

Seismic vulnerability assessment of a historical building in Tunisia

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Abstract. A methodology for the seismic vulnerability assessment of historical monuments is presented in this paper. The ongoing work has been conducted in Tunisia within the framework of the FP6 European Union project (WIND-CHIME) on the use of appropriate modern seismic protective systems in the conservation of Mediterranean historical buildings in earthquake-prone areas. The case study is the five-century-old Zaouia of Sidi Kassem Djilizi, located downtown Tunis, the capital of Tunisia. Ambient vibration tests were conducted on the case study using a number of force-balance accelerometers placed at selected locations. The Enhanced Frequency Domain Decomposition (EFDD) technique was applied to extract the dynamic characteristics of the monument. A 3-D finite element model was developed and updated to obtain reasonable correlation between experimental and numerical modal properties. The set of parameters selected for the updating consists of the modulus of elasticity in each wall element of the finite element model. Seismic vulnerability assessment of the case study was carried out via three-dimensional time-history dynamic analyses of the structure. Dynamic stresses were computed and damage was evaluated according to a masonry specific plane failure criterion. Statistics on the occurrence, location and type of failure provide a general view for the probable damage level and mode. Results indicate a high vulnerability that confirms the need for intervention and retrofit.

Keywords: ambient vibration testing; output-only modal identification; enhanced frequency domain identification technique; finite element model updating and seismic vulnerability.

1. Introduction

The WIND-CHIME Project, sponsored by the European Union, aims at the preservation and conservation of Mediterranean historical buildings in earthquake-prone areas by means of appropriate modern seismic protective systems. The building under investigation is the Zaouia of Sidi Kassem Djilizi, located near downtown Tunis.

The present study aims at assessing the seismic vulnerability of the selected building. The adopted methodology consisted in the following tasks. Ambient vibration tests were conducted to measure the acceleration at selected locations in the building. The complex non-stationary nature of the unmeasured excitation requires the use of robust output-only modal identification techniques such as the Frequency Domain Decomposition which was applied to accurately extract the modal signature of the building. A finite element model of the Zaouia was developed using estimates of material properties based on the

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measured characteristics of stone and mortar. Seismic vulnerability assessment of the building was carried out via three-dimensional time-history dynamic analyses of the structure subject to a Tunis area strong motion earthquake record. The latter was scaled to a peak ground acceleration specified by the seismic hazard analysis of the site.

2. Case study description and finite element modeling

The Zaouia of Sidi Kassem Djilizi (Saint) is a temple built in the 16th century near downtown Tunis by a Moslem refugee of Andalousian origin named of Kassem Djilizi. The building is not only a charming monument but also one of a big artistic value. Its architecture is a mixture of Islamic and Spanish architecture. The roofing is covered by green and glazed tiles surmounting a richly decorated square mausoleum called the Alhambra. The monument contains pretty ceilings and a collection of remarkable ceramic tiles and black marble surrounding the tomb where the Saint was buried. The four-side pyramidal minaret contains the grave of its builder Sidi Kassem al Djilizi (Fig. 1). The monument suffers from degradation (cracks) that may have been caused over time by earthquakes, water infiltration, foundation settlement and induced car traffic vibrations. The historical building was first restored in the early 90's by the Spanish Government and it is used now as ceramics museum.

The Zaouia building, which covers a total area of about 1410 m², is composed of 25 rooms and a central patio. Fig. 2 shows a 3-D view of the building. As shown in this figure, the monument can be subdivided into four major blocs: (i) the mosque and the “Beaux Arts” Workshops (top left), (ii) the patio bloc (center), (iii) the mausoleum where the Saint is buried (top front) and (iv) the ceramics museum and the administration offices (top right). The load carrying elements consist of walls with massive stones interconnected through a weak mortar-like material. The main flooring and roofing systems consist of brick vaults with an earth filling standing on the wall with or without steel joists.

The monument comprises various types of bearing elements, mostly masonry walls, 50 cm thick, made of natural stones. The roofs of the large rooms are built in crossed vaults supported by cylindrical columns (Fig. 3) braced by wood beams, whereas in small rooms and corridors, the roofs are made of masonry supported by wood beams overlaid by wood sheets. The rooms' floors are covered with colored ceramics and the patios' floor with marble. Most of the walls are painted with lime, whereas,



Fig. 1 Photograph of the case study seen from the patio

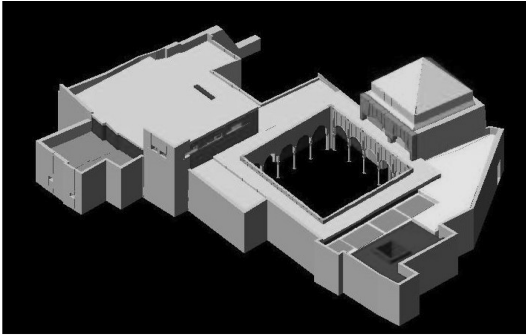


Fig. 2 Three-dimensional Auto-CAD model of the case study



Fig. 3 A typical cross vaulted roof

the master room walls are covered with ceramics and sculptured panels.

A series of experimental tests were conducted to determine the mechanical properties of stone and mortar samples extracted from the monument. Based on compression tests and indirect tensile tests performed on stone specimen, bloc compressive and tensile strengths are evaluated at $f_{bc} = 18$ MPa and $f_{bt} = 4.9$ MPa, respectively. For mortar, compressive and tensile strengths are estimated at $f_{mc} = 2.7$ MPa and $f_{mt} = 0.42$ MPa, respectively. Equivalent properties for masonry are determined based on available empirical rules (Syrmakizis, *et al.* 1995). A key parameter is the compressive strength f_{wc} which is estimated at 4.5 MPa based on values given by several alternative formulae, ranging between 2 and 6 MPa. The modulus of elasticity is given by $E_{wc} = 1000 f_{wc} = 4500$ MPa. Poisson's ratio is taken as 0.16. Finally, the tensile strength is estimated at $f_{wt} = (2/3) f_{mt} = 0.28$ MPa.

A three dimensional finite element model of the building was elaborated with the SAP2000 and FEMTools computer programs based on a detailed geometric model. The model, shown in Fig. 4, uses a total of 5589 shell and beam elements. The material behavior is assumed to be linear elastic, isotropic and homogeneous. The real behavior actually does not obey these assumptions. The masonry is essentially non-homogeneous and anisotropic at the macroscopic scale, with different mechanical characteristics along directions parallel and normal to bed joints, and the behavior is no longer linear elastic near failure. Nevertheless, the adopted simplified behavior remains a meaningful approximation for the purpose of vulnerability analysis. It should be noted that, under small amplitude ambient vibrations, the linear elasticity assumption is realistic and can justifiably be adopted for model updating based on ambient measurements.

3. Ambient vibration testing, modal identification and model updating

3.1. Optimum sensor location and test planning

For large structures, ambient vibration tests with output-only measurements are preferred over forced vibration tests where both excitation and response are measured. In ambient vibration testing, the measured response is representative of the actual operating conditions of the structure which vibrates due to natural excitation loads such as wind, micro-tremors, traffic and human activities.

An important task in ambient vibration testing is to optimize the placement of the sensors while

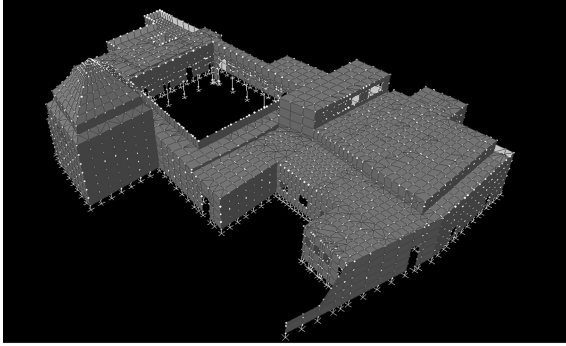


Fig. 4 Finite element model of the case study

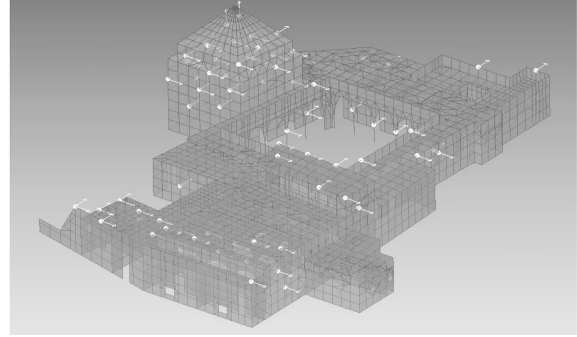


Fig. 5 Optimal sensor locations and directions obtained using SEAMAC method (shown in white dots)

minimizing their numbers and maximizing the amount of modal test data. The number and location of the sensors should reflect the vibrational modes dominating the dynamic response of the structure. Because these locations have a strong influence on the quality and amount of modal test data, and hence on correlation with FEA results, careful pretest planning is needed.

The first step in the pretest analysis is the development of the baseline FE model, using the SAP2000 and FEMTools model. The model is used to find indicative values of the dominant frequencies of the structure in the frequency band of interest as well as their distribution and the corresponding mode shapes. These mode shapes are used to select optimal locations and directions for simultaneous excitation of different modes and to find a minimum set of degrees of freedom where measurements are required to enable unique pairing with analytical mode shapes. The second step in the pretest analysis consists in selecting the modes in the frequency band of interest based on energy considerations. The last step is to find the optimal measurement locations and directions. This can be based on a large choice of criteria, including observability, linear independence of modes, effect of sensor elimination on Modal Assurance Criteria, kinetic energy and others. The commercially available finite element code FEMTools, 2007, contains a number of these methods which may be classified into two categories: (a) Sensor placement metrics methods which are based on the observability of target modes using information on modal displacement or energy (kinetic or strain); and (b) Sensor elimination methods which iteratively eliminate sensors from the set of candidates in a way to optimally maintain linear independence between mode shapes.

The method used in this study is the Sensor Elimination method using the Modal Assurance Criterion (SEAMAC) which belongs to the second category. This method uses the off-diagonal terms of the Modal Assurance Criterion (MAC) matrix as elimination criterion.

The Modal Assurance Criterion (Chopra 2001) is commonly used to compare mode shapes. Each term in the MAC matrix measures the squared cosine on the angle between two mode shapes i and j ; i.e., $MAC_{ij} = \cos^2 \theta_{ij}$, and is calculated as:

$$MAC_{ij} = \frac{(|\{\psi_i\}^t \{\psi_j\}|)^2}{(\{\psi_i\}^t \{\psi_i\})(\{\psi_j\}^t \{\psi_j\})} \quad (1)$$

Ideally, when $MAC_{ij} = 0$, the i^{th} and j^{th} mode shapes ψ_i and ψ_j are orthogonal to each other.

Hence, the mode shapes are most linearly independent when all off-diagonal terms of MAC are zero. The MAC between all possible combinations of analytical and test modes are stored in the MAC

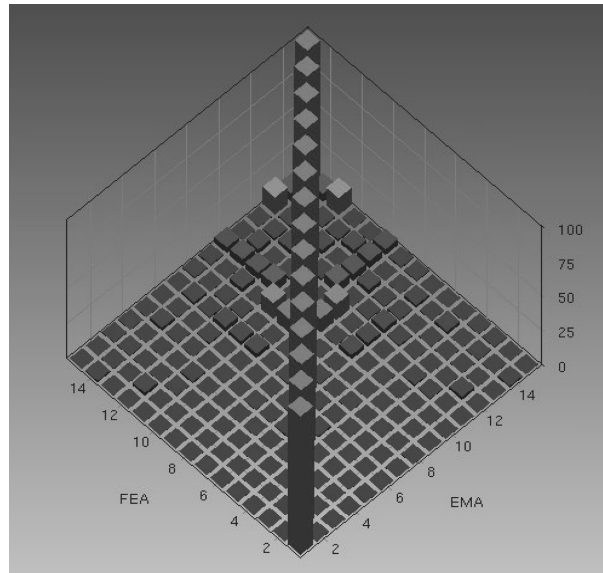


Fig. 6 MAC Plot of Finite Element Mode Shapes (FEMS) versus Simulated Experimental Mode Shapes (SEMS) using optimal sensor locations and directions obtained using SEAMAC method

matrix. Computation of MAC is fast and does not require the mass and stiffness matrices. The off-diagonal terms of the MAC matrix provide a mean to check linear independence between modes. Two mode shapes with a value equal to 1 indicate identical modes (or perfect correlation). MAC values are calculated by multiplying numerically and experimentally obtained modal displacements at paired DOFs. A sufficient number of common DOFs is required to prevent spatial aliasing. The SEAMAC algorithm tests the removal of each candidate sensor DOF. The removal that results in the minimum largest off-diagonal term of MAC is then carried out, and the process is repeated for the remaining candidate sensor DOFs.

Using the SEAMAC algorithm, FEMTools, essentially, reduces the baseline finite element model into a pretest model based on a small number of simulated sensor locations, as shown in Fig. 5. The program then computes the frequencies and mode shapes based on this simplified model. The resulting mode shapes are called Simulated Experimental Mode Shapes (SEMS). In addition, FEMTools computes the frequencies and mode shapes of the complete finite element model. The resulting mode shapes, called Finite Element Mode Shapes (FEMS), are then compared with the Simulated Experimental Mode Shapes in order to determine the effectiveness of the proposed sensor locations to capture the important dynamic properties of the structure. The correlation between the FEMS and SEMS modes can be quantified using the Modal Assurance Criterion (MAC) (Chopra 2001). A MAC value of 100% means perfect correlation between two vectors, while a 0% value means that the two vectors are completely uncorrelated. The obtained MAC plot is shown in Fig. 6.

In practice, not all the suggested sensor positions are accessible and measurable. Consequently, some suggested sensor locations were slightly modified and other sensors were added to the original ones. These sensors were added to ensure a sufficient number of measurement points to reconstruct the building shape. The final optimal set of sensor locations, represented in Fig. 7, contains 145 sensor positions.

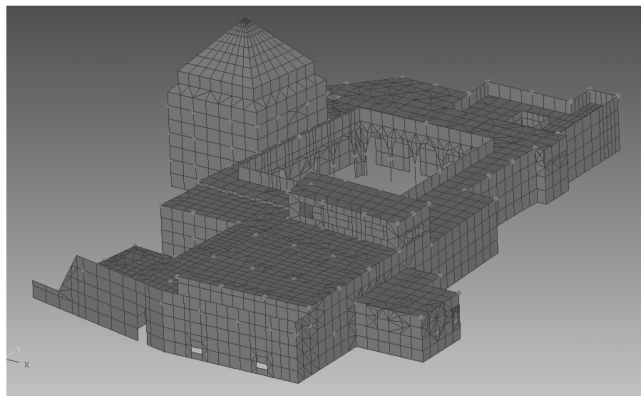


Fig. 7 Final optimal sensor locations (shown in light gray dots)



Fig. 8 Picture of the data acquisition system and measurement equipment

3.2. Ambient vibration testing

Ambient vibration tests were conducted on the case study using a sixteen-channel data acquisition system with nine force-balance uniaxial accelerometers (Fig. 8). The sensors are capable of measuring accelerations of up to ± 0.25 g with a resolution of 0.1g. Signal conditioners were used to improve the quality of the signals by filtering undesired frequency contents. Vibration experiments were conducted at a sampling frequency of 100 Hz with all channels set for a low-pass filter of 40 Hz.

The sensors were placed at selected locations of the building to measure the horizontal components of the acceleration at each point. Two accelerometers were utilized as reference sensors at one point on the mausoleum and the remaining sensors were used as roving sensors. A total of forty-four setups were accomplished to provide sufficient data for accurate identification of the modal signature.

3.3. Output-only modal identification

The fundamental assumption in the analysis of ambient vibrations is that the inputs causing motion have white noise characteristics in the frequency range of interest. This assumption implies that the input loads are not driving the system at any particular frequency and therefore any identified frequency associated with significant strong response reflects structural modal response. However, in reality, some of the ambient disturbances, such as an adjacent machine operating at a particular frequency, may drive

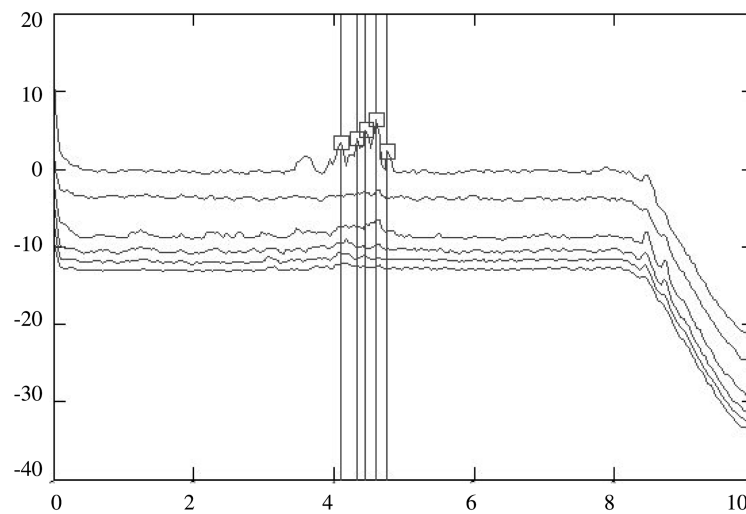


Fig. 9 Average of normalized singular values of spectral density matrices of all data sets using FDD algorithm

the structure at that frequency. In this case, the deformed shapes of the structure at such driving frequency are called Operational Modes. This means that a crucial requirement on methods that analyze ambient vibration data is the ability to distinguish the natural structural modes from any imposed operational modes.

Such output-only modal identification techniques include the EFDD method (Brincker and Andersen 2000, Brincker, *et al.* 2000b) and the Stochastic Subspace Identification (SSI) methods (Van Overschee and De Moor 1996). These methods were recently applied to buildings and bridges (Brincker and Andersen 2000, Brincker, *et al.* 2000a).

In the present study, the EFDD technique, which is a refinement of the Frequency Domain Decomposition (FDD) technique, was applied to extract the modal signature of the historic building using Artemis Extractor Program, 2007. Theory and description of both FDD and EFDD techniques can be found in (El-Borgi, *et al.* 2005).

Fig. 9 shows the average of normalized singular values of spectral density matrices of all data sets

Table 1 Measurement-based estimates of natural periods versus computed natural periods before and after updating

Mode	Measured natural period (Sec)	Computed natural period before updating (Sec)	Relative error before updating (%)	Computed natural period before updating (Sec)	Relative error after updating (%)
1	0.212	0.188	-11.3%	0.213	0.5
2	0.099	0.173	74.7%	0.099	0.0
3	0.077	0.161	109%	0.079	2.6

Table 2 Measurement-based estimates of identified natural periods and damping ratios using EFDD technique

Mode	Estimated from measurements using EFDD technique	
	Natural period (Sec)	Damping ratio (%)
1	0.212	2.8
2	0.099	1.6
3	0.077	1.2

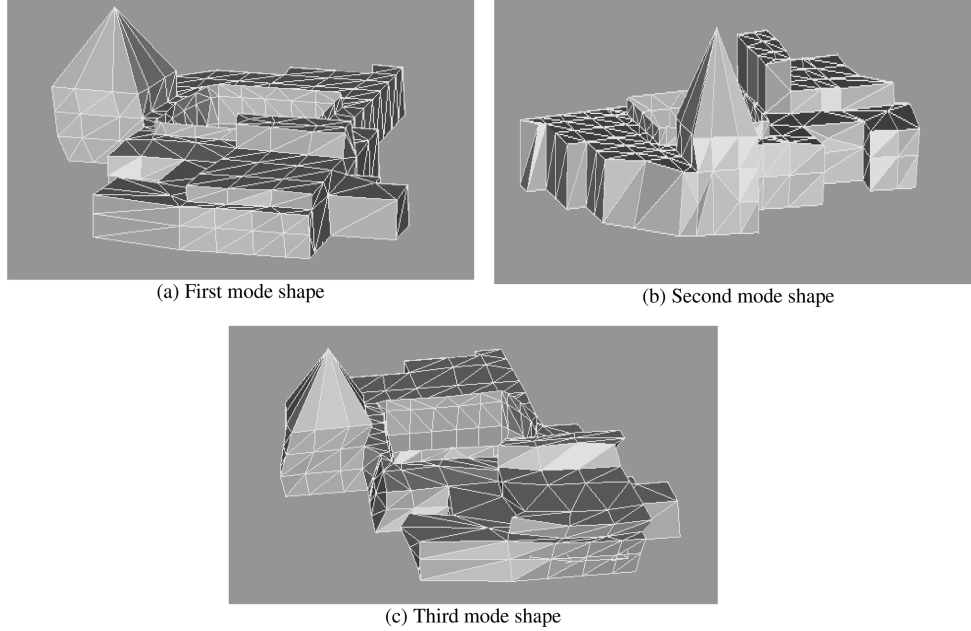


Fig. 10 First three modes of vibration identified from measurements using the EFDD technique

using EFDD technique. Table 2 shows the measurement-based estimates of the natural periods and damping ratios of the first three modes using EFDD. The damping ratios estimated by the EFDD technique show a relative variability and range approximately in the interval between 1% and 3%, which seem to be reasonable values. Fig. 10 shows the first three modes of vibration identified from measurements using the EFDD technique.

3.4. Model updating

Finite element calculation of the modal signature using the masonry characteristics reported in Section 2 yielded the three fundamental periods $T_1 = 0.188$ sec, $T_2 = 0.173$ sec and $T_3 = 0.161$ sec, which are, respectively, 11.3%, 74.7% and 109% away from experimentally identified periods (Table 1). This clearly indicates the need for model updating.

The objective of finite element model updating is to obtain a reasonable correlation between experimental and numerical modal properties. The parameter selected for updating consists of the masonry modulus of elasticity in each element of the finite element model. Finite element model updating is a nonlinear iterative procedure, implemented in FEMTools, 2007, based on the following matrix equation:

$$\{\Delta R\} = [S]\{\Delta P\} \quad (2)$$

where $\{\Delta P\} = \{P\} - \{P^0\}$ in which

- $\{P\}$ is a vector containing parameters from the numerical model. For the current case, these parameters are the Young's modulus of each element;
- $\{P^0\}$ are the starting values of the parameters;
- $\{\Delta R\} = \{R^e\} - \{R\}$ in which
- $\{R\}$ is a vector containing responses from the model. For the current case, they correspond to the

numerical modes (frequencies) that are paired with the corresponding experimental ones and numerical mode shapes;

- $\{R^e\}$ is the vector associated with the reference response test data; and $[S]$ is the sensitivity matrix which contains the gradients of the responses R with respect to parameters P

$$[S] = S_{ij} = \frac{\partial R_i}{\partial P_j} \quad (3)$$

The updated values of parameters P are obtained from Eqs. (5) and (6) as follows:

$$\{P\} = \{P^0\} + [G](\{R^e\} - \{R\}) \quad (4)$$

where $[G]$ is the gain matrix computed following Bayesian estimation theory as

$$[G] = [C_p][S]^T([C_R] + [S][C_p][S]^T)^{-1} \quad (5)$$

in which $[C_p]$ are weighting matrices that express the analyst's confidence in $\{P^0\}$ and the Preference responses test data $\{R^e\}$.

Iterations are continued until error functions satisfy a convergence criterion based on a comparison between computed and measured frequencies.

After performing the model updating, the computed natural periods for the first three modes are $T_1 = 0.212$ sec, $T_2 = 0.099$ sec and $T_3 = 0.079$ sec which are, respectively, 0.5%, 0.0% and 2.6% away from experimentally identified periods (Table 1). This indicates an almost perfect match for the first three natural periods.

4. Seismic vulnerability analysis

A recent seismic hazard analysis of the Grand Tunis area indicates that the peak ground acceleration (PGA) is 0.22 g for a 500-year return period earthquake (Kacem, *et al.* 2001). The latest earthquake that caused damage in the Grand Tunis area occurred in December 1970 with a magnitude of 5.1 on the Richter scale.

Seismic vulnerability assessment of the building was carried out via three-dimensional time response dynamic analysis of the structure subject to a Tunis area strong motion earthquake record: the Ezzahra Earthquake which occurred on April 24th, 2000. The North-South component of the earthquake is utilized and the magnitude of the peak ground acceleration (PGA) is scaled up to the peak values of 0.22 g for a relatively strong earthquake (Fig. 11) and 0.10 g for a moderate earthquake. The record has duration of 16.63 sec with a sampling frequency of 0.005 sec.

For each value of the PGA, dynamic stresses are computed for the finite element model under a combination of dead load, service loads and earthquake excitations. Moreover, four configurations of earthquake actions are considered for each loading combination. The first consists of the full earthquake acting along one reference direction and a 30% fraction of the same earthquake acting on a perpendicular direction. The second configuration is defined in a similar way except that the weaker earthquake is applied in the opposite direction. The two other configurations are defined by rotating the first two by 90°.

For failure analysis a masonry specific plane failure criterion is adopted (Syrmakezis *et al.* 1995). The

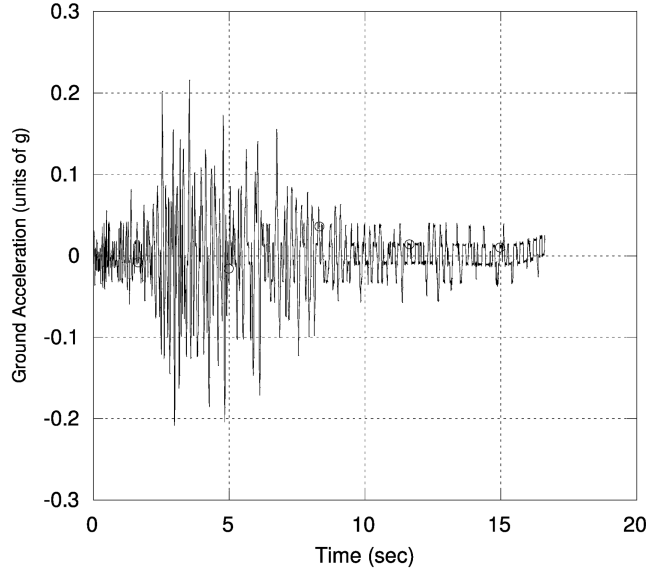


Fig. 11 Ezzahra (suburb of Tunis) earthquake, April 24th, 2000, NS component, scaled to a peak ground acceleration of 0.22 g

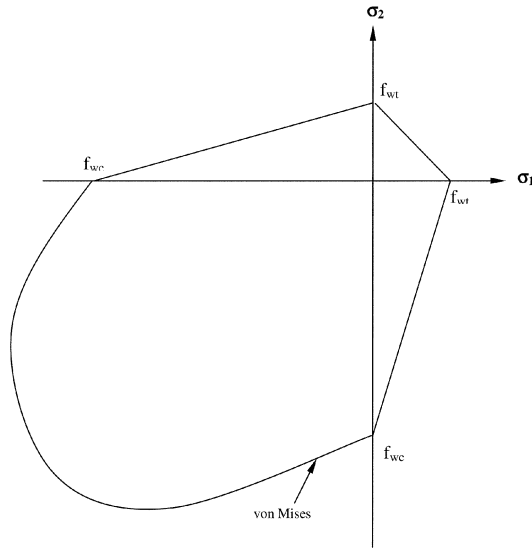


Fig. 12 Masonry failure criterion

failure line is made up of a von Mises criterion for states of biaxial compression (BC). For states of biaxial tension (BT) and biaxial tension-compression (BTC) the line is completed by linear interpolation from failure states of pure tension and pure compression. A further approximation is made by assuming the same biaxial failure criterion when the shearing stress is not vanishing. The failure function, shown schematically in Fig. 12, can be expressed piecewise as follows:

$$F_1(\sigma) = \sqrt{J_2} - f_{wc} \quad (6a)$$

Table 3 Results of vulnerability analysis under Tunis earthquakewith a PGA of 0.22 g

	Damaged joints	Total number of joints	Percentage (%)
Whole structure	4253	4409	96
Biaxial tension-compression (BTC)	2462		56
Biaxial tension (BT)	1772	4409	40
Biaxial compression (BT)	19		0.4

$$F_2(\sigma) = \sigma_1 + \sigma_2 - f_{wt} \quad (6b)$$

$$F_3(\sigma) = -\sigma_1 + \sigma_2 \frac{f_{wc}}{f_{wt}} - f_{wc} \quad (6c)$$

$$F_4(\sigma) = \sigma_1 - \sigma_2 \frac{f_{wt}}{f_{wc}} - f_{wt} \quad (6d)$$

where J_2 is the second invariant of the deviatoric stress tensor and σ_1 and σ_2 are the principal stresses.

Damage is evaluated according to the adopted masonry failure criterion by considering, at each joint, the envelope of the yield function over the complete set of loading cases. A joint is damaged when its state of stress is outside the failure criterion. The ratio of number of damaged joints to the total number of joints is taken as a measure of seismic vulnerability. Statistics of damage in the building as a whole are summarized in Table 3. Overall vulnerability of the Zaouia subjected to a 0.22 g earthquake is estimated at 96%, with biaxial tension-compression as the predominant type of failure (56%), followed by biaxial tension (40%) and only 0.4% for biaxial compression. According to the calculated seismic vulnerability, the Sidi Kassem Djilizi Zaouia clearly is in need for protection against earthquake hazard.

5. Conclusions

This paper summarizes work conducted within the framework of a European Commission project promoting the use of appropriate modern seismic protective systems in the conservation of Mediterranean historical buildings in earthquake-prone areas. The case study is the five century old Zaouia of Sidi Kassem Djilizi, located in the Capital of Tunisia.

Ambient vibration tests were performed on the Zaouia and a series of experimental tests were conducted to determine the mechanical properties of stone and mortar samples extracted from the monument. The complex non-stationary nature of the unmeasured excitation, due essentially to wind, traffic, micro-tremors and human activity, requires the use of output-only modal identification techniques that must be robust with respect to this nonstationarity. Hence, the Enhanced Frequency Domain Decomposition (EFDD) was applied to extract the modal signature of the studied historic building. The measurement-based estimates of the natural periods of the first three modes using the EFDD technique were found to be in very close agreement. On the other hand, the damping ratios estimated by the EFDD technique showed a relative variability and ranged in the interval between 1% and 3%.

A preliminary finite element model was elaborated based on the measured estimates of the material properties of stone and mortar samples which bear a significant degree of uncertainty. Finite element calculation of the modal signature using these characteristics yielded the three fundamental periods which are, respectively, 11.3%, 74.7% and 109% away from experimentally identified periods, which

clearly indicates the need for model updating. Finite element updating was carried out based on the first three periods identified by the EFDD technique for the purpose of obtaining a reasonable correlation between experimental and numerical modal properties. Model updating is a nonlinear iterative procedure in which the updating parameter is the masonry modulus of elasticity in each element of the finite element model. A close match was obtained between the measured and computed fundamental periods.

With a calibrated model of the building, seismic vulnerability assessment was carried out via three-dimensional time response dynamic analysis of the structure subject to a Tunis area strong motion earthquake record. The seismic excitation was scaled to a PGA of 0.22 g, representing relatively strong earthquakes. Dynamic stresses were computed and damage was evaluated according to a masonry specific plane failure criterion. The estimated damage statistics indicate that the building in its current state is severely vulnerable and requires intervention.

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