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Structural damage detection using decentralized controller design method

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Abstract. Observer-based fault detection and isolation (FDI) filter design method is a model-based method. By carefully choosing the observer gain, the residual outputs can be projected onto different independent subspaces. Each subspace corresponds to the monitored structural element so that the projected residual will be nonzero when the associated structural element is damaged and zero when there is no damage. The key point of detection filter design is how to find an appropriate observer gain. This problem can be interpreted in a geometric framework and is found to be equivalent to the problem of finding a decentralized static output feedback gain. But, it is still a challenging task to find the decentralized controller by either analytical or numerical methods because its solution set is, generally, non-convex. In this paper, the concept of detection filter and iterative LMI technique for decentralized controller design are combined to develop an algorithm to compute the observer gain. It can be used to monitor structural element state: healthy or damaged. The simulation results show that the developed method can successfully identify structural damages.

Keywords: structural damage detection; decentralized control; iterative LMI technique.

1. Introduction

Existence of structural damage in civil engineering infrastructures, such as high-rise buildings, long-span bridges, offshore platforms, etc., may greatly influence the overall performance of the system or even lead to disastrous consequences. Therefore, detecting structural damages, which are caused by earthquakes, impacts, or explosions immediately after the event or monitoring long-term deterioration due to environmental changes and human uses, is very important for structural maintenance. This leads to the field of research known as structural damage detection, or structural health monitoring, or alternatively, fault detection, isolation and identification.

In the past decades, numerous approaches to the problem of Failure Detection and Isolation (FDI) in dynamic systems have been developed (Doebling, *et al*. 1996, Sohn, *et al*. 2003, Kim, *et al.* 2007,

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Glaser, *et al.* 2007, Shin and Oh 2007, Gao and Spencer 2007). Among them are two major FDI philosophies: physical redundancy and analytical redundancy. Physical redundancy is achieved simply through hardware replication. Unlike physical redundancy, analytical redundancy, which implies the inherent redundancy contained in the static and dynamic relationship among the system inputs and measured outputs (Frank 1990), is a model-based method and has gained increasing consideration world-wide recently. Analytical redundancy methods have many advantages over physical redundancy methods, for example the replication of identical hardware components (actuator/sensor) is more expensive, restricted, and sometimes difficult to implement in practice (Dharap, *et al.* 2006, Koh, *et al.* 2005a, 2005b).

There are many FDI methods based on analytical redundancy approaches. Among them, the Beard-Jones detection filter (BJDT) has received increasing consideration world-wide recently. In their pioneering work done in the early seventies, Beard (1971) and Jones (1973) found that with the proper choice of filter feedback gains, the filter residual will have directional characteristics that can be easily associated with different faults. The BJDT filter design method has been successively improved by many people, e.g. Massoumnia (1986), White (1987), Douglas (1996, 1999) and Liberatore (2002). In particular, an important geometric interpretation of the BJDT filter has been developed by Massoumnia (1986). Douglas (1996, 1999) extended this geometric interpretation and found that the problem of finding a detection filter gain is equivalent to that of finding a constant static decentralized output feedback controller, and then Youla parameterization was used to obtain those detection filters. In this paper, this equivalent static decentralized output feedback controller design problem is solved by iterative linear matrix inequalities (ILMI) method, which is introduced by Cao, *et al*. (1998).

Decentralized control is widely used in large-scale systems, e.g. electric power networks, socioeconomic systems and large-scale space stations, which are usually geographically distributed. Centralized control of such systems is either uneconomical or unreliable due to long-distance information transfer between local control stations. Decentralized control only uses the locally measurements to compute control inputs, which reduces the risk of data losing and time delay during long-distance information transfer. However, it is a challenging task to find decentralized controllers by either analytical or numerical methods because its solution set is generally non-convex. Recently, linear matrix inequalities (LMIs) approach has been proposed to solve the decentralized stabilization and H_∞ control problem, for example Cao, *et al.* (1998), Scorletti and Duc (2001), Zhai, *et al.* (2001) and reference therein. Cao, *et al.* (1998) studied the static output feedback decentralized stabilization problem, proposed an iterative LMI (ILMI) algorithm to obtain the decentralized feedback gain and extended the idea to static output feedback stabilization with guaranteed H_∞ performance. Scorletti and Duc (2001) modeled the linear time invariant (LTI) system with decentralized controllers as an interconnection of subsystems. Dissipative concept and LMI approach were combined to design the controller for each closed-loop subsystem. Zhai, *et al.* (2001) considered the dynamic decentralized output feedback H_∞ control problem and reduced it to a feasibility problem of a bilinear matrix inequality (BMI) which was solved by using the idea of the homotopy method.

In this paper, the concept of detection filter design (Douglas 1996, 1999) and the ILMI method for decentralized controller design (Cao, *et al.* 1998) are combined to develop an algorithm for structural damage detection and isolation. The healthy system is assumed to be known a priori and observable. The state observer is constructed to generate the residual outputs, which contain the structural damage information. Once the system is damaged, the residual outputs will significantly deviate from zero even if the data on the intact system is absent. It is the property of the observer-based damage detection methods. By carefully choosing the observer gain, the residual outputs can be projected onto different independent subspaces. Each subspace is related to the monitored structural element. The problem of finding the observer gain is then converted to that of finding a static decentralized output feedback controller. Finally, the ILMI approach is used to find the stable observer gain which places the poles of the closed-loop system in the left side of some specified negative number to improve FDI system performance. The simulation results show that the developed method can successfully identify structural damage.

This paper is organized as follows. Section 2 is divided into two parts. The first part explains the iterative LMI method for decentralized controller design problem; the second part describes the observer-based detection filter design method and how to apply iterative LMI procedure to find the detection gain for structural damage detection and isolation. Section 3 introduces two examples to illustrate the applicability of the presented FDI method. Finally, the conclusion is made in Section 4.

2. Mathematical formulations

2.1. Decentralized static output feedback controller design using ILMI approach

This section gives a brief review of decentralized static output feedback controller design based on iterative LMI approach (Cao, *et al.* 1998). Consider a linear-time-invariant (LTI) system with q control channels

$$\begin{aligned} \dot{x} &= Ax + \sum_{i=1}^{q} B_i u_i \\ z &= C_1 x \\ y_i &= C_{2i} x, \qquad\qquad i = 1, 2, \ldots, q \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, z is the controlled output, u_i and y_i are the control input and the measured output of channel i (i = 1,2,...,q). In general, they are matrices instead of scalars. B_i is an $n \times n_{ui}$ input influence matrix, C_1 is an $n_z \times n$ controlled output influence matrix, C_{2i} is an $n_{yi} \times n$ measured output influence matrix.

The decentralized static output feedback control law is characterized as

$$u_i = F_i y_i, \qquad\qquad i = 1, 2, \ldots, q \tag{2}$$

where F_i is an $n_{ui} \times n_{yi}$ constant output feedback gain. The controlled input u_i only depends on y_i, which is part of the whole measurements y. In this paper, the observer-based structural damage detection problem can be converted to the same form as Eq. (1) and Eq. (2). Thus, the proposed iterative LMI method (Cao, *et al*. 1998) for the decentralized static output feedback controller design can be applied for structural damage detetion.

If the input u and output y are stacked as: $u = [u_1^T, u_2^T, \ldots, u_q^T]^{\mathrm{T}}$, $y = [y_1^T, y_2^T, \ldots, y_q^T]^{\mathrm{T}}$, then the system (1) and control law (2) can be rewritten as a closed-loop form as

$$\begin{aligned} \dot{x} &= (A + BF_D C_2)x \\ z &= C_1 x \end{aligned} \tag{3}$$

where: $B = [B_1\ B_2\ \dots\ B_q],\ C_2 = [C_{21}^T\ C_{22}^T\ \dots\ C_{2q}^T]^T,\ F_D = \begin{bmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_q \end{bmatrix}$

Notice that the static output feedback gain F_D is a block diagonal matrix in spite of a full matrix as seen in centralized control. The closed-loop system (3) is stabilizable if there exists a block diagonal matrix F_D such that all eigenvalues of $(A + BF_DC_2)$ are in the left complex plane. However, even for centralized static output feedback control it is very difficult to find a stable controller since the solution set is non-convex (Ghaoui, *et al.* 1997). Obviously, it is more difficult to obtain the decentralized static output feedback controller F_D because of its block diagonal constraint. Cao, *et al.* (1998) presented an iterative LMI approach to solve this problem; they proved the following necessary and sufficient condition for decentralized static output feedback stability.

There exists a decentralized static output feedback controller F_D such that the closed-loop system is stable and all poles of the closed-loop system are placed to the left of $\mathrm{Re}(s) = \alpha/2$ in the complex plane if and only if there exist two symmetric and positive definite matrix $P > 0$ and $X > 0$ with compatible dimensions satisfying the matrix inequality.

$$\begin{bmatrix} A^TP + PA - XBB^TP - PBB^TX + XBB^TX - \alpha P & (B^TP + F_DC_2)^T \\ (B^TP + F_DC_2) & -I \end{bmatrix} < 0 \tag{4}$$

where α is a negative number.

The matrix inequality (4) is non-linear since there exist non-linear terms, such as XBB^TP, PBB^TX and XBB^TX. Thus, the difficulty lies in how to find a block diagonal matrix F_D such that the non-linear matrix inequality (4) is satisfied with $P > 0$ and $X > 0$. It can not be solved directly using LMI technique. This is the reason why the iterative LMI approach is needed. The iterative algorithm consists of the following steps.

Step 1: Select $Q > 0$, and solve the following Riccati equation:

$$A^TP + PA - PBB^TP + Q = 0 \tag{5}$$

Assume the solution is X and set $i = 1$.

Step 2: Substitute X into the matrix inequality (6) and solve the generalized eigenvalue problem for α_i.

$$\begin{bmatrix} A^TP_i + P_iA - XBB^TP_i - P_iBB^TX + XBB^TX - \alpha P_i & (B^TP_i + F_DC_2)^T \\ (B^TP_i + F_DC_2) & -I \end{bmatrix} < 0 \tag{6}$$

$$P_i = P_i^T > 0 \tag{7}$$

Step 3: Substitute X and α_i into the matrix inequality (6) and solve the optimization problem for P_i and F_D: Minimize trace (P_i) subjected to the LMI constraints (6) and (7).

Step 4: If α_i or all eigenvalues of $(A + BF_DC)$ are less than some specified negative number μ, F_D is a stabilized decentralized static output feedback gain.

Stop.

Step 5: If $\|X - P_i\| < \delta$, a pre-determined tolerance, go to *Step 6*, else set $X = P_i$ and $i = i + 1$, then go to *Step 2*.

Step 6: The system may be not stabilizable via decentralized static output feedback gain. Stop.

2.2. Detection filter problem

The state-space model of a LTI dynamic system with q failure modes can be modeled by (Beard 1971, Jones 1973, Massoumnia 1986, White and Speyer 1987, Douglas 1996 and 1999).

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_u f(t) + \sum_{i=1}^{q} F_i m_i(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8}$$

where $x \in x$, $f \in u$ and $y \in y$ with $n = \dim(x)$, $p = \dim(u)$ and $m = \dim(y)$. A is an $n \times n$ system state transmission matrix, B_u is an $n \times q$ input influence matrix, f is an $q \times 1$ input force vector, and C is an $m \times n$ output influence matrix. F_i is an $n \times 1$ fault direction vector, $i = 1,2,...,q$, q is the number of fault directions and $m_i(t)$ is the ith arbitrary scalar function of time. When no faults occur, $m_i(t) = 0$. The fault directions F_i can be used to model actuator, sensor and component faults. It is not necessary for F_i to be a vector (Douglas 1993). It can have 2 or more columns. But, the basic procedure is the same. In this paper, we make it rank one for simplicity. Also, the stiffness matrix of some civil structures can be simplified and decomposed to be the combination of rank one fault vectors, as shown in the examples. A detailed treatment of all three failures can be found in Beard (1971) and Jones (1973). Consider the following full-order observer

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B_u f(t) - L(y(t) - C\hat{x}(t)) \\ r(t) &= C\hat{x}(t) - y(t) \end{aligned} \tag{9}$$

where $L \in \mathbf{R}^{n \times m}$ is the observer gain matrix, $r(t)$ is the residual outputs. Define the state estimation error $\varepsilon(t) = \hat{x}(t) - x(t)$, then the error dynamics system becomes

$$\begin{aligned} \dot{\varepsilon}(t) &= (A + LC)\varepsilon(t) - \sum_{i=1}^{q} F_i m_i(t) \\ r(t) &= C\varepsilon(t) \end{aligned} \tag{10}$$

The solution of $r(t)$ is

$$r(t) = Ce^{(A+LC)t}\varepsilon(0) - C\sum_{i=1}^{q} \int_0^t e^{(A+LC)(t-\tau)} F_i m_i(\tau) d\tau$$

$$= Ce^{(A+LC)t}\varepsilon(0) - \sum_{i=1}^{q} \int_0^t \left[CF_i + C(A+LC)F_i(t-\tau) + C(A+LC)^2 F_i \frac{(t-\tau)^2}{2!} + ... \right] m_i(\tau) d\tau \tag{11}$$

$$= Ce^{(A+LC)t}\varepsilon(0) + \sum_{i=1}^{q} [\alpha_{i0}(t)CF_i + \alpha_{i1}(t)C(A+LC)F_i + \alpha_{i2}(t)C(A+LC)^2 F_i + ...]$$

where $a_{i0}(t)$, $a_{i1}(t)$, ... are functions of time. If the system is observable and L is chosen such that (A + LC) is stable, i.e., all eigenvalues of (A + LC) stay in the left side of the imaginary axis in the complex plane, then the steady-state solution for $r(t)$ becomes

$$r(t) = \sum_{i=1}^{q}[\alpha_{i0}(t)CF_i + \alpha_{i1}(t)C(A+LC)F_i + \alpha_{i2}(t)C(A+LC)^2F_i + ...] \tag{12}$$

Eq. (12) shows that, in steady-state and in the absence of disturbances and modeling errors, the residual $r(t)$ is nonzero when any of faults has occurred. But, it is not enough. We also want to know which fault has occurred and this is what a detection filter is designed to do. In the papers by Massoumnia (1986) and Douglas (1999), the detection filter problem is interpreted in a geometric approach, where a set of (C, A)-invariant subspaces ω_i are found first and then L is the end product of an observer design algorithm. When $m_i(t) \neq 0$, the residual $r(t)$ remains in the output subspace $C\omega_i$. Furthermore, the output subspace $C\omega_1$, $C\omega_2$, ..., $C\omega_q$ are independent so that $r(t)$ has a unique representation $r(t) = z_1 + z_2 + \cdots + z_q$ with $z_i \in C\omega_q$. The fault is identified by projecting $r(t)$ onto each of the output subspaces $C\omega_i$. The following statement summarizes the above detection filter problem in the geometric framework.

Given the LTI system (8), the detection problem is to find a set of n-dimensional subspaces Ω_i, i=1, 2, ..., q, such that the following conditions are satisfied.

Subspace invariance:

$$(A + LC)\omega_i \subseteq \omega_i \tag{13}$$

Fault inclusion

$$F_i \subseteq \omega_i \tag{14}$$

Output separability

$$C\omega_i \cap \sum_{j \neq i}^{q} C\omega_j = \varnothing \tag{15}$$

for a matrix L with dimension $n \times m$.

Douglas (1996 and 1999) showed that the subspaces ω_i (i = 1, 2, ..., q) are usually chosen as a set of mutual detectable, minimal unobservability subspaces or so-called detection spaces. These subspaces are dependent on system matrices A, C and F_i and satisfy conditions (13), (14) and (15) such that the spectrum of (A + LC) may be placed arbitrarily. Given a set of detection spaces, the detection filter gain L can then be characterized easily as follows.

Let ω_1, ω_2, ..., ω_q be a set of (C, A)-invariant subspaces that solve the detection filter problem and let the W_i: $\omega_i \rightarrow x$ be the insertion map. Let P_i: $\omega_i \rightarrow \omega_i$ be any projection where Ker(P_i) = Ker($C\omega_i$) and let $\hat{F}_i$ decompose P_i as $\hat{F}_iF_i^T = P_i$ and $\hat{F}_iF_i^T = I$. Let H_i: $y \rightarrow y$ be another projection where Im(H_i) = $C\omega_i$ and let $\tilde{H}_i$ be the associated natural projection that satisfies $\tilde{H}_iCT_i\hat{F}_i = I$ and $CY_i\hat{F}\tilde{H}_i = H_i$. The kernel of H_i and $\tilde{H}_i$ are satisfy

$$\sum_{j \neq i} CW_j \in \mathrm{Ker}(H_i) = \mathrm{Ker}(\tilde{H}_i) \tag{16}$$

Also define the projection

$$H_0 = \left(I - \sum_{i=1}^{q} H_i\right) \tag{17}$$

and the associated natural projection $\tilde{H}_0$. Then, the detection filter gain L can be parameterized as follows

$$L = \sum_{i=1}^{q}(-AW_i\hat{F}_i + W_i\alpha_i)\tilde{H}_i + \beta\tilde{H}_0 \tag{18}$$

for some β: $\mathrm{Im}(H_0) \rightarrow x$ and α_i: $C\omega_i \rightarrow \omega_i$ where $i = 1, 2, ..., q$.

In Eq. (18), α_i (i = 1, 2, ..., q) and β are free parameters with compatible dimensions. These parameters should be chosen so that $(A + LC)$ is stable. This is the only requirement to ensure that the residual $r(t)$ will stay in output subspace $C\omega_i$ when $m_i(t)$ is nonzero. Douglas (1993) substituted Eq. (18) into the error dynamic system Eq. (10), finding that it is equivalent to the problem of decentralized static output feedback controller, shown as follows.

$$\dot{\varepsilon}(t) = \hat{A}\varepsilon(t) - \sum_{i=1}^{q}F_i m_i(t) + W_1u_1 + \cdots + W_qu_q + u_0 \tag{19}$$

$$\begin{cases} \tilde{z}_1(t) = \tilde{H}_1C\varepsilon(t) \\ \qquad\vdots \\ \tilde{z}_q(t) = \tilde{H}_qC\varepsilon(t) \end{cases} \tag{20}$$

$$\begin{cases} y_1(t) = \tilde{H}_1C\varepsilon(t) \\ \qquad\vdots \\ y_q(t) = \tilde{H}_qC\varepsilon(t) \\ y_0(t) = \tilde{H}_0C\varepsilon(t) \end{cases} \tag{21}$$

$$\begin{cases} u_1(t) = K_1y_1(t) \\ \qquad\vdots \\ u_q(t) = K_qy_q(t) \\ u_0(t) = K_0y_0(t) \end{cases} \tag{22}$$

where

$$\hat{A} = A + \sum_{i=1}^{q}(-AW_i\hat{F}_i)\tilde{H}_iC$$

The $\tilde{z}_1,..., \tilde{z}_q$ are system outputs and corresponding to the detection filter natural failure indications. $y_1,..., y_q$ are system observations; $u_1,..., u_q$ are system controlled inputs; and $\tilde{z}_1$, $\tilde{z}_2,...,\tilde{z}_q$ are filtered outputs. The K_1, ..., K_q and K_0 are constant decentralized controller gains which correspond to the detection filter gain parameters $\alpha_1,..., \alpha_q$ and β and determine the closed-loop properties of the detection filter. Actually, Eq. (20) and Eq. (21) show that y_i (i = 1, 2,..., q) corresponds to the ith filtered output $\tilde{z}_i$. Define

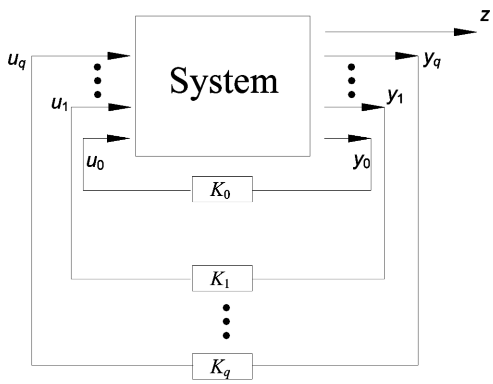

Fig. 1 Decentralized control diagram

$$\hat{B} = [T_1 \cdots T_q\ I];\ \hat{C} = \begin{bmatrix} \tilde{H}_1 C \\ \vdots \\ \tilde{H}_q C \end{bmatrix};\ \mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_q \\ u_0 \end{bmatrix};\ \tilde{\mathbf{z}} = \begin{bmatrix} \tilde{z}_1 \\ \vdots \\ \tilde{z}_q \end{bmatrix};\ \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_q \\ y_0 \end{bmatrix};\ \mathbf{F_D} = \begin{bmatrix} K_1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & K_q & 0 \\ 0 & \cdots & 0 & K_0 \end{bmatrix}$$

The equivalent decentralized static output feedback control problem becomes:

$$\dot{\varepsilon}(t) = \hat{A}\varepsilon(t) + \hat{B}\mathbf{u} - \sum_{i=1}^{q} F_i m_i(t) \tag{23}$$

$$\tilde{\mathbf{z}} = \hat{C}\varepsilon(t) \tag{24}$$

$$\mathbf{u} = \mathbf{F_D}\mathbf{y} = \mathbf{F_D}\tilde{\mathbf{z}} \tag{25}$$

Substituting Eqs. (24) and (25) into Eq. (23) yields the following closed-loop system

$$\dot{\varepsilon}(t) = (\hat{A} + \hat{B}\mathbf{F_D}\hat{C})\varepsilon(t) - \sum_{i=1}^{q} F_i m_i(t) \tag{26}$$

The system (23)~(25) is a decentralized static output feedback control problem (shown in Fig. 1). The task is to find a set of unknown controllers $K_0, K_1, \ldots, K_q$ such that the closed-loop system is stable. As long as the closed-loop system is stable, i.e., all eigenvalues of $(A + LC)$ are in the left complex plane, the effect of initial condition on the state estimation error $\varepsilon(t)$ (see Eq. (10)) will approach zero when time approaches infinity. But, if the poles of the closed-loop system are very close to the imaginary axle, it will take a long time to damp the effect of initial condition. As we know, the system output $z_i(t)$ will be nonzero when the ith fault occurs, i.e., $m_i(t) \neq 0$. It is better that $z_i(t)$ is mainly caused by faults, otherwise false alarm may happen. Therefore, we hope that the closed-loop poles stay in the left of some negative number which is not very small. The decentralized controllers $K_0, K_1, \ldots, K_q$ can now be solved by iterative LMI approach introduced in Section 2.1.

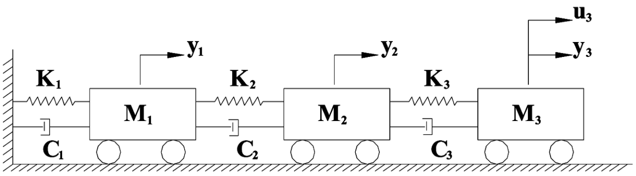

Fig. 2 One-input (u_1), three output (y_1, y_2, y_3) spring-mass-damper system

3. Simulation example

3.1. Example 1: 3-DOF spring-mass-damper system

As shown in Fig. 2, a three degree-of-freedom (DOF) spring-mass-damper system is used to demonstrate the robust filter design method presented in this paper. This example looks simple, but very instructive. It gives people the idea how to apply the proposed method step by step, so that readers can understand and practice the developed method for their own problems. Stiffness of each spring is 1000 N/m and mass at each node is 1 kg. Proportional Rayleigh damping is considered, i.e., $C = M + 0.001\,K$, where M, C, K are system mass, damping and stiffness matrix, respectively. The equation of motion of 3-DOF structure is

$$M\ddot{X}(t) + C\dot{X}(t) + (K + \Delta K)X(t) = F(t) \tag{27}$$

where $F(t)$ is the excitations and ΔK is the change of the stiffness matrix. In this case, ΔK can be expressed as

$$\Delta K = \begin{bmatrix} \Delta k_1 + \Delta k_2 & -\Delta k_2 & 0 \\ -\Delta k_2 & \Delta k_2 + \Delta k_3 & -\Delta k_3 \\ 0 & -\Delta k_3 & \Delta k_3 \end{bmatrix} \tag{28}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Delta k_1 [1\ 0\ 0] + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \Delta k_2 [1\ -1\ 0] + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \Delta k_3 [0\ 1\ -1]$$

where Δk_1, Δk_2 and Δk_3 are changes of the spring stiffness for Spring 1, 2, and 3, respectively. They are unknown and time-variant. Post-multiplying ΔK by $X(t)$, we have

$$\Delta K X(t) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Delta k_1 [1\ 0\ 0] X(t) + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \Delta k_2 [1\ -1\ 0] X(t) + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \Delta k_3 [0\ 1\ -1] X(t) \tag{29}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} m_1(t) + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} m_2(t) + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} m_3(t)$$

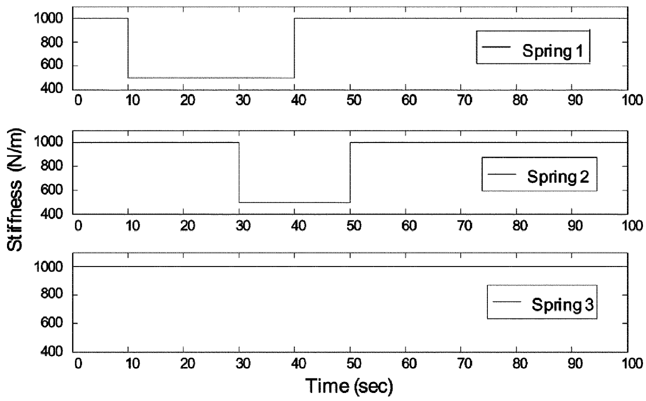

Fig. 3 Stiffness changes of Spring 1 and 2

where $m_1(t)$, $m_2(t)$ and $m_3(t)$ represent the time-variant parameters due to changes of spring stiffness.

Assume that the system has one input locating at mass 3 and three outputs at mass 1, 2 and 3, as shown in Fig. 2. Only displacement measurement is considered in this paper. Suppose that Spring 3 is always healthy and the stiffness changes for Spring 1 and 2 are shown in Fig. 3. The stiffness of Spring 1 is reduced 50% during 10s~40s, so is Spring 2 during 30s~50s. The state-space representation for this system is

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -2000 & 1000 & 0 & -3 & 1 & 0 \\ 1000 & -2000 & 1000 & 1 & -3 & 1 \\ 0 & 1000 & -1000 & 0 & 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_3(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} m_1(t) \\ m_2(t) \end{Bmatrix} \tag{30}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} x(t) \tag{31}$$

Two fault vectors are $F_1 = [0\ 0\ 0\ 1\ 0\ 0]^T$ and $F_2 = [0\ 0\ 0\ 1\ \text{-}1\ 0]^T$. The corresponding detection spaces are $W_1 = [F_1 \ \ AF_1]$ and $W_2 = [F_2 \ \ AF_2]$. H_i, $\hat{F}_i$, $\tilde{H}_i$, H_0 and $\tilde{H}_0$ used in this example are listing below.

$$H_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, H_2 = \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, H_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \tilde{H}_1 = [1\ 1\ 1], \tilde{H}_2 = [0\ 1\ 1]$$

$$\tilde{H}_0 = [0\ 0\ 1], \hat{F}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \hat{F}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

When applying iterative LMI to compute the decentralized controller, we require that the closed-loop poles remain in the left of -0.3. The obtained decentralized controller gain F_D is shown as follows.

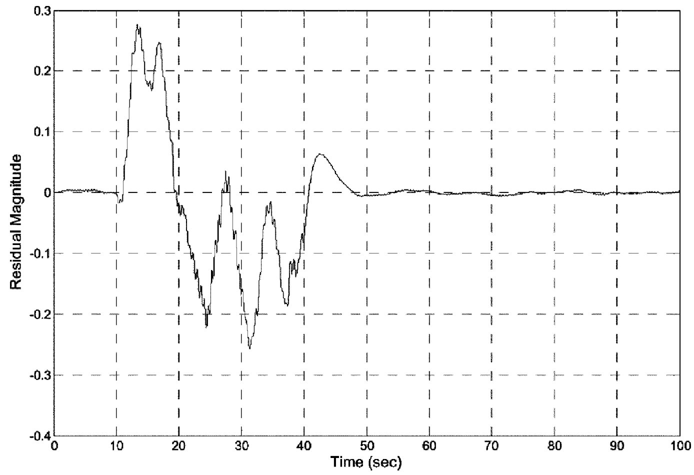

Fig. 4 The residual output for Spring 1

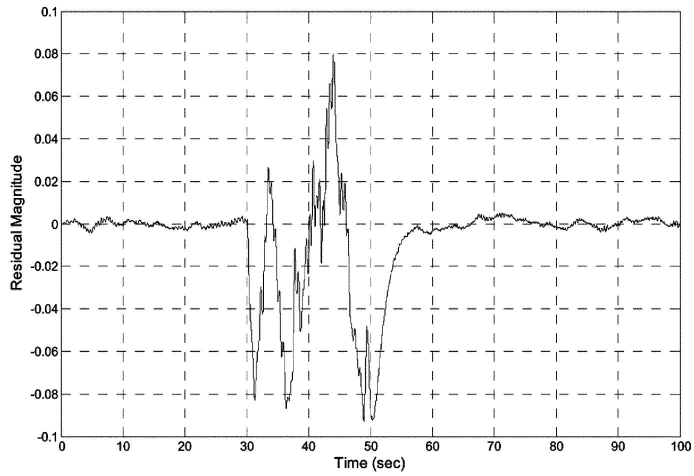

Fig. 5 The residual output for Spring 2

$$K_1 = \begin{bmatrix} -0.3316 \\ -0.7525 \end{bmatrix}, K_2 = \begin{bmatrix} 0.8813 \\ 1.947 \end{bmatrix} \text{ and } K_0 = \begin{bmatrix} 155.2 \\ 314.1 \\ -2242 \\ 6.013 \\ 108.3 \\ 83.65 \end{bmatrix}$$

With these decentralized static output feedback controller gains, the actual poles of the closed-loop system are: -0.376 $\pm j$0.436, -0.717, -1.23, -2.99 and -2242. They are all in the left of -0.3 which is obtained by running the iterative LMI algorithm. Fig. 4 and Fig. 5 represent residual histories associated with Spring 1 and 2, where 1% rms white noise is applied to three measurement sensors. Compared to the true damage situation (Fig. 3), the damages of spring 1 and 2 are clearly identified and

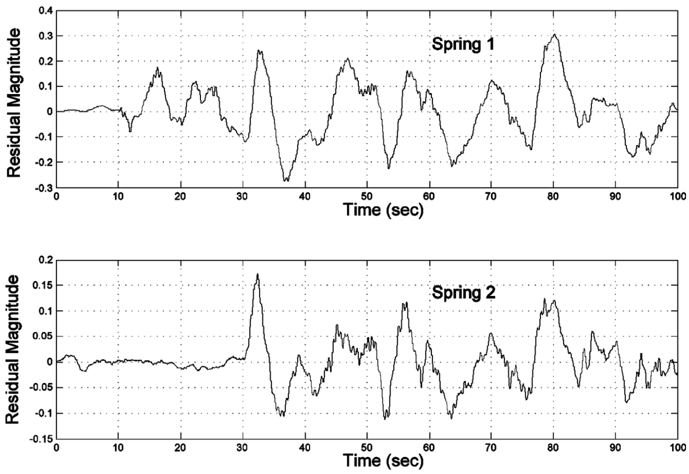

Fig. 6 The residual outputs for Spring 1 and 2 without stiffness recovery after damage

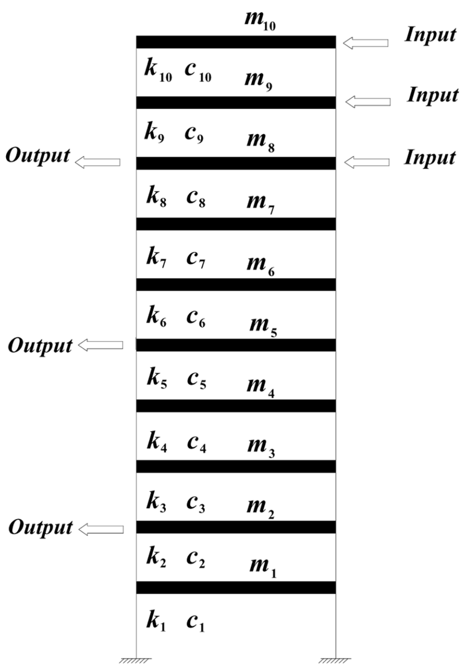

Fig. 7 10-story shear type building

correctly isolated even if those damages are involved during 30s~40s. The magnitudes of residuals in these two figures are different because we use different projection matrices to isolate two faults from the output residual. But, the signal when the associated spring is damaged significantly deviates from that when the spring is healthy. From Fig. 4 and 5, we can find that the residual will not approach zero immediately when there is no fault, but it will take a couple of seconds to damp out the effect of initial conditions. In this simulation, we assume that the stiffness of damaged springs will come back to the healthy state. But, it is not true, practically. Therefore, the residuals will always be large once structural elements are damaged, as shown in Fig. 6 where stiffness of Spring 1 and 2 did not recover after damage. In this sense, the structural fault can also be identified even if the closed-loop poles are close to the imaginary axle.

3.2. Example 2: Ten-story shear type building

As shown in Fig. 7, it is a ten-story shear type building with 3 inputs and 3 outputs. Three inputs locate at the 8th, 9th and 10th story, and three outputs at the 2nd, 5th and 8th story. Measured outputs are displacement. Stiffness and mass of each story is 1000 N/m and 1 kg, respectively. Proportional Rayleigh damping is considered, i.e., $C = 0.1M + 0.01K$, where M, C, K are system mass, damping and stiffness matrix, respectively.

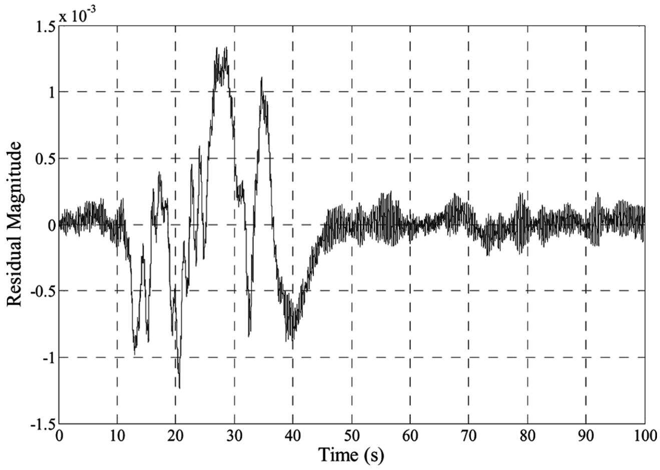

Fig. 8 The residual output for Story 1

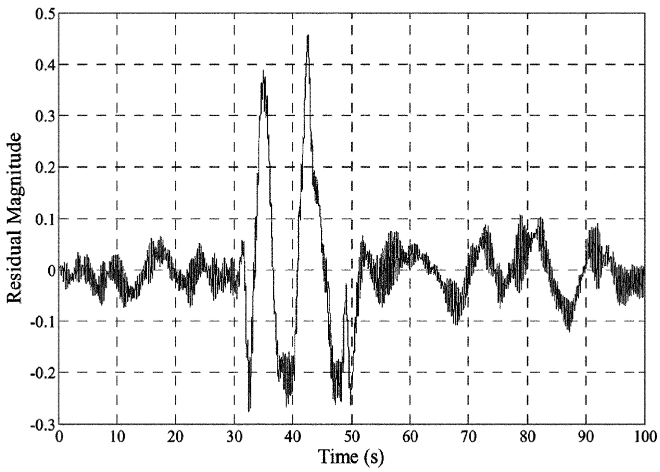

Fig. 9 The residual output for Story 5

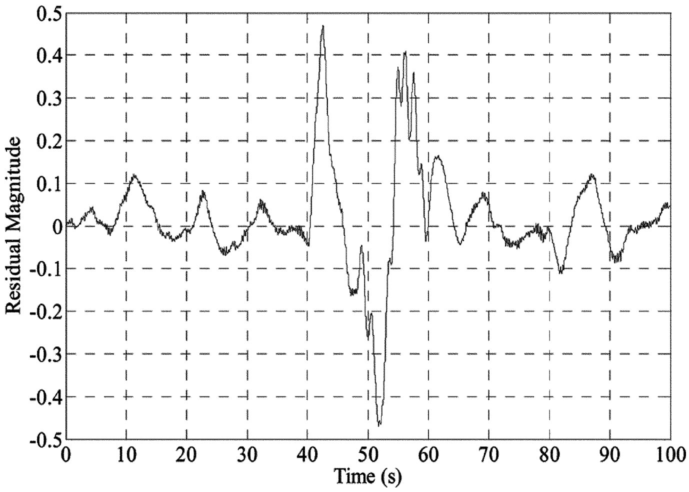

Fig. 10 The residual output for Story 8

Suppose that after a moderate earthquake cracks only happen on the surface of shear walls in the first, fifth and eighth stories. Thus, in the simulation program, the stiffness of the first story is reduced 50% between 10s~40s, so is the fifth story between 30s~50s and the eighth story between 40s~60s. 5% rms noise is included in all measurements. The closed-loop poles are required to remain in the left of -0.3. After obtaining W_i, $\hat{F}_i$, $\tilde{H}_i$ ($i = 1, 2, 3$) and substituting them into iterative LMI algorithm, we have the following decentralized controller.

$$F_D = \begin{bmatrix} -131.154 & 0 & 0 \\ -372.016 & 0 & 0 \\ -315.400 & 0 & 0 \\ 0 & -0.634 & 0 \\ 0 & -0.942 & 0 \\ 0 & 0 & -0.5145 \\ 0 & 0 & -0.8510 \end{bmatrix}$$

The most left pole of the closed-loop system is -0.425 ± j0.577, which is in the left of -0.3. The corresponding residual outputs are shown in Figs. 8, 9 and 10. Clearly, the story damages are identified correctly. Thus, it can be concluded that the proposed method can be applied for structural damage detection and isolation.

4. Conclusions

In this paper, the observer-based fault detection and isolation problem is studied using detection filter concept and iterative LMI approach. The detection filter can not only detect the occurrence of structural damages, but also tell which one has damaged. The geometric interpretation of detection filter discloses the characteristics of observer gain L. The problem of finding detection filter gain is equivalent to that of finding a decentralized static output feedback controller gain which is a non-linear problem. In this paper, iterative LMI approach is used to find the decentralized controller and apply it to structural damage detection and isolation. The simulation examples show that the algorithm present in this paper can realize structural damage detection and isolation goals.

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