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Damage detection in plates based on pattern search and Genetic algorithms

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Abstract. This paper is aimed at presenting two methods on the basis of pattern search and genetic algorithms to detect and estimate damage in plates using the modal data of a damaged plate. The proposed methods determine the damages of plate structures using optimization of an objective function by pattern search and genetic algorithms. These methods have been applied to two numerical examples, namely four-fixed supported and cantilever plates with and without noise in the modal data and containing one or several damages. The obtained results clearly reveal that the proposed methods can be viewed as a powerful and reliable method for structural damage detection in plates using the modal data.

Keywords: damage detection; plate; pattern search algorithm; genetic algorithm; modal data.

1. Introduction

Plates as prominent component of structures are widely used in different field of engineering applications such as civil, mechanical engineering, aerospace and so on. Evaluating of structural condition of plates is one of the most important aspects of health monitoring. Hence, a great deal of attention has been drawn to study of damage identification of plate like-structures over the few past decades. Existence of damage causes a change in structural physical characteristics, mostly in stiffness and damping at the location of damage which leads to alteration in dynamic parameters of a structure. This idea has been considered as basic concept behind of vibration based techniques for damage identification (Doebling *et al.* 1996, Salawu 1997).

Numerous approaches and various response parameters for localizing damage on structures have been proposed. However, the focus of the majority of these studies was centered in one-dimensional elements. Comparatively few researches have been reported in the literatures aimed at studying damage identification in plate like structures. Cawley and Adamas (1979) probably for the first time, extend a method for damage detection of plate using the changes in the frequency response function. Many effective methods for damage detection of plate have been presented based on the fact of extending the methods proposed for one-dimensional structural element (Cornwell *et al.* 1999, Yoon *et al*. 2005, Hadjileontiadis and Douka 2007). Wavelet analysis has been also widely

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employed in the area of two dimensional problems (Wang and Deng 1999, Douka *et al.* 2004, Chang and Chen 2004, Rucka *et al.* 2006). The significance of wavelet analysis is that it has wider practical application that can be applied to response with transient, nonlinear and non-stationary behavior. Bayissa *et al.* (2008) presented damage detection technique in a concrete plate model and in steel plate girder of bridge structure using continuous wavelet transform. Also Fan and Qiao (2009) have studied damage detection of a cantilever plate with different types of damages using continuous wavelet transform which only requires mode shape of the damaged plate and is superior in noise resistance and reliable with limited sensors data. Bagheri *et al*. (2009) have recently developed a new method based on curvelet transform to localize damage on plate like structures, while curvelet transform had been developed as robust transform to overcome inherent shortcomings of traditional multiscale presentation like wavelets. The methods mentioned above were mostly involved with localizing damage and couldn't provide a reliable feature in quantifying severity of damages.

In recent years, genetic algorithms (GAs) have been developed and promisingly applied to the field of damage identification. GAs have been applied to detect the damage location and severity by optimizing an error function. Unlike conventional search methods, GAs use set of points instead of a single point to search solution space. Furthermore, the stochastic nature and parallel computational framework of GAs lead to overcome the difficulties of conventional methods in converging to an absolute optimal solution. Mares and Surace (1996) used GAs to locate and identify structural damage from measured natural frequencies and mode shapes. Chou and Ghaboussi (2001) developed a method using GAs to detect structural damage. Optimizing objective functions was obtained using terms of static measured displacements. Another method is also proposed applying GA for damage detection based on the changes in frequencies and mode shapes of a beam (Perera and Torres 2006). A novel technique is also developed by Casciati (2008) based on deferential evolutionary algorithm to identify damages in structures. In proposed method objective function is formed as difference between measured and generated modal data. Lately, Yun *et al*. (2009) presented a method including two stages for damage identification. The subset selection method has been applied to detect multiple damage location in the first stage then; using GA the quantity of damages severity was determined.

Pattern search (PS) method is considered as a kind of prominent direct search strategies, which was first introduced by Box in 1957 (Box 1957) and Hooke and Jeeves (1961) in early 1960s. PS approach is a method of optimization in which there is no need for information about the gradient of objective function (Lewis *et al.* 2000). Unlike other heuristic algorithms, such as PS and GAs possesses a flexible and well-balanced operator to boost and adopt the global and fine tune local search. PS has been widely used in different field of engineering as robust and promising method to tackle barriers of traditional methods (Swann 1972, Lewis and Torczon 1999, Al-Sumait *et al.* 2007, Biondi *et al.* 2006).

In this research work, two new methods for localizing and also estimating the severity of structural damage are introduced. The damage identification is carried out through applying PS and GAs to minimize objective function derived from dynamic characteristics of damaged plate. Numerical examples show that the proposed method can be considered as a flexible and robust approach in damage identification of plates.

2. The proposed methods

In this section, the proposed methods for detection and estimation of damage in plate structures are described in detail. The objective function using the modal data is first formulated. Finally, the objective function is optimized by PS and GAs for localization and quantification of damage.

2.1 Objective function

The modal characteristics of a plate without damage are described by the equations

$$[\mathbf{K}^{ud} - \lambda_i \mathbf{M}]\mathbf{\Phi}_i = 0 \quad i = 1, 2, \ldots, n \tag{1}$$

where, $\mathbf{M}$ and $\mathbf{K}^{ud}$ are mass and undamaged stiffness matrices, respectively; λ_i is square of the natural frequency of the plate corresponding to mode shape $\mathbf{\Phi}_i$; and n is total number of obtained mode.

The inverse procedure for damage detection of plate that uses modal data to characterize damages requires a forward method, which is supposed to be dependent on parameters derived from damage quantification and sensitive to perturbation in the response function of plate as much as possible.

Assigning certain representation to stiffness of damaged area is a difficult problem because dimension and geometry of damage is unknown. Therefore, one of the simplest techniques to determine damage-induced alteration in stiffness is the degradation in Young's modulus of the element as follows

$$E_j^d = E_j - \Delta E_j \tag{2}$$

where, E_j^d and E_j are the damaged and undamaged Young's modulus of the jth element in the finite element model of the plate, respectively; and ΔE_j is the amount of decrease in Young's modulus induced by damage in jth element.

Degradation in the Young's modulus of the element can be defined as follows

$$\Delta E_j = \alpha_j E_j \tag{3}$$

where, α_j indicates the local damage severity at the jth element in the finite element model whose values are between 0 for an element without damage and 1 for a ruptured element.

Moreover, it is assumed that no change would occur after damage in the mass matrix, which seems to be reasonable in most real problems.

Thus, Eq. (1) for a damaged plate can be written as follows

$$[\mathbf{K}^d - \lambda_i^d \mathbf{M}^d]\mathbf{\Phi}_i^d = 0 \quad i = 1, 2, \ldots, n \tag{4}$$

where, $\mathbf{K}^d$ and $\mathbf{M}^d$ are the damaged stiffness and mass matrices, respectively; λ_i^d and $\mathbf{\Phi}_i^d$ are square of the ith natural frequency and the ith mode shape of the damaged plate, respectively.

The inverse problem of damage search can be carried out by applying the measured modal data of damaged structure to the Eq. (4) which is described in detail below. The problem can be formulated as optimization problem of objective function while using some transforms as a direct inversion to obtain solution is impossible most of the time.

Localizing and quantifying damage is a difficult problem, requiring sophisticated optimization procedure. In typical optimization problem, there may be lots of locally optimal layout, therefore a downhill-proceeding algorithm in which steadily declining value of objective function is created in iterations, may be stuck into a locally optimal point instead of global optimal solution. For that reason, authors adopted global search algorithms like GA and PS to this optimization problem which lead to characterizing damage.

The PS and GAs approach attempt to find the best solution to a given problem by minimizing an objective function. In any optimization process, there should be an objective function according to which effectiveness of solution can be evaluated. Then, the key point in a minimization problem is the objective function.

The general expression for the objective function is

$$F = f(\alpha_1, \alpha_2, \ldots, \alpha_e) \tag{5}$$

To construct the objective function, all kinds of the structural output which are highly sensitive to the damage severities should be used.

In the process of substituting the measured modal parameters of the damaged plate in Eq. (4), vectors can be defined as follows

$$\mathbf{E}_i(\alpha_1, \alpha_2, \ldots, \alpha_e) = [\mathbf{K}^d - \lambda_i^m \mathbf{M}^d]\mathbf{\Phi}_i^m \quad i = 1, 2, \ldots, p \tag{6}$$

where, λ_i^m and $\mathbf{\Phi}_i^m$ are square of the ith natural frequency and the ith mode shape from measurements, respectively; and p is number of mode for damage identification.

Therefore, the problem of damage detection can be formulated as an optimization problem. The objective is to minimize the following objective function

$$F = \sum_{i=1}^{p} \|\mathbf{E}_i(\alpha_1, \alpha_2, \ldots, \alpha_e)\|^2, 0 \le \alpha_j \le 1 \tag{7}$$

where, $\| \; \|$ represents the Euclidean length of $\mathbf{E}_i$.

2.2 Optimization using genetic algorithm

The GA is an optimization and search technique based on genetics and natural selection principles. The study of GAs was originated in the mid 1970s (Holland 1975) and has developed into a powerful optimization approach. Excellent introductions to GAs was given by Goldberg (1989).

Some of the advantages of a GA are (Haupt *et al.* 2004):

- Optimizes with continuous or discrete variables
- Derivative information isn't required
- Simultaneously searches from a wide sampling of the objective function
- Deals with a large number of variables
- Is well suited for parallel computers
- Optimizes variables with extremely complex objective function
- Provides a list of optimum variables and not only a single solution

These advantages are intriguing and produce stunning results while traditional optimization approaches fail miserably.

The GA approach attempts to find the best solution to a given problem by minimizing an objective function. This function is used to provide a measure of how individuals have performed in the problem domain. In the case of a minimization problem, the fit individuals will have the lowest numerical value of the associated objective function (Goldberg 1989).

In order to minimize Eq. (7), an initial population of randomly generated candidate solutions, encoded as chromosomes are considered, then by applying the principle of survival of the fittest, acceptable approximations as solution are produced. At each generation, a new population is generated by numerical processes of selection, crossover, and mutation with the purpose of improving the best fitnesses.

Process of selection specifies how the GA chooses parents for the next generation. The utilized selection function is *stochastic uniform* that lays out a line in which each parent corresponds to a section of the line of length proportional to its scaled value.

Crossover specifies how the GA combines two individuals (parents), to form a crossover child for the next generation. *Scattered* is chosen as the crossover function that creates a random binary vector and selects the genes where the vector contains a 1 from the first parent, and the genes where the vector contains a 0 from the second parent, and combines the genes to form the child.

Also, Mutation in the GA makes small random changes in the individuals in the population to create mutated children. Mutation provides genetic diversity and enables the GA to search a broader space. The selected mutation function, *Gaussian*, adds a random number taken from a Gaussian distribution with mean 0 to each entry of the parent vector.

This process leads to the evolution of populations of individuals that are better suited to their

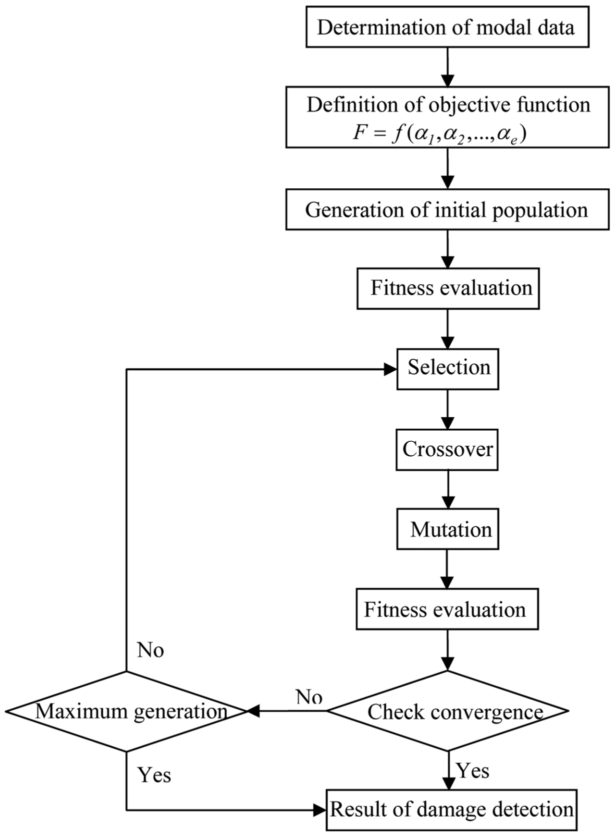

Fig. 1 Flowchart of the damage detection method using genetic algorithm

environment than the individuals that were created, just as in the natural adaptation. The algorithm progresses to reach an optimum solution for the damage detection problem. Number of *Generations* has been considered as the stopping criterion that determines the maximum number of iterations the GA will perform. Fig. 1 shows the flowchart of the proposed method for estimation and localization of the damage via GA.

After terminating the genetic algorithm, another minimization function is utilized. For this purpose, we used the Optimization Toolbox function *fminunc* in MATLAB® (2005) to perform unconstrained minimization. This routine implement sequential quadratic programming (SQP) to minimize the nonlinear cost function subjected to linear and nonlinear equality and inequality constraints. SQP converts a nonlinear minimization to a linear minimization using a Hessian matrix of cost function and gradient of nonlinear constraints.

2.3 Optimization using pattern search method

PS is subclass of direct search methods which was first introduced in 1950s (Box 1950). This technique is appropriate tools to deal with verity of optimization problems where typical optimization approaches are not so effective. The main notion is to produce a sequence of iterate which consider the behavior of objective function at a pattern of points, all of which lies on a logical lattice without utilizing any information about derivatives including, gradient and second-order derivatives of objective function.

The PS method can briefly be explained in a way that starts by establishing set of points called mesh around the given point which it could be computed from pervious step of iteration or the initial starting point provided by user. The mesh is created by adding scalar multiple set of vectors called pattern to the current point, then it searches set of points (mesh) around the current point of parameters to find a point where the objective function has lower value. After a point with lower objective function value is detected, the algorithm sets the point as its current point and iteration can be consider successful. Then the algorithm steps the next iteration with extended mesh size which is induced by expansion factor. If algorithm could not find a point that improves objective function, the iteration called unsuccessful. The current points stay same in the next iteration and the mesh size decreases caused by contraction factor (Lewis and Torczon 2002).

The PS optimization algorithm stops when any of following situations occurs (Coelho and Mariani 2006):

- The number of iteration or evaluation of objective function reaches the max value.
- The mesh size becomes less than mesh tolerance.
- Distance between two successful points obtained in two consecutive iterations is less that the given tolerance.
- Alteration in the improvement of objective function is less than the function tolerance.

The optimization problem is formulated as minimization of objective function. The PS method applies to Eq. (7) to find optimal solution which leads to localizing and quantifying damage in the considered plates. The flowchart of the proposed method for damage localization and estimation using PS method is shown in Fig. 2.

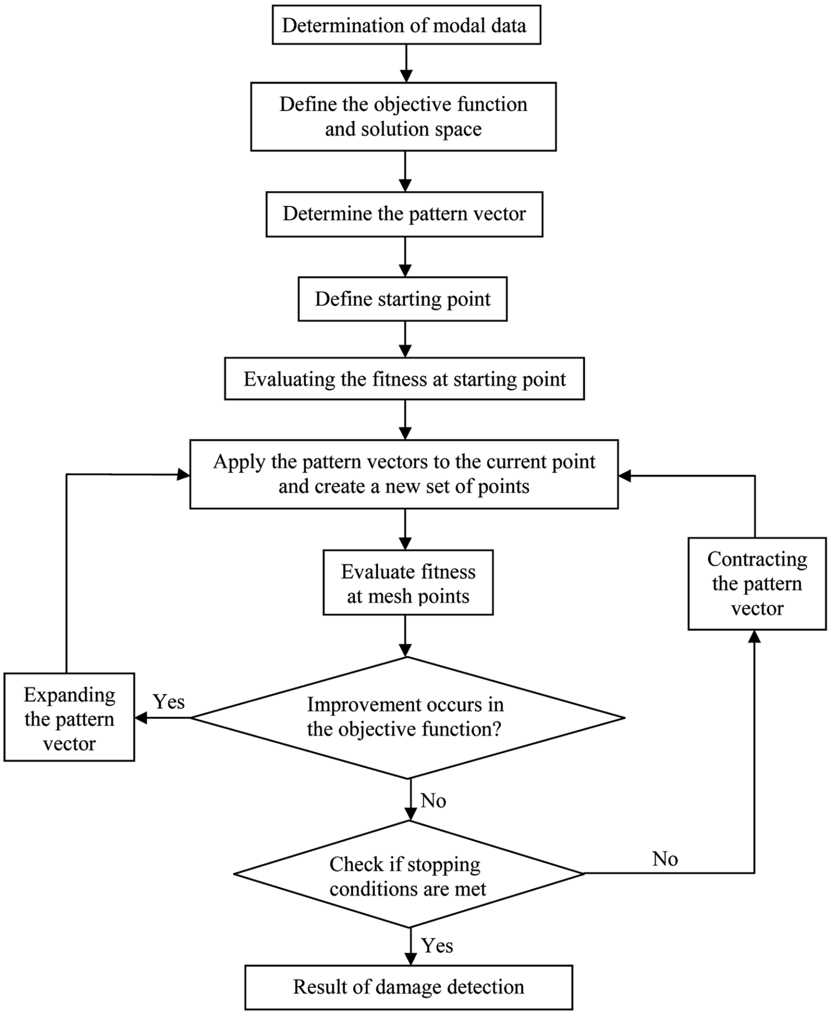

Fig. 2 Flowchart of the damage detection method using pattern search method

3. Numerical examples

In this section, the feasibility of the proposed methods is validated through some numerically simulated damage identification tests. A four-fixed supported plate and a cantilever plate are considered as case studies with two different scenarios of damages for each of them.

3.1 Four-fixed supported plate

Consider a four-fixed supported plate in which the finite-element model consists of 16 elements and 25 nodes, as shown in Fig. 3. The numerical studies are carried out within the MATLAB® (2005) environment, whose use in the solution of finite element problems.

A concrete plate with the thickness of $t = 0.15$ m and the material properties include Young's modulus of E=20 GPa, mass density of $\rho = 2400$ kg/m^3 and Poisson's ratio of $\mu = 0.2$.

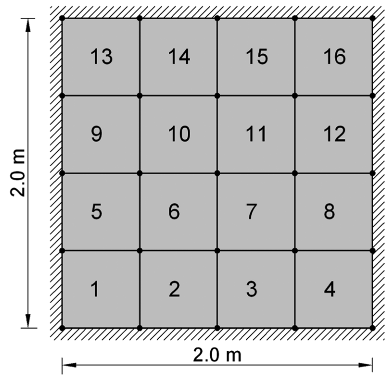

Fig. 3 The four-fixed supported plate with finite-element model

Classical displacements approach was employed which state the local displacement as function of nodal displacement depends on modal function. Nodal displacement can be related to element strain by compatibility matrix **B** which contains derivates of mode shape function. The stress-strain constitutive relation for linear materials commonly is known as Hooke's law. According to expressed law, strain and stress are related by matrix **D** of material constant. Using the principle of virtual work, element stiffness matrix is given by

$$\boldsymbol{k}_e = \int_{V_e} \boldsymbol{B}_e^T \boldsymbol{D}_e \boldsymbol{B}_e dV \tag{8}$$

where V_e is the volume of *e*th element.

Element mass matrix is defined

$$\boldsymbol{M}_e = \int_{V_e} \rho \boldsymbol{N}_e^T \boldsymbol{N}_e dV \tag{9}$$

$\mathbf{N}_e$ stands for shape function in the Equation above.

By defining Transform matrix $\mathbf{T}_e$, the global stiffness and mass matrix can be obtained

$$\boldsymbol{K} = \sum_{e=1}^{m} \boldsymbol{T}_e^T \boldsymbol{k}_e \boldsymbol{T}_e \tag{10}$$

$$\boldsymbol{M} = \sum_{e=1}^{m} \boldsymbol{T}_e^T \boldsymbol{M}_e \boldsymbol{T}_e \tag{11}$$

In the finite-element model, the damage is represented as the elements with reduction in Young's modulus. In order to qualify damage detection methods, two different patterns from the aspect of number and damage severity were employed. In this numerical example, the two following damage patterns have been considered as:

1. The damage severity of element 6 is 0.1.
2. The damage severity of elements 6 and 14 are 0.1 and the damage severity of element 12 is 0.05.

The natural frequencies and mode shapes of damaged state of plate were analytically determined and used in the proposed method for the case of four-fixed supported plate.

Table 1 Input parameters for the GA

Population size	50
Generations	300
Function tolerance	1.0E-05
Nonlinear constrain tolerance	1.0E-05
Stall generation	50

Table 2 Input parameters for the PS

Maximum iteration	1600-3000
Maximum function evaluations	30000-100000
Bind tolerance	0.001
X tolerance	1.00E-06
Function tolerance	1.00E-06
Nonlinear constrain tolerance	1.00E-06

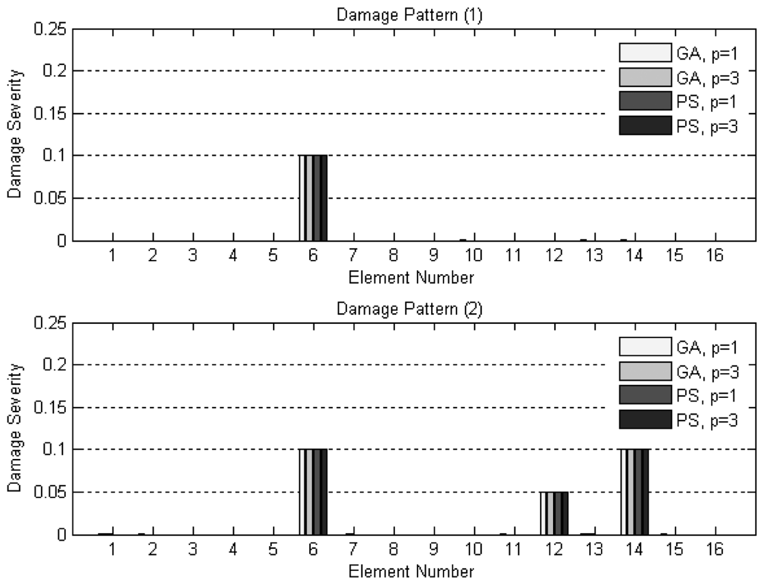

Fig. 4 The obtained results for two damage patterns of the four-fixed supported plate

Damage in the plate can be determined using the proposed method. The GA and PS input parameters adopted for the following analyses are summarized in Tables 1 and 2. Fig. 4 shows the identified damage distribution in plate for two considered patterns, the proposed methods based on GAs and PS, which are described before are employed. It can be clearly find out that the damage locations and severity of damage are precisely obtained for two different patterns considered. Results can be considered reasonably good even using one mode rather than three modes.

Both proposed methods efficiently predict the damage location and quantities when modal data were not contaminated with noise. To be more suited with the real dynamic cases, another examination has been performed in which the natural frequencies with different level of noise are utilized to damage identification considering the same patterns mentioned before. Figs. 5 to 7

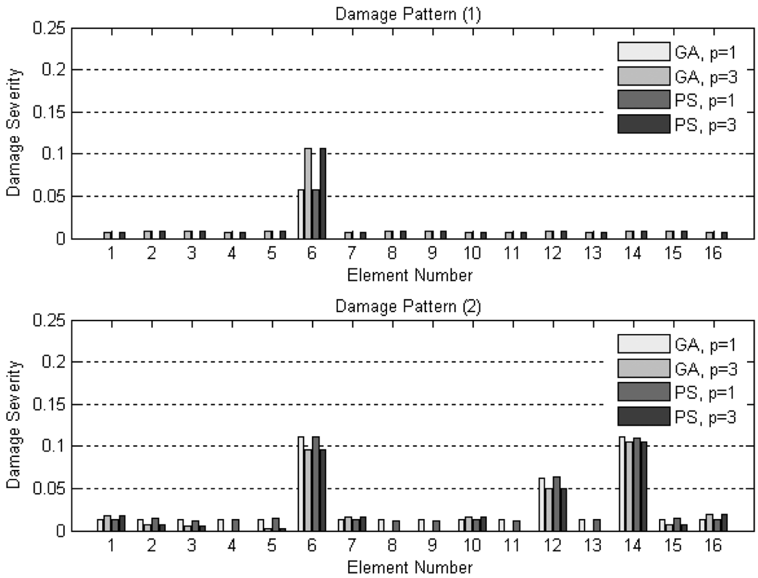

Fig. 5 The obtained results for two damage patterns of the four-fixed supported plate with 3% noise

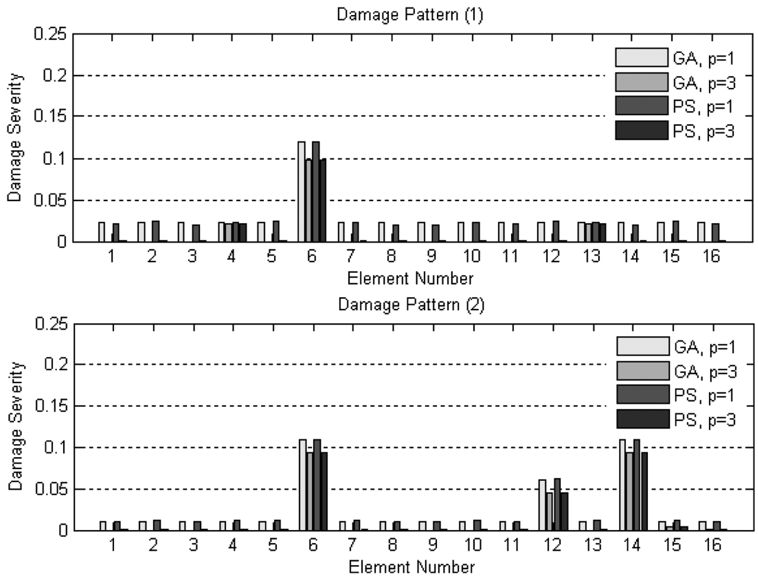

Fig. 6 The obtained results for two damage patterns of the four-fixed supported plate with 5% noise

illustrate that proposed methods are robust and promising methods in detecting and quantifying of various damage pattern with different level of noise up to 8%. Although some undamaged elements are detected as damaged by mistake which value of damage is very low, the damaged elements are properly detected with correct value of severity. Note that better estimations are obtained when the number of applied modes increases to three.

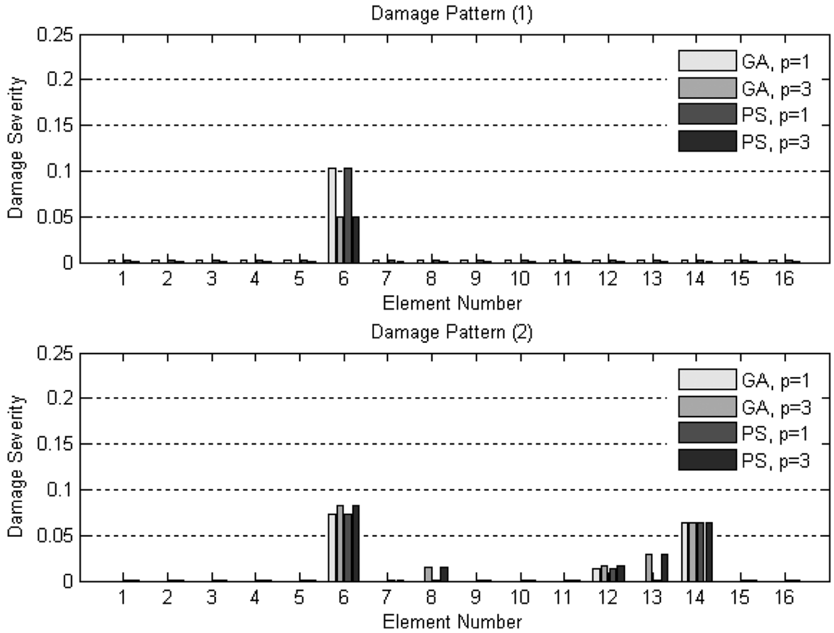

Fig. 7 The obtained results for two damage patterns of the four-fixed supported plate with 8% noise

3.2 Cantilever plate

A cantilever plate as illustrated in Fig. 8 with finite-element model consists of 18 elements and 42 nodes are considered. The thickness of considered plate is $t = 0.15$ m and the material properties of the concrete include Young's modulus, mass density and Poisson's ratio are same the four-fixed supported plate. The boundary conditions and the dimensions of plate are different in this example comparing to previous case study in order to demonstrate effectiveness of the proposed methods for various cases.

The three damage patterns for this case are:

1. The damage severity of element 3 is 0.1.
2. The damage severity of elements 3, 11 and 13 are 0.1, 0.05 and 0.2, respectively.
3. The damage severity of elements 3 and 15 are 0.1.

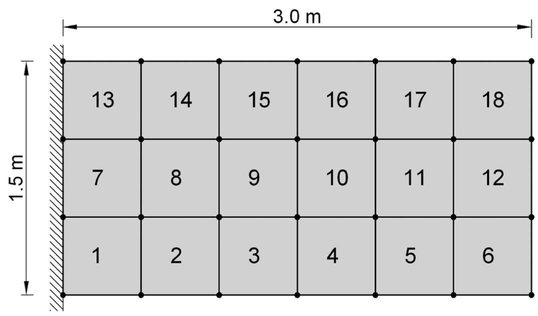

Fig. 8 The cantilever plate with finite-element model

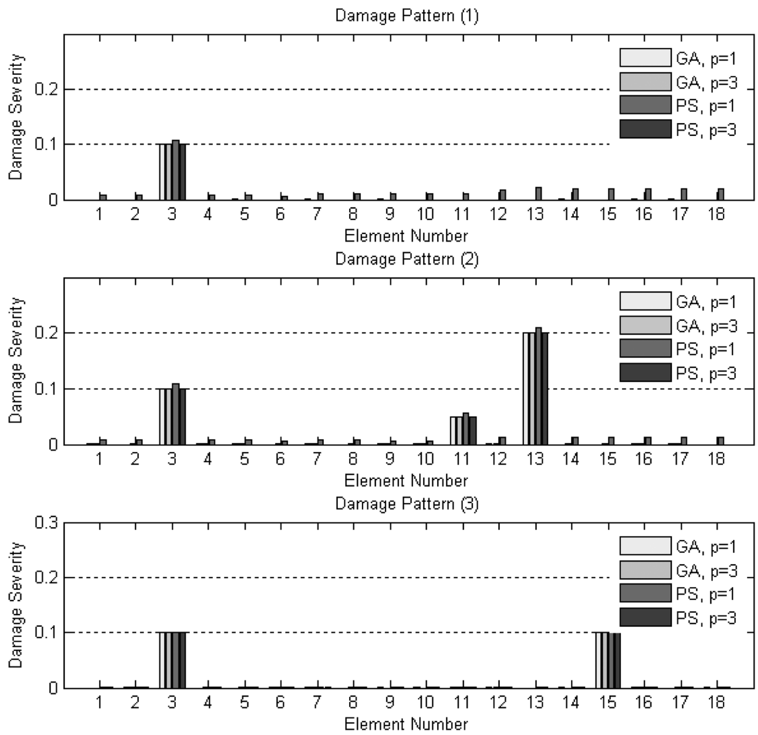

Fig. 9 The obtained results for three damage patterns of the cantilever plate

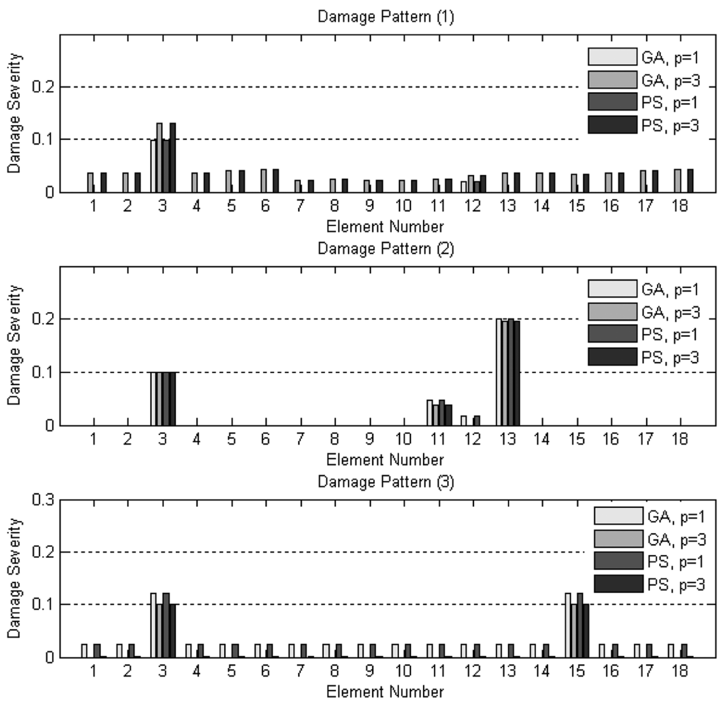

Fig. 10 The obtained results for three damage patterns of the cantilever plate with 3% noise

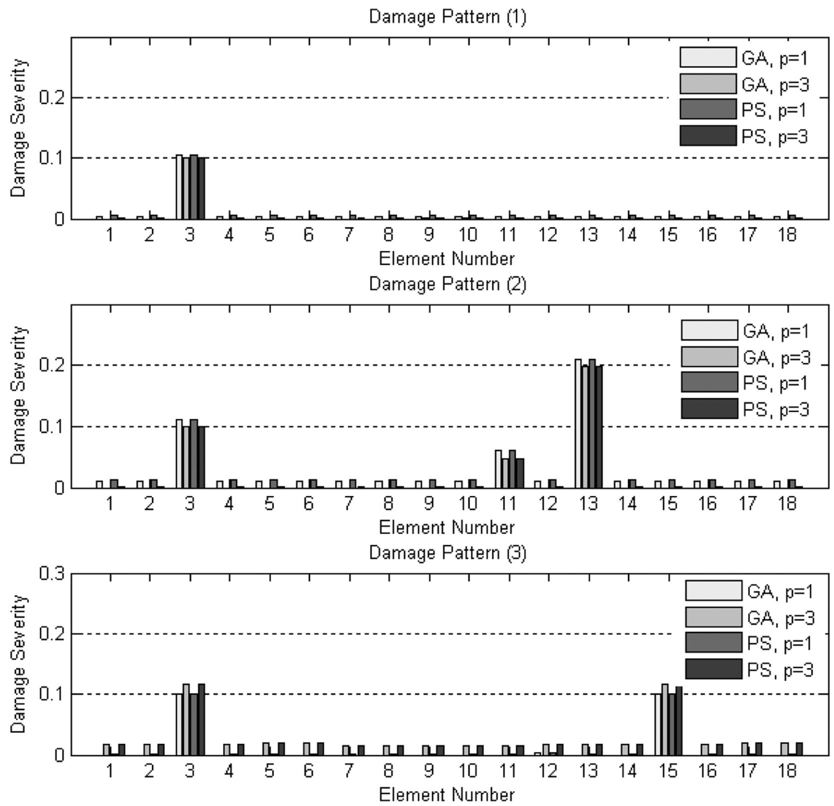

Fig. 11 The obtained results for three damage patterns of the cantilever plate with 5% noise

After determination of the modal parameters of the damaged plate, the proposed method was applied to detect and quantify the damage in the cantilever plate. Fig. 9 shows the identified damaged elements with levels of severity using both of the introduced methods. It can be derived from the figure that element number 3 is damaged with severity level of 0.1 considering the first pattern of damage. Using three modes instead of one mode leads to the same accuracy to some extent in the obtained result. The proposed method based on GAs seems to be more reliable than PS based procedure because of undamaged elements that are incorrectly detected damage, although level of damage is quiet low.

The modal data experimentally carried out usually are contaminated with noise, therefore to evaluate the stability and robustness of proposed methods, it is essential to consider natural frequencies with different levels of noise. Figs. 10 to 12 illustrate the same example investigated above contaminated with noise here, considering the patterns mentioned already. As it is noted before by increasing mode numbers the accuracy in detecting and quantifying damages increase but when the model is contaminated with noise there are some extra perturbation in other elements. A reason for this is participating of noise in there different modes, in other word noise put its influence three times more in comparison with when only one mode is used in the methods. Besides because the noise has random property, sometimes less perturbation might be seen but generally amount of damage estimated properly by increasing mode numbers.

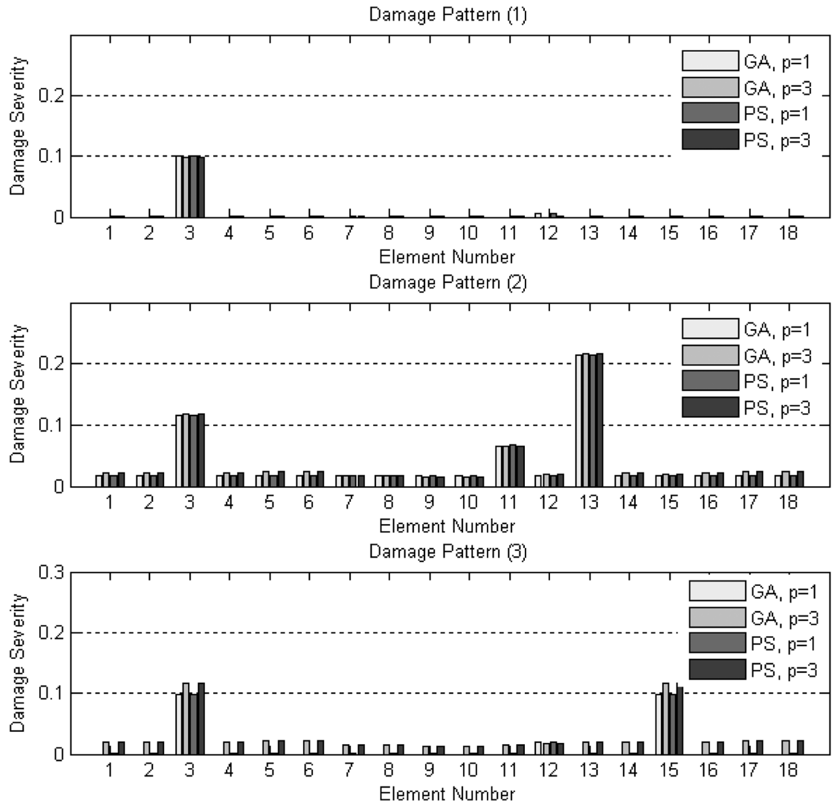

Fig. 12 The obtained results for three damage patterns of the cantilever plate with 8% noise

4. Conclusions

This study presented two methods for detection and estimation of structural damage in plates using the GA and PS. The proposed methods are aimed at localizing and quantifying the damage severity in the elements of plate only using modal data of damaged plate via an optimization problem. In the first method, estimating the damages identification in the plate is conducted by optimizing an objective function applying the GA, while second method employs PS to determine the damages by optimizing the objective function that created and used in the first method.

In illustrative examples, the proposed methods are applied to a four-fixed supported plate and a cantilever plate. The obtained results indicated that the both proposed method are promising procedures to damage identification. In some cases using GA has produced superior results to method using PS method. Therefore, GA can be characterized as a powerful tool for structural damage detection and estimation in plate structures.

References

Al-Sumait, J.S., Al-Othman, A.K. and Sykulski, J.K. (2007), “Application of pattern search method to power system valve-point economic load dispatch”, *Int. J. Elect. Power,* **29**(10), 720-730.

Bagheri, A., Ghodrati Amiri, G. and Seyed Razaghi, S.A. (2009), "Vibration-based damage identification of plate structures via curvelet transform", *J. Sound. Vib*., **327**(3-5), 593-603.

Bayissa, W., Haritos, N. and Thelandersson, S. (2008), "Vibration-based structural damage identification using wavelet transform", *Mech. Syst. Signal Pr.,* **22**(5), 1194-1215.

Biondi, T., Ciccazzo, A., Cutello, V., D'Antona, S., Nicosia, G. and Spinella, S. (2006), "Multi-objective evolutionary algorithms and pattern search methods for circuit design problems", *J. Univers. Comput. Sci*., **12**(4), 432-449.

Box, G.E.P. (1957), "Evolutionary operation: A method for increasing industrial productivity", *J. Appl. Stat*., **6**(2), 81-101.

Casciati, S. (2008), "Stiffness identification and damage localization via differential evolution algorithms", *Struct. Health Monit*., **15**(3), 436-449.

Cawley, P. and Adams, R.D. (1979), "Defect location in structures by a vibration technique", *Proceedings of the American Society of Mechanical Engineers Design Engineering Technical Conference*, St. Louis.

Chang, C.C. and Chen, L.W. (2004), "Damage detection of a rectangular plate by spatial wavelet based approach", *Appl. Acoust.,* **65**(8), 819-832.

Chou, J. and Ghaboussi, J. (2001), "Genetic algorithm in structural damage detection", *Comput. Struct*., **79**(14), 1335-1353.

Coelho, L.S. and Mariani, V.C. (2006), "Combining of chaotic differential evolution and quadratic programming for economic dispatch optimization with valve-point effect", *IEEE T. Power. Syst.,* **21**(2), 989-96.

Cornwell, P., Doebling, S.W. and Farrar, C.R. (1999), "Application of the strain energy damage detection method to plate-like structures", *J. Sound. Vib*., **224**(2), 359-374.

Doebling, S.W., Farrar, C.L., Prime, M.B. and Shevitz, D.W. (1996), "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review", Los Alamos National Laboratory Report, LA-13070-MS.

Douka, E., Loutridis, S. and Trochidis, A. (2004), "Crack identification in plates using wavelet analysis", *J. Sound. Vib*., **270**(1-2), 279-295.

Fan, W. and Qiao, P. (2009), "A 2-D continuous wavelet transform of mode shape data for damage detection of plate structures", *Int. J. Solids Struct*., **46**(25-26), 4379-4395.

Goldberg, D. (1989), *Genetic algorithms in search, optimization and machine learning*, Addison-Wesley, Reading, Mass.

Hadjileontiadis, L.J. and Douka, E. (2007), "Crack detection in plates using fractal Dimension", *Eng. Struct.,* **29**(7), 1612-1625.

Haupt, R.L., and Haupt, S.E. (2004), *Practical Genetic Algorithms*, John Wiley & Sons, Hoboken, NJ.

Holland, J. (1975), *Adaptation in natural and artificial systems*, MIT Press, Cambridge, Mass.

Hooke, R. and Jeeves, T.A. (1961), "Direct search solution of numerical and statistical problems", *J. ACM*, **8**(2), 212-229.

Lewis, R.M., Torczon, V. and Trosset, M.W. (2000), "Direct search methods: then and now", *J. Comput. Appl. Math.*, **124**(1-2), 191-207.

Lewis, R.M. and Torczon, V.J. (1999), "Pattern search algorithms for bound constrained minimization", *SIAM J. Optimiz*., **9**(4), 1082-1099.

Lewis, R.M. and Torczon, V. (2002), "A globally convergent augmented Lagrangian pattern search algorithm for optimization with general constraints and simple bounds", *SIAM J. Optimiz*., **12**(4), 1075-1089.

Mares, C. and Surace, C. (1996), "Application of genetic algorithms to identify damage in elastic structures", *J. Sound. Vib*., **195**(2), 195-215.

MATLAB (2005), *Matlab User Manual*, Mathwork Inc., Lowell, MA, U.S.A.

Perera, R. and Torres, R. (2006), "Structural damage detection via modal data with genetic algorithms", *J. Struct. Eng- ASCE,* **132**(9), 1491-1501.

Rucka, M. and Wilde, K. (2006), "Application of continuous wavelet transform in vibration based damage detection method for beams and plates", *J. Sound. Vib*., **297**(3-5), 536-550.

Salawu, O.S. (1997), "Detection of structural damage through changes in frequency: a review", *Eng. Struct.*, **19**(9), 718-723.

Swann, W.H. (1972), *Direct search methods, in Numerical Methods for Unconstrained Optimization*, (Ed. W.

Murray), Academic Press, London, New York.
Wang, Q. and Deng, X. (1999), “Damage detection with spatial wavelets”, *Int. J. Solids Struct*., **36**(23), 3443-3468.
Yoon, M., Heider, K.D., Gillespie, J.W., Ratcliffe, C.P. and Crane, R.M. (2005), “Local damage detection using the two-dimensional gapped smoothing method”, *J. Sound. Vib*., **279**(1-2), 119-139.
Yun, G.J., Ogorzalek, K.A., Dyke, S.J. and Song, W. (2009), “A Two-Stage Damage Detection Approach Based on Subset Selection and Genetic Algorithms”, *Smart Struct. Syst.*, **5**(1), 1-21.

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