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Health monitoring of multistoreyed shear building using parametric state space modeling

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Abstract. The present work utilizes system identification technique for health monitoring of shear building, wherein Parametric State Space modeling has been adopted. The method requires input excitation to the structure and also output acceleration responses of both undamaged and damaged structure obtained from numerically simulated model. Modal parameters like eigen frequencies and eigen vectors have been extracted from the State Space model after introducing appropriate transformation. Least square technique has been utilized for the evaluation of the stiffness matrix after having obtained the modal matrix for the entire structure. Highly accurate values of stiffness of the structure could be evaluated corresponding to both the undamaged as well as damaged state of a structure, while considering noise in the simulated output response analogous to real time scenario. The damaged floor could also be located very conveniently and accurately by this adopted strategy. This method of damage detection can be applied in case of output acceleration responses recorded by sensors from the actual structure. Further, in case of even limited availability of sensors along the height of a multi-storeyed building, the methodology could yield very accurate information related to structural stiffness.

Keywords: structural health monitoring; system identification; state space modeling; modal parameters; damage detection; earthquake ground acceleration.

1. Introduction

Structural health monitoring (SHM) of civil structures has received significant attention by researchers in recent days. It is very much important to check the state of a structure soon after it undergoes a major earthquake. If a structure experiences a force larger than what it is designed to withstand, the basic properties of the structure e.g. stiffness, natural frequencies, damping ratio etc. does not remain the same and sometimes the degradation in the stiffness is so severe that the structure may not remain safe to be used again. Though the damage detection in structure can also be done visually or through localized experimental methods, the limitations of these methods are that they require knowledge about the location of the damage prior to the experiment and the portion of the structure must be easily accessible to carry out the experiment. Moreover, visual inspection is often less than trivial as structural elements are often covered by nonstructural elements like walls and facades. As a result, there are large numbers of ongoing research activities in the field of SHM throughout the world to find out efficient method for monitoring the state of a structure. However,

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the studies of SHM based on modal properties of structure have received a major thrust in recent years.

A large number of techniques have been adopted over the years for damage identification. The recent state-of-the-art surveys of SHM as applied to civil engineering applications were carried out by Doebling, *et al.* (1996, 1998). A numerically simulated benchmark problem was developed by the joint IASC–ASCE Task Group on Structural Health Monitoring, which could be excited by ambient vibration. A large number of system identification techniques based studies were conducted on this benchmark model. Natural excitation technique in conjunction with eigen value realization algorithm (Caicedo, *et al.* 2004), Hilbert–Huang transformation (Yang, *et al*. 2004), damage index method (Barros and Rodriguez 2004), flexibility based damage characterization technique (Bernal and Gunes, 2004), two-stage structural health monitoring methodology using Bayesian system identification (Yuen, *et al.* 2004), statistical model updating methodology (Lam, *et al.* 2004) etc. were some of the methods, which were applied by the researcher for finding out damages in structures. All of these methods are non- parametric methods of system identification applied to SHM. The approaches do not require knowledge of the excitation and hence it is applicable to the general problem, where the excitation is from ambient sources, which is generally not measurable.

On the other hand, there are different types of parametric models in system identification like Auto Regressive eXogenous model (ARX), Auto Regressive Moving Average eXogenous model (ARMAX), Box-Jenkins model, State Space Model etc. Most of the researches carried out using parametric models were confined to the evaluation of damping and frequency of damaged/undamaged structures as an indicative of damaged state of the structure. However, in this paper, the authors have explained and implemented parametric modeling in order to locate the damage in a structure efficiently and accurately. Some researches on parametric modeling in recent years were also carried out by using active sensing (Lynch 2004, Fasel, *et al*. 2005) for damage detection in structures. Lynch (2004) introduced the concept of linear classifications of poles obtained from the parametric identification of a structure. The poles of ARX time-series models describing modal frequencies and damping ratios were plotted upon the discrete-time complex plane and perception linear classifiers were employed to determine if poles of the structural element in an unknown state (damaged or undamaged) could be distinguished from those of the undamaged structure. However, this method is not capable of locating the damage in a realistic structural system. The detection of the location and severity of the damage is as important as evaluating its presence. Thus, in this study, the method of detecting the location and evaluation of the extent of the damage in a structure using parametric State Space model has been carried out. In the State Space form, the relationship between the input, noise and output signals is written as a system of first order differential or difference equations using an auxiliary state vector $x(t)$ (Ljung 1987). Unlike other parametric models, the State Space model has the advantage wherein the state vectors provide more insight to the physical state of the system. The state vector $x(t)$ has the physical significance like positions, velocities etc. and the outputs can be expressed as the known combinations of the states. Among the various parametric state space modeling algorithms (e.g. algorithm based on block Hankel matrix with Markov parameters), numerical algorithms for subspace state space system identification (N4SID), introduced by Overschee and Moor (1993) has been used in this study. The major advantage with N4SID is that it is non-iterative with no non-linear optimization part involved. Further, unlike classical identification, the estimation of a state space system from data measured does not need the initial condition to be zero. These are some of the reasons for adopting State Space Modeling based on N4SID algorithm as the parameterized method in damage detection of the structure in the present work.

In this paper, the objective is to identify damage through the detection of stiffness changes in multistoreyed shear building. The focus here is to monitor the state of the structure using Parametric State Space modeling. Considering floor acceleration response data from all the floor levels (assuming sensors are located at all the floors) of a numerically simulated building in both undamaged and damaged state as output and earthquake ground motion data as input, parametric system identification using State Space Model of the structure has been carried out. The extracted modal matrix for the entire structure has been utilized for the evaluation of structural stiffness matrix using least square technique. Thus, by applying this strategy to both the undamaged and damaged floor acceleration response data, the identification of the damaged location as well as the evaluation of the extent of damage can be very efficiently carried out through the comparison of structural stiffness matrices obtained corresponding to both the cases. Further, in the case of limited number of sensors connected to selected floors, partial modal matrix is only available. The identification of structural parameters for such cases have been carried out following the strategy suggested by Chakraverty (2005) and Yuan, *et al*. (2004).

2. Parametric state space modeling

2.1. Fundamental equations

The equation of motion for a finite dimensional linear dynamic system with $\mathbf{M}$, ζ and $\mathbf{K}$ as mass, damping and stiffness matrices respectively can be expressed by the state equation as:

$$\mathbf{M}\ddot{\mathbf{a}} + \zeta\dot{\mathbf{a}} + \mathbf{K}\mathbf{a} = \mathbf{f}(a, t) \tag{1}$$

where $\ddot{\mathbf{a}}$, $\dot{\mathbf{a}}$ and $\mathbf{a}$ are the vectors of acceleration, velocity and displacement respectively and $\mathbf{f}(a, t)$ is the forcing function at any time t over a specific location.

The system defined by Eq. (1) can be represented in State Space form as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t) \tag{2}$$

where,
$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\zeta \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \dot{\mathbf{a}} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{B}_2 \end{bmatrix}$$

$$\mathbf{f}(a, t) = \mathbf{B}_2\mathbf{u}(t)$$

and $\mathbf{u}(t)$ is the input of the State Space Model.

If the response of the dynamic system is measured by the n output quantities using sensors like accelerometer, velocity meter, and displacement meter etc, a matrix output equation can be written as

$$\mathbf{y}(t) = \mathbf{C}_a\ddot{\mathbf{a}} + \mathbf{C}_v\dot{\mathbf{a}} + \mathbf{C}_d\mathbf{a} \tag{3}$$

where $\mathbf{C_a}$, $\mathbf{C_v}$ and $\mathbf{C_d}$ are the output influence matrix for acceleration, velocity and displacement respectively. The output influence matrices basically indicate the location of appropriate sensor. A unity will be put corresponding to a particular degree of freedom, where sensor has been placed.

Substituting Eqs. (1) and (2) in Eq. (3) and after rearranging, it can be written as

$$\mathbf{y}(t) = \mathbf{Cx}(t) + \mathbf{Du}(t) \tag{4}$$

where

$$\mathbf{C} = [\mathbf{C_d} - \mathbf{C_a}\mathbf{M}^{-1}\mathbf{K} \quad \mathbf{C_v} - \mathbf{C_a}\mathbf{M}^{-1}\zeta]$$

$$\mathbf{D} = \mathbf{C_a}\mathbf{M}^{-1}\mathbf{B_2}$$

Since modern sensors used for collection of output response data are generally digital, Eqs. (2) and (4) must be represented in discrete time.

Considering equally spaced time given by 0, Δt, $2\Delta t$,……, $(k+1)\Delta t$, where Δt is the constant sampling time interval, the solution of Eq. (2) can be represented in a discrete time as

$$\mathbf{x}[(k+1)\Delta t] = e^{\mathbf{A}\Delta t}\mathbf{x}(k\Delta t) + \left[\int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau\, \mathbf{B}\right]\mathbf{u}(k\Delta t) \tag{5}$$

Eq. (5) can be written in a form given by,

$$\mathbf{x}(k+1) = \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,\mathbf{u}(k) \tag{6}$$

Similarly, Eq. (4) can also be written as

$$\mathbf{y}(k) = \mathbf{C}\,\mathbf{x}(k) + \mathbf{D}\,\mathbf{u}(k) \tag{7}$$

The numerical algorithms for subspace state space system identification (N4SID), introduced by Overschee and Moor (1993) has been used in this study for the evaluation of system matrices.

z transformation of $\mathbf{x}(k+1)$ is obtained as

$$z[\mathbf{x}(k+1)] = \sum_{k=0}^{\infty}\mathbf{x}(k+1)z^{-k} = z\sum_{k=0}^{\infty}\mathbf{x}(k+1)z^{-(k+1)}$$

$$= z\left[\sum_{k=-1}^{\infty}\mathbf{x}(k+1)z^{-(k+1)} - \mathbf{x}(0)\right]$$

$$= z\left[\sum_{k'=0}^{\infty}\mathbf{x}(k')z^{-k'} - \mathbf{x}(0)\right]$$

$$z[\mathbf{x}(z) - \mathbf{x}(0)] \tag{8}$$

where $z = e^{s\Delta t}$ and $k' = k + 1$.

Considering $\mathbf{x}(0) = 0$ and applying z transformation to Eq. (6) and Eq. (7) and using Eq. (8),

$$\mathbf{x}(z) = (z\mathbf{I\text{-}A})^{-1}\mathbf{Bu}(z) \tag{9}$$

$$\mathbf{y}(z) = \mathbf{Cx}(z) + \mathbf{Du}(z) \tag{10}$$

Using the parameters defined in Eq. (9) into Eq. (10), we get

$$\begin{aligned}\mathbf{y}(z) &= [\mathbf{C}[z\mathbf{I\text{-}A}]^{-1}\mathbf{B} + \mathbf{D}]\mathbf{u}(z)\\ &= \left[\mathbf{D} + \sum_{k=0}^{\infty} z^{-(k+1)}\mathbf{CA}^k\mathbf{B}\right]\mathbf{u}(z)\\ &= \mathbf{H}(z)\,\mathbf{u}(z)\end{aligned} \tag{11}$$

where $\mathbf{H}(z)$ is the system transfer function relating the input and output.

The values of z for which $\mathbf{H}(z)$ is infinity are called poles. For a stable system, all poles must have a magnitude <1 and should be located within the unit circle.

The jth pole of the system is given by

$$z_j = e^{(-\xi_j\omega_j \pm i\,\omega_j\sqrt{1-\xi_j^2})\Delta t} \tag{12}$$

where ξ_j and ω_j are damping ratio and frequency of the jth mode of vibration. The frequency and damping ratio can be determined as follows:

$$\omega_j = \frac{1}{\Delta t}\sqrt{\ln^2 r_j + \theta_j^2} \tag{13}$$

$$\xi_j = -\frac{\ln r_j}{\sqrt{\ln^2 r_j + \theta_j^2}} \tag{14}$$

where $r_j = |z_j|$, the magnitude; and $\theta_j = \tan^{-1}[\mathrm{Im}\, g(z_j)/\mathrm{Re}(z_j)]$, the phase angle of the jth pole.

2.2. Extraction of the mode shape matrix

The evaluated system matrix **A** has been utilized for the evaluation of eigenvalues (λ) and their corresponding eigenvectors (φ). It has been found that the evaluated eigen values (λ) represent the poles of the structure. Hence, eigen vector (φ) corresponding to the eigen values of the matrix **A** must also represent the modal displacement vector of the structure. While Eq. (13) and (14) are used for the determination of the natural frequencies and the damping ratios of the structure, the stability of the system is observed from the value of pole from Eq. (12). However, the modal matrix (Θ) corresponds to the non-physical state of the structure and hence, the **C** matrix is used to transform the computed

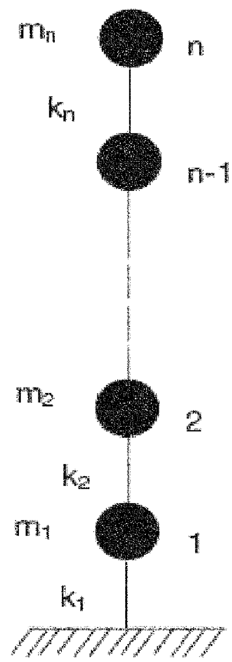

Fig. 1 Lumped mass model of a multistoreyed shear building

eigenvector from the non physical state to the mode shape vector at the structural floor level, where the response data have been measured. Thus, the modal displacement vector for the structure corresponding to all the modes can be calculated as

$$\Gamma = \mathbf{C}\,\Theta \tag{15}$$

3. Mathematical model to calculate the stiffness matrix

The structure has been assumed to be a shear building with lumped mass at each floor level as shown in Fig. 1. Two cases have been considered depending on the availability of sensors at all the floor levels or not. The modal matrix will accordingly be either complete or only representative of those floor displacements, where sensors are attached.

The global mass and stiffness matrices can be represented as:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & \dots & 0 & 0 \\ 0 & m_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & m_{n-1} & 0 \\ 0 & 0 & \dots & 0 & m_n \end{bmatrix} \tag{16}$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & & & 0 \\ -k_2 & k_1 + k_2 & \ddots & & \\ & & \ddots & & \\ & \ddots & & k_{n-1} + k_n & -k_n \\ 0 & & & -k_n & k_n \end{bmatrix} \tag{17}$$

3.1. Case 1: When the full mode shape matrix is available

If sensors are attached at each floor level, the mode shape matrix corresponding to entire structure is available. The following steps will be carried out for the evaluation of structural stiffness matrix.

Step 1: For shear building, the characteristics equation can be written as:

$$(\mathbf{K} - \lambda_i \mathbf{M}) \begin{Bmatrix} \phi_i^{(1)} \\ \phi_i^{(2)} \\ \phi_i^{(3)} \\ \cdots \\ \cdots \\ \phi_i^n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ \cdots \\ 0 \end{Bmatrix} \tag{18}$$

where $\lambda_i = \omega_i^2$ is the i^{th} Eigen value and

θ_i^r, r = 1, 2, 3……n is the i^{th} mode shape at the r^{th} floor level.

Step 2: The equation can be expanded and rearranged as

$$\Delta_{\mathbf{i}} \mathbf{k} = \Lambda_{\mathbf{i}} \tag{19}$$

where

$$\Delta_{\mathbf{i}} = \begin{bmatrix} \phi_i^{(1)} & \phi_i^{(1)} - \phi_i^{(2)} & 0 & \ldots & 0 \\ 0 & \phi_i^{(2)} - \phi_i^{(1)} & \phi_i^{(2)} - \phi_i^{(3)} & \ldots & 0 \\ \cdot & \cdot & \cdot & \ldots & \ldots \\ \cdot & \cdot & \cdot & \ldots & \ldots \\ 0 & 0 & \ldots & \phi_i^{(n-1)} - \phi_i^{(n-2)} & \phi_i^{(n-1)} - \phi_i^{(n)} \\ 0 & 0 & \ldots & 0 & \phi_i^{(n)} - \phi_i^{(n-1)} \end{bmatrix} \tag{20}$$

$$\mathbf{k} = \begin{Bmatrix} k_1 \\ k_2 \\ . \\ . \\ k_n \end{Bmatrix} \tag{21}$$

$$\Lambda_\mathbf{i} = \begin{Bmatrix} \phi_i^{(1)}\lambda_i m_1 \\ \phi_i^{(2)}\lambda_i m_2 \\ . \\ . \\ \phi_i^{(n)}\lambda_i m_n \end{Bmatrix} \tag{22}$$

3.2. Case 2: When partial mode shape matrix is available due to limited number of sensors

If there are limited numbers of sensors and sensors are attached at the first and n^{th} floor of the building, the following method due to Chakraverty (2005) and Yuan, *et al*. (2004) is adopted for finding out the stiffness matrix. Details of the method are available in Chakraverty (2005) and Yuan, *et al*. (2004), which have been implemented in the present work with some correction in the formulation. The procedure has been enumerated below in step-wise form.

Step 1: The initial value of θ, which is the ratio between the mass and stiffness of the n^{th} floor is assumed and the following matrix $\mathbf{A}_\theta$ is calculated.

$$\mathbf{A}_\theta = \begin{bmatrix} (2-\lambda\theta) & -1 & 0 & \dots & \dots & 0 \\ -1 & (2-\lambda\theta) & & \dots & & 0 \\ \dots & \dots & & \dots & & \dots \\ \dots & \dots & & \dots & & \dots \\ 0 & \dots & & -1 & (2-\lambda\theta) & -1 \\ 0 & \dots & \dots & 0 & -1 & (2-\lambda\theta) \end{bmatrix} \tag{23}$$

Step 2: The unknown mode shape $\{\Theta_\mathbf{u}\}$ is calculated as

$$\{\Theta_\mathbf{u}\} = (\mathbf{A}_\theta^\mathbf{T}\mathbf{A}_\theta)^{-1}(\mathbf{A}_\theta^\mathbf{T}\mathbf{Z}) \tag{24}$$

where,

$$\Theta_\mathbf{u} = \begin{Bmatrix} \phi_i^{(2)} \\ \phi_i^{(3)} \\ . \\ . \\ \phi_i^{(n-1)} \end{Bmatrix} \tag{25}$$

$$\mathbf{Z} = \begin{Bmatrix} \phi_i^{(1)} \\ 0 \\ . \\ . \\ \phi_i^{(n)} \end{Bmatrix} \tag{26}$$

Step 3: The stiffness and mass of each floor is calculated by taking two order modes λ_i and λ_j as:

$$\{\mathbf{b}^{/}\} = ([\mathbf{B}^{/}]^T[\mathbf{B}^{/}])^{-1}[\mathbf{B}^{/}]^T \begin{Bmatrix} 0 \\ 0 \\ . \\ . \\ \lambda_i\phi_i^{(n)} \\ \lambda_j\phi_j^{(n)} \end{Bmatrix} \tag{27}$$

where, $\lambda_i = \omega_i^2$ and $\lambda_j = \omega_j^2$ are the ith and jth eigen values respectively and

$$\mathbf{b}^{/} = \begin{Bmatrix} k_1 \\ m_1 \\ k_2 \\ m_2 \\ . \\ . \\ k_{n-1} \\ m_{n-1} \end{Bmatrix}$$

where k_1, k_2, …..k_{n-1} are the stiffness and m_1, m_2, ……, m_{n-1} are mass of each floor.

$$\mathbf{B}^{/} = \begin{bmatrix} \phi_i^{(1)} & -\lambda_i\phi_i^{(n)} & \phi_i^{(1)}-\phi_i^{(2)} & 0 & & \dots & & 0 \\ \phi_j^{(1)} & -\lambda_j\phi_j^{(n)} & \phi_j^{(1)}-\phi_j^{(2)} & 0 & & \dots & & 0 \\ 0 & 0 & \phi_i^{(2)}-\phi_i^{(1)} & -\lambda_i\phi_i^{(n)} & \phi_i^{(2)}-\phi_i^{(3)} & \dots & & 0 \\ 0 & 0 & \phi_j^{(2)}-\phi_j^{(1)} & -\lambda_j\phi_j^{(n)} & \phi_j^{(2)}-\phi_j^{(3)} & \dots & & 0 \\ & & 0 & & & & & \\ \dots & & \dots & & & \dots & & \dots \\ & & & & & & \phi_i^{(n-1)}-\phi_i^{(n-2)} & -\lambda_i\phi_i^{(n-1)} \\ & & & & & & \phi_j^{(n-1)}-\phi_j^{(n-2)} & -\lambda_j\phi_j^{(n-1)} \end{bmatrix}$$

The mass of the nth floor is considered as one and the stiffness of the nth floor k_n is calculated as

$$k_n = \frac{\lambda_i \phi_i^n}{\phi_i^{(n)} - \phi_i^{(n-1)}} = \frac{\lambda_j \phi_j^n}{\phi_j^{(n)} - \phi_j^{(n-1)}} \tag{28}$$

Step 4: The Holzer Criteria which was first used by Chakraverty (2005) in system identification problems is applied to find out the value of θ and steps 1 to 3 are repeated till the Holzer's Remainder Function becomes zero.

However, the limitation of this method is that it can be applied only when the floors from 2 to n are standard floors.

4. Results and discussions

The parametric system identification technique has been applied to a five storey symmetric R.C. building with assumed stiffness and mass matrices. Using the adopted strategy, the structural stiffness matrix has been re-evaluated and compared. Further, damages have been introduced and using the adopted methodology, the identification study has been carried out in terms of location and extent of damages. The issue of limited availability of sensors along the height of the building has also been addressed.

4.1 Identification of the stiffness matrix of the structure in undamaged state

Stiffness and mass matrices of the five storey symmetric R.C. structure have been considered as below:

$$\mathbf{K} = (1.0\text{E} + 006) \times \begin{bmatrix} 4.5 & -1.5 & 0 & 0 & 0 \\ -1.5 & 3.0 & -1.5 & 0 & 0 \\ 0 & -1.5 & 3.0 & -1.5 & 0 \\ 0 & 0 & -1.5 & 3.0 & -1.5 \\ 0 & 0 & 0 & -1.5 & 1.5 \end{bmatrix} \text{ (N/m)}$$

$$\mathbf{M} = \begin{bmatrix} 6000 & 0 & 0 & 0 & 0 \\ 0 & 6000 & 0 & 0 & 0 \\ 0 & 0 & 6000 & 0 & 0 \\ 0 & 0 & 0 & 6000 & 0 \\ 0 & 0 & 0 & 0 & 6000 \end{bmatrix} \text{ Kg}$$

Identification of structural stiffness matrix through parametric system identification technique has been carried out by considering the El-Centro, 1940 (N-S component) and Northridge, 1994 earthquake (longitudinal) acceleration data with a sampling period 0.02 second as input and the corresponding acceleration responses of the different floors of the structure as output. The computation of acceleration response has been carried out using Matlab, where the numerical model has been simulated using appropriate element stiffness. Time integration has been carried out using average acceleration Newmark-β method

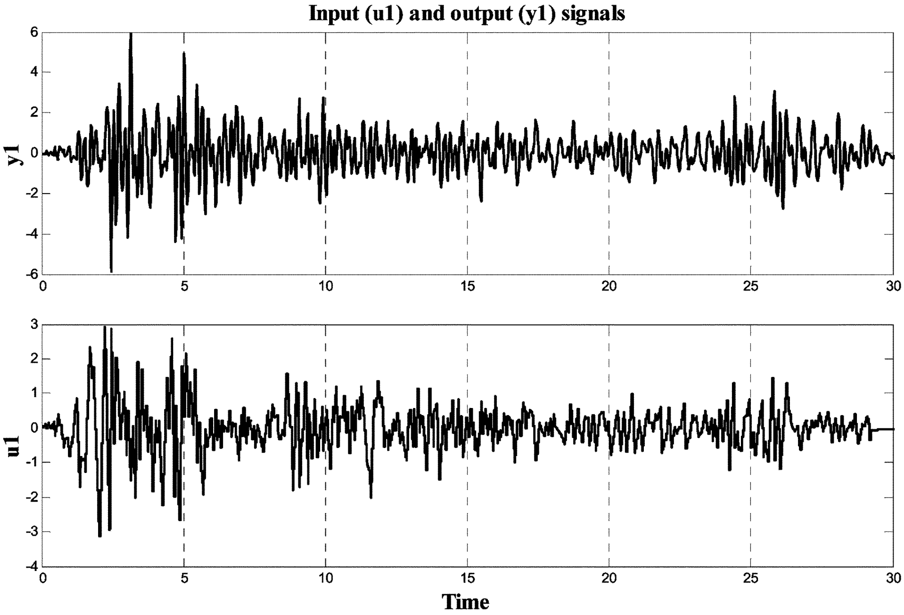

Fig. 2 El Centro Earthquake, 1940 (N-S component) acceleration (u1 m/sec^2) and first floor acceleration response (y1 m/sec^2)

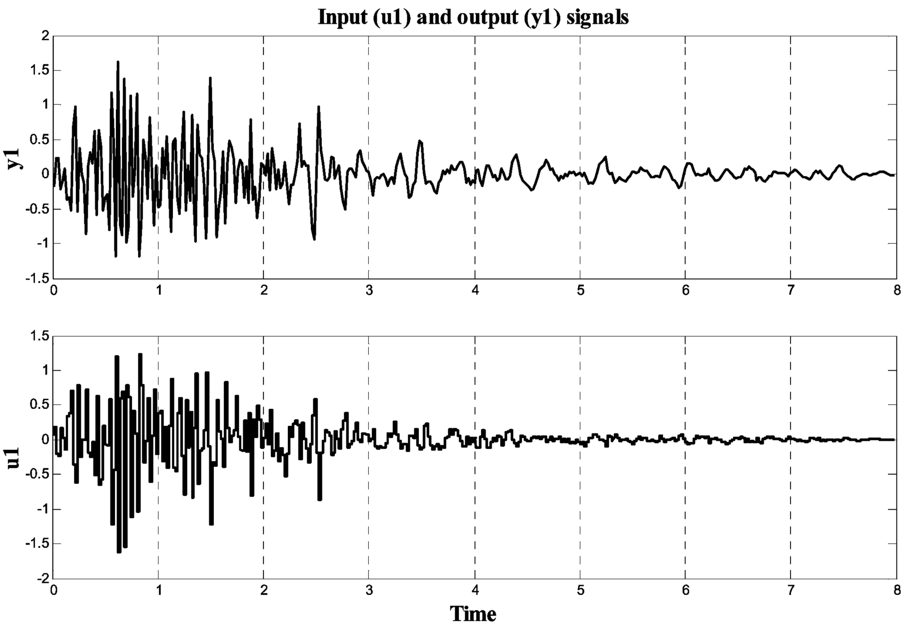

Fig. 3 Northridge Earthquake, 1994 (longitudinal) acceleration (u1 m/sec^2) and first floor acceleration response (y1 m/sec^2)

in order to obtain the acceleration response of different floors under different seismic excitations. The input/output plots from first floor corresponding to the El-Centro and Northridge excitation are as shown in Fig. 2 and Fig. 3 respectively. Noise in the sensor is simulated by adding a stationary

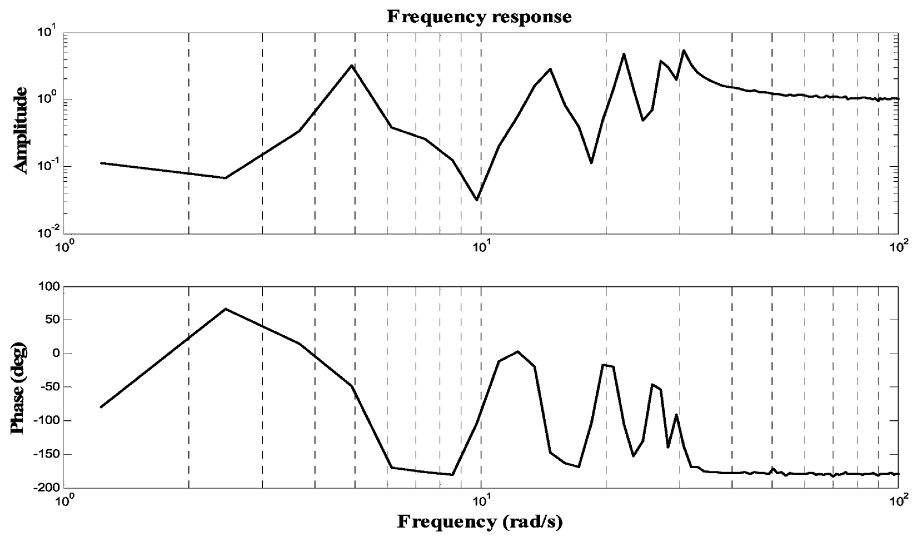

Fig. 4 Frequency response plot corresponding to El-Centro (N-S component) Earthquake 1940

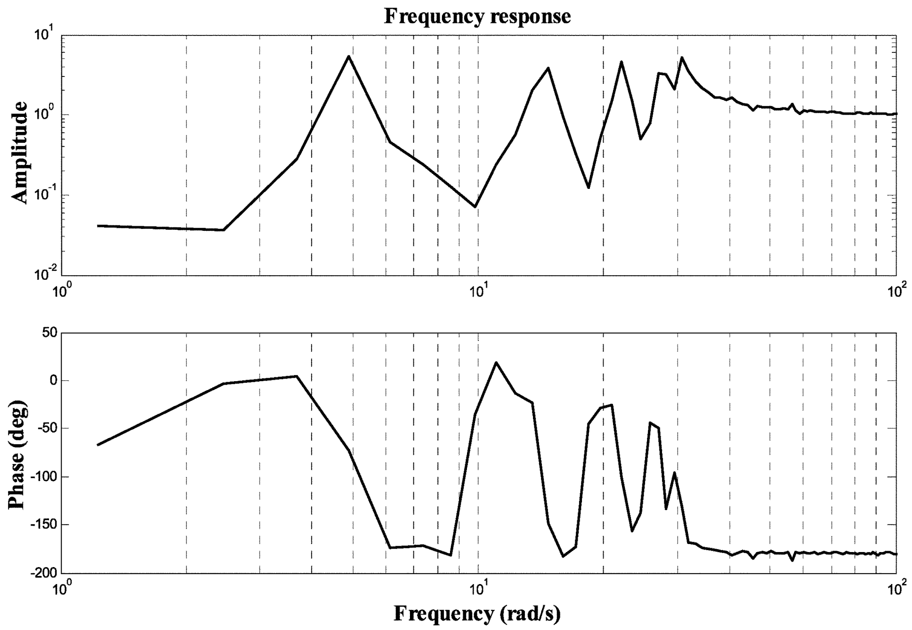

Fig. 5 Frequency response plot corresponding to Northridge Earthquake 1994 (longitudinal)

broadband signal to the response. The Parametric State Space Model has been formulated using IDENT toolbox of Matlab, where N4SID algorithm has been utilized.

In order to find out the order of the state space model, a spectral model, which is nothing but the frequency response plot, has been created (Fig. 4 and 5) for each of the acceleration responses corresponding to the El-Centro and Northridge excitation. The order of the state space model should be adequate in order to describe the dynamic properties of the system explicitly. From Figs. 4 and 5, it is evident that there are five distinct peaks in the amplitude vs. frequency plot and since poles exist as pair of complex

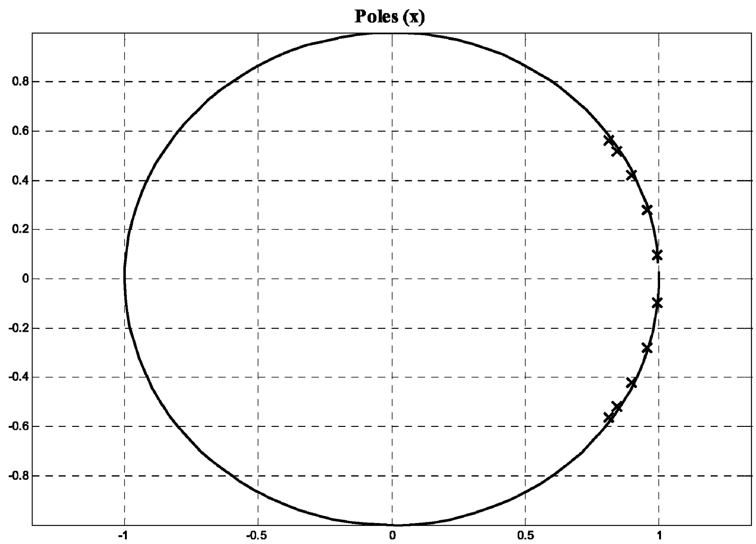

Fig. 6 Poles (x) within the unit circle for El Centro Earthquake 1940, (N-S component) ground acceleration

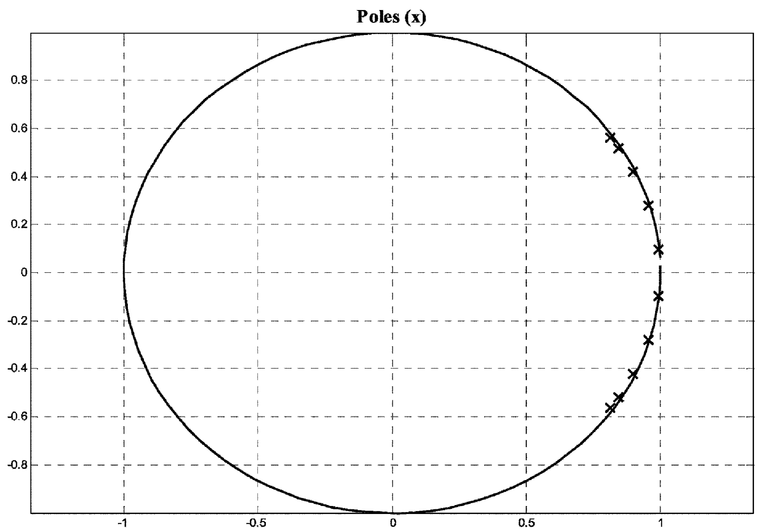

Fig. 7 Poles (x) within the unit circle for Northridge Earthquake 1994 (longitudinal) ground acceleration

conjugate corresponding to each frequency, the order of the model has been considered as ten in order to extract the five predominant natural frequencies of the five storey shear building. Poles are shown in Figs. 6 and Fig. 7 corresponding to El Centro (N-S component) and Northridge earthquake (longitudinal) respectively.

The poles of the transfer function have been plotted on the complex plane using the polar coordinates according to Eq. (12). Poles within the unit circle in the complex plane refer to stable poles of the system, while poles falling outside the unit circle destabilize the dynamic system. Thus, as observed from both Figs. 6 and 7, five numbers of complex conjugate poles marked as '*x*' have been found to be lying within the unit circle, which signifies that the system is stable.

Table 1 Comparison of actual and identified frequencies

Mode No.	Actual Frequency (rad/sec)	Identified Frequency (rad/sec) corresponding to El Centro N-S Earthquake Data	Identified Frequency (rad/sec) corresponding to Northridge Earthquake Data
1	4.94	4.94	4.94
2	14.35	14.30	14.26
3	22.36	22.00	21.99
4	28.18	27.46	27.56
5	31.23	30.30	30.27

The above procedure is repeated for each of the other four floors in which input is taken as the El Centro (N-S component) and Northridge (longitudinal) ground acceleration and outputs are taken as second, third, fourth and fifth floor acceleration response data.

The frequencies as calculated by using Eq. (13) based on pole of the system have been compared with the frequencies from the numerically simulated model and are shown in Table 1.

The State Space Model is then presented in the command window of Matlab for the extraction of mode shapes. From the **A** matrix, mode shape vectors $\mathbf{\Phi_E}$ and $\mathbf{\Phi_N}$ are calculated using Eq. (15) for El Centro 1940 (N-S component) and Northridge Earthquake 1994 (longitudinal) data respectively. Thus,

$$\mathbf{\Phi_E} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2.90 & 2.18 & 1 & -0.18 & -0.9 \\ 4.52 & 1.56 & -1 & -0.79 & 0.72 \\ 5.70 & -0.34 & -1 & 1.11 & -0.46 \\ 6.31 & -1.96 & 1 & -0.51 & 0.16 \end{bmatrix}$$

and

$$\mathbf{\Phi_N} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2.94 & 2.18 & 1 & -0.18 & -0.91 \\ 4.51 & 1.55 & -0.98 & -0.78 & 0.71 \\ 5.70 & -0.34 & -1 & 1.11 & -0.46 \\ 6.14 & -1.91 & 0.98 & -0.50 & 0.16 \end{bmatrix}$$

Using these mode shape matrices, the stiffness matrices are calculated as $\mathbf{K_{ident\_elcentro}}$ and $\mathbf{K_{indent\_Northridge}}$ corresponding to El Centro 1940 (North-South Component) and Northridge Earthquake 1994 (Longitudinal) Data respectively.

$$\mathbf{K_{ident\_El\ Centro}} = (1.0E + 006) \times \begin{bmatrix} 4.55 & -1.52 & 0 & 0 & 0 \\ -1.52 & 3.04 & -1.52 & 0 & 0 \\ 0 & -1.52 & 3.04 & -1.52 & 0 \\ 0 & 0 & -1.52 & 3.04 & -1.52 \\ 0 & 0 & 0 & -1.52 & -1.52 \end{bmatrix} \text{(N/m)}$$

Table 2 Comparison of actual and identified stiffness

Floor No.	Actual Stiffness (N/m)	Identified Stiffness (N/m) corresponding to El Centro N-S Earthquake Data	Identified Stiffness (N/m) corresponding to Northridge Earthquake Data
1	3.0×10^6	3.03×10^6	2.97×10^6
2	1.5×10^6	1.52×10^6	1.48×10^6
3	1.5×10^6	1.52×10^6	1.48×10^6
4	1.5×10^6	1.52×10^6	1.48×10^6
5	1.5×10^6	1.52×10^6	1.48×10^6

$$\mathbf{K}_{\mathbf{ident\_Northridge}} = (1.0\text{E} + 006) \times \begin{bmatrix} 4.45 & -1.48 & 0 & 0 & 0 \\ -1.48 & 2.96 & -1.48 & 0 & 0 \\ 0 & -1.48 & 2.96 & -1.48 & 0 \\ 0 & 0 & -1.48 & 2.96 & -1.48 \\ 0 & 0 & 0 & -1.48 & -1.48 \end{bmatrix} \text{(N/m)}$$

The comparison of floor wise distribution of stiffness has been shown in Table 2. It is very clearly evident that the stiffness matrix evaluated through the system identification technique corresponding to different ground excitations agrees very closely with the actual stiffness of the structure.

4.2. Identification of the stiffness matrix of the structure when the numbers of sensors are limited

It is realistic to assume that the sensors may be made available at only a few key locations (floors) and the limited output from those sensors should be utilized for the identification process. In the present study, it is assumed that there are only three sensors available for the identification process. One sensor is used to measure ground acceleration and other two sensors are attached at the first floor and top floor of the shear building to collect the acceleration response data. The Parametric State Space Modeling strategy has been applied for the data collected at the first floor and top floor. Table 3 shows first two identified frequencies and the corresponding components of modal displacements at first and top floor of the structure.

The methodology mentioned in Section 3 (case 2) has been used to identify the complete modal matrix, which is utilized for the evaluation of stiffness matrix. The stiffness matrix evaluated through

Table 3 First two Identified Frequencies and corresponding modal displacement vectors

Frequency	First Floor Displacement	Top Floor Displacement
4.94	1	6.313
14.30	1	-1.962

Table 4 Identified stiffness of the structure with limited sensor data

	k_1 (N/m)	k_2 (N/m)	k_3 (N/m)	k_4 (N/m)	k_5 (Kg)	m_1 (Kg)	m_2 (Kg)	m_3 (Kg)	m_4 (Kg)	m_5 (Kg)
True Value	3.0×10^6	1.5×10^6	1.5×10^6	1.5×10^6	1.5×10^6	6000	6000	6000	6000	6000
True Value/6000	5×10^2	2.5×10^2	2.5×10^2	2.5×10^2	2.5×10^2	1	1	1	1	1
Identified Value	4.99×10^2	2.49×10^2	2.49×10^2	2.49×10^2	2.49×10^2	0.997	0.997	0.997	0.997	0.997

system identification technique with only two sensors have been shown in Table 4. It is observed that the order of accuracy in the estimation of stiffness is very high, even with a limited numbers of sensors.

4.3. Identification of the stiffness matrix of the structure in an unknown state

The stiffness matrix of the structure as obtained through identification technique corresponding to undamaged state is compared with all such stiffness matrix of the structure, which is obtained at any future state adopting similar identification strategy. This enables the analyst to ascertain the unknown state of the structure and to verify whether the structure has undergone any damage after any seismic event. Thus, considering the $\mathbf{K}_{\mathbf{ident\_El\ Centro}}$ as the stiffness matrix of the structure at undamaged state, the stiffness matrix at an unknown state (damaged and undamaged) can be calculated and compared.

In the present study, damage has been introduced in the first floor of the structure by changing the stiffness of the first floor from 3.0×10^6 N/m to 1.0×10^6 N/m. This damaged state of the structure will be considered as the unknown state in order to verify the application of the Parametric State Space modeling in the damage identification problem. The theoretical value of stiffness matrix corresponding to this damaged state $\mathbf{K}_{\mathbf{damaged}}$ should be:

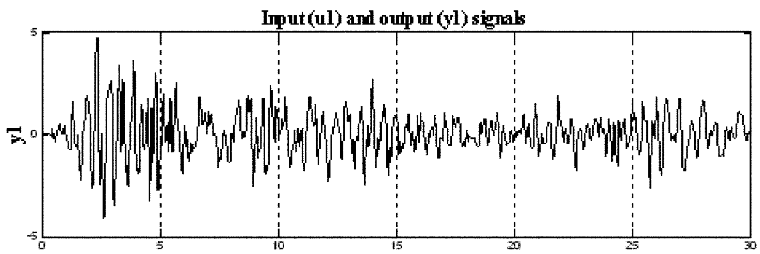

Fig. 8 First floor acceleration response (y1 m/sec^2) for the unknown state corresponding to El Centro Earthquake 1940 (N-S component) motion

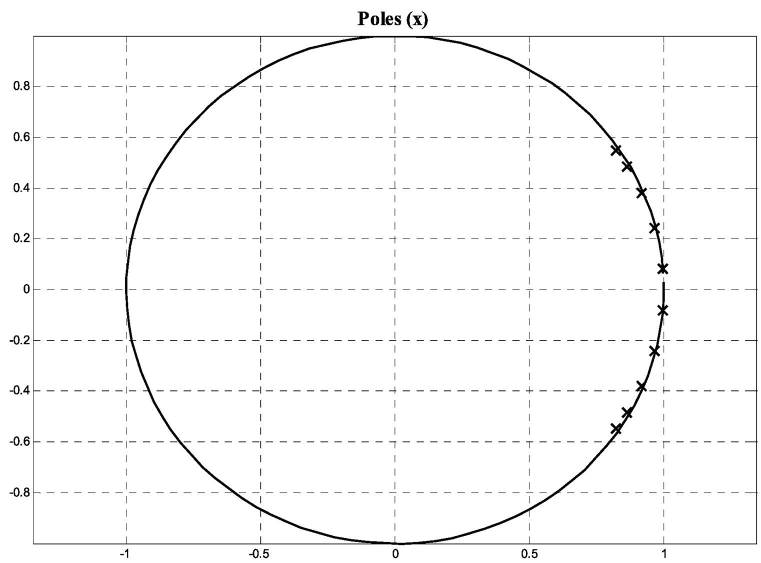

Fig. 9 Poles (x) of unknown state within the unit circle corresponding to El Centro (N-S component) Earthquake

Table 5 Comparison of actual and identified frequencies corresponding to the unknown state

Mode No.	Evaluated Frequency (rad/sec) corresponding to the undamaged state using El Centro Data	Evaluated Frequency (rad/sec) corresponding to the unknown state using El Centro Data
1	4.94	4.14
2	14.30	12.33
3	22.00	19.70
4	27.46	25.60
5	30.30	29.34

$$\mathbf{K}_{\mathbf{damaged}} = (1.0\text{E} + 006) \times \begin{bmatrix} 2.50 & -1.50 & 0 & 0 & 0 \\ -1.50 & 3.00 & -1.50 & 0 & 0 \\ 0 & -1.50 & 3.00 & -1.50 & 0 \\ 0 & 0 & -1.50 & 3.00 & -1.50 \\ 0 & 0 & 0 & -1.50 & 1.50 \end{bmatrix} \text{(N/m)}$$

The stiffness matrix $\mathbf{K}_{\mathbf{damaged}}$ is used in the Matlab program to get the responses of all the five floors corresponding to El Centro 1940 (N-S component) earthquake ground acceleration with a sampling period of 0.02 second. These responses are further utilized to verify the efficiency of the above-discussed procedure in locating the damaged floor. Thus, the damaged stiffness, which has been evaluated using the identification technique is considered as the stiffness at the unknown state $\mathbf{K}_{\mathbf{unknown}}$ and compared with $\mathbf{K}_{\mathbf{indent\_ElCentro}}$, the identified stiffness matrix of the structure at undamaged state

Fig. 8 shows a typical acceleration response of the first floor of the structure at an unknown state. The poles are calculated and displayed in the Fig. 9. The frequencies calculated by using the Eq. (13) have been displayed in Table 5.

It can be very clearly concluded from Table 5 that there are degradations in the structure since the frequencies of the structure at the unknown state are lower than the frequencies of the undamaged state.

The mode shape matrix of the structure in the unknown state $\Phi_{\mathbf{unknown}}$ has been obtained as

$$\Phi_{\mathbf{unknown}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1.61 & 1.06 & 0.07 & -1.07 & -1.99 \\ 2.09 & 0.47 & -0.97 & -0.20 & 2.28 \\ 2.43 & -0.41 & -0.47 & 1.23 & -1.79 \\ 2.61 & -1.03 & 0.78 & -0.70 & 0.69 \end{bmatrix}$$

Using this mode shape matrix, the stiffness matrix corresponding to the unknown state is calculated and compared with the theoretical values of the individual floor stiffnesses (Table 6).

The overall stiffness matrix of the unknown state is obtained as

$$\mathbf{K}_{\mathbf{unknown}} = (1.0\text{E} + 006) \times \begin{bmatrix} 2.496 & -1.496 & 0 & 0 & 0 \\ -1.496 & 2.992 & -1.496 & 0 & 0 \\ 0 & -1.496 & 2.992 & -1.496 & 0 \\ 0 & 0 & -1.496 & 2.992 & -1.496 \\ 0 & 0 & 0 & -1.496 & -1.496 \end{bmatrix} \text{(N/m)}$$

Table 6 Identified stiffness of the structure corresponding to the unknown state

Floor No.	Theoretical value of damaged floor stiffness (N/m)	Evaluated Stiffness (N/m) for the unknown State corresponding to El Centro N-S Earthquake Data
1	1.0×10^6	1.000×10^6
2	1.5×10^6	1.496×10^6
3	1.5×10^6	1.496×10^6
4	1.5×10^6	1.496×10^6
5	1.5×10^6	1.496×10^6

The identified stiffness matrix of the structure at the undamaged state as obtained earlier

$$\mathbf{K}_{\mathbf{ident\_El\ Centro}} = (1.0\mathrm{E}+006) \times \begin{bmatrix} 4.55 & -1.52 & 0 & 0 & 0 \\ -1.52 & 3.04 & -1.52 & 0 & 0 \\ 0 & -1.52 & 3.04 & -1.52 & 0 \\ 0 & 0 & -1.52 & 3.04 & -1.52 \\ 0 & 0 & 0 & -1.52 & -1.52 \end{bmatrix} \text{ (N/m)}$$

Thus, the percentage change in each element of the stiffness matrix is obtained as

$$\frac{\mathbf{K}_{\mathbf{ident}} - \mathbf{K}_{\mathbf{unknown}}}{\mathbf{K}_{\mathbf{ident}}} \times 100 = \begin{bmatrix} 45.14 & 1.5 & 0 & 0 & 0 \\ 1.5 & 1.5 & 1.5 & 0 & 0 \\ 0 & 1.5 & 1.5 & 0 & 0 \\ 0 & 0 & 1.5 & 0 & 1.5 \\ 0 & 0 & 0 & 1.5 & 1.5 \end{bmatrix} \%$$

A similar observation has also been obtained while considering the Northridge excitation and hence the detailed results are not repeated. Thus, the identification of unknown state could be carried out very accurately and the adopted procedure can be regarded as a reliable methodology for damage identification.

Table 7 First two identified frequencies and modal displacement vectors corresponding to unknown state

Frequency	First floor displacement	Top floor displacement
4.14	1	2.61
12.33	1	-1.03

Table 8 Identified stiffness of the structure in the unknown state with limited sensor data

	k_1 (N/m)	k_2 (N/m)	k_3 (N/m)	k_4 (N/m)	k_5 (Kg)	m_1 (Kg)	m_2 (Kg)	m_3 (Kg)	m_4 (Kg)	m_5 (Kg)
True Value	1.0×10^6	1.5×10^6	1.5×10^6	1.5×10^6	1.5×10^6	6000	6000	6000	6000	6000
True Value/6000	1.67×10^2	2.5×10^2	2.5×10^2	2.5×10^2	2.5×10^2	1	1	1	1	1
Identified Value	1.63×10^2	2.45×10^2	2.45×10^2	2.45×10^2	2.49×10^2	0.971	0.978	0.978	0.978	0.978

4.4. Identification of the stiffness matrix of the structure at the damaged state when the numbers of sensors are limited

The case of limited sensors (three sensors) are considered again for the identification process of the damaged structure. Table 7 shows first two identified frequencies and the corresponding components of modal displacements at first and top floor of the structure extracted from the data collected by the sensors mounted at first and top floors.

The methodology as mentioned in Section 3 (case 2) has been used to extract the complete modal matrix, which is utilized for the evaluation of stiffness matrix. The stiffness matrix evaluated through system identification technique with only two sensors have been shown in Table 8. It is again observed that the order of accuracy in the estimation of stiffness is very high, even with a limited numbers of sensors. However, it can be noted that the degradation in stiffness has occurred in first floor and hence the first floor may be identified as the damaged floor.

5. Conclusions

The Parametric State Space modeling has been utilized for the identification of damage in a structure. The studies have been conducted by seismically exciting a numerically simulated model of a multistoreyed shear building and evaluating acceleration response at floor levels. A very accurate estimation of the structural stiffness could be done for undamaged as well as damaged state of the structure and irrespective of number of available sensors for acceleration data. The procedure is directly useful to be applied for the real time structure subjected to earthquake excitation and structural damages can be very efficiently identified by collecting acceleration response data from some sensors located at different floor levels. The Holzer's Criteria could be used to overcome the problem of limited number of sensors for the extraction of full modal matrix of a structure.

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