Smart Structures and Systems, Vol. 7, No. 2 (2011) 103-116
DOI: http://dx.doi.org/10.12989/sss.2011.7.2.103

Trajectory tracking and active vibration suppression of a smart Single-Link flexible arm using a composite control design

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(Received December 4, 2009, Accepted September 20, 2010)

Abstract. This paper is concerned with the trajectory tracking and vibration suppression of a single-link flexible arm by using piezoelectric materials. The dynamics of a single flexible arm with PZT patches as sensor and actuator is derived using extended Hamilton's principle. Resulting equations show that the coupled beam dynamics including beam vibration and its rigid in-plane rotation takes place in two different time scales. By using singular perturbation theory, the system dynamics is divided into two subsystems. Then, a composite control scheme is elaborated that makes the orientation of the arm track a desired trajectory while suppressing its vibration. The proposed controller has two parts: one is a tracking controller designed for the slow (rigid) subsystem, and the other one is a stabilizing controller for the fast (flexible) subsystem. The outputs considered for the system are angular position of the hub and voltage of the sensor mounted on the structure. To avoid requiring further measurements of beam vibration and also angular velocity of the hub for the fast and slow control laws, respectively, two sliding mode observers for estimating the unknown states are also designed.

Keywords: flexible arm; piezoelectric material; variable structure control; singular perturbation theory; vibration suppression.

1. Introduction

Recent developments in sensor/actuator technologies have enticed many researchers to model and use the capabilities of smart materials such as: shape memory alloys, magnetorheological materials, electrorheological materials and especially piezoelectric materials in order to suppress the vibration of flexible structures. Although the piezoelectric effect was discovered for the first time by the Curie brothers, Pierre and Jacques in 1880, its application for vibraton control of smart structures was proposed by Baily and Hubbard (1985) and also Crawley and de Luis (1987), Crawley and Anderson (1990) in 1980's. Since then there have been extensive investigations dealing with the modeling of flexible structures with embedded piezoceramics as sensors and actuators with different approaches (Benjeddou 2000, Franco Correia *et al.* 2000). In these studies different control methods are presented for the case of single-link flexible arm.

Lin and Huang (1999) proposed an algorithm for vibration control of beam-plates with bonded piezoelectric sensors and actuators based on Lyapunov energy function. The equations of motion for

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a beam-plate structure bonded with pairs of piezoelectric sensors or actuators were derived using the Hamilton's principle and a finite element method (FEM) was used for the analysis purpose.

Jalili *et al.* (2002), presented an adaptive nonlinear feedback gain for the PZT input voltage and a simple PD controller for the base input force of a flexible cantilever beam with a mobile base while the overall stability of the system was guaranteed using the Lyapunov approach. Sun *et al*. (2004) presented a combined PD feedback control for rigid body motion control of a single link flexible arm and an L-type (linear velocity) feedback control depending on the linear velocity at the tip, for the distributed piezoelectric polymer (PVDF) of a single-link flexible arm. Global stability of the system was investigated using Lyapunov approach. In their study the effect of piezoelectric actuator was only considered as a bending moment which appeared in boundary conditions. Dadfarnia *et al.* (2004) designed a Lyapunov-based controller for a flexible cantilever beam with a mobile base and bonded PZT actuators on the surface of the flexible beam. Lin and Nien (2005) proposed an optimal closed loop control algorithm for vibration control of a smart beam with embedded piezoelectric materials used as sensor and actuator. Fei (2005) carried out the first mode vibration suppression of a flexible steel cantilever beam with bonded piezoelectric actuators and strain gage as sensor using strain rate feedback (SRF) control and an optimized PID compensator and compared their efficiency. Bandyopadhyay and Mehta (2005) designed a second order sliding mode controller (SSMC) based on super twisting algorithm for vibration suppression of the smart structure with piezoelectric materials as sensor and actuator bonded to the structure and the FEM technique was used for the simulation. Hu and Ma (2005) used the variable structure control (VSC) to design the switching logic for the thruster firing of a flexible spacecraft during attitude maneuver in order to achieve desired angular positions while the effect of piezoelectric materials mounted on the structure was neglected. Then separately the technique of positive position feedback (PPF) control using the smart materials as sensors and actuators for eliminating the micro vibrations was used without investigating the overall stability of the system. In another study, Hu *et al.* (2006) proposed a generalized scheme based on the output feedback sliding mode control (OFSMC) and active vibration control techniques using piezoceramics as actuator/sensor where two separate control loops were adopted. The first technique used piezoceramics as sensors and actuators by designing positive position feedback (PPF) compensators which added damping to the flexible structures. The second feedback loop was designed based on an output feedback sliding mode control (OFSMC) design for the thruster firing of a flexible spacecraft. Using the same process as described, since then different control designs for the thruster firing of a flexible spacecrafts during attitude maneuver have been proposed by Hu and Ma (2008), Zhu *et al.* (2008a) such as adaptive variable structure control (Hu and Ma 2008), robust control based on backstepping (Zhu *et al*. 2008a) and variable structure control based on time-varying sliding surface with various control designs for using piezoceramics as actuator/sensor such as strain rate feedback control, positive position feedback control, linear quadratic regulator (LQR) based on PPF, and modal velocity feedback (Shan 2007, Zhu *et al*. 2008b), (Hu 2009a, 2009b).

To add to the aforementioned bulk of literature in this field, trajectory tracking and vibration suppression of a single-link flexible arm by using piezoelectric materials are considered in this study. First, the flexible-rigid dynamics of the transverse vibration and the rotational motion has to be taken into account. It can be shown that the flexible and rigid motions take place in two different time scales. So, dynamics of the beam can be separated into flexible and rigid subsystem dynamics, by using the singular perturbation theory (Kokotovic *et al.* 1999). The main goal of this work is to design a composite controller for this system; the proposed controller consists of a state feedback variable structure control (VSC) for the slow subsystem and a classical PD control for the fast subsystem. Since the output signals in hand are assumed to be the hub angle and the sensor voltage, the unknown states of the system required for the state feedback controller are estimated by two sliding mode observers. The main advantages of the proposed method comparing to the referenced studies, mentioned previously, can be noted as:

1) By utilizing the proposed method role of each controller is specifically determined for performing the maneuver and vibration suppression of the system and there is no need for designing complex controllers/observers for the system.
2) The proposed controller can be straightforwardly utilized for the case of multilink flexible structure
3) Various control methods for multilink rigid manipulator can be readily implemented on the system
4) Considering the observer for estimating the unknown states of the system which is needed in practical cases where accurate measurement of all the state variables of a given system is not often possible due to various limitations.

The rest of the paper is organized as follows. In Section two, dynamic modeling of a single link flexible arm with embedded piezoelectric materials as sensor and actuator is presented and singular perturbation is applied to the system. In Section three, according to the two separated subsystems, a VSC for slow subsystem and a classical PD control for fast subsystem are designed and the sliding mode observers used for estimating the unknown states of the system are presented. Simulation results and relevant conclusions are presented and discussed in Sections four and five, respectively.

2. Mathematical modeling of the smart flexible arm

2.1 Problem description

In this section, the dynamical model of a smart single-link flexible arm with bonded PZT patches as sensor and actuator is presented. Fig. 1 shows the flexible arm model under consideration for this study, which consists of a flexible link of length L, situated in horizontal plane and attached to the rotor of a motor with PZT actuators and sensors bonded on its surfaces (Shan 2007). The reference frame and the local frame attached to the hub are X_0OY_0 and X_1OY_1, respectively. The joint angle at the hub is $\theta(t)$ and the transverse displacement of the arm at point x with respect to the local frame X_1OY_1 is $w(x,t)$. Herein t denotes time and $x \in [0,L]$ is the link coordinate. Moreover, the following assumptions are made:

(1) The link is considered to be an Euler-Bernoulli beam and the axial deformation is neglected;
(2) The PZT layer is homogeneous and electric field is unidirectional;
(3) The PZT patches are perfectly bonded to the link.

2.2 Governing equations

The equations of motion are derived using extended Hamilton's principle (Clark *et al.* 1998). The kinetic energy of the link and PZT sensors/actuators, T, can be presented as

$$T = \frac{1}{2}I_h\dot{\theta}^2 + \frac{1}{2}\int_0^L \rho_l \dot{\vec{r}}^2 dx \frac{1}{2}\int \rho_p \dot{\vec{r}}^2 dx \tag{1}$$

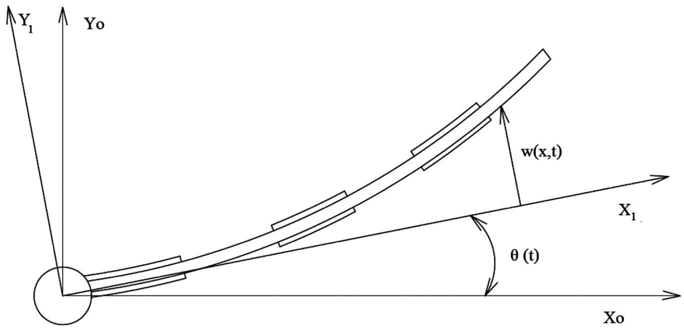


Fig. 1 A single-link flexible arm with bonded piezoelectric materials

where $\vec{r}$ is the position vector of point x on the link with respect to the reference frame, I_h is hub's moment of inertia, and ρ_l and ρ_p are mass per unit length of the link and piezoelectric patches, respectively. The potential energy of the link can be written as

$$V_L = \frac{1}{2}\int_0^L E_L I_L\left(\frac{\partial^2 w}{\partial x^2}\right)dx \tag{2}$$

where E_L and I_L are link's modulus of elasticity and moment of inertia. According to the assumption of Euler-Bernoulli beam and unidirectional electric field (Ballas 2007), the sum of potential and electrical energy of the piezoelectric materials, W_e and V_p, and their constitutive equations can be simplified as follows

$$W_e - V_p = \frac{1}{2}\int_V (E_3 D_3 - T_1 S_1)dV = \frac{1}{2}w_p\int_{xp}^{x_p+L_p}\int_{y_p}^{y_p+t_p}(E_3 D_3 - T_1 S_1)dydx \tag{3}$$

where w_p is the width, x_p is the starting point in x direction, L_p length, t_p thickness, y_p starting point from neutral axis of piezoelectric actuator/sensor and (Ballas 2007)

$$T_1 = c_{11}^E S_1 - e_{13}E_3$$
$$D_3 = e_{31}S_1 + \varepsilon_{33}^S E_3 \tag{4}$$

where T_1 represents longitudinal stress, S_1 represents strain, E_3 is perpendicular electric field, D_3, represents electric displacement of the piezoelectric material, c_{mn}, e_{mk} and ε_{ik} are the elements of the modulus of elasticity, the piezoelectric modulus and the permittivity matrices of the piezoelectric material, respectively. By replacing T_1 and D_3 from constitutive equation in the Eq. (5) and using $S_1 = -y\frac{\partial^2 w}{\partial x^2}$ we have

$$-V_p + W_e = \frac{1}{2}\int_{xp}^{x_p+L_p}\int_{y_p}^{y_p+t_p}\left(-c_{11}^E\left(y\frac{\partial^2 w}{\partial x^2}\right)^2 + e_{13}E_3\left(-y\frac{\partial^2 w}{\partial x^2}\right) + e_{13}\left(-y\frac{\partial^2 w}{\partial x^2}\right)E_3 + \varepsilon_{33}^S E_3^2\right)dydx \tag{5}$$

The work done by the torque, τ, can be expressed as

$$W = \tau\theta \tag{6}$$

2.3 Assumed mode method

Utilizing assumed mode method (Meirovitch 2001), defining $w(x, t)= \sum_{j=1}^{n} \varphi_j(x)q_j(t) = \{\varphi\}\{q\}$ where $\varphi_j(x)$ and $q_j(t)$ are the vector of assumed mode shape functions and the vector of generalized flexible coordinates of the arm, respectively, and applying the extended Hamilton's principle, the equations of motion of a smart structure can be expressed as

$$\begin{bmatrix} M_{\theta\theta} & M_{\theta q} \\ M_{q\theta} & M_{qq} \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{qq} \end{bmatrix} \begin{Bmatrix} \theta \\ q \end{Bmatrix} = \begin{Bmatrix} \tau \\ -[B_{pa}]\{v_{pa}\} \end{Bmatrix}$$

$$\{v_{ps}\} = [C_{ps}]\{q\} \tag{7}$$

where $\begin{bmatrix} M_{\theta\theta} & M_{\theta q} \\ M_{q\theta} & M_{qq} \end{bmatrix}$ is the mass matrix, $\begin{bmatrix} 0 & 0 \\ 0 & K_{qq} \end{bmatrix}$ is the stiffness matrix, τ is the torque at the hub, v_{pa} is the voltage of the piezoelectric actuator, v_{ps} is the voltage of the piezoelectric sensor, $[B_{pa}]$ and $[C_{ps}]$ are the matrices which contain piezoelectric parameters.

2.4 Decomposition of the equations of motion by singular perturbation theory

The underlying idea of singular perturbation approach is to decouple the system dynamics into the slow and fast subsystems. Control design may then proceed for each lower-order subsystem, and the results are combined to yield a composite controller for the original system. In order to be able to apply this method, a new parameter z is introduced as follows (Kokotovic *et al.* 1999)

$$[K_{qq}]\{q\} = k_m[\bar{K}_{qq}]\{q\} = \{z\} \rightarrow [\bar{K}_{qq}]\{q\} = \frac{1}{k_m}\{z\} \rightarrow [\bar{K}_{qq}]\{q\} = \varepsilon^2\{z\} \tag{8}$$

where k_m is the smallest coefficient of the stiffness matrix, K_{qq}, and is the singular perturbation parameter. Comparing the magnitude of elements of B_{pa} matrix, B_{pai}, with $\frac{1}{k_m}$, it can be concluded that $O(B_{pai}) = O(\varepsilon)$; hence $B_{pa} = \varepsilon B_{pa}$. Substituting the new variables in equations of motion of the system and applying singular perturbation method, the slow and fast subsystems are obtained as follows (Canudas *et al.* 1997)

$$\ddot{\theta}_s = (H_{\theta\theta} - H_{\theta q}[H_{qq}]^{-1}H_{q\theta})\tau_s$$

$$\{z_s\} = [H_{qq}]^{-1}H_{q\theta}\tau_s \tag{9}$$

and

$$\varepsilon^2\{\ddot{z}_f\} = [\bar{K}_{qq}](-H_{qq}\{z_f\} + H_{q\theta}\tau_f - H_{qq}\varepsilon[B_{pa}]\{v_{pa_f}\}) \tag{10}$$

where $H = M^{-1}$ and $z_f = z\text{-}z_s$. As expected, it can be observed that the piezoelectric actuator does not have a major effect on the slow subsystem. On the other hand, using the motor torque for controlling both slow and fast subsystems has a counteracting effect and tuning controller parameters will become a demanding task as one of them may cause the controller torque to increase while the other one enforce the controller torque to decrease. Therefore, we utilize the piezoelectric actuator to stabilize the fast subsystem and use motor torque only to stabilize the slow subsystem.

3. Composite control system design

A composite control strategy is employed to control the slow and fast subsystem in order to stabilize the whole system. In the present paper a VSC and a classical PD control are designed for slow and fast subsystems, respectively, along with a variable structure observer in order to observe the unknown states of the system which are required for the controller algorithm.

3.1 Variable structure control system for the slow subsystem

In order to have the desired tracking performance, a linear sliding mode controller is designed for the slow subsystem due to its distinguished features and its ability to result in a robust control which is thoroughly insensible to parameter uncertainties and external disturbances. The controller design procedure for the slow subsystem of the single-link flexible arm as expressed in Eq. (9) can be outlined in two steps. First a sliding surface which represents the desired system dynamics is designed and then the controller structure is designed such that the states of the system reach the sliding surface.

The sliding surface can be represented by

$$\sigma(\theta) = \lambda e_\theta + \dot{e}_\theta, e_\theta = \theta - \theta_d \tag{11}$$

where λ is a design parameter and θ_d is the desired angle of the hub. In order to design the controller, the method of equivalent control is employed (DeCarlo *et al.* 1988). The structure of the controller is considered as

$$\tau = \tau_{eq} + \tau_n = (H_{\theta\theta} - H_{\theta q}[H_{qq}]^{-1}H_{q\theta})^{-1}(\ddot{\theta}_d - \lambda\dot{e}_\theta) - k \cdot sat(\sigma(\theta)) \tag{12}$$

where τ_{eq} is the equivalent control, τ_n is the nonlinear part of the controller, k is a positive constant,

$$sat(\sigma(\theta)) = \begin{cases} 1 & if \quad \sigma(\theta) > \alpha \\ \sigma(\theta) & if \quad -\alpha < \sigma(\theta) < \alpha \\ -1 & if \quad \sigma(\theta) < -\alpha \end{cases} \tag{13}$$

and α is a positive constant. For the following Lyapunov candidate

$$V_{Lyapunov}(\theta, t) = \sigma^T\sigma \tag{14}$$

It can be seen that

$$V_{Lyapunov}(\theta, t) < 0, \text{ when } \sigma \neq 0 \tag{15}$$

3.2 Classical control system for the fast subsystem

By decomposing the system into slow and fast subsystems, singular perturbation approach has provided the opportunity of a separate controller design for piezoelectric actuator in order to overcome undesirable vibrations of the flexible arm. In our study a PD control is designed for control of the fast subsystem which is exerted by the piezoelectric actuator.

3.3 Sliding mode observer design

The need for observer design is due to various limitations such as high cost of measurement instruments or difficulties associated with mounting or utilizing the sensors, hence accurate measurement of all the state variables of a given system is not often viable. Due to such complexities in measuring the full states of the system two state observers are designed to estimate the unknown states of the system. In the case of a flexible single link arm the angular position of the hub can be monitored using an encoder, and the angular velocity of the hub can be estimated by the sliding mode observer (Martinez and Nakano 2008). For the fast subsystem the voltage of the piezoelectric actuator is considered as the output.

4. Simulation results and discussion

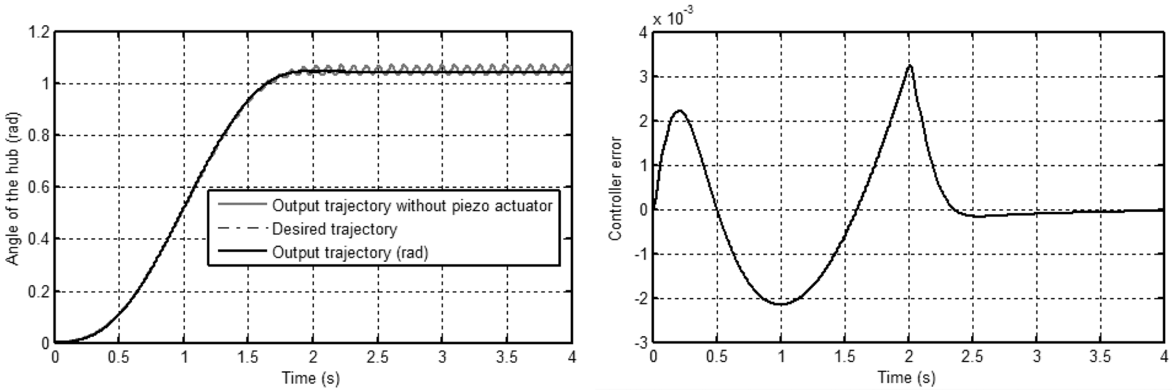
The proposed controller is used to control a single link flexible arm with two piezoelectric materials as sensor and actuator with the properties pointed out in Table 1, which are placed on the opposite sides of the arm. The actuator is placed on top surface and sensor is bonded at the same position but on the other surface of the arm and it is supposed that the actuator and sensor are located just at the base of the link, $x_p = 0$. In order to study the structural system behavior with the proposed controller a maneuver of 60 degree angle rotation of the hub is considered. The desired trajectory is designed according to the initial and final conditions of the structure before and after the maneuver, as a fifth order polynomial

$$\theta_d(t) = \left[6\left(\frac{t}{t_f}\right)^5 - 15\left(\frac{t}{t_f}\right)^4 + 10\left(\frac{t}{t_f}\right)^3\right](\theta_f - \theta_0) + \theta_0 \tag{16}$$

in which the constants are defined using the fact that angular velocity and angular acceleration are needed to be zero at the beginning and end of the maneuver, and θ_0, θ_f are the initial and final angular positions, respectively and t_f is the desired time so that the flexible arm reaches its final destination. In the maneuver considered for simulation the parameters are defined as $\theta_0 = 0$, $\theta_f = 60$ and $t_f = 2(s)$. The composite controller presented in section 3 is applied to the motor at the hub and also the piezoelectric actuator mounted on the arm. For controlling the slow subsystem which is associated with the rigid body rotation of the system the sliding mode controller is applied, while the angular velocity of the system is estimated using a sliding mode observer and the classical controller is applied to the PZT actuators. The output trajectory of the hub orientation angle and angular velocity is demonstrated in Figs. 2(a) and 3(a), respectively. Considering the controller error, Figs. 2(b) and 3(b), for position and

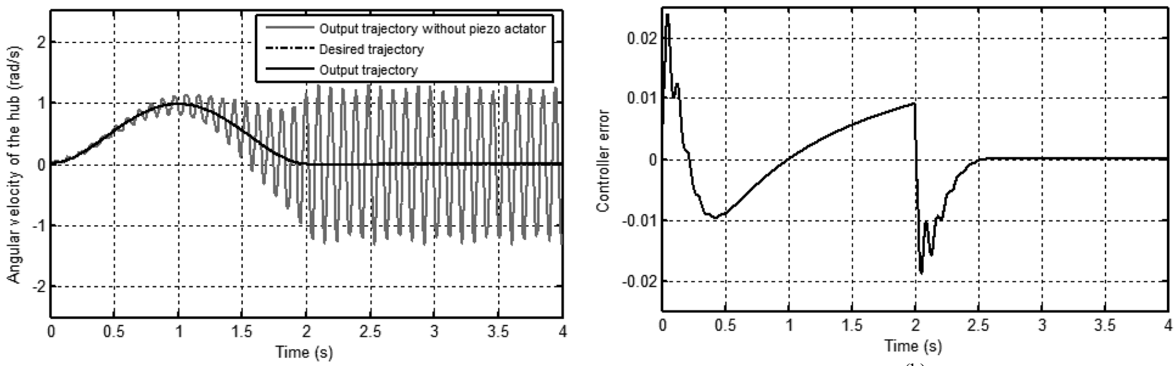
Table 1 Characteristics of the arm model (Martinez and Nakano 2008)

Parameters	Arm	PZT
Modulus (N/m^2)	76×10^9	63×10^9
Length (m)	1	.2
Width (m)	0.05	0.05
Thickness (mm)	4	0.75
Density (kg/m^3)	2840	7600
Piezoelectric constant (m/V)		-110×10^{-12}



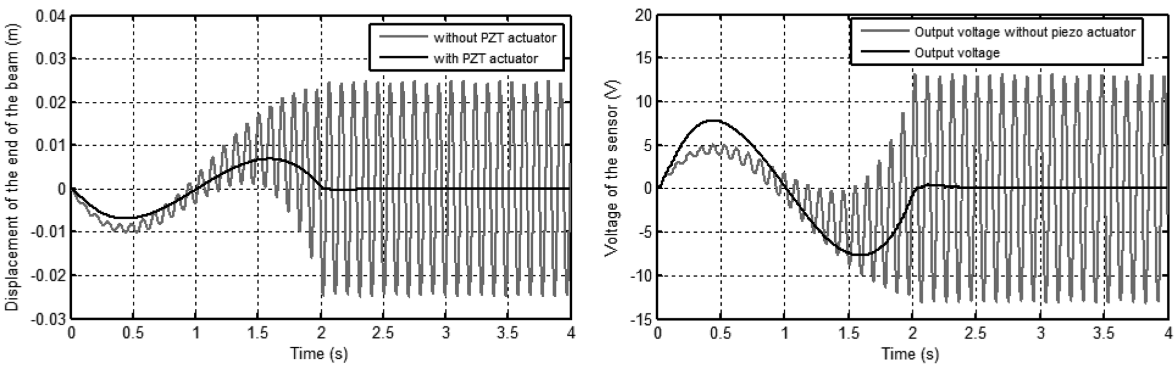
(a) (b)

Fig. 2 (a) Tracking performance of the hub orientation, (b) controller error in tracking the desired angle



(a) (b)

Fig. 3 (a) Tracking performance of the hub angular velocity, (b) controller error in tracking the desired angular velocity

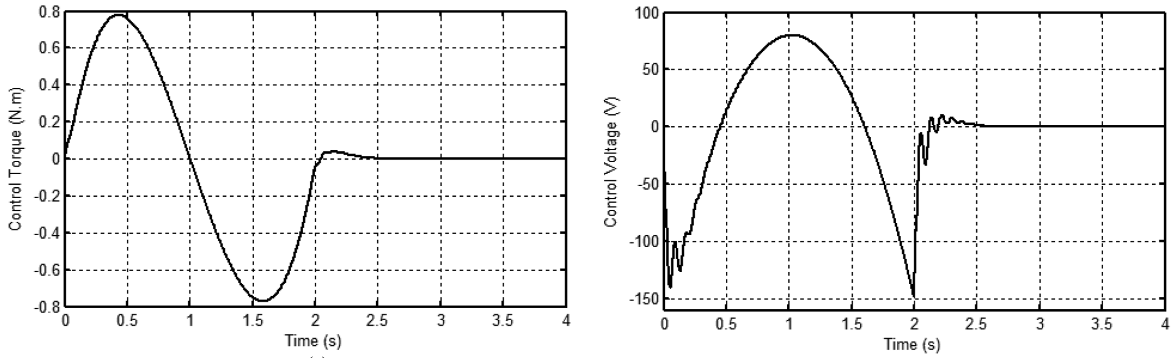


(a) (b)

Fig. 4 Time history of: (a) the displacement of the arm tip, (b) the piezoelectric sensor voltage

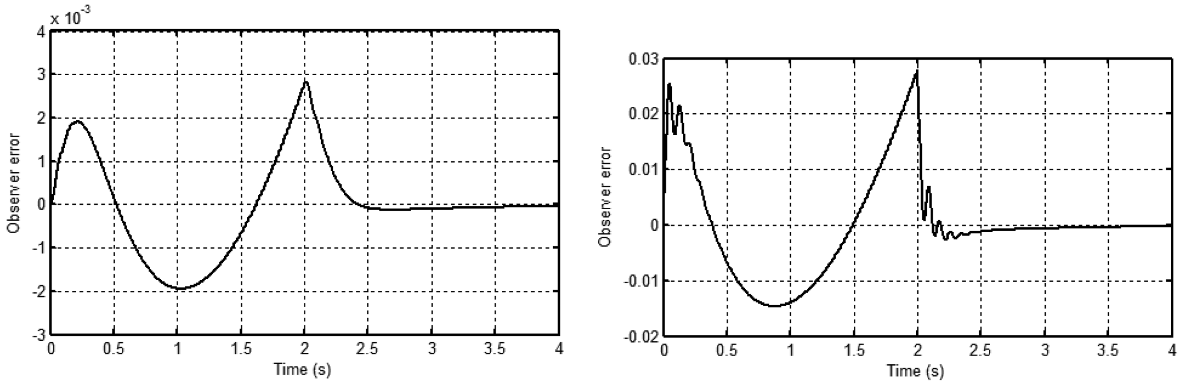
velocity respectively, the controller shows a good performance in tracking the desired trajectory.

In order to study the behavior of the structure with and without piezoelectric, the time histories of the displacement of the arm tip, Fig. 4(a), and the piezoelectric sensor voltage, Fig. 4(b), are shown.



(a) (b)

Fig. 5 Time history of the controller: (a) hub torque, (b) piezoelectric actuator voltage



(a) (b)

Fig. 6 Estimation error of the observer for: (a) the hub angle, (b) the hub angular velocity

It can be observed that vibrations of the arm are damped out in a rather short period of time right after the maneuver of the structure ends at t=2(s) using PZT actuator which has a significant role in damping the vibrations of the flexible system. The torque exerted at the hub and the applied the voltage of the piezoelectric actuator are presented in Fig. 5.

The controller proposed in this paper for the flexible structure is a state feedback control, thus a full-state observer is used in order to be able to overcome the problem of unknown states of the system. The performance of the sliding mode observer for the slow subsystem can be seen in the Fig. 6. The observer error in estimating the states of the slow system is presented and it can be seen that the observer has a great performance. As it can be observed the observer error has greater magnitude at t=2(s) which is significantly a result of the induced vibrations of the decelerating system.

Since the first generalized coordinate is the most important mode (state) of the system and the flexible behavior of the other modes resembles the first one's but with smaller values, in Figs. 7 and 8 the performance of the sliding mode observer for estimating the first and fourth (time rate of changes of the first) generalized coordinates of the system is presented. In these figures, first the actual states and their estimated values are presented; then for further clarification the observer errors in estimating these states are illustrated. In order to fully demonstrate the performance of the

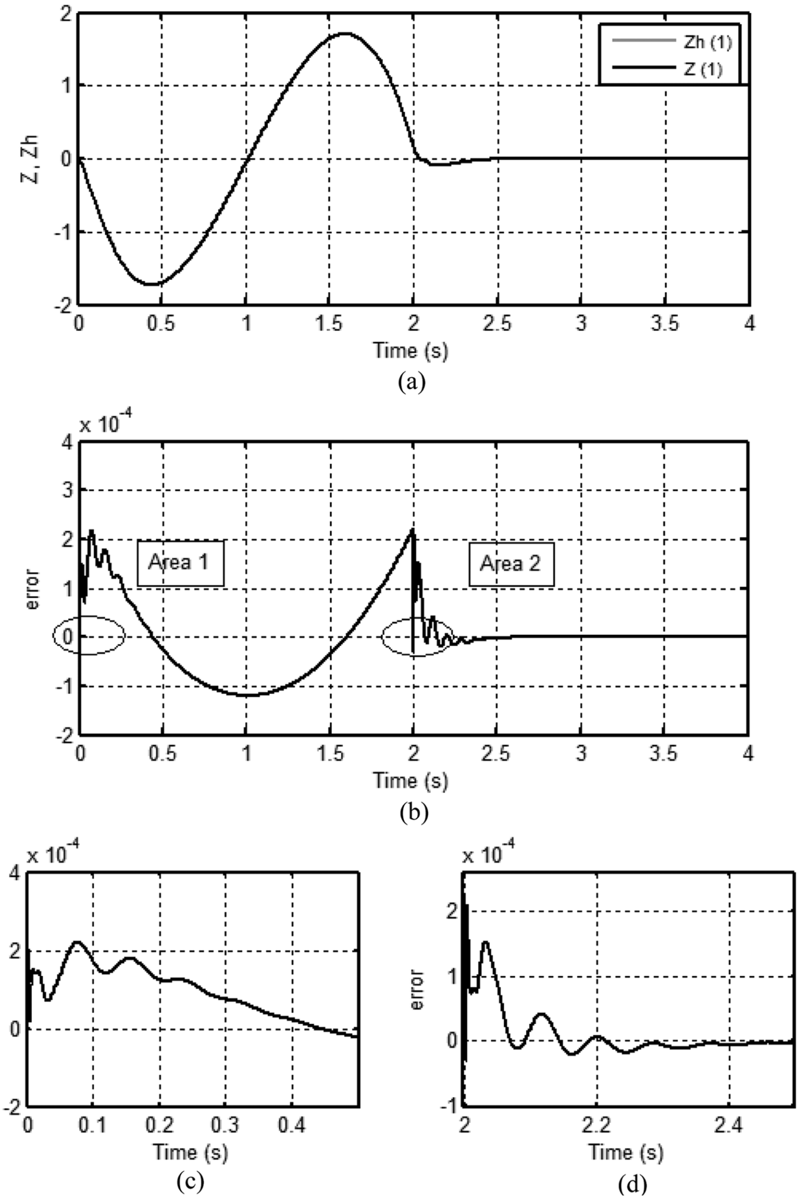


Fig. 7 (a) Time histories of the actual and estimated magnitude of the first fast state $(z)_1$, (b) observer error for estimating $(z)_1$, (c) Zone I, (d) Zone II

observer, in two important areas where the states have faster variations, zoomed views with more details are also presented. Furthermore, from these figures, it can be seen that observer errors increase at the beginning of the maneuver and also at t=2(s) where the maneuver ends. This is a result of the accelerating and decelerating effects of slow subsystem, rigid body motion, which cause induced vibrations on the system.

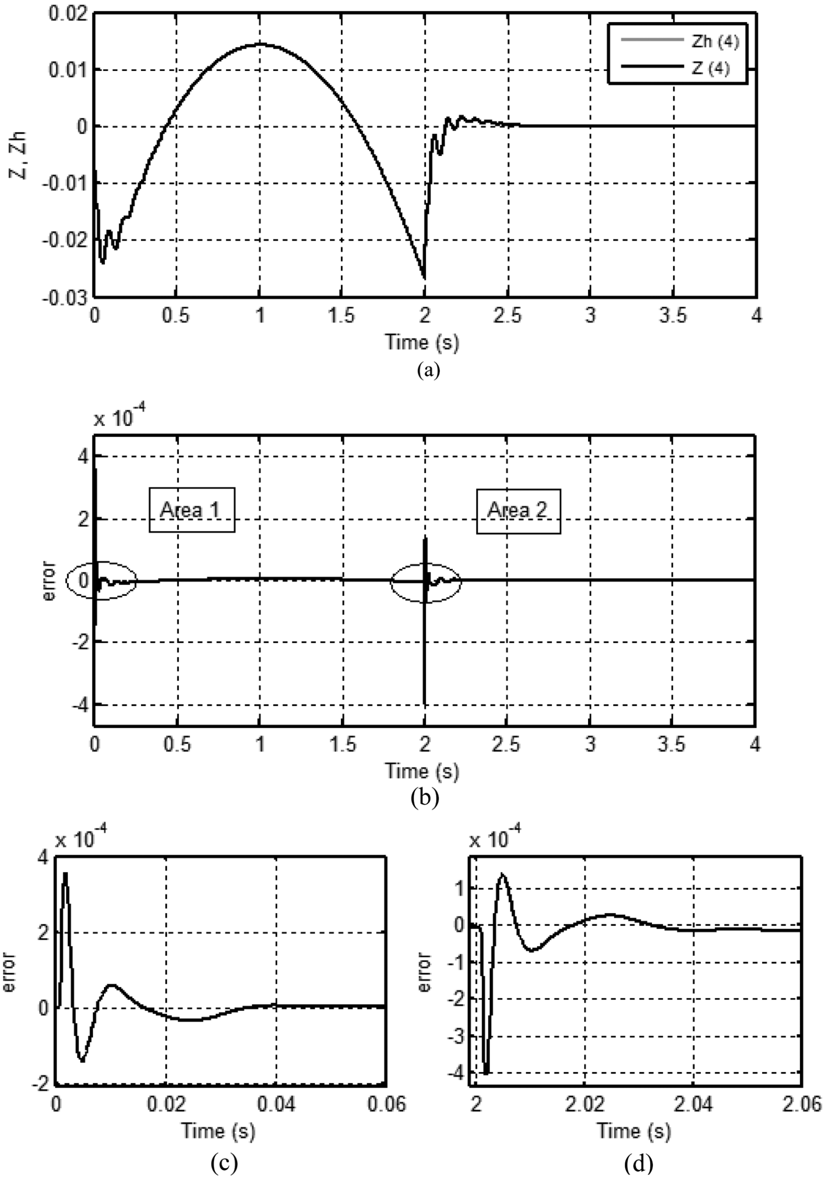


Fig. 8 (a) Time histories of the fourth fast state $Z_4 = \varepsilon(\dot{z})_1$ and its estimated magnitude Zh_4, (b) observer error for estimating $\varepsilon(\dot{z})_1$, (c) Zone I, (d) Zone II

5. Conclusions

Equations of motion of a flexible single link flexible arm with bonded piezoelectric materials were derived using extended Hamilton's principle. Then, the equations of motion of the system were divided into slow and fast subsystems by using singular perturbation concept where the slow subsystem happened to represent equations of motion of a rigid single-link arm with actuator torque at the hub and the fast subsystem included the flexibility of the system and had PZT actuator as its actuator. Method of separation was based on different time scales of rigid and flexible motions. The method allowed us to design control signals for each subsystem, individually, and to combine the control inputs to introduce a composite control input. Utilization of the singular perturbation method reduced the complexity of the system and consequently resulted in a less complicated control problem. Simulation results demonstrated that the proposed composite controller, which consisted of a VSC for slow subsystem and a classical PD control and also the designed sliding mode observers for estimating the unknown states of the system, could effectively accomplish the trajectory tracking and vibration suppression of the flexible structure.

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Nomenclature

x : The link longitudinal coordinate
$\theta(t)$: Hub's joint angle
$w(x,t)$: Transverse displacement (deflection) of the arm (link) at point x
L : Length of the link
W_m : Virtual work due to magnetic terms
T : Kinetic energy of the system
W_e : Electrical energy of the system
V : Potential energy of the system
W : Non-conservative work
$\vec{r}$: Position vector of point
I_h : Hub's moment of inertia
ρ_l : Mass per unit length of the link
ρ_p : Mass per unit length of the piezoelectric patches
V_L : Potential energy of the link
E_L : Modulus of elasticity of the link
I_L : Moment of inertia of the link
V_p : Potential energy of the piezoelectric patches
x_p : Starting point of piezoelectric actuator/sensor in x direction
L_p : Length of piezoelectric actuator/sensor
t_p : Thickness of piezoelectric actuator/sensor
y_p : Starting point of piezoelectric actuator/sensor from neutral axis of the arm (link)
w_p : Width of piezoelectric actuator/sensor
T_{ij} : Stress tensor
S_{ij} : Strain tensor
E_i : Electric field vector
D_i : Electric displacement vector
c_{mn} : Modulus of elasticity of the piezoelectric patches
e_{mk} : Piezoelectric modulus (coefficient) tensor
ε_{ik} : Permittivity tensor of the piezoelectric patches
τ : Torque exerted at the hub
$\varphi_j(x)$: Assumed mode shape functions

$q_j(t)$: Generalized flexible coordinates of the arm (link)
v_{pa} : Voltage of the piezoelectric actuator
v_{ps} : Voltage of the piezoelectric sensor
k_m : Smallest coefficient of the stiffness matrix
ε : Singular perturbation parameter
$\sigma(\theta)$: Sliding surface
θ_d : Desired angle of the hub's rotation

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