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Probabilistic distribution of displacement response of frictionally damped structures excited by seismic loads

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Abstract. Accurate peak response estimation of a seismically excited structure with frictional damping system (FDS) is very difficult since the structure with FDS shows nonlinear behavior dependent on the structural period, loading characteristics, and relative magnitude between the frictional force and the excitation load. Previous studies have estimated the peak response of the structure with FDS by replacing a nonlinear system with an equivalent linear one or by employing the response spectrum obtained based on nonlinear time history and statistical analysis. In case that earthquake excitation is defined probabilistically, corresponding response of the structure with FDS becomes to have probabilistic distribution. In this study, nonlinear time history analyses were performed for the structure with FDS subjected to artificial earthquake excitation generated using Kanai-Tajimi filter. An equation for the probability density function (PDF) of the displacement response is proposed by adapting the PDF of the normal distribution. Coefficients of the proposed PDF are obtained by regression of the statistical distribution of the time history responses. Finally, the correlation between the resulting PDFs and statistical response distribution is investigated.

Keywords: frictional damping system; probabilistic density function; nonlinear system analysis; estimation of peak displacement.

1. Introduction

Frictional Damping System (FDS), one of the vibration-energy-dissipating systems using the friction phenomenon, has been used to control the motion of large-scale structures excited by dynamic loads such as earthquake and wind (Soong and Dargush 1997). FDS, which is available in the recent market, can be classified into two categories. One is the friction damper that can act a fixed amount of friction force and the other is the magneto-rheological (MR) damper, of which friction force can be modulated semi-actively according to the magnitude of applied magnetic field (Chopra 2001, Ying *et al*. 2009). FDS is known to increase significantly the structural capacity of dissipating vibration energy, but has little influence on the natural vibration periods (Chopra 2001). Accordingly, extensive researches associated with FDS have been conducted analytically and experimentally. Pall and Marsh proposed a friction device located at the intersection of cross bracing (Pall and Marsh 1982). A rotational friction damper, which is installed simply using cable, was developed for the seismic protection and its performance was evaluated both experimentally and numerically (Mualla and Belev 2002). Bhaskararao

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and Jangid evaluated the seismic performance of two MDOF (multi-degree-of-freedom) structures connected with friction dampers and proposed the solution of the equation of motion (Bhaskararao and Jangid 2006). Fu and Cherry proposed a design procedure of the friction dampers using a force modification factor (Fu and Cherry 2000), and Lee *et al*. found the optimal slip load distribution for the seismic retrofitting of a damaged building based on the numerical model of a bracing-friction damper system (Lee *et al.* 2008). Ciampi *et al.* performed a simple approach for determining the distribution of stiffness and strength of the FDS within the elastic and inelastic structures and proposed the design methodology of a MDOF structure with a bracing-friction damper system (Ciampi *et al*. 1995). Also, the study on the semi-active friction damper with a variable slip force was conducted for obtaining better control performance over the passive one (Kori and Jangid 2008).

The accurate peak response estimation of a seismically excited structure with FDS is very difficult since the structure with FDS shows nonlinear behavior dependent on the structural period, loading characteristics and relative magnitude between the frictional force and the excitation load (Lee *et al*. 2007). In case that earthquake load is defined with probabilistic characteristics, the corresponding response of the structure with FDS becomes to have specific probabilistic distribution (Cai *et al.* 2000). However, aforementioned previous studies mostly did not define the characteristics of the probability distribution of structural response estimated, while they provided mean and standard deviation of the peak displacements, which were simply obtained by statistical analysis.

Park and Min investigated the response of nonlinear systems using probability density function (PDF) and estimated the peak response based on the PDF of an equivalent linear system replacing nonlinear one considering the characteristics of FDS realistically (Park and Min 2005). Cai and Lin obtained the PDF of a structure with FDS by solving a Fokker-Plank equation (Cai and Lin 1998). However, because they assumed that input excitation was white noise and an equivalent nonlinear system was utilized to replace the original one, the resulting PDF might be different from that of the actual structural response induced by earthquake excitation.

In this study, nonlinear time history analyses were performed for the structure with FDS subjected to artificial earthquake excitation generated using Kanai-Tajimi filter (Soong and Grigoriu 1993). An equation representing the PDF of the displacement response is proposed by adapting the normal distribution accounting for the influence of the friction force and the natural frequency. And then coefficients of the proposed PDF are determined by regression analysis of the statistical distribution of the time history response. Finally, correlation between the proposed PDF and statistical PDF is investigated.

2. Modeling

2.1 Definition of the input excitation

In this study, in order to simulate the earthquake loads, the absolute acceleration of a single degree of freedom (SDOF) system, which is modeled by so called Kanai-Tajimi filter and excited by white noise excitation, is used as the ground acceleration (Soong and Grigoriu 1993). The Kanai-Tajimi filter can be represented by the following equation of motion of a SDOF system.

$$\ddot{x}_f + 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f = -w \tag{1}$$

$$E[w(t)w(t+\tau)] = 2\pi S_w\delta(\tau) \tag{2}$$

where, x_f, ω_f and ξ_f, respectively, denote the displacement, the natural frequency, the damping ratio of the SDOF system, and w is the white noise, S_w is the power spectral density of w, and $\delta(\tau)$ is dirac delta function.

In Kanai-Tajimi filter, the natural frequency and damping ratio of the filter should be changed with respect to the soil type, and $\omega_f = 15.6$ rad/s and $\xi_f = 60\%$ are used for simulating earthquakes on the rock site (Soong and Grigoriu 1993). S_w of input excitation is a critical parameter in estimating the peak response of the structure with/without FDS. In this study, S_w is determined to provide a target RMS value of ground absolute acceleration. When S_w is given, variances of the displacement and velocity of the SDOF system in Eq. (1) become

$$\sigma_{xf}^2 = \frac{\pi S_w}{2\xi_f \omega_f^3} \quad \sigma_{\dot{x}f}^2 = \frac{\pi S_w}{2\xi_f \omega_f} \tag{3}$$

And the resulting standard deviation of the absolute acceleration based on Eq. (3) is

$$\sigma_{\ddot{x}_a}^2 = \frac{\pi S_w}{2\xi_f}[4\xi_f^2\omega_f^2 + 1] \tag{4}$$

So, S_w is obtained as

$$S_w = \frac{2\xi_f \sigma_{\ddot{x}_a}^2}{\pi(4\xi_f^2\omega_f^2 + 1)} \tag{5}$$

In case that the structural response is a narrow band stochastic process governed by resonance content, the power spectral density function $S_{\ddot{x}_a}$ of the filtered white noise is presented as follows (Lutes and Sarkani 1997).

$$S_{\ddot{x}_a} = \frac{1 + 4\xi_f^2\beta^2}{[(1-\beta^2)^2 + 4\xi_f^2\beta^2]} S_w \tag{6}$$

where β is a frequency ratio ω_n/ω_f and ω_n means natural frequency of the structures.

2.2 Modeling of the structure with FDS

In this section, an equation of a SDOF system with the FDS which is modeled by simple Coulomb friction is given as

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s = -m\ddot{x}_f - f\,\mathrm{sgn}(\dot{x}_s) \tag{7}$$

where, m_s, c_s, k_s and f denote structural mass, damping, stiffness and frictional force of FDS, respectively. And Fig. 1 shows the model of a SDOF system on a friction surface. The Eq. (8) is reconstructed in the form of mass-normalized SDOF system.

$$\ddot{x}_s + 2\xi_s\omega_n\dot{x}_s + \omega_n^2 x_s = -\ddot{x}_f - f_f \ \mathrm{sgn}(\dot{x}_s) \tag{8}$$

where, ξ_s and ω_n are, damping ratio and radial natural frequency of the structure. $f_f = f/m$ is the mass-normalized friction force.

The control performance of structure with FDS varies with regard to relative magnitude between the

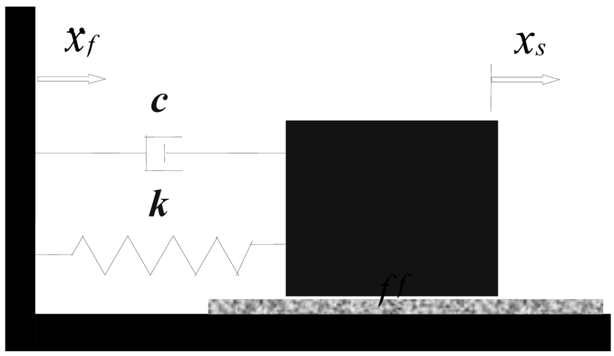

Fig. 1 Model of a SDOF system on the friction surface

external load and friction force, and the structural properties such as period and damping ratio having strong influence on seismic responses (Lee *et al*. 2007). Lee *et al*. showed that the ratio of the friction force to the story shear force is strongly related to the control performance of the FDS (Lee *et al.* 2005). The friction force can be expressed using non-dimensional variable (ρ_O).

$$\rho_O = f_f / f_s \tag{9}$$

where, f_s is the RMS value of the story shear force of the SDOF structure without FDS and is given by

$$f_s = k_s |x_o|_{RMS} \tag{10}$$

where, $|x_o|_{RMS}$ denotes the RMS displacement, which can be estimated by the following equation using $S_{\ddot{x}_a}$ which was obtained in section 2.1.

$$|x_o|_{RMS} = \sqrt{\frac{\pi S_{\ddot{x}_a}}{2\xi_s \omega_n^3}} \tag{11}$$

3. Probabilistic density function

3.1 Equivalent nonlinear system method

The PDF of structural responses with FDS cannot be obtained analytically in the exact closed-form solution due to their strong nonlinearity. In this section, the equivalent nonlinear system method is introduced in order to estimate the joint PDF of a SDOF structure with FDS. Cai and Lin proposed this method to obtain an approximate solution of the joint PDF for a nonlinear SDOF system, which does not belong to a class of generalized stationary potential, and presented an example for the case of Coulomb damping (Cai and Lin 1998). They replaced the nonlinear SDOF system with an equivalent nonlinear system which belongs to the class of generalized stationary potential. That approximation procedure was developed for nonlinear systems excited by white noise. In this study, the procedure proposed by Cai and Lin is adopted and applied to the system excited by white noises filtered with the Kanai-Tajimi filter (Cai and Lin 1998, Soong and Grigoriu 1993). The resulting PDF of equivalent linear system replacing the nonlinear one with FDS is given as

$$\phi(H) = \frac{1}{\pi S_w}\left[2\xi_s\omega_n H - \frac{4}{\pi}f_f\sqrt{2H}\right] \tag{12a}$$

$$P(x,\dot{x}) = C\exp[-\phi(H)] \tag{12b}$$

$$H = \frac{1}{2}(\dot{x}_s^2 + \omega_n^2 x_s^2) \tag{12c}$$

where, H denote energy function of the nonlinear system. The normalization coefficient C is given as

$$C^{-1} = \frac{2\pi^3(S_{\ddot{x}_a})^2}{\omega_n\{(4/\pi)f_f\}^2} \tag{13}$$

3.2 New probability density function based on normal distribution

The PDF proposed by Cai and Lin was obtained by solving the Fokker-Planck equation, partial differential equation (Cai and Lin 1998). However, because they assumed the conditions that input load was white noise and equivalent linear system could replace the nonlinear one, the resulting PDF might be different from that of the structural response induced by earthquake loads.

The PDF of normal distribution using mean value and standard deviation of a random variable is given as

$$P(X) = \frac{1}{\sigma_X\sqrt{2\pi}}\exp\left[\frac{-(X-m)^2}{2\sigma_X^2}\right] \tag{14}$$

The PDF of the structure with FDS is proposed as follows to consider the variation of PDF according the normalized friction force ρ and the natural period T_n by modifying the PDF of the normal distribution.

$$P(X) = \frac{1}{(1-\alpha\rho)\sigma_X\sqrt{2\pi}}\exp\left[\frac{-(X-m)^2}{2(1-\alpha\rho)^2\sigma_X^2}\right] \tag{15}$$

where, α is a parameter to reflect the effect of the natural period of the structure. α is obtained by numerical curve fitting technique using LSQCURVEFIT function in MATLAB 7.1.

$$\alpha(T_n) = 0.5774 - 0.1251T_n \tag{16}$$

4. Statistical verification of PDF approximation

The validity of the PDF proposed in this study for the structure with FDS is statistically verified according to the friction force and the natural period of the structure in this section. The structure is assumed to have damping ratio ξ_s of 5%, and natural periods T_n of 0.2, 0.5, 1.0, 2.0 second. Numerical simulations are conducted with an excitation signal having duration second of 300 times as long as the structural period. In this study, the standard deviation of $\ddot{x}_a$, ($\sigma_{\ddot{x}_a}$), is assumed to be 0.1g to generate excitation load. Time data corresponding to the interval from 0 second to the second of 30 times as long

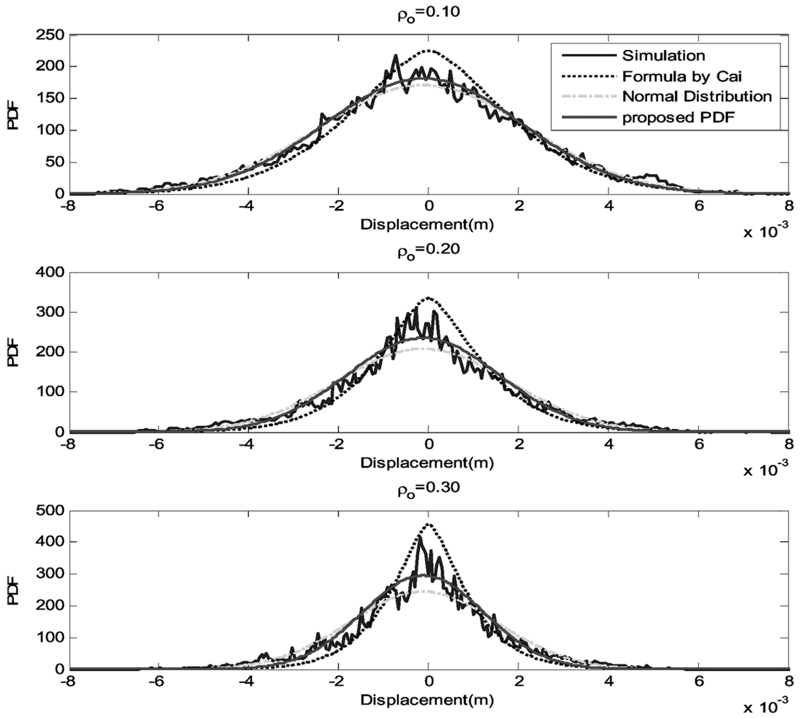

Fig. 2 Comparison on probabilistic density function of displacement response of SDOF structure ($T_n = 0.2$)

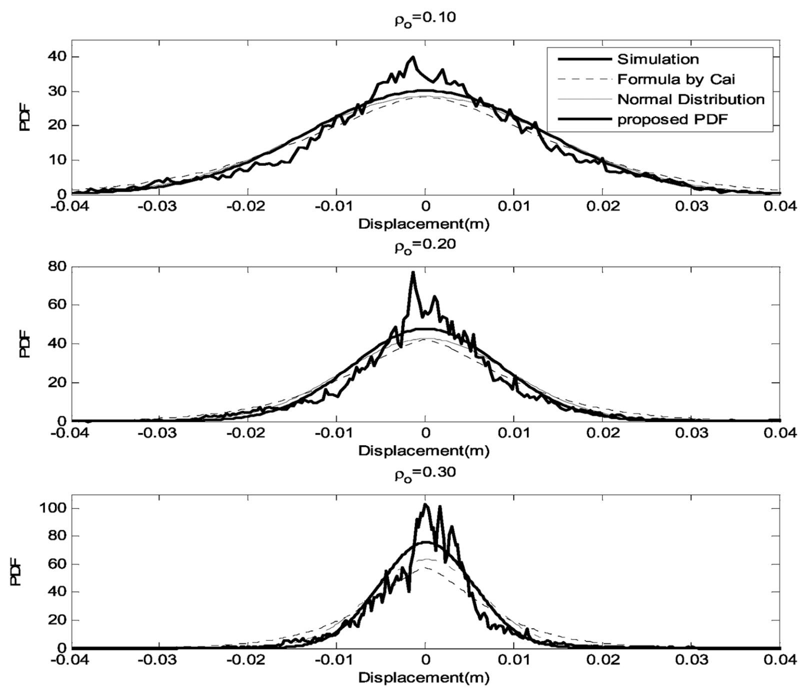

Fig. 3 Comparison on probabilistic density function of displacement response of SDOF structure ($T_n = 0.5$)

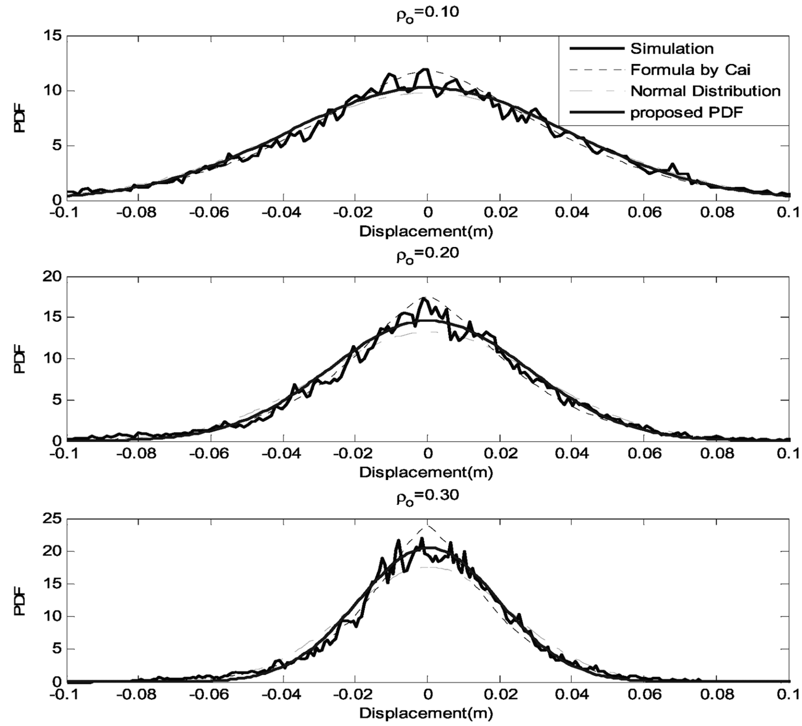

Fig. 4 Comparison on probabilistic density function of displacement response of SDOF structure (T_n = 1.0)

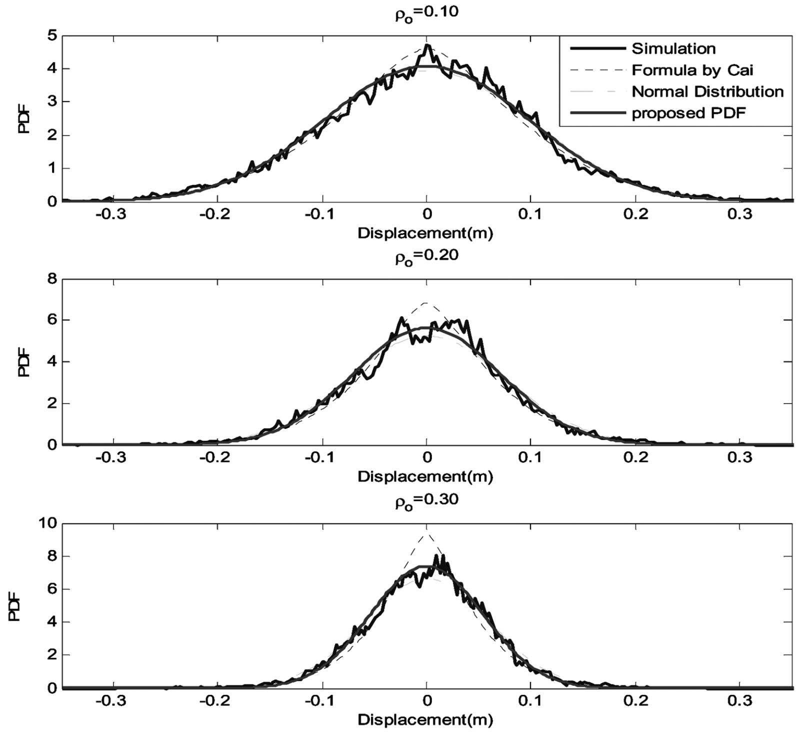

Fig. 5 Comparison on probabilistic density function of displacement response of SDOF structure (T_n = 2.0)

Table 1 Coefficients of determination of probability density functions according to 3 cases

T_n (second)	ρ_o	Uncontrolled	0.1	0.2	0.3
0.2	Case 1	0.9957	0.9887	0.9730	0.9506
	Case 2	0.9877	0.9559	0.9076	0.8781
	Case 3	0.9957	0.9897	0.9774	0.9603
0.5	Case 1	0.9912	0.9851	0.9695	0.9467
	Case 2	0.9914	0.9887	0.9620	0.9091
	Case 3	0.9912	0.9886	0.9814	0.9668
1.0	Case 1	0.9937	0.9905	0.9811	0.9784
	Case 2	0.9933	0.9922	0.9865	0.9851
	Case 3	0.9937	0.9928	0.9882	0.9874
2.0	Case 1	0.9952	0.9827	0.9757	0.9808
	Case 2	0.9953	0.9945	0.9836	0.9684
	Case 3	0.9952	0.9866	0.9845	0.9900

·Case 1: Normal Distribution (Gaussian)
·Case 2: PDF proposed by Cai and Lin
·Case 3: PDF proposed in this study

as the structural period is discarded in the calculation of PDF, in order to exclude the initial non-stationary response and extract the stationary one.

Figs. 2, 3, 4 and 5 respectively show the PDF of the SDOF systems of which structural periods are 0.2, 0.5, 1.0 and 2.0 second. The frictional force varies with $\rho_o = 0.1$, 0.2 and 0.3. Figs. 2 and 3 indicate that the proposed PDF modifying normal distribution fits the distribution of the displacement of frictionally damped SDOF structures for various ρ_o with considerable accuracy, while PDF proposed by Cai and Lin shows comparatively large error especially for the structure with $T_n = 0.2$ and $T_n = 0.5$ second (Cai and Lin 1998). Furthermore, PDF proposed by Cai and Lin requires complicated procedure in order to obtain the solution of the Fokker-Planck equation, and employs various assumptions on input excitation and equivalent system, while the proposed PDF simply requires the RMS displacement of the structure without FDS and the information on friction force and structural period (Cai and Lin 1998).

Table 1 lists the coefficients of determination of the investigated PDFs. It is observed that the proposed PDF in this study (case 3) provides higher values of coefficients of determination than the PDF proposed by Cai and Lin (case 2) for most structural periods and especially for larger ρ_O, which indicates that the proposed PDF can capture the characteristics of the PDF with respect to the friction force and structural period, exactly (Cai and Lin 1998).

5. Conclusions

In this study, a probabilistic density function of structural response was proposed in order to estimate the peak displacement of a SDOF structure with frictional damping system subjected to earthquake excitation defined in probabilistic sense such as earthquake. And the verification of the PDF was performed by nonlinear time history analysis. To simulate the earthquake excitation defined in probabilistic sense, white noise filtered by Kanai-Tajimi filter was generated, and its power spectral density was employed in the PDF (Soong and Grigoriu 1993).

The proposed PDF using the estimated RMS displacement was updated based on the normal distribution PDF which could be simulated by standard deviation of random variable. The proposed PDF for the SDOF structure with FDS, of which coefficients were obtained by curve fitting regression, is compared with PDF proposed in the previous study by Cai and Lin (Cai and Lin 1998). Compared to the PDF proposed by Cai and Lin requiring various assumption and complicate calculation, the proposed PDF shows better accuracy especially in the natural period of structure is 0.5 second (Cai and Lin 1998). And for the natural period of 1.0 and 2.0 seconds, both PDFs have coefficient of determination higher than 0.95.

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References

Bhaskararao, A.V. and Jangid, R.S. (2006), "Seismic Analysis of Structures connected with friction dampers", *Eng. Struct.*, **28**(5), 690-703.

Cai, G.P., Huang, J.Z., Feng, S. and Chao, W. (2000), "Modified sliding-mode bang-bang control for seismically excited linear structures", *Earthq. Eng. Struct. D.*, **29**(11), 1647-1657.

Cai, G.Q. and Lin, Y.K. (1998), "A new approximate solution technique for randomly excited non-linear oscillators", *Int. J. Nonlinear Mech.*, **23**(5/6), 409-420.

Chopra, A.K. (2001), *Dynamics of Structures; Theory and Applications to Earthquake Engineering*, Prentice Hall, Upper Saddle River, New Jersey.

Ciampi, V., De Angelis, M. and Paolacci, F. (1995), "Design of yielding or friction-based dissipative bracing for seismic protection of buildings", *Eng. Struct.*, **17**(5), 381-391.

Fu, Y. and Cherry, S. (2000), "Design of Friction Damped Structures Using Lateral Force Procedure", *Earthq. Eng. Struct. D.*, **29**(7), 989-1010.

Kori, J.G. and Jangid, R.S. (2008), "Semi-active friction dampers for seismic control of structures", *Smart Struct. Syst.*, **4**(4), 493-515.

Lee, S.H., Min, K.W., Chung, L., Lee, S.K., Lee, M.K., Hwang, J.S., Choi, S.B. and Lee, H.G. (2007), "Bracing Systems for Installation of MR dampers in a building structure", *J. Intel. Mat. Syst. Str.*, **18**(11), 1111-1120.

Lee, S.H., Min, K.W., Lee, R.J., Lee, H.G., Sung, K.G. and Kim, D.K. (2005), "Preliminary design procedure of MR dampers for controlling seismic response of building structures", *Int. J. Mod. Phys. B*, **19**(7), 1492-1498.

Lee, S.K., Park, J.H., Moon, B.W., Min, K.W., Lee, S.H. and Kim, J. (2008), "Design of a bracing-friction damper system for seismic retrofitting", *Smart Struct. Syst.*, **4**(5), 685-696.

Lutes, L.D. and Sarkani, S. (1997), *Stochastic analysis of structural and mechanical vibrations*, Prentice Hall, Upper Saddle River, New Jersey.

Mualla, I.H. and Belev, B. (2002), "Performance of steel frames with a new friction damper device under earthquake excitation", *Eng. Struct.*, **24**(3), 365-371.

Pall, A.S. and Marsh, C. (1982), "Seismic Response of Friction Damped Braced Frames", *J. Struct. Div.*, **108**(6), 1313-1323.

Park, J.H. and Min, K.W. (2005), "Bounded nonlinear stochastic control based on the probabilistic distribution for the SDOF oscillator", *J. Sound. Vib.*, **281**(1), 141-153.
Soong, T.T. and Dargush, G.F. (1997), *Passive Energy Dissipation Systems in Structural Engineering*, John Wiley & Sons Ltd, Chichester, England.
Soong, T.T. and Grigoriu, M. (1993), *Random Vibration of Mechanical Structural System*, Prentice Hall, Englewood Cliff, New Jersey.
Ying, Z.G., Ni, Y.Q. and Ko, J.M. (2009), "A semi-active stochastic optimal control strategy for nonlinear structural systems with MR dampers", *Smart Struct. Syst.*, **5**(1), 69-79.

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