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H_∞ filter design for offshore platforms via sampled-data measurements

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Abstract. This paper focuses on the H_∞ filter design problem for offshore steel jacket platforms. Its objective is to design a full-order state observer for offshore platforms in presence of unknown disturbances. To make the method more practical, it is assumed that the measured variables are available at discrete-time instants with time-varying sampling time intervals. By modelling the sampling intervals as a bounded time-varying delay, the estimation error system is expressed as a time-delay system. As a result, the addressed problem can be transformed to the problem of stability of dynamic error between the system and the state estimator. Then, based on the Lyapunov-Krasovskii Functional (LKF), a stability criterion is obtained in the form of Linear Matrix Inequalities (LMIs). According to the stability criterion, a sufficient condition on designing the state estimator gain is obtained. In the end, the proposed method is applied to an offshore platform to show its effectiveness.

Keywords: offshore platforms; H_∞ filter; sampled-data; Lyapunov–Krasovskii functional method; time-delay

1. Introduction

The most typical form of fixed offshore structures is the steel jacket platforms (Sakthivel *et al.* 2015). These flexible megastructure platforms play an important role for storage, transportation, drilling, oil and gas extraction in the ocean (Wilson 2003). Being in the hostile environment of the sea can cause various disturbances, including wave (Zhang *et al*. 2016), ice (Wang *et al*. 2013), wind (Raheem 2014), and earthquake (Park *et al.* 2011). These external loads cause structural vibration and oscillation which make an uncomfortable environment for whose working on them. Therefore, many research papers have been published and proposed various control approaches to attenuate these vibrations and increase the safety and the stiffness of the structure (Abdallah *et al.* 1993, Li *et al.* 2003, Zhang *et al.* 2011, Zhang and Tang 2013, Jang *et al.* 2014, Teng *et al.* 2014, Zhang *et al*. 2014, Sakthivel *et al*. 2015, Zhang *et al*. 2016). For this purpose, several passive and active control methods have been proposed (Zhang *et al.* 2015, Zhang *et al*. 2017).

Passive control methods introduce energy dissipation devices to attenuate the vibrations (Li and Hu 2002, Aly 2014). These methods can affect only a narrow structural frequency, while the ocean is so complicated and the sea wave frequency spectrum is a wide band (Yang 2014). Hence, active control methods attract more attention since they can improve control performances over a wide frequency range. Terro *et al.* (1999) designed a multi-loop feedback control for offshore steel jacket platforms, which include an inner loop an outer loop to regulating the linear part and overcome the self-excited hydrodynamic forces of the platform dynamics, respectively. Li *et al*. (2003) and Luo and Zhu (2006) introduced an H_2 active control and nonlinear stochastic optimal control, respectively, to attenuate the vibrations for offshore platforms subjected to wave loading. Also, Zhang *et al*. (2013), Zhang *et al.* (2014) used sliding mode H_∞ control for offshore steel jacket platforms which subjected to nonlinear self-excited wave force and external disturbance. Zhang *et al.* (2016) investigates an event-triggered H_∞ reliable control for these platforms under network environment. While, all the aforementioned results have considered delayed or non-delayed state feedback, the state vector is not usually available in practice. In this paper, an H_∞ filter is designed to estimate the states of the offshore steel jacket platforms in spite of unknown disturbances. State estimation for these platforms makes the previous published papers more practical.

In practice, it is difficult to measure the continuous information because of using digital hardware and communication technology (Chen *et al.* 2016). Due to this reason, discrete measurements usually are available for estimation and control (Sakthivel *et al.* 2015). In recent years, much attention is focused on sampled-data control and estimation due to its practical applications (Liu *et al.* 2011, Rakkiyappan *et al.* 2015, Ge and Han 2016). Therefore, it is assumed in this paper that output measurements are available at discrete-time instants. Time intervals between two consecutive samplings are not necessarily constant. In other words, the sampling time is assumed to be time-varying.

The organization of this paper is as follows. In Section 2, offshore steel jacket platform model is given. In Section 3, based on a LKF, some criteria are given to design an H_∞

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filter for state estimation for the offshore steel jacket platform. Section 4 provides the simulation results. Finally, section 5 concludes the paper.

Notations. Throughout this paper, R^n denotes the *n*-dimensional Euclidean space and $R^{n\times m}$ is the set of real $n\times m$ matrices. $\mathbf{P}>0$ means that $\mathbf{P}$ is a real positive definite and symmetric matrix. Identity matrix is denoted by $\mathbf{I}$ and $\mathbf{A}^T$ represents the transpose of the real matrix $\mathbf{A}$. Symmetric terms in a symmetric matrix are denoted by $*$.

2. Offshore steel jacket platform model and preliminaries

Dynamic equations of an offshore steel jacket platform with Active Mass Damper (AMD) mechanism (Fig. 1), where the first dominant vibration mode of the structure is considered, is as follows (Yang 2014, Sakthivel *et al*. 2015)

$$\begin{aligned} m_1\ddot{z}_1(t) &= -\left(m_1\omega_1^2+m_2\omega_2^2\right)z_1(t)+m_2\omega_2^2 z_2(t)-2\left(m_1\xi_1\omega_1+m_2\xi_2\omega_2\right)\dot{z}_1(t)\\ &\quad +2m_2\xi_2\omega_2\dot{z}_2(t)+f(t)-u(t),\\ m_2\ddot{z}_2(t) &= m_2\omega_2^2 z_1(t)+2m_2\xi_2\omega_2\dot{z}_1(t)-m_2\omega_2^2 z_2(t)-2m_2\xi_2\omega_2\dot{z}_2(t)+u(t), \end{aligned} \tag{1}$$

where $z_1(t)$ and $z_2(t)$ are displacement of the deck motion of the offshore platform and the AMD, respectively. The parameters m_i, ω_i, and ξ_i, ($i=1,2$), are the mass, natural frequency, and the damping ratio of the platform ($i=1$) and the AMD ($i=2$), respectively. The active control signal is represented by $u(t)$, and $f(t)\in L_2[0,\infty)$ is the external wave force acting on the offshore structure which belongs to limited energy signals.

Let define the state vector as

$$\mathbf{x}(t)=[x_1(t)\quad x_2(t)\quad x_3(t)\quad x_4(t)]^T=[z_1(t)\quad z_2(t)\quad \dot{z}_1(t)\quad \dot{z}_2(t)]^T$$

then the Eq. (1) can be written as

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}u(t)+\mathbf{D}f(t), \tag{2}$$

where

$$\mathbf{A}=\begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\left(\omega_1^2+\omega_2^2\dfrac{m_2}{m_1}\right) & \omega_2^2\dfrac{m_2}{m_1} & -2\left(\xi_1\omega_1+\xi_2\omega_2\dfrac{m_2}{m_1}\right) & 2\xi_2\omega_2\dfrac{m_2}{m_1}\\ \omega_2^2 & -\omega_2^2 & 2\xi_2\omega_2 & -2\xi_2\omega_2 \end{bmatrix},$$

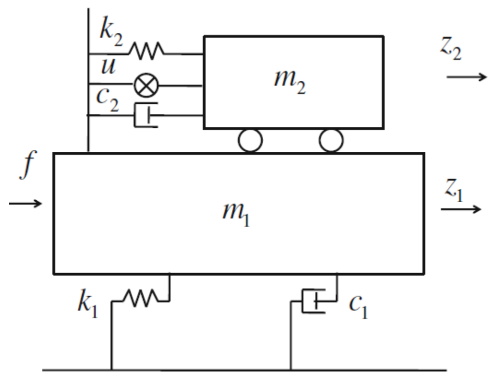


Fig. 1 A reduced model of offshore steel jacket platform with AMD mechanism (Yang 2014)

$$\mathbf{B}=\begin{bmatrix} 0\\ 0\\ -\dfrac{1}{m_1}\\ \dfrac{1}{m_2} \end{bmatrix},\ \mathbf{D}=\begin{bmatrix} 0\\ 0\\ \dfrac{1}{m_1}\\ 0 \end{bmatrix}.$$

The measured output vector is

$$\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t), \tag{3}$$

where the matrix $\mathbf{C}$ depends on the sensors types and their installations which will be discussed in the simulation section. It is assumed that $(\mathbf{A},\mathbf{C})$ is completely observable (Dorf and Bishop 1992). Based on this assumption, a full-order filter to estimate the states of the platform is considered as follows

$$\begin{cases} \dot{\hat{x}}(t)=\mathbf{A}\hat{x}(t)+\mathbf{B}u(t)+\mathbf{L}(y(t)-\hat{y}(t))\\ \hat{y}(t)=\mathbf{C}\hat{x}(t) \end{cases} \tag{4}$$

where $\hat{\mathbf{x}}(t)\in R^4$ and $\hat{\mathbf{y}}(t)\in R^2$ are the estimation of the state and output vectors, respectively. The matrix $\mathbf{L}\in R^{4\times 2}$ is the filter gain to be designed. It is assumed that the output vector $\mathbf{y}(t)$ is measurable and available at discrete-time instants $0=t_0<t_1<\cdots<t_k<\cdots$

$$\mathbf{y}(t)=\mathbf{y}(t_k),\quad t_k\le t<t_{k+1}, \tag{5}$$

where t_k is the updating instant time of the Zero-Order-Hold (ZOH) and satisfies $\lim_{k\to\infty}t_k=\infty$. Moreover, the sampling period is assumed to be bounded by a known constant $h>0$, that is, $t_{k+1}-t_k\le h$ for every integer $k\ge 0$.

Remark 1. In a practical point of view, it is difficult to measure the continuous information because of using digital hardware and communication technology. Hence, assuming sampled-data measurements is more practical. Moreover, the sampling periods considered here are time-varying and the only requirement is to be bounded.

By defining a saw-tooth function as follows

$$m(t)=t-t_k,\quad t_k\le t<t_{k+1},$$

it would be clear to see that

$$0\le m(t)<h$$

Therefore, the dynamic filter (4) can be written a

$$\begin{cases} \dot{\hat{x}}(t)=\mathbf{A}\hat{x}(t)+\mathbf{B}u(t)+\mathbf{LC}(x(t-m(t))-\hat{x}(t-m(t)))\\ \hat{y}(t)=\mathbf{C}\hat{x}(t) \end{cases} \tag{6}$$

Let $\mathbf{e}(t)=\mathbf{x}(t)-\hat{\mathbf{x}}(t)$ and $\bar{\mathbf{y}}(t)=\mathbf{y}(t)-\hat{\mathbf{y}}(t)$. Then the dynamic filter error can be given by

$$\begin{cases} \dot{\mathbf{e}}(t)=\mathbf{A}\mathbf{e}(t)-\mathbf{LC}\mathbf{e}(t-m(t))+\mathbf{D}f(t),\\ \bar{\mathbf{y}}(t)=\mathbf{C}\mathbf{e}(t). \end{cases} \tag{7}$$

In the following section, a sufficient condition is provided to guarantee that the filter error $\mathbf{e}(t)$ satisfies the following H_∞ performance index

$$\int_0^\infty \|\mathbf{e}(t)\|^2 dt \le \gamma^2 \int_0^\infty \|f(t)\|^2 dt, \tag{8}$$

for any non-zero $f(t) \in L_2[0,\infty)$, where $\|.\|$ denotes the Euclidean norm and $\gamma > 0$ is the H_∞ performance level.

Remark 2. The introduced filter (4) is the well-known Luenberger observer (Dorf and Bishop 1992). Here, the gain matrix $\mathbf{L}$ is designed in such a way that, in addition to considering the sampled-data measurements, the performance index (8) is also met.

The following lemma will be used in this paper.

Lemma 1. ((Jensen Integral Inequality). Assume that the vector function $\boldsymbol{\omega}:[0,r] \to R^n$ is well defined for the following integrations. For any symmetric matrix $\mathbf{R} \in R^{n\times n}$ and a scalar $r > 0$, one has

$$r\int_0^r \boldsymbol{\omega}^T(s)\mathbf{R}\boldsymbol{\omega}(s)ds \ge \left(\int_0^r \boldsymbol{\omega}(s)ds\right)^T \mathbf{R}\left(\int_0^r \boldsymbol{\omega}(s)ds\right)$$

Remark 3. Recently, some new integral inequalities have been proposed (Zhang and Han 2008, Seuret and Gouaisbaut 2013, Zhang and Han 2013, Zhang and Han 2017). However, Jensen integral inequality in Lemma 1 can provide a *simpler* form for the bound of the integral term involved. Thus, in this paper, the Jensen integral inequality is still used to bound some related integral terms.

3. Main results

In this section, the sampled-data H_∞ filter for the offshore steel jacket platform is considered. The following theorem provides a sufficient condition to guarantee that the filter error $\mathbf{e}(t)$ satisfies the H_∞ performance index (8) with a known filter gain $\mathbf{L}$.

Theorem 1. For given $\gamma > 0$, $h > 0$, and $\mathbf{L} \in R^{4\times 2}$, the filter error $\mathbf{e}(t)$ satisfies the H_∞ performance index (8), if there exist 4×4 positive definite matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{W}$, such that the following LMI holds

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Sigma}_1 & \boldsymbol{\Sigma}_2 & 0 & \mathbf{PD} & \mathbf{W} & h\mathbf{A}^T\mathbf{P} \\ * & -2\mathbf{P} & \mathbf{P} & 0 & 0 & -h\mathbf{C}^T\mathbf{L}^T\mathbf{P} \\ * & * & \boldsymbol{\Sigma}_3 & 0 & -\mathbf{W} & 0 \\ * & * & * & -\gamma^2\mathbf{I} & 0 & h\mathbf{D}^T\mathbf{P} \\ * & * & * & * & -\mathbf{R} & 0 \\ * & * & * & * & * & -\mathbf{P} \end{bmatrix} < 0, \tag{9}$$

where $\boldsymbol{\Sigma}_1 = \mathbf{PA} + \mathbf{A}^T\mathbf{P} + \mathbf{Q} + h^2\mathbf{R} - \mathbf{P} + \mathbf{I}$, $\boldsymbol{\Sigma}_2 = -\mathbf{PLC} + \mathbf{P}$, $\boldsymbol{\Sigma}_3 = -\mathbf{Q} - \mathbf{P}$.

Proof. Consider the following Lyapunov-Krasovskii functional for the error system (7)

$$V(t) = \sum_{i=1}^{5} V_i(t), \tag{10}$$

where

$$V_1(t) = \mathbf{e}^T(t)\mathbf{Pe}(t), \quad V_2(t) = \int_{t-h}^{t} \mathbf{e}^T(s)\mathbf{Qe}(s)ds,$$

$$V_3(t) = h\int_{-h}^{0}\int_{t+\theta}^{t} \mathbf{e}^T(s)\mathbf{Re}(s)dsd\theta,$$

$$V_4(t) = h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{\mathbf{e}}^T(s)\mathbf{P}\dot{\mathbf{e}}(s)dsd\theta,$$

$$V_5(t) = \left(\int_{t-h}^{t} \mathbf{e}^T(s)ds\right)\mathbf{W}\left(\int_{t-h}^{t} \mathbf{e}(s)ds\right).$$

Taking the derivative of $V_1(t)$ and $V_2(t)$ with respect to t yields

$$\dot{V}_1(t) = 2\mathbf{e}^T(t)\mathbf{P}\dot{\mathbf{e}}(t) = 2\mathbf{e}^T(t)\mathbf{P}(\mathbf{Ae}(t) - \mathbf{KCe}(t-m(t)) + \mathbf{D}f(t)), \tag{11}$$

$$\dot{V}_2(t) = \mathbf{e}^T(t)\mathbf{Qe}(t) - \mathbf{e}^T(t-h)\mathbf{Qe}(t-h). \tag{12}$$

The third term of (10) becomes

$$\dot{V}_3(t) = h^2\mathbf{e}^T(t)\mathbf{Re}(t) - h\int_{t-h}^{t} \mathbf{e}^T(s)\mathbf{Pe}(s)ds. \tag{13}$$

Using Lemma 1, (13) can be written as the following

$$\dot{V}_3(t) \le h^2\mathbf{e}^T(t)\mathbf{Re}(t) - \left(\int_{t-h}^{t} \mathbf{e}^T(s)ds\right)\mathbf{R}\left(\int_{t-h}^{t} \mathbf{e}(s)ds\right). \tag{14}$$

Taking the derivative of $V_4(t)$ with respect to t yields

$$\dot{V}_4(t) \le h^2\dot{\mathbf{e}}^T(t)\mathbf{P}\dot{\mathbf{e}}(t) - h\int_{t-h}^{t} \dot{\mathbf{e}}^T(s)\mathbf{P}\dot{\mathbf{e}}(s)ds, \tag{15}$$

which is equivalent to

$$\dot{V}_4(t) \le h^2\dot{\mathbf{e}}^T(t)\mathbf{P}\dot{\mathbf{e}}(t) - h\int_{t-h}^{t-m(t)} \dot{\mathbf{e}}^T(s)\mathbf{P}\dot{\mathbf{e}}(s)ds - h\int_{t-m(t)}^{t} \dot{\mathbf{e}}^T(s)\mathbf{P}\dot{\mathbf{e}}(s)ds. \tag{16}$$

Using Lemma 1, (16) can be written as the following

$$\begin{aligned} \dot{V}_4(t) &\le h^2\dot{\mathbf{e}}^T(t)\mathbf{P}\dot{\mathbf{e}}(t) - \left(\int_{t-h}^{t-m(t)} \dot{\mathbf{e}}^T(s)ds\right)\mathbf{P}\left(\int_{t-h}^{t-m(t)} \dot{\mathbf{e}}(s)ds\right) \\ &\quad - \left(\int_{t-m(t)}^{t} \dot{\mathbf{e}}^T(s)ds\right)\mathbf{P}\left(\int_{t-m(t)}^{t} \dot{\mathbf{e}}(s)ds\right) = h^2\dot{\mathbf{e}}^T(t)\mathbf{P}\dot{\mathbf{e}}(t) \\ &\quad - (\mathbf{e}^T(t-m(t)) - \mathbf{e}^T(t-h))\mathbf{P}(\mathbf{e}(t-m(t)) - \mathbf{e}(t-h)) \\ &\quad - (\mathbf{e}^T(t) - \mathbf{e}^T(t-m(t)))\mathbf{P}(\mathbf{e}^T(t) - \mathbf{e}^T(t-m(t))). \end{aligned} \tag{17}$$

Finally, the fifth term of (10) becomes

$$\dot{V}_5(t) = 2(\mathbf{e}^T(t) - \mathbf{e}^T(t-\tau))\mathbf{W}\left(\int_{t-h}^{t} \mathbf{e}(s)ds\right). \tag{18}$$

By setting

$$\xi(t) = \begin{bmatrix} \mathbf{e}^T(t) & \mathbf{e}^T(t-m(t)) & \mathbf{e}^T(t-h) & f^T(t) & \int_{t-h}^{t} \mathbf{e}^T(s)ds \end{bmatrix}^T, \tag{19}$$

and substituting $\dot{\mathbf{e}}(t)$ from (7) into (17), and considering (8), it can be obtained from (11)-(19) that

$$\dot{V}(t) + \mathbf{e}^T(t)\mathbf{e}(t) - \gamma^2 f^T(t)f(t) \le \zeta^T(t)\boldsymbol{\Xi}\zeta(t), \tag{20}$$

where

$$\mathbf{\Xi}=\begin{bmatrix}\mathbf{\Sigma}_1 & \mathbf{\Sigma}_2 & 0 & \mathbf{PD} & \mathbf{W}\\ * & -2\mathbf{P} & \mathbf{P} & 0 & 0\\ * & * & \mathbf{\Sigma}_3 & 0 & -\mathbf{W}\\ * & * & * & -\gamma^2\mathbf{I} & 0\\ * & * & * & * & -\mathbf{R}\end{bmatrix}+\mathbf{UP}^{-1}\mathbf{U}^T, \qquad (21)$$

and $\mathbf{\Sigma}_1$, $\mathbf{\Sigma}_2$, $\mathbf{\Sigma}_3$ are defined in (9), and $\mathbf{U}=\begin{bmatrix}h\mathbf{PA} & -h\mathbf{PKC} & 0 & h\mathbf{PD} & 0\end{bmatrix}^T$.

Using the Schur complement, (21) can be written as (9). If we can show that $\mathbf{\Xi}<0$, then from (20), one yields

$$\dot{V}(t)+\mathbf{e}^T(t)\mathbf{e}(t)-\gamma^2 f^T(t)f(t)<0 \qquad (22)$$

By the fact that $V(t)=0$ for zero initial condition ($\mathbf{x}(0)=0$), integrating both side of (22) from 0 to ∞ yields

$$\int_0^\infty\left[\mathbf{e}^T(t)\mathbf{e}(t)-\gamma^2 f^T(t)f(t)\right]dt\le 0 \qquad (23)$$

which indicates that the H_∞ performance (8) is guaranteed. This completes the proof.

In order to obtain the H_∞ filter gain $\mathbf{L}$, the following theorem can be applied.

Theorem 2. For given $\gamma>0$, $h>0$, the filter error $\mathbf{e}(t)$ satisfies the H_∞ performance index (8), if there exist 4×4 positive definite matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{W}$, and general matrix $\mathbf{Y}\in R^{4\times2}$, such that the following LMI holds

$$\mathbf{\Xi}=\begin{bmatrix}\mathbf{\Sigma}_1 & \bar{\mathbf{\Sigma}}_2 & 0 & \mathbf{PD} & \mathbf{W} & h\mathbf{A}^T\mathbf{P}\\ * & -2\mathbf{P} & \mathbf{P} & 0 & 0 & -h\mathbf{C}^T\mathbf{Y}^T\\ * & * & \mathbf{\Sigma}_3 & 0 & -\mathbf{W} & 0\\ * & * & * & -\gamma^2\mathbf{I} & 0 & h\mathbf{D}^T\mathbf{P}\\ * & * & * & * & -\mathbf{R} & 0\\ * & * & * & * & * & -\mathbf{P}\end{bmatrix}<0, \qquad (24)$$

where $\bar{\mathbf{\Sigma}}_2=-\mathbf{YC}+\mathbf{P}$, and $\mathbf{\Sigma}_1$, $\mathbf{\Sigma}_3$ are defined in (9). Moreover, the gain matrix $\mathbf{L}$ in (6) is given by $\mathbf{L}=\mathbf{P}^{-1}\mathbf{Y}$, if the LMI (24) is feasible.

Proof. The proof is straightforward by substituting $\mathbf{Y}=\mathbf{PL}$ in (9).

Remark 4. To obtain the gain matrix $\mathbf{L}$ from Theorem 2, for a given $h>0$, choose a relatively large value for the performance level γ, e.g., $\gamma=1$. If there is a feasible solution for (24) and correspondingly for $\mathbf{L}$, then decrease the selected amount of γ and do this to the value that there is a feasible solution. Otherwise, increase γ to obtain a feasible solution.

4. Simulation results

For evaluating the results, a typical offshore steel jacket platform is considered and the parameters are given (Sakthivel *et al.* 2015). To simulate the vibration and disturbances, an external wave force is generated and applied to the platform. Then, the H_∞ filter is designed and simulated to estimate the states of the platform.

The parameters of the offshore steel jacket platform are given in Table 1. Taking these parameters into account, the matrices in model (2) are obtained as

$$\mathbf{A}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -4.2290 & 0.0403 & -0.0899 & 0.0080\\ 4.0297 & -4.0297 & 0.8030 & -0.8030\end{bmatrix},$$

$$\mathbf{B}=10^{-4}\times\begin{bmatrix}0\\0\\-0.0013\\0.1278\end{bmatrix}, \quad \mathbf{D}=10^{-6}\times\begin{bmatrix}0\\0\\0.1278\\0\end{bmatrix}$$

The following two conditions are considered for the matrix $\mathbf{C}$:

Filter 1: Two sensors are installed on the platform to measure the displacement and velocity motions of the deck, i.e., the matrix $\mathbf{C}$ is equal to

$$\mathbf{C}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Filter 2: Velocity motion of the AMD is measurable, i.e. $\mathbf{C}=\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}$.

According to the above matrices, $(\mathbf{A},\mathbf{C})$ is completely observable for both two cases. Therefore, the system is fully observable.

A wave force is generated as external disturbance which is shown in Fig. 2. Considering the H_∞ performance level $\gamma=0.0001$ and $h=0.38$, using Theorem 2, the filter gains $\mathbf{L}_1$ and $\mathbf{L}_2$ are calculated as Filter 1 and 2, respectively

$$\mathbf{L}_1=\begin{bmatrix}0.8755 & 0.1263\\ 0.9404 & -0.4993\\ -0.5318 & 0.8628\\ 2.4220 & 0.5753\end{bmatrix}, \qquad \mathbf{L}_2=\begin{bmatrix}0.1429\\ 0.0645\\ 0.0251\\ 0.8961\end{bmatrix}$$

Table 1 Parameters of the offshore steel jacket platform (Kazemy 2017)

Symbol	Value	Unit
m_1	7,825,307	kg
m_2	78,253	kg
ω_1	2.0466	rad/s
ω_2	2.0074	rad/s
ξ_1	0.02	-
ξ_2	0.2	-

Table 2 Performance of both two filters

Filter Type	M_{e1}	M_{e2}	M_{e3}	M_{e4}	J_{e1}	J_{e2}	J_{e3}	J_{e4}
Filter 1	0.054	0.049	0.088	0.079	0.0246	0.0370	0.0496	0.0798
Filter 2	0.114	0.143	0.181	0.223	0.0580	0.0675	0.1161	0.1336

Sampling time instants are generated which shown in Fig. 3. Time intervals between sampling instants are time-varying and bounded by $h = 0.38$. For initial state vector $\mathbf{x}(0) = [0.2, -0.2, -0.2, 0.2]^T$, Fig. 4 shows the states of the offshore platform and the estimated values by the both H_∞ filters, Filter 1 and 2. It is obvious that the estimated states converge to real states rapidly. The state estimation errors for Filter 1 and 2 are plotted in Figs. 5 and 6, respectively. Clearly, Filter 1 is much better than Filter 2, which suggests that the sensor installation on the platform is better than installation on the AMD. To investigate the filters performances, both the peak value and the root mean square (RMS) of estimation errors are defined as

$$M_{ei} = \max\left\{\left|e_i(t)\right|, t \in [T_0, T]\right\}, J_{ei} = \left(\frac{1}{T}\int_0^T e_i^2(t)dt\right)^{1/2}, i = 1, \ldots, 4$$

where $T = 40$ is the simulation time and $T_0 = 5$ has been chosen in such a way that the effects of the transient time are over. The quantitative performances for both filters are presented in Table 2, which shows that the performance of Filter 1 is about two times better than Filter 2. It is worth to mention that the estimation errors do not converge to zero until the external disturbance goes to zero.

The main purpose of state estimation is to use these estimated states for state-feedback controller. As mentioned before, several papers have studied and designed state-feedback controller for such a system. The final control scheme with considering the proposed filter is drawn in Fig. 7. In this figure, $u(t) = \mathbf{K}\hat{\mathbf{x}}(t)$ is the control signal and $\mathbf{K} \in R^4$ is the state feedback controller gain. Kazemy (Kazemy 2017) has proposed a design procedure to obtain the gain matrix $\mathbf{K}$. In order to complete the simulation, the gain matrix

$$\mathbf{K} = 10^7 \times [1.2781, 0.0147, 0.3269, -0.0087]$$

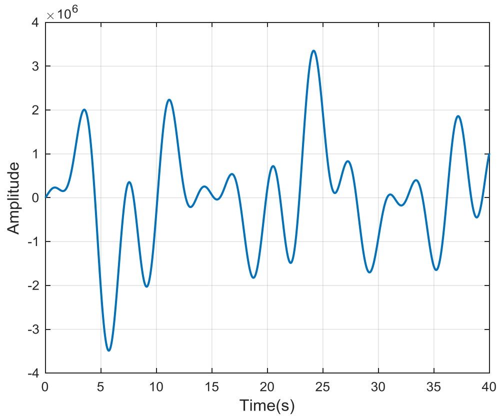


Fig. 2 Wave force applied to the offshore platform

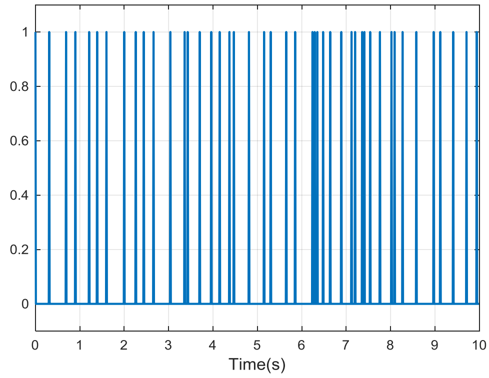


Fig. 3 Sampling time instants

Table 3 Performance of the controllers

Controller Type	M_{y1}	M_{y2}	J_{y1}	J_{y2}
No Control	0.2088	0.3539	0.1017	0.1971
Case I	0.0861	0.1089	0.0457	0.0814
Case II	0.1086	0.1502	0.0552	0.0986

is taken from (Kazemy 2017). Fig. 8 shows the displacement and velocity motions of the platform for the case of under control and without control. This figure implies that the controller decreases the displacement and velocity motions of the platform significantly. Note that the controller uses the estimated states by Filter 1, instead of real state variables. This will decrease the cost of construction and maintenance of the control hardware. If the controller uses the real states values directly, the performance of the controller will be improved which is expected. The Fig. 9 has been provided to compare the results for two situations:

Case I: when the controller uses real states, i.e., $u(t) = \mathbf{K}\mathbf{x}(t)$.

Case II: when the controller uses estimated states by Filter 1, i.e., $u(t) = \mathbf{K}\hat{\mathbf{x}}(t)$.

As Fig. 9 shows, the performance of the controller that uses measured states is slightly better, but there is no much difference between them. For numerical comparison of the results of controllers, the peak value and the root mean square (RMS) of measured outputs are defined as

$$M_{yi} = \max\left\{\left|y_i(t_k)\right|, t \in [T_0, T]\right\}, J_{yi} = \left(\frac{1}{T}\int_0^T y_i^2(t_k)dt\right)^{1/2}, i = 1, 2$$

where T and T_0 are defined before. Using these indexes, the efficiency of controllers is shown in the Table 3. Therefore, the performance of the controller which uses the estimated states is acceptable. This means that the proposed filter is applicable for implementation, in order to reduce the number of usage sensors.

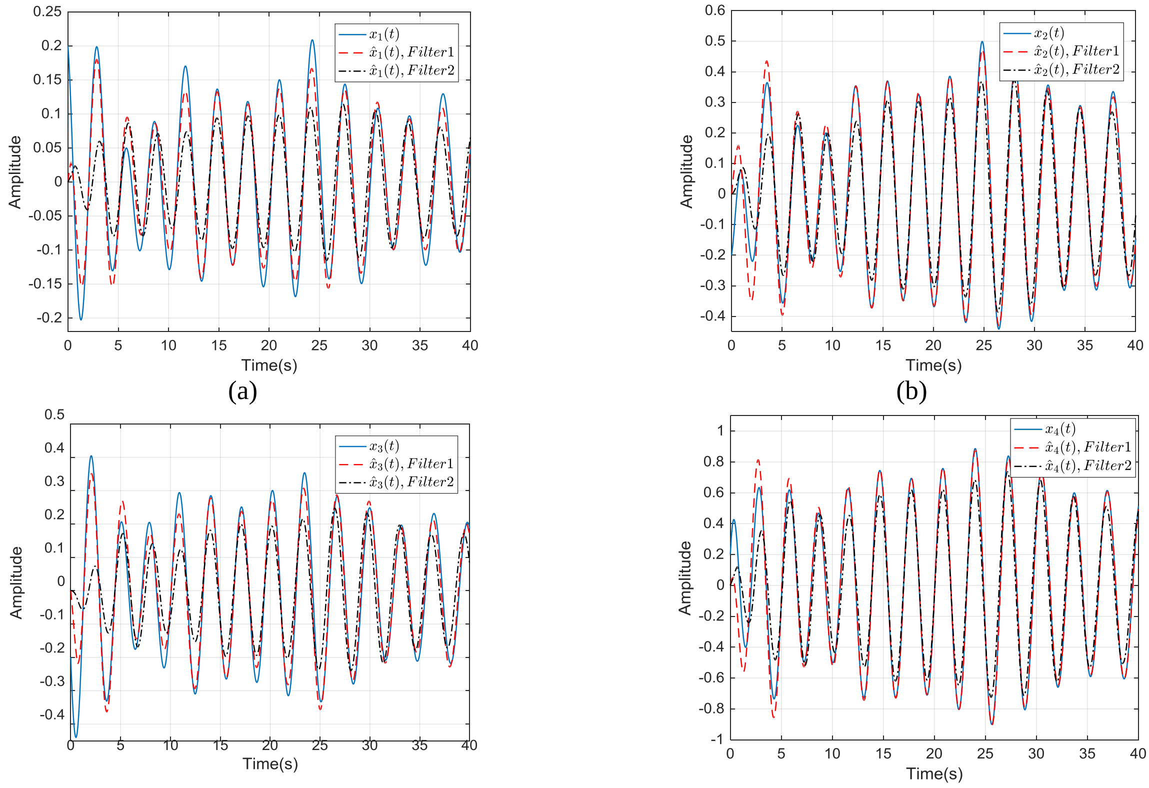


Fig. 4 The states of the offshore platform and the estimated values by the both H_∞ filters

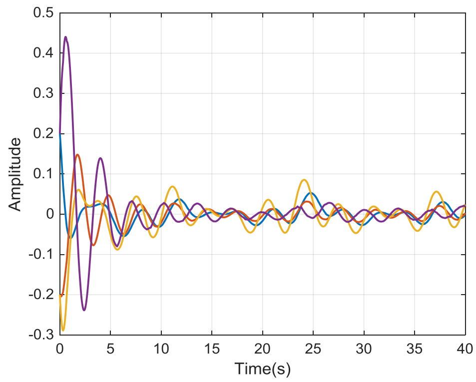


Fig. 5 The state estimation errors for Filter 1

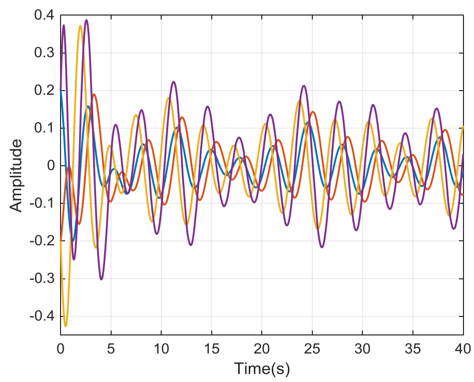


Fig. 6 The state estimation errors for Filter 2

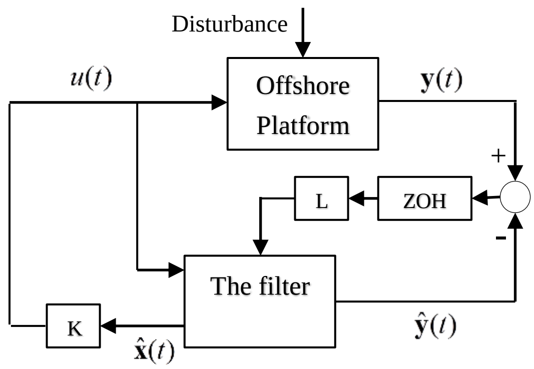
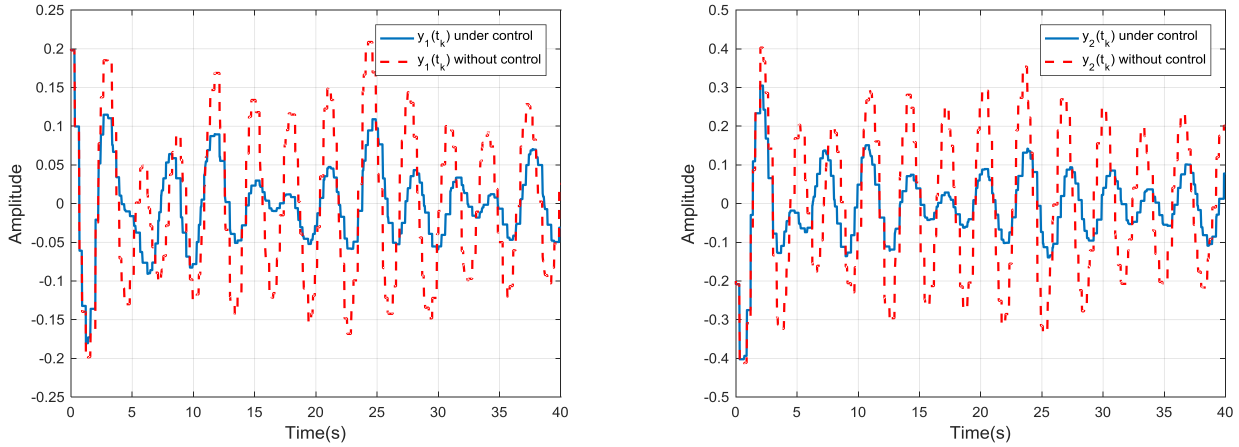


Fig. 7 The state feedback control scheme

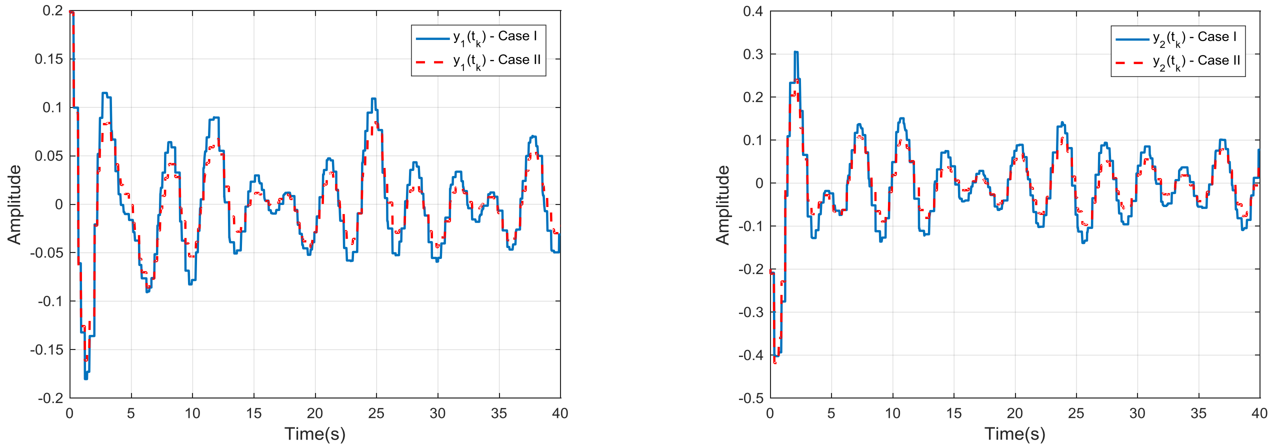
4. Conclusions

This paper has considered the problem of designing H_∞ filter for offshore steel jacket platforms. In order to make the method more practical, it is assumed that the measured variables are available at discrete time instants with time-varying sampling intervals. The sampling intervals are modelled as a bounded time-varying delay. A full-state estimator is considered and a state error dynamic has been derived. Based on the LKF method, a stability criterion has been obtained in the form of LMIs. According to the stability criterion, a sufficient condition for designing



(a) (b)

Fig. 8 The displacement, (a), and velocity, (b), of the platform for the case of under control and without control



(a) (b)

Fig. 9 The difference between using real states and estimated states on the displacement and velocity of the controlled platform

the state estimator gain has been obtained. In the end, the proposed method has been applied to a reduced mode offshore platform to show the effectiveness of the proposed results.

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