Smart Structures and Systems, Vol. 3, No. 3 (2007) 373-384 DOI: http://dx.doi.org/10.12989/sss.2007.3.3.373

Development and application of a vision-based displacement measurement system for structural health monitoring of civil structures

Jong Jae Lee*

Department of Civil & Environmental Engineering, Korea Advanced Institute of Science and Technology, Guseong-dong, Yuseong-gu, Daejeon, Korea

Yoshio Fukuda and Masanobu Shinozuka

Department of Civil & Environmental Engineering, University of California Irvine, Irvine, CA, 92697, USA

Soojin Cho and Chung-Bang Yun

Department of Civil & Environmental Engineering, Korea Advanced Institute of Science and Technology, Guseong-dong, Yuseong-gu, Daejeon, Korea

(Received October 31, 2006, Accepted November 28, 2006)

Abstract. For structural health monitoring (SHM) of civil infrastructures, displacement is a good descriptor of the structural behavior under all the potential disturbances. However, it is not easy to measure displacement of civil infrastructures, since the conventional sensors need a reference point, and inaccessibility to the reference point is sometimes caused by the geographic conditions, such as a highway or river under a bridge, which makes installation of measuring devices time-consuming and costly, if not impossible. To resolve this issue, a vision-based *real-time* displacement measurement system using digital image processing techniques is developed. The effectiveness of the proposed system was verified by comparing the load carrying capacities of a steel-plate girder bridge obtained from the conventional sensor and the present system. Further, to simultaneously measure multiple points, a synchronized vision-based system is developed using master/slave system with wireless data communication. For the purpose of verification, the measured displacement by a synchronized vision-based system (LVDT) from a laboratory test.

Keywords: displacement measurement; vision-based system; digital image processing; load carrying capacity; time synchronization; multi-point measurement.

1. Introduction

Civil infrastructures are exposed to various external loads such as traffic, earthquakes, gusts, and possible wave loads during their lifetime. The structures may deteriorate as time passes in unexpected ways, which may lead to structural damage requiring costly repair/replacement and even may result in human

^{*}Corresponding Author, E-mail: jongjae74@kaist.ac.kr

374 Jong Jae Lee, Yoshio Fukuda, Masanobu Shinozuka, Soojin Cho, and Chung-Bang Yun

losses. Consequently, structural health monitoring has become an important research topic for continuous assessment and evaluation of structural safety and integrity. Because of the current lack of sensors suitable for monitoring of large structures, displacement has not been the popular response quantity to be measured and monitored, even though it is one of the most important descriptor of structural behavior under all the potential disturbances of natural and man-made origin. This is due to the inaccessibility and the huge size of the civil infrastructures. A linear variable differential transformer (LVDT), the most frequently employed contact-type sensor for displacement measurement, needs stable reference points plummeted to earth from the measured points, however for civil infrastructures it is very difficult to find the place suitable for the sensor installation. To overcome the prescribed problems, some promising advanced sensors emerged in the meantime. Examples are Global Positioning System (GPS) (Roberts and Dodson 1998, Çelibi 2000, Nakamura 2000, Knecht and Manetti 2001, Xu, et al. 2002) and Laser Doppler vibrometer (Nassif, et al. 2005), other than the proposed vision-based system. The use of GPS at the level of resolution we require is a very expensive proposition. The Laser Doppler vibrometers perform very well but they are also much more expensive to use than the proposed visionbased sensor and the laser intensity may become dangerously strong for the distance we envision for remote sensing. Double integration of an acceleration record is a rather unreliable method to obtain a corresponding displacement record even when we use super-precision accelerometers, which by itself is an expensive proposition (Faulkner, et al. 1996, Park, et al. 2005). Indeed, there is no cost-effective displacement sensor system to remotely measure the displacement of large structures, which is reliable, accurate, easy to use, and having real-time capability.

On the other hand, a vision-based system has offered an effective alternative to displacement measurement. With rapid advancement in electronics and computer technology, optical devices have become affordable, and they are expected to be more so in the future. Stephen, et al. (1992) successfully utilized a visual tracking system to measure the very low frequency movement of the Humber Bridge in the UK. Transputer-based parallel processing techniques were employed to track the motion of multiple, independent objects in real-time. A transputer is a kind of fast and powerful microprocessor. For object tracking, templates of the user-selected object features are extracted from the initial frame and a template matching operation using the minimum least squares error scheme is applied repeatedly (Stephen, et al. 1993). Olaszek (1999) developed a method for the investigation of the dynamic characteristic of bridges based on the photogram-metric principle using additional reference system (Olaszek 1999). The reference point concurrently measured with a target point is used to exclude the effect of translational movement at image capturing camera. However, a specially manufactured optical device should be employed to concurrently capture two distant points (reference and target points) using one image capturing device. Whabeh, et al. (2003) deployed the high-fidelity video camera with a resolution of 520 lines and a capability of 450 digital zoom, and the targets consisted of black steel sheets 28 inches high by 32 inches wide, on which two high-resolution red lights (LED) were mounted, to measure displacement of the Vincent Thomas Bridge located in San Pedro, California (Wahbeh, et al. 2003). Sophisticated signal processing techniques, including optical data reduction and a nonlinear Gaussian regression curve fit to determine the center of the high-intensity red spot, were applied to the entire optically recorded data of 30 minutes in off-line manner due to computational time. Lee and Shinozuka (2006) developed a real-time displacement measurement system using digital image processing techniques, which is highly cost-effective and easy to implement, still maintains the advantages of dynamic measurements with high resolution (Lee and Shinozuka 2006). The performance of the proposed system was verified through a shaking table test and a field test on a bridge with open-box girders.

In this study, the vision-based system was employed for structural health monitoring application. For bridges, load carrying capacity is one of the widely used indices for bridge rating and maintenance. Load carrying capacity can be evaluated using the displacement data from the truck loading test with known axle weights. In the case of ordinary bridges, not the long-span bridges, displacement by the loading truck is very small in millimeter-order, so the accurate measurement of displacement with high resolution during the loading test is absolutely required. To show the applicability of the vision-based system to the evaluation of load carrying capacity, the truck loading tests were carried out on a steel-plate girder bridge. Conventional and vision-based displacement sensors were installed and the load carrying capacities of the bridge obtained from both systems were compared.

The existing vision-based methods cannot measure multiple displacements simultaneously. For a laboratory test on a small specimen, it is possible to measure the whole displacement field by capturing the shape of test specimen in a scene. However, it is very difficult to do so in large civil infrastructures especially using the commercial digital video camcorder, since the accuracy of measured displacement depends on the number of pixels of the charged coupled device (CCD) image sensor. The CCD image sensor in a commercial digital camcorder provides the resolution of several hundreds pixels in height and width. To accurately measure the dynamic displacement at a measured point using a commercial digital camcorder, the full range of a captured frame with several hundreds of pixels should be utilized to trace the dynamic displacement of a specific location. Therefore, in this study, to measure dynamic displacements at multiple points of a structure without losing the benefit of using commercial digital camcorders, the synchronized vision-based system was developed, which is highly cost-effective and easy to implement, but still maintains the advantage of real-time measurement of dynamic displacements at multiple locations with a high level resolution. By employing several camcorders and laptops with wireless communication, each target can be captured individually in a scene of a camcorder, and the *real-time* multiple measurements can be possible with time synchronization. Basic technical backgrounds and verification examples of the proposed synchronized vision-based system for dynamic displacement measurements are described in the later section.

2. Real time displacement measurement using image processing techniques

A displacement measurement system using digital image processing techniques is composed of (1) hardware including a target object, a telescopic lens, a digital camcorder, a IEEE1394 port, and a laptop computer and (2) software including continuous image capturing, a target recognition algorithm, calculation of a trigonometric transformation matrix from pre-captured images, and the actual displacement calculation from on-line image data.

In this study, we deployed a commercial digital camcorder with 30x optical zooming capability, a resolution of 720 by 480 pixels, and frame rate of 30 frames a second. A telescopic lens with 8x optical zooming capability was installed on the camcorder to trace the target more far away from the camcorder. A laptop computer with Pentium M 1.6 GHz processor and 512 MB RAM was used to process the data. The total cost of this system is less than \$2000 including a laptop computer, which is very economical while pursuing real time displacement measurement of a flexible structure.

A target designed as in Fig. 1 has four white spots with known geometry and black background. The horizontal (L_x) and vertical length (L_y) are predetermined considering the expected maximum displacement to be measured and the performance of hardware including a digital video camcorder and a telescopic lens. To recognize the white spots on the target, a threshold for the black and white image is calculated



Fig. 1 Target design and basic calculations

based on the brightness of background and target region as

$$\theta = median[\mu_B + 3\sigma_B, \mu_T - 3\sigma_T]$$
⁽¹⁾

where, μ_B and σ_B are average and standard deviation of brightness in background region, and μ_T and σ_T are average and standard deviation of brightness in target region. The centroids of four white spots can be located from the black and white image. Then the direction vectors $([x_1 y_1]^T, [x_2 y_2]^T)$ corresponding to horizontal and vertical direction are decided. The target should be properly installed at a measurement location before measurement; the direction vectors decided by four white spots should orient horizontal and vertical directions of interests. The trigonometric transformation matrix (**T**) and scaling factors (SF_x, SF_y) are calculated as

$$\mathbf{T} = \begin{bmatrix} X_1 & X_2 \\ Y_1 & Y_2 \end{bmatrix}^{-1}, \quad SF_x = L_x / \sqrt{x_1^2 + y_1^2}, \quad SF_y = L_y / \sqrt{x_2^2 + y_2^2}$$
(2)

where, $[X_1 Y_1]^T = [x_1 x_1]^T / \sqrt{x_1^2 + y_1^2}$ and $[X_2 Y_2]^T = [x_2 x_2]^T / \sqrt{x_2^2 + y_2^2}$. Actual displacement $([d_x d_y]^T)$ is calculated using the number of pixels of target movement $([x y]^T)$, which is based on the movement of the center of four centroids, and transformation matrix and scaling factors as

$$\begin{bmatrix} d_x \ d_y \end{bmatrix}^T = \begin{bmatrix} SF_x \ 0 \\ 0 \ SF_y \end{bmatrix} \mathbf{T} \begin{bmatrix} x \ y \end{bmatrix}^T$$
(3)

For the purpose of real time measurement and visualization, image processing should be carried out within 1/30 second, since most commercial video camcorders support the frame rate of 30 frames a second. While capturing image frames, the displacement of the target is calculated using image processing techniques, which include target recognition, calculation of the number of pixel movement, calculation of the actual displacement using transformation matrix and scaling factors, and display and storage of the calculated displacement. The quantity of information to be processed in real-time manner depends on the number of pixels per frame and the number of frames per second. Region of interest (ROI) for target recognition does not need to cover the whole image frame. And it is needed to trace only white spot regions to reduce the quantity of information to be processed in real-time manner. In this study, four white spot regions were selected as shown in Fig. 1. Auto-tracking algorithm for selecting moving ROI in consecutive frames was also incorporated in the proposed system.

Basically, the proposed vision-based system can capture in-plane motion of the target - two

displacements and one rotation. The in-plane displacements can be calculated by equation (3), and the rotation can also be calculated by the rotational change of the direction vectors. In-plane rotation of the target does not affect the location of center of four centroids, even though it makes each centroid rotated. Since this study focuses to measure displacement, rotation of the target is out of the scope of this study. Moreover, it is required to deploy a high-precision camera for measuring rotation, since the magnitude of rotation is relatively small compared with displacement. As imaging devices are being evolved so fast, the rotation information will be affordable soon.

3. Application to evaluation of load carrying capacity of bridges

The load carrying capacity defined by Eq. (4) is a commonly-used index for assessing the integrity of bridges (Ministry of Construction and Transportation 2005, 1996).

$$P = P_r \times RF \times K_{\delta}(or \ K_{\varepsilon}) \times K_i \times K_t \times K_r \tag{4}$$

where P_r is design live load; RF is rating factor determined by structural analysis using the FE model of a bridge; K_t and K_r are correction factors for traffic volume and pavement roughness, which are empirically estimated by structural engineers; and K_i is impact correction factor determined by vehicle running tests or given by design value. Deflection correction factor (K_δ) is generally estimated using the displacement from static or quasi-static loading tests using a loading truck with known axle loads and the displacement obtained from the FE model by simulating the same truck loading as

$$K_{\delta} = \frac{\delta_{calculated}^{initial FEM}}{\delta_{measured}}$$
(5)

where $\delta_{calculated}^{initial FEM}$ is a static deflection obtained from the FE model of a bridge, and $\delta_{measured}$ is a measured static deflection from the static or quasi-static loading test.

For measuring deflection, a reference point to which plummeted from measured point at the bridge member is essential, but there may be a ditch, valley, river, or a road with passing traffic under the bridge, which makes the measurement of deflection difficult. Therefore, the vision-based system, which does not require the reference point, can be a good alternative for the evaluation of load carrying capacity of bridges. The only requirement is that the camera is perfectly stationary.

The proposed vision-based system was applied to the evaluation of load carrying capacity of a steelplate girder bridge. The target bridge is a 40 m single-span bridge with five steel girders conjointly with a concrete slab. A quasi-static loading test with a speed of 3 km/h and vehicle running test with a speed of 50 km/h were performed using trucks with the loads of 15, 30 and 40 tons. The vibrations are measured at the center of the span by three different sensors including a contact-type displacement transducer based on wire connection with the sampling rate of 1000 Hz, a laser vibrometer with 100 Hz sampling rate, and a digital camcorder with 30 frames a second. Fig. 2 shows the experimental setups for the loading test and Fig. 3 shows the comparison of responses measured by 3 types of sensors when different weights of trucks were running. The camera with a telescopic lens was placed at 20 m apart from the target, and captured an image of 3 cm-height with 480 pixels in vertical direction. Therefore, the approximate accuracy was found to be 0.0625 mm/pixel. Although the mechanical error is relatively small, e.g., 0.06 mm in 20 m distance, seismic disturbance can cause significant errors in actual applications. In this study, the error



Fig. 3 Example of test results on a target bridge

range was 0.1 mm including such effects. In the comparison, the displacements measured by vision-based system are found to be close to those measured by the laser vibrometer with low noise level.

The compared deflections and deflection correction factors are shown in Table 1. Using the measured and calculated deflection of the target bridge, deflection correction factors were estimated by Eq. (5). Measured deflections by the vision-based system were found to be very close with those by the laser vibrometer. Accordingly, estimated deflection correction factors by the vision-based system also showed good agreement with those by the laser vibrometer with less than 3% error. The load carrying capacities of the target bridge were evaluated as Table 2 using the estimated deflection correction factors based on the given design live load ($P_r = 432 \text{ kN}$), the rating factor of the target bridge (RF = 1.40), and other assumed factors (K's = 1.0). For the conservative evaluation, the smallest value was used as the representative value of the deflection correction factors obtained from various truck loading tests. The load carrying capacity evaluated from the vision-based system was almost same as that from the laser vibrometer with less than 1% error, which proved the performance of the vision-

Sensor Type	$\delta_{\it calculated}^{\it initial\ FEM}$	Deflection(mm)			Deflection correction factor (K_{δ})		
		15 ton	30 ton	40 ton	15 ton	30 ton	40 ton
Laser	1.038 (15 ton)	0.973	1.800	2.608	1.067	1.133	1.059
Vision	2.039 (30 ton) 2.762 (40 ton)	0.976	1.854	2.573	$1.064 \\ (0.3\%)^{*}$	1.100 (-2.9%)	1.073 (1.3%)

Table 1 Comparison of estimated deflection correction factors (K_{δ}) obtained from the quasi-static loading tests(mm)

*Errors compared with laser vibrometer are shown in the parentheses.

Table 2 Result of load carrying capacity evaluation (P)

е	P_r	RF	K_{δ}	Other K's	Р
Laser Vibrometer	432 kN	1.40	1.059	1.0	640 kN
Vision	432 kN	1.40	1.064	1.0	644 kN (0.625%)

*Error compared with laser vibrometer is shown in the parenthesis.

based system for the evaluation of load carrying capacity of a bridge. From this result, it can be concluded that performance evaluation of a bridge with harsh surroundings can be carried out easily using the proposed system.

4. Multi-points measurement using a synchronized vision-based system

A vision-based displacement measurement system is composed of a target object, a telescopic lens, a digital camcorder, a IEEE 1394 port, and a laptop computer. This system is efficient to measure the displacement of a flexible structure, but it is restricted to single point measurement. In this study, to measure the displacements of multiple locations simultaneously, the synchronized vision-based system is developed. This system can be also applied to measure three-dimensional motion of a structure using two current in-plane measurement systems, which are timely synchronized, and geometrically correlated.

Fig. 4 shows the schematics of the proposed vision-based system for measuring multiple points of a structure. Each slave system has individual setups consisting of a camcorder with a telescopic lens and a laptop computer with a wireless network card. At first, each camcorder with a telescopic device connected to a slave computer takes a motion picture of the target marked. Meanwhile, the displacement of the target is calculated locally using image processing techniques, which were already installed and



Fig. 4 Vision-based multi-points measurement system



Fig. 5 Display windows of vision-based multi-points measurement system

have been independently running at each slave computer. The calculated displacement is transmitted to master computer periodically, for example, every 5 seconds, using wireless data communication. Display and storage of the calculated displacement is carried out in master computer when it receives the calculated displacement from each slave system. Display windows in master and two slave computers, for example, are shown in Fig. 5.

For simultaneous measurement and monitoring of displacements at multiple locations of a structure, time synchronization is the core issue to be resolved. In this study, the system clock of each slave computer is synchronized with the system clock of master computer by wireless networking before running measurement. First, master computer sends its local time information to each slave computer. Then, the slave computer sets its clock to the master computer, and sends its time information back to master computer. Time delay, which is the time required for wireless data communication between master and slave computers, is calculated at the mater computer. Finally, each time clock at slave computer is corrected by the averaged time delay after several communication steps. Time synchronization using networking is found to be affordable for displacement measurement of a flexible structure, since millisecond accuracy in time synchronization, in general, can be achieved over networking.

5. Verification through laboratory tests

380

To verify the performance of the proposed method, a laboratory test was performed using a shaking table test at University of California, Irvine. Fig. 6 shows the test setups and the photos taken by a camcorder with a telescopic lens. Two camcorders are installed 15m away from the target aiming at the same target. The displacement of a target under the excitation frequencies of 1 Hz and 2 Hz was measured, and the measured displacement by image processing techniques was compared with the data from a contact-type sensor, a linear variable differential transformer (LVDT). The proposed method gave close results to a conventional sensor as shown in Fig. 7 showing with less than 3% error in maximum values.



Fig. 8 A shear building model

As an application example, experimental modal analysis was carried out using simultaneously measured displacement data. First, free vibrations at 2^{nd} and 4^{th} floor were measured by LVDTs and the proposed vision-based system, respectively. Fig. 8 shows the test setups. Each camcorder installed at



Fig. 10 Modal parameter identification using frequency domain decomposition technique

15 m away from the target (as shown in Fig. 6) aims at each target. Fig. 9 shows the measured displacement by both systems. Then, modal parameters were identified using the frequency domain decomposition (FDD) technique (Otte, *et al.* 1990, Brincker, *et al.* 2000). FDD technique was originally used to extract the operational deflection shapes in mechanical vibrating systems. It utilizes the singular value decomposition for the spectral matrix of output responses, as shown in Eq. (6). The natural frequencies are evaluated from singular value (SV) functions of the spectral matrix in the FDD method.

$$\mathbf{S}_{vv}(\omega) = \mathbf{U}(\omega)\mathbf{s}(\omega)\mathbf{V}(\omega) \tag{6}$$

where $\mathbf{S}_{yy}(\omega) \in \mathbf{R}^{Nm \times Nm}$ is the spectral matrix for output responses $y(t) \in \mathbf{R}^{Nm}$, $\mathbf{s}(\omega) \in \mathbf{R}^{Nm \times Nm}$ is a diagonal matrix containing the singular values of its spectral matrix, and $\mathbf{U}(\omega)$, $\mathbf{V}(\omega) \in \mathbf{R}^{Nm \times Nm}$ are corresponding unitary matrices. *Nm* is the number of measuring points. The general multi-DOF system can be transformed to the single DOF system close to its natural frequencies by means of singular value decomposition. Then the mode shape can be estimated as the first column vector of the unitary matrix of $\mathbf{U}(\omega)$, since the first singular value may include the structural mode close to its natural frequencies. However, in the

382

closely spaced modes, the peak of the largest singular values at one natural frequency indicates the structural mode, and an adjacent second singular value may indicate the close mode.

Fig. 10 shows the results of modal parameter identification using FDD method. The identified 1st natural frequency is 2.31 Hz, which is identically estimated from LVDTs and vision-based system, and the corresponding mode shapes have shown less than 1% error as shown in Fig. 10b.

6. Concluding remarks

In this study, a vision-based system for dynamic displacement measurement was introduced. As a SHM application of the present system, loading tests were carried out on a steel-plate girder bridge with conventional and vision-based displacement measurement systems. First, the displacement data from a contact-type displacement transducer, a laser vibrometer, and the vision-based system excited by loading trucks were compared, and the data from the vision-based system showed very good agreement with those from the laser vibrometer with low-level noise. Therefore, the load carrying capacity evaluated by the vision-based system showed very good agreement (less than 1% error) with that by the laser vibrometer, which is a good application example of the vision-based system for the structural health monitoring of the civil infrastructure. For dynamic displacement measurements at multiple locations, a synchronized vision-based dynamic measurement system was developed. The effectiveness and applicability of the synchronized system were verified through laboratory tests; shaking table tests and free vibration tests. In the 1st laboratory test, the measured displacement by the synchronized visionbased system was compared with the data from a contact-type sensor, a linear variable differential transformer (LVDT), and showed close results to conventional sensors with less than 3% error in maximum values. In the free vibration tests on a 4-story shear building model, the 1st natural frequency is identically estimated from LVDTs and the synchronized vision-based system, and the corresponding mode shapes have shown less than 1% error.

The innovative features of the proposed synchronized vision-based system can be summarized as follows.

- Dynamic measurement with high resolution and cost-effectiveness
- Multi-points measurement
- Real-time measurement and visualization
- Easy installation and easy operation

Hence, it is expected that this technology will provide the sensor and structural health monitoring community with a powerful addition of an advanced sensor to the current list. Note that the current vision-based system cannot measure the out-plane motion. However, three-dimensional motion of a structure can be measured using two in-plane measurement systems, which are timely synchronized, and geometrically correlated. This can be done by incorporating the proposed synchronized vision-based system. Two perpendicular targets are attached to the measurement point, and each sub-system with a camera aims at each target. Three-dimensional motion can be visualized in the master computer by combining two measured in-plane motions.

Acknowledgements

This work was supported by the Korea Research Foundation Grand funded by the Korean Government (MOEHRD)(KRF-2005-214-D00376) and supported by Smart Infra-Structure Technology Center (SISTeC)

at KAIST, which is sponsored by Ministry of Science and Technology (MOST) and Korea Science and Engineering Foundation (KOSEF). Their supports are immensely appreciated.

References

- Brincker, R., Zhang, L. and Andersen, P. (2000), "Modal identification from ambient response using frequency domain decomposition", *Proceedings of 18th International Modal Analysis Conference*, 625-630, San Antonio, TX, USA.
- Celibi, M. (2000), "GPS in dynamic monitoring of long-period structures", Soil Dyn. Earthq. Eng., 20, 477-483.
- Faulkner, B. C., Furman, W. B., Thomas, T. B. and Wallace, T. M. (1996), "Determination of bridge response using acceleration data", Report No. FHWA/VA-97-R5 1996; Virginia Department of Transportation.
- Knecht, A. and Manetti, L. (2001), "Using GPS in structural health monitoring", Proc. SPIE's 8th Annu. Symp. on Smart Structures and Materials, Newport Beach, CA.
- Lee, J. J. and Shinozuka, M. (2006), "A vision-based system for remote sensing of bridge", NDT&E Inter., 39(5), 425-431.
- Ministry of Construction and Transportation (1996), A guide to Safety Inspection and Detailed Safety Assessment, Seoul, Korea, 1996.
- Ministry of Construction and Transportation (2005), Standard Design Specification of Highway Bridges, Seoul, Korea, 2005.
- Nakamura, S. (2000), "GPS measurement of wind-induced suspension bridge girder displacements", J. Struct. Eng., 126, 1413-1419.
- Nassif, H. H., Gindy, M., and Davis, J. (2005), "Comparison of laser Doppler vibrometer with contact sensors for monitoring bridge deflection and vibration", *NDT&E Inter.*, **38**, 213-218.
- Olaszek, P. (1999), "Investigation of the dynamic characteristic of bridge structures using a computer vision method", *Measurement*, **25**, 227-236.
- Otte, D., Van de Ponseele, P. and Leuridan, J. (1990), "Operational shapes estimation as a function of dynamic loads", *Proceedings of the 8th International Modal Analysis Conference*, 413-421.
- Park, K. T., Kim, S. H., Park, H. S. and Lee, K. W. (2005), "The determination of bridge displacement using measured acceleration", *Eng. Struct.*, 27, 371-378.
- Roberts, G. W. and Dodson, A. H. (1998), "A remote bridge health monitoring system using computational simulation and GPS sensor data", *Seismol. Soc. Am.* 25, 299-308.
- Stephen, G. A., Brownjohn, J. M. W. and Taylor, C. A. (1993), "Visual monitoring of the humber bridge", *Eng. Struct.*, **15**, 197-208.
- Wahbeh, A. M., Caffrey, J. P. and Masri, S. F. (2003), "A vision-based approach for the direct measurement of displacements in vibrating systems", *Smart Mater. Struct.* 12, 785-794.
- Xu, L., Guo, J. J. and Jiang, J. J. (2002), "Time-frequency analysis of a suspension bridge based on GPS", J. Sound Vib., 254(1), 105-116.