for the same control effort. It is possible to model both the actuation mechanism in a unified way if two electric fields i.e., one along the direction of polarization and another perpendicular to the direction of polarization are substituted in the constitutive equations of piezoelectricity (Raja *et al.* 2002).

Direction of the stresses applied by piezo-actuators on the host structure is a function of: (i) the angle '$\phi$' between the applied electric field and the poling axis and (ii) fibre '$\psi$' orientation in case

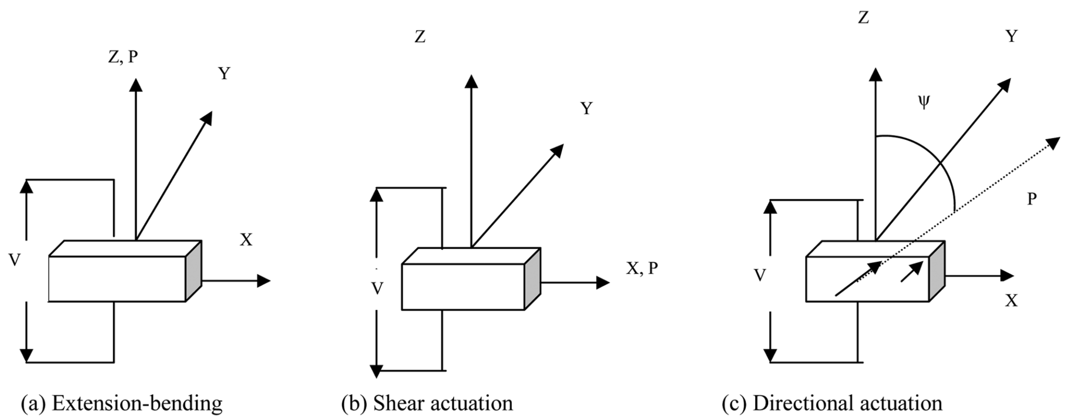


Fig. 5 Various piezoelectric actuation techniques being realized by altering the direction between the applied electric potential 'V' and the poling direction 'P'

of composite piezoelectric material (Fig. 5(c)). Piezo-actuators where $d_{31} \neq d_{32}$ are referred to as anisotropic piezo-actuators. Such anisotropic piezo-actuators possess directional actuation characteristics. Therefore while modeling a smart structure with anisotropic piezoelectric material, '$\psi$' and '$\phi$' can not be ignored (Raja *et al.* 2004, Wang *et al.* 2001). Ignorance of this anisotropic property of such piezo-actuators will cause underestimation of control voltages (Shaik *et al.* 2008).

### 2.3 Modal sensors/actuators

Many times, only some particular structural modes need to be controlled. For such a control of a structure, the mathematical model (say finite element model) is usually converted into modal domain by using following orthonormal modal transformation.

$$\{x\}_{nx1} = [U]_{nxm}\{\eta\}_{mx1} \tag{33}$$

where '$n$' is the total number of modes of the system and '$m$' is the number of modes to be considered (Sharma *et al.* 2007). This converts coupled differential equations into uncoupled ones. Now, it becomes quite natural to develop a sensor technology which senses only a particular mode. Such a sensor is called modal-sensor and its signal is manipulated, amplified, and then fed back to so called modal-actuator. Use of independent modal sensor/actuator reduces the required number of sensors/actuators, dimensionality of the controller and required control energy (Meirovitch *et al.* 1983). These can be used for independent control of structural modes i.e., independent modal space control (Meirovitch *et al.* 1988). Interaction between the actuator and structural vibrations under closed circuit conditions induces modal coupling and hence higher modes get excited (Proulx and Cheng 2000). This is called spillover effect and it degrades the performance of active vibration control. To avoid these effects, spatially distributed piezoelectric modal sensors/actuators for beam structure can be used (Lee and Moon 1990). Spillover effects can also be minimized by optimal placement of piezoelectric sensors and actuators on a smart structure (Gupta *et al.* 2010). Spatially distributed self-sensing piezoelectric actuators can also be used for modal control of a cantilevered beam (Tzou and Hollkamp 1994). Design of spatially distributed modal sensor/actuator is based upon structural mode shapes and is used for simple structures only. Modal sensors/actuators using an array of piezoelectric patches and feedback gain design can be used for complex structures (Chen and Shen 1997). Such sensors/actuators are called discrete modal sensor/actuator. Also, same array of piezoelectric patches can be given multiple weights and can act as a modal sensor/actuator for multiple modes simultaneously and independently. These can be realized using different design and implementation techniques (Fripp and Atalla 2001).

## 3. Techniques to model real life environmental effects

Performance of piezoelectric sensors and actuators is sensitive to thermal, hygro, non-linearities, fatigue and ageing effects etc. A smart structure instrumented with piezoelectric patches is intended to work in real-life environment where these effects are encountered. Also, electromechanical interaction of piezoelectric sensors and actuators in real life environment has not been well understood. This is the real and actual potential hindrance to achieve AVC in real-life situations. Therefore, mathematical models robust to these effects are desired. In the following sub-sections, techniques to model thermal, hygro, non-linearities, fatigue and ageing effects in piezoelectric sensors and actuators are discussed.

### 3.1 Temperature effects

This effect is the most encountered effect during the operation of a piezo smart structure. Piezoelectric materials are processed and standardized at a stress-free temperature called 'reference temperature'. Eqs. (31) and (32) are applicable when operating temperature, termed as 'ambient temperature', is same as 'reference temperature'. Mostly, researchers have attempted AVC at 'reference temperature' using these constitutive equations (Crawley and de Luis 1987, Baz and Poh 1988, Hwang and Park 1993, S.M. and Lee 1994b, Kim *et al.* 1996). However in real-life situation, piezo smart structure may have to work at 'ambient temperature' other than 'reference temperature'. It would induce thermal stresses in the piezoelectric material. The theory of piezoelectricity, with inclusion of thermal effects is known as piezothermoelasticity and is well established in literature (Mindlin 1961, Tiersten 1971, Mindlin 1974, Nowacki 1978, Nowacki 1982, Sunar and Rao 1992). Piezothermoelastic studies have also been extended to design smart piezo structures operating at 'ambient temperatures' other than 'reference temperature' (Tzou and Ye 1993, Sunar and Rao 1997). Most of the research is concentrated on developing piezothermoelastic finite element models of smart piezo structures (Rao and Sunar 1993, Tzou and Howard 1994, Bao *et al.* 1998, Raja *et al.* 1999, Balamurugan and Narayanan 2001, Gornandt and Gabbert 2002, Jiang and Li 2008). Developments in piezothermoelasticity in relevance to smart piezo structures are also well documented (Saravanos and Heylinger 1999, Sunar and Rao 1999, Tauchert *et al.* 2000). When 'ambient temperature' is other than 'reference temperature', thermal effect can affect the response of smart piezo structure in three distinct ways: (i) thermal strain effect i.e., introduction of thermal stresses in smart structure as a result of different thermal expansion coefficients (ii) pyroelectric effect i.e., generation of electric displacement / voltage on piezoelectric material and, (iii) change in elastic, piezoelectric & dielectric properties of smart structure. AVC performance is influenced by thermal strain and pyroelectric effects (Tzou and Ye 1994, Tzou and Ye 1996, Friswell *et al.* 1997, Narayanan and Balamurugan, 2003, Jiang and Li 2007). These effects induce only static displacement in a piezo smart structure (Kumar *et al.* 2008, Roy and Chakraborty 2009). Influence of thermal stresses upon isotropic substrate material can be ignored in case of steel, aluminum and titanium alloys within temperature range of ±200°C.

Piezo smart structure can be excited by giving a thermal load i.e., subjecting it to temperature change for some time and can be controlled by simple negative velocity feedback (Chandershekhara and Tanneti 1995, Raja *et al.* 2004, Kumar *et al.* 2008). Piezo smart structure subjected to heat flux are analyzed using the concept of entropy which is a function of strain, electric field and temperature. Helmholtz free energy (34) can be formulated as (Giannopoulos and Vantomme 2006)

$$F_H = \frac{1}{2}K_{ij}\varepsilon_i\varepsilon_j - e_{ij}E_i\varepsilon_k - \frac{1}{2}\in_{lk}E_lE_k - p_lE_l\theta - \alpha_i\varepsilon_i\theta - \frac{1}{2}\beta\theta^2 \tag{34}$$

where $\beta$ is the ratio of heat capacity to ambient temperature. Variation of Helmholtz free energy *w.r.t.* mechanical displacements, electric field and temperature would give equation of motion of a smart structure, sensor-actuator equation and heat transfer equation respectively.

In all piezothermoelastic studies of smart piezo structures referred above, influence of temperature on active structural vibration control has been theoretically investigated using linear piezoelectric constitutive equations

$$\{D\} = [e]\{\varepsilon\} + [\in]\{E\} + \{p\}\Delta T \tag{35}$$

$$\{\sigma\} = [c]\{\varepsilon\} - [e]^T\{E\} - [c]\{\alpha\}\Delta T \tag{36}$$

Constitutive Eqs. (35) and (36) address thermal strain and pyroelectric effects only and changes in elastic, piezoelectric & dielectric properties of smart piezo structure at 'ambient temperatures' other than 'reference temperature' are ignored. It has been experimentally found that piezoelectric sensor and actuator perform differently at different ambient temperatures (Birman 1996, Schulz *et al.* 2003, Li *et al.* 2008, Yang and Xiang 2007). Dynamic transfer function between voltage applied to actuator and the voltage measured by sensor changes with temperature and repeated thermal cycles cause reduction in frequency, stiffness, capacitance and dynamic performance of sensor (Bronowicki *et al.* 1996). Reduction in stiffness with increase in temperature causes increase in vibration amplitude of the actuator (Yang and Xiang 2007). Change in temperature is not attributed only to the environmental conditions but smart material manufacturing processes can also induce temperature changes. Manufacturing process of thermoplastic composites embedded with PZT patches involves higher temperature treatments. It causes significant losses in PZT generated voltage even within safe temperature limits. However, these losses can be minimized if patch location is at about $1/4^{th}$ of the composite plate thickness (Elsoufi *et al.* 2007). These experimental findings can not be explained by constitutive Eqs. (35) and (36). In these constitutive equations, piezoelectric stress coefficients ($e_{31}$, $e_{33}$, $e_{15}$) & permittivity ($\in_{33}$) are measured at 'reference temperature' only and are assumed not to vary away from 'reference temperature'. In other words, piezoelectric stress coefficients & permittivity are assumed constant at all temperatures. Sensing and actuation behaviour of piezoelectric materials depend upon their strain/stress coefficients which are different at different ambient temperatures (Jaffe *et al.* 1971). Experimental characterization of piezoceramic PZT-5H (Fig. 6) reveals that: (i) magnitude of piezoelectric strain coefficient ($d_{31}$) increases linearly with temperature & (ii) permittivity ($\in_{33}$) increases non-linearly with temperature (Wang *et al.* 1998). These linear variations of '($d_{31}$)' cause as much as 75% decrease in piezoelectric strain output by actuator within temperature range of -150 to 100 °C (Wang *et al.* 1998). Therefore while actively controlling structural vibrations at temperatures away from reference temperature, influence of temperature on piezoelectric strain coefficient ($d_{31}$) & permittivity ($\in_{33}$) can not be ignored.

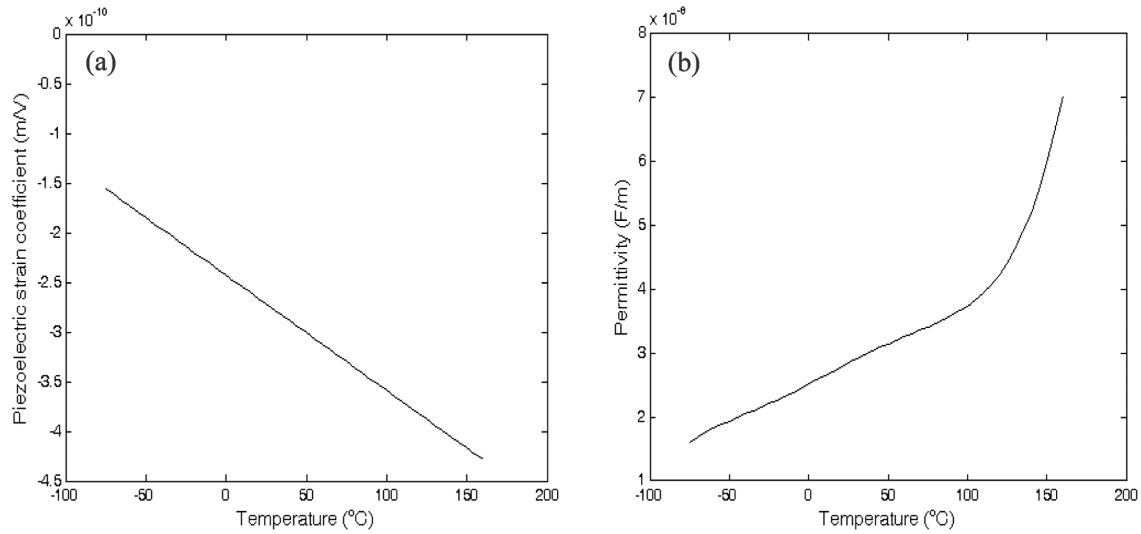


Fig. 6 Temperature dependence of piezoelectric strain coefficient ($d_{31}$) & permittivity ($\in_{33}$)

Constitutive equations which take into account the temperature dependence of '$d_{31}$' and '$\in_{33}$', should be used. When temperature dependence of '$d_{31}$' and '$\in_{33}$' is considered, augmented constitutive equations are (Wang *et al.* 1998)

$$\varepsilon_{ij} = s_{ijkl}^{E,T}\sigma_{kl} + d_{nij}^{T}E_n + \bar{d}_{nij}E_n\Delta T + \alpha_{ij}^{E}\Delta T \tag{37}$$

$$D_n = d_{nij}^{T}\sigma_{ij} + \bar{d}_{nij}\sigma_{ij}\Delta T + \bar{\in}_{nm}E_m\Delta T + p_n^{\sigma}\Delta T \tag{38}$$

In augmented constitutive Eqs. (37) and (38), temperature dependent parts '$\bar{d}_{31}$' & '$\bar{\in}_{33}$' have been experimentally measured at ambient temperatures as shown in Fig. 6 (Wang *et al.* 1998). Simulation studies using these augmented constitutive equations show that (i) higher stresses are induced in PZT-5H at temperature higher than 'reference temperature' and, stresses are reduced at temperature lower than 'reference temperature' (Joshi *et el.* 2003) & (ii) power consumption in a smart piezo structure is different at different 'ambient temperatures' (Apte and Gauguli 2009). Therefore, experimental findings are explained if augmented equations are used. If temperature dependence of '$d_{31}$' and '$\in_{33}$' is not included in constitutive equations then: (i) vibration response of the smart structure would be wrongly estimated by sensor (ii) actuator response would also be wrongly estimated (iii) controller would apply wrong control voltages on actuator and, (iv) performance of AVC would not be maintained and it can even fail if ambient temperatures are substantially away from reference temperature. Such temperature variations would be frequently encountered in real life situations where AVC is to be used in near future. Therefore, it is recommended that temperature dependence of '$d_{31}$' and '$\in_{33}$' be included in constitutive equations. Since, accurate analytical relationship for temperature dependence of '$d_{31}$' and '$\in_{33}$' is not available so it can be found by experimentally characterizing the piezoelectric material being used as sensor and actuator. Temperature dependent relations for permittivity, elastic compliance and piezoelectric constants are also derived using thermo dynamical considerations (Luck *et al.* 1998). Sensitivity analysis can be used to find out the impact of variation in the elements of elastic constant matrix, piezoelectric constant matrix and dielectric constant matrix on smart structural response. For example, if piezoelectric polarized in $z$-direction is subjected to electric field along the same direction then, change in $e_{31}$ has most impact on the response of smart structure as compared to change in $e_{33}$ and $e_{15}$ (Perry *et al.* 2008).

### *3.2 Hygro effects*

Moisture is another environmental factor which affects the AVC performance. Moisture/hygro effect can modify structural stiffness and piezoelectric behavior resulting in different control performance. In situations where effect of moisture and temperature is appreciable, moisture-temperature dependent form of the constitutive Eqs. (39) and (40) can be used (Smittakorn and Heyliger 2000).

$$\sigma_{ij} = c_{ijkl}(\varepsilon_{kl} - \alpha_{kl}\Delta T - \beta_{kl}\Delta\gamma) - e_{kij}E_k \tag{39}$$

$$D_j = e_{jkl}\varepsilon_{kl} + \in_{jk}E_k + p\Delta T + \chi\Delta\gamma \tag{40}$$

Using the principal of virtual work, finite element equations of motion can be derived. These equations of motion have one extra stiffness matrix due to stresses developed by temperature & moisture and one extra load vector due to hygrothermal loading (Raja *et al.* 2004).

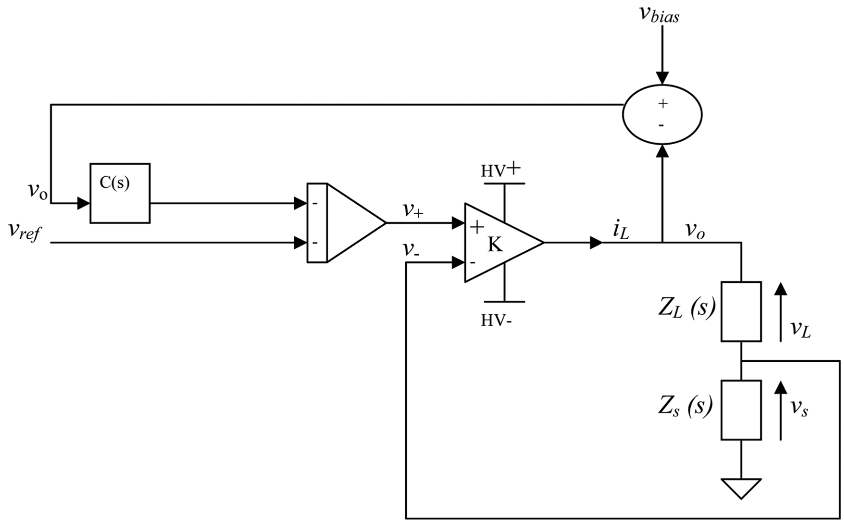


Fig. 7 A schematic of a compliance feedback current amplifier.

### 3.3 Non-linearities, fatigue and ageing effects

Piezoelectric materials exhibit non-linearities called hysteresis when driven by high electric field or when high mechanical stress is applied. It non-linearly increases the dielectric, piezoelectric and elastic coefficients of the piezoelectric material (Alberada *et al.* 2000, Garcia *et al.* 2001, Masys *et al.* 2003). Hysteresis effects are attributed to intrinsic contributions due to elastic deformation of the crystal cell and extrinsic contributions due to rotation of dipoles formed by the presence of defects. Intrinsic contributions are perfectly linear while extrinsic contributions are non-linear. These non-linearities can be introduced in the piezoelectric constitutive equations (Perez *et al.* 2004)

$$\varepsilon = [s_{int} + s_{ext}\, g(\sigma)](1 - \nu)\sigma + 2[d_{int} + d_{ext}\, f(E)]E \tag{41}$$

$$D = [d_{int} + d_{ext}\, g(\sigma)]\sigma + 2[\in_{int} + \in_{ext} f(E)]E \tag{42}$$

where $s_{int}/d_{int}/\in_{int}$ & $s_{ext}/d_{ext}/\in_{ext}$ are compliance $(s_{11})$ / piezoelectric strain coefficient $(d_{31})$ / permittivity $(\in_{33})$ due to intrinsic & extrinsic effects and $g(\sigma)$ & $f(E)$ are monotonous increasing functions of $\sigma$ & $E$ respectively. Need for an accurate constitution equation incorporating hysterisis effects can be avoided if current or charge source is used instead of voltage source to drive actuators (Fleming and Moheimani 2004). By regulating the charge or current, fivefold reduction in hysterisis can be achieved (Tataka *et al.* 1989, Ge and Jouaneh 1996). Such a control scheme is referred to as 'compliance feedback charge amplifier' (Fig. 7) (Fleming and Moheimani 2004). It is capable of reducing hysterisis and vibration as well.

In compliance feedback charge amplifier, role of high voltage operation amplifier '$K$' is to equate the input signal '$q_{ref}$' to the voltage across the sensing capacitance '$C_s$'. Load impedance '$Z_L(s)$' experiences a charge '$q$' proportional to '$q_{ref}$'. Inverted reference voltage '$v_{ref}$' is maintained across the sensing impedance '$Z_s(s)$'. Charge gain of the amplifier is '$C_s$' coulombs per volt. Controller '$C(s)$' regulates the output voltage '$v_0$' to zero in the additional feedback loop to prevent DC offsets. DC offsets arise due to charging of load capacitor as a result of uncontrolled nature of output/

compliance voltage. The additional output voltage feedback loop comprising of '$C(s)$', effectively estimates and rejects all sources of DC offset. Signal '$v_{bias}$' forces a non-zero constant offset voltage across the load. This feature is used for electrical pre-stressing of piezoelectric actuators to allow bi-polar (charge and current) actuation. (Vautier and Moheimani 2005). This compliance feedback charge amplifier is capable of providing high accuracy, low frequency and high frequency regulation of current or charge. State space equations of a piezo smart structure driven by a voltage source are

$$\begin{aligned} \{\dot{s}\} &= A\{s\} + Bv \\ \{y\} &= C_y\{s\} \\ V_p &= C_v\{s\} \end{aligned} \tag{43}$$

whereas state space equations of piezo smart structure driven by a charge source become

$$\begin{aligned} \{\dot{s}\} &= (A - BC_y)\{s\} + \left(\frac{B}{C_p}\right)q \\ \{y\} &= C_y\{s\} \\ V_p &= C_v\{s\} \end{aligned} \tag{44}$$

where $\{s\}$ is state vector of the system, $A$ is system state matrix, $B$ is control matrix, $v$ is control input voltage, $q$ is control input charge, $\{y\}$ is sensor output, $C$ is state output matrix, $V_p$ is induced voltage in the piezo-patch and $C_p$ is the capacitance of piezo-patch. From the Eqs. (43) and (44), we observe that system state matrix $A$ changes when piezo actuator is driven by charge instead of voltage source. Therefore, dynamics of the piezo smart structure change when we use electric charge instead of voltage source to drive actuator (Vautier and Moheimani 2005). The performance of AVC is also influenced by repeated fatigue loading of the actuator. However, it can be ignored if the strain levels do not cross 60% of the static strain limit (Bronowicki *et al.* 1996). Piezoelectric materials suffer a long term ageing after polarization even without external stresses (Li *et al.* 2008). Ageing effect can be estimated by experimental characterization of the ageing process. $d_{31}$ coefficient and dielectric constant of piezoceramic PZT-5H has ageing rate of -4.4% and -1.34% per time decade respectively after polarization without any external stresses (Glynne-Jones *et al.* 2001). Therefore, AVC design should also incorporate ageing process over the life time of device.

## 4. Techniques to control structural vibrations

Applying suitable modeling techniques as discussed in previous sections, final equation of motion of smart piezo structure becomes

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \tag{45}$$

Eq. (45) presents a coupled system of equations. Analysis of such a system would become easier if these coupled equations are converted into uncoupled equations. In that case, single degree of freedom system tools can be used to analyze the actual multi degree of freedom system. The process of analyzing multi degree of freedom system using single degree of freedom system tools is known as 'modal analysis'. Using transformation Eq. (33), coupled equations of motion are converted into uncoupled ones. Uncoupled equations of motion are used to control the desired vibration modes using appropriate control law. In order to control the response of a system using modern control theories, we need to convert uncoupled equations of motion into a state-space model. A state space model represents a system in the form of first order differential equations. The variables are called 'state variables' (say $\{s\}$) and set of values of these state variables at any time '$t$', is called 'state of the system' at time '$t$' (Gopal 2008). The state of the system at any time '$t$' is represented by the state equation

$$\{\dot{s}\} = A\{s\} + Bu \tag{46}$$

It relates the first derivative $\{\dot{s}\}$ of the state variables with variable themselves and the input '$u$'. The output of the system is a function of $\{s\}$ and is given by

$$\{y\} = C\{s\} \tag{47}$$

where $[A]$ is system state matrix, $[B]$ is control matrix and $[C]$ is output matrix. Eqs. (46) and (47) form 'state space model' of smart structure. To implement control laws, modal vectors are required which cannot be measured by a single sensor patch. However, modal vectors can be estimated using an observer/estimator. A computer program which estimates/observes full state vector is called a state estimator/observer (Gopal 2008). Suitable observer can be constructed using matrices $[A]$, $[B]$ and $[C]$ of the 'state space model'. Kalman (Sharma *et al.* 2005) and Luenberger (Bruant *et al.* 2001) observers have been used by researchers for AVC.

Once we have obtained: (i) the equation of motion (ii) matrices $[A]$, $[B]$ & $[C]$ and (iii) modal vectors of piezo smart structure, we can control its vibration response using suitable control technique. Depending upon the nature of piezo smart structure, many control techniques have been used by researchers in AVC viz. Independent Modal Space Control (Meirovitch 1989, Lee and Yao 2003), Modified Independent Modal Space Control (Baz and Poh 1988), Efficient Modal Control (Singh *et al*. 2003), Negative Velocity Feedback Control (Yang and Bian 1996, Chen *et al.* 1997), Output Feedback (Lim *et al.* 1999), Optimal Control (Han *et al*. 1997, Balamurugan and Narayanan 2001, Zhabihollah *et al.* 2007), Positive Position Feedback Control (Friswell and Inman 1999), Pole Placement Technique (Manning *et al.* 2000, Kumar and Khan 2007), Lyapunov Functional Control (Miller *et al.* 1995, Lin and Huang 1999), LMS algorithm (Amant and Cheng 2000), Sliding Mode Control (Kim and Wang 1993, Choi *et al.* 1995), Neural Networks (Rao *et al*. 1994, Smyser and Chandershekhara 1997), Fuzzy Logic Control (Sharma *et al.* 2007), $\mu$-synthesis Control (Li *et al.* 2003) and $H_2/H$ Control (Wang and Huang 2002, Kar *et al.* 2002) etc. Out of these control techniques, optimal control has been chiefly used in AVC. It provides a simple and powerful tool for designing multivariable systems and, controls several modes of the structure. It requires sufficiently accurate model of the system and observer to estimate the states of the system. Present day research is directed towards designing robust and adaptive controllers for AVC which can perform satisfactorily in complex structures and perturbed conditions as well. Therefore fuzzy logic, neural networks, sliding mode, H based controllers etc. are of interest to present researchers.

To implement the state-of-the-art discussed in previous sections, researcher and engineers need appropriate software to carry out simulations and experiments. Simulation analysis of a mathematical model derived using appropriate modeling technique and control law can be carried out by writing computer code in MATLAB (Zhabihollah *et al.* 2007), MATHEMATICA (Huang and Sun 2001), VAPAS (Yu and Hodges 2004), DYNADID2D (Bruant *et al.* 2001) etc. Commercial finite element packages such as ANSYS/Multiphysics (Abramovich and Pletner 1997, Karagulle *et al.* 2004, Jiang and Li 2007, Malgaca and Karagulle 2009, Benjeddou 2009), ABAQUS (Sze and Pan 1999, Sze and Yao 2000b), MSC/NASTRAN (Reaves and Horta 2003), COSAR (Marinkovic *et al.* 2007) etc. are also available to model piezo smart structure. Control action simulations can be carried out in ANSYS (Karagulle *et al.* 2004, Malgaca and Karagulle 2009), MATLAB/Simulink (Han *et al.* 1997), MATLAB/Control System Toolbox (Kumar and Singh 2006), SCILAB (Blanguernon *et al*. 1999, Bruant *et al.* 2001) etc. To implement AVC control algorithm experimentally, LabVIEW (Sharma *et al.* 2005, Zhabihollah *et al.* 2007, Malgaca and Karagulle 2009) is used.

## 5. Conclusions

In this review article, state-of-the-art for three important aspects of mathematical modeling of actively controlled piezo smart structures viz. (i) techniques to write equations of motion (ii) techniques to implement sensor-actuator design and (iii) techniques to model real life environment effects, have been reviewed. Aim of presenting these techniques in a unified way is to enable (i) a practicing engineer to pick the most suitable philosophy for his AVC application and (ii) a research student to know how the field has progressed and thus find the real & actual potential hindrances in the development of this field. These techniques are extracted from landmark works in the field. As of now, AVC has been applied on simple beam and plate structures. Authors conclude that it is possible to develop smart structures which: (i) are complex in nature (ii) capture structural dynamics well (iii) have appropriate sensor-actuator design (iv) take into consideration environmental effects and, (v) use robust vibration control technique. Therefore, researchers should try to model real-life complex three dimensional smart mechanical piezo structures. If this happens, this field of AVC would be serving the mankind in coming years.

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## Nomenclature

| | |
|---|---|
| $[M]$ | : Mass matrix |
| $[C]$ | : Damping matrix |
| $[K]$ | : Stiffness matrix |
| $\{x\},\{\dot{x}\},\ \{\ddot{x}\}$ | : Displacement, velocity & acceleration vectors |
| $u$, $v$, $w$ | : Displacements along x, y & z-axis |
| $f$ | : Force |
| $T$ | : Traction |
| $M$ | : Moment |
| $L$ | : Lagrangian |
| $T$ | : Kinetic energy of the system |
| $U$ | : Potential energy of the system |
| $W$ | : Work done by external force and electric effect |
| $\rho$ | : Density |
| $Y$ | : Young's modulus of elasticity |
| $s_{ij}$ | : Compliance |
| $v$ | : Poisson's ratio |
| $\varepsilon^{s+}$ | : Strain value at the left end |
| $\varepsilon^{s-}$ | : Strain value at the right end |
| $l$ | : Length |
| $b$ | : Width |
| $t$ | : Thickness |
| $dof$ | : Degrees of freedom |
| $\{\varepsilon_{ij}\}$ | : Normal strain vector in jth direction in the plane perpendicular to $i^{th}$direction |
| $\{\gamma_{ij}\}$ | : Shear strain vector in jth direction in the plane perpendicular to $i^{th}$ direction |
| $\{\sigma_{ij}\}$ | : Normal stress vector in jth direction in the plane perpendicular to $i^{th}$ direction |
| $\{\tau_{ij}\}$ | : Shear stress vector in jth direction in the plane perpendicular to $i^{th}$ direction |
| $\lambda_i$ | : $i^{th}$ eigen value |
| $\psi$, $\zeta$, $\phi$ | : Assumed displacement shapes in x, y & z-directions |
| $h$, $f$, $g$ | : Generalized coordinates of plate response in x, y & z-directions |
| $q$ | : Generalized coordinate |
| $Q$ | : Generalized force |
| $\{E\}$ | : Electric field vector |
| $\{D\}$ | : Electric displacement |
| $\{c\}$ | : Elasticity matrix |
| $d_{ij}$ | : Piezoelectric strain constant |
| $e_{ij}$ | : Piezoelectric stress constant |
| $[\in]$ | : Permittivity matrix |
| $[\Phi]$ | : Complex modal matrix of eigen vectors |
| $\theta$ | : Temperature |
| $\Delta T$ | : Change from ambient temperature |
| $\Delta\gamma$ | : Change in moisture concentration |
| $H$ | : Enthalpy |
| $F_H$ | : Helmholtz free energy |
| $\alpha_{kl}$ | : Coefficient of thermal expansion |
| $\beta_{kl}$ | : Coefficient of moisture expansion |
| $p$ | : Pyro electric constant |
| $\chi$ | : Moisture electric constant |
| $V$ | : Voltage applied to piezo-actuator |
| $\Phi$ | : Electric potential |

**superscripts**

| | |
|---|---|
| $p$ | : piezoelectric |
| $s$ | : substrate |
| $i$ | : in the ith direction |

**subscripts**

| | |
|---|---|
| $T$ | : Transpose |