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# The buckling of piezoelectric plates on pasternak elastic foundation using higher-order shear deformation plate theories

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**Abstract.** In this article, an exact analytical solution for mechanical buckling analysis of magnetoelectroelastic plate resting on pasternak foundation is investigated based on the third-order shear deformation plate theory. The in-plane electric and magnetic fields can be ignored for plates. According to Maxwell equation and magnetoelectric boundary condition, the variation of electric and magnetic potentials along the thickness direction of the plate is determined. The von Karman model is exploited to capture the effect of nonlinearity. Navier's approach has been used to solve the governing equations for all edges simply supported boundary conditions. Numerical results reveal the effects of (i) lateral load, (ii) electric load, (iii) magnetic load and (iv) higher order shear deformation theory on the critical buckling load have been investigated. These results must be the analysis of intelligent structures constructed from magnetoelectroelastic materials.

**Keywords:** buckling; piezoelectric; plates; shear deformation theories

## 1. Introduction

The plate elements are used in civil, mechanical, aeronautical and marine structures. the researches on plates have received great attention, and a variety of plate theories has been introduced based on considering the transverse shear deformation effect. The classical plate theory (CPT), which neglects the transverse shear deformation effect, to overcome the limitation of CPT, many shear deformation plate theories which account for then transverse shear deformation effect have been developed. First order shear deformation theories (FSDTs) are based on the assumption that straight lines which are normal to neutral surface before deformation remain straight but not necessarily normal to the deformed neutral surface.

To overcome the limitations of classical plate theory and first order shear deformation theory, a many higher-order shear deformation plate theories which involve the higher-order terms in power series of the coordinate normal to the middle plane, have been proposed. An extensive review of laminated plate theories can be found in such as Levinson (1980), Bhimaraddi and Stevens (1984), Reddy (1984), Ren (1986), Kant and Pandya (1988) and Mohan et al. (1994). A good review of these models for the investigation of laminated plates is found in (Noor and Burton 1989a, b, Reddy 1990, 1993, Mallikarjuna and Kant 1993, Dahsin and Xiaoyu 1996). Reddy (1984) proposed a HSDT with cubic distributions for axial displacements. Based on Reddy's theory, Xiang *et al*. (2011) developed a n-order shear deformation theory. Mallikarjuna and Kant (1993) and Kant and Khare (1997) employed also HSDTs with cubic distributions for axial displacements as in the article by Reddy (1984). Recently, a new class of HSDTs is proposed by many researchers such as Shahrjerdi *et al*. (2011), Bouderba *et al*. (2013), Viswanathan *et al*. (2013), Ait Amar Meziane *et al*. (2014), Belabed *et al*. (2014), Ahmed (2014), Swaminathan and Naveenkumar (2014), Nedri *et al*. (2014), Hamidi *et al*. (2015), Kar *et al*. (2015), Hebali *et al*. (2014), Mahi *et al*. (2015), Saidi *et al*. (2016), Bennoun *et al*. (2016), Bourada *et al*. (2016) and Tounsi *et al*. (2016), Panda and Singh (2009, 2010, 2011, 2013), Panda and Katariya (2015), Katariya and Panda (2016).

In other hand, piezoelectric and piezomagnetic materials are a new class of smart materials which exhibit a coupling between mechanical, electric and magnetic fields and because of the ability of converting energy among these three energy forms, these materials have direct application in sensors and actuators, control of vibrations in structures, etc. Magnetoelectroelastic mechanics has absorbed much attention of researchers Avellaneda and Harshe (1994), Benveniste (1995), Li and Dunn (1998), Achenbach (2000), Wu and Huang (2000), Aboudi (2001), Priya *et al*. (2007).

In addition to the above, piezoelectric materials are one of the most common subgroups of smart materials currently being used in structures to control deformation, vibration, buckling, etc. Shen (2001) presented the thermal postbuckling of shear-deformable laminated plates with piezoelectric actuators under uniform temperature rise using a perturbation technique. Also, a theoretical framework for analyzing the buckling and postbuckling response of composite laminates and plates with piezoactuators and

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sensors has been presented by Varelis *et al.* (2004).

Recently, Panda and Singh (2010), investigated the buckling and post-buckling behaviours of a laminated composite spherical shallow shell panel embedded with shape memory alloy (SMA) fibres under a thermal environment. The nonlinear finite element analysis of thermal post-buckling vibration of laminated composite shell panel embedded with shape memory alloy fibre was presented by Panda and Singh (2013a). Panda and Singh (2013b,c) investigated the post-buckling and large amplitude free vibration analysis of laminated composite doubly curved panel embedded with shape memory alloy fibres subjected to thermal environment. Panda and Singh (2015a,b) presented the large amplitude free vibration behaviour of laminated composite spherical shell panel and doubly curved composite panels embedded with the piezoelectric layer using a numerical approach. Various analytical or numerical studies have been carried out for these PZT and Magnetostrictive materials which include studies on the large amplitude by Singh *et al.* (2016a, b), Dutta *et al.* (2017), Singh and Panda (2016), Suman *et al.* (2016).

To the best of authors' knowledge, however, the buckling problem of magnetoelectroelastic plate resting on a Pasternak foundation has not been considered. Hence, in this paper the buckling load of magnetoelectroelastic plates resting on elastic foundations are investigated by using a third-order shear deformation plate theory. According to Maxwell equations and magnetoelectric boundary conditions, the variation of electric and magnetic potentials along the thickness direction of the plate is determined.

## 2. Mathematical formulations

The strain displacement relations are (Reddy 2004)

$$\begin{aligned} \varepsilon_{xx} &= u_{,x} + \frac{1}{2} w_{,x}^2, \\ \varepsilon_{yy} &= v_{,y} + \frac{1}{2} w_{,y}^2, \\ \gamma_{xy} &= u_{,y} + v_{,x} + w_{,x} w_{,y}, \\ \gamma_{yz} &= v_{,z} + w_{,y}, \\ \gamma_{xz} &= u_{,z} + w_{,x}, \end{aligned} \tag{1}$$

where $\varepsilon_{xx}$ and $\varepsilon_{yy}$ are the normal strains and $\gamma_{xy}$, $\gamma_{yz}$, and $\gamma_{xz}$ are the shear strains. Here, $u$, $v$, and w are the plate displacements parallel to the coordinates ($x$,$y$,$z$), and a comma indicates the partial derivative.

In this research work, further simplifying supposition are considered to the third-order shear deformation plate theory, the displacement field is assumed to be (Reddy 2004)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) - c_1 z^3(\phi_x + w_{0,x}) \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) - c_1 z^3(\phi_y + w_{0,y}) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

where $u_0$, $v_0$, and $w_0$ represent the displacements on the mid-plane ($z$=0) of the plate, and $\phi_x$ and $\phi_y$ are the mid-plane rotations of transverse normal about the y and x axes, respectively. Here, $c_1 = \frac{4}{3h^2}$, where the traction- free boundary conditions on the top and bottom faces of the laminated plate are satisfied.

Substituting Eqs. (2) into the strain displacement relations (1) gives the kinematic relations as :

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + z^3 \begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(0)} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + z^2 \begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix} \tag{3}$$

where

$$\begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{(3)} \\ \varepsilon_{yy}^{(3)} \\ \gamma_{xy}^{(3)} \end{Bmatrix} = -c_1 \begin{Bmatrix} \phi_{x,x} + w_{0,xx} \\ \phi_{y,y} + w_{0,yy} \\ \phi_{x,y} + \phi_{y,x} + 2w_{0,xy} \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} = \begin{Bmatrix} \phi_y + w_{0,y} \\ \phi_x + w_{0,x} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz}^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix} = -3c_1 \begin{Bmatrix} \phi_y + w_{0,y} \\ \phi_x + w_{0,x} \end{Bmatrix}$$

For the transversely isotropic magnetoelectroelastic solid, the constitutive equations can be defined as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} - \begin{bmatrix} 0 & 0 & f_{31} \\ 0 & 0 & f_{31} \\ 0 & f_{24} & 0 \\ f_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} h_{11} & 0 & 0 \\ 0 & h_{22} & 0 \\ 0 & 0 & h_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} - \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \tag{6}$$

$$\begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & f_{15} & 0 \\ 0 & 0 & f_{24} & 0 & 0 \\ f_{31} & f_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} - \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \tag{7}$$

where $C_{ij}$, $e_{ij}$, $f_{ij}$ and $g_{ij}$ denote elastic, piezoelectric, piezomagnetic and magnetoelectric constant respectively; hij and μij are dielectric and magnetic permeability coefficients, respectively. $\sigma_{ij}$ is the stress component; $D_i$ and $B_i$ are the electric displacement and magnetic induction respectively. $E_i$ and $H_i$ are the electric and magnetic field intensities respectively.

The electric and magnetic intensities can be defined as gradients of the scalar electric and magnetic potentials $\psi$ and $\varphi$, respectively,

$$\begin{aligned} E &= -\nabla\psi \\ H &= -\nabla\varphi \end{aligned} \tag{8}$$

### *2.1 Governing equations*

The starin energy of the magnetoelectroelastic plate can be expressed as

$$U = \frac{1}{2}\int_\Omega \int_{-h/2}^{h/2} (\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{yz}\varepsilon_{yz} + \sigma_{xz}\varepsilon_{xz} + \sigma_{xy}\varepsilon_{xy} - D_xE_x - D_yE_y - D_zE_z - B_xH_x - B_yH_y - B_zH_z)\, d\Omega dz \tag{9}$$

Since the magnetoelectroelastic layer is thin, the in-plane electric and magnetic field can be ignored, i.e., $E_x$=$E_y$=0 and $H_x$=$H_y$=0. Substituting Eqs. \ (1)- (7) to Eq. (9).

The stress resultants are related to the stresses by the equations

$$\begin{aligned} \begin{Bmatrix} N_{\alpha\beta} \\ M_{\alpha\beta} \\ P_{\alpha\beta} \end{Bmatrix} &= \int_{-h/2}^{h/2} \sigma_{\alpha\beta} \begin{Bmatrix} 1 \\ z \\ z^3 \end{Bmatrix} dz \\ \begin{Bmatrix} Q_\alpha \\ R_\alpha \end{Bmatrix} &= \int_{-h/2}^{h/2} \sigma_{\alpha\beta} \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz \end{aligned} \tag{10}$$

where $\alpha$ and $\beta$ take the symbols x and y.

The external virtual work due to surrounding elastic medium can be written as

$$\delta W = \int_\Omega q\delta w_0 d\Omega \tag{11}$$

where $q$ is related to the Pasternak foundation and transverse load, which can be expressed as

$$q = k_w w_0 - k_g\nabla^2 w_0 + (N_{xm} + N_{xe} + N_{xa})\frac{\partial^2 w_0}{\partial x^2} + (N_{ym} + N_{ye} + N_{ya})\frac{\partial^2 w_0}{\partial y^2} \tag{12}$$

The principle can be expressed in analytical form as

$$\delta U - \delta W = 0 \tag{13}$$

when substituting Eqs. (9)- (12) into (13), integrating by parts the governing equations as

$$\mathrm{N_{xx,x}} + \mathrm{N_{xy,y}} = 0 \tag{14}$$

$$\mathrm{N_{yy,y}} + \mathrm{N_{xy,x}} = 0 \tag{15}$$

$$\mathrm{Q_{x,x}} + \mathrm{Q_{y,y}} - 3c_1(\mathrm{R_{x,x}} + \mathrm{R_{y,y}}) + c_1(\mathrm{P_{xx,xx}} + 2\mathrm{P_{xy,xy}} + \mathrm{P_{yy,yy}}) + k_w w_0 - k_g\nabla^2 w_0 + (\mathrm{N_{xm}} + \mathrm{N_{xe}} + \mathrm{N_{xa}})\frac{\partial^2 w_0}{\partial x^2} + (\mathrm{N_{ym}} + \mathrm{N_{ye}} + \mathrm{N_{ya}})\frac{\partial^2 w_0}{\partial y^2} = 0 \tag{16}$$

$$\mathrm{M_{xx,x}} + \mathrm{M_{xy,y}} - \mathrm{Q_x} + 3c_1\mathrm{R_x} - c_1(\mathrm{P_{xx,x}} + \mathrm{P_{xy,y}}) = 0 \tag{17}$$

$$\mathrm{M_{xy,x}} + \mathrm{M_{yy,y}} - \mathrm{Q_y} + 3c_1\mathrm{R_y} - c_1(\mathrm{P_{xy,x}} + \mathrm{P_{yy,y}}) = 0 \tag{18}$$

$$\begin{aligned} \mathrm{D_{z,z}} &= 0 \\ \mathrm{B_{z,z}} &= 0 \end{aligned} \tag{19}$$

$$\begin{aligned} \mathrm{N_{xm}} &= \mathrm{P} \\ \mathrm{N_{xe}} &= e_{31}V_0 \\ \mathrm{N_{xa}} &= f_{31}\Omega_0 \\ \mathrm{N_{ym}} &= \lambda\mathrm{P} \\ \mathrm{N_{ye}} &= e_{31}V_0 \\ \mathrm{N_{ya}} &= f_{31}\Omega_0 \end{aligned} \tag{20}$$

In wich $P$ is the mechanical load along x direction and $\lambda$ is the lateral load parameter.

Substituting Eqs. (1) into Eq. (19), the following two equations can be derived

$$e_{31}[\phi_{x,x} - 3c_1z^2(\phi_{x,x} + w_{0,xx})] + e_{31}[\phi_{y,y} - 3c_1z^2(\phi_{y,y} + w_{0,yy})] - h_{33}\frac{\partial^2\psi}{\partial z^2} - g_{33}\frac{\partial^2\varphi}{\partial z^2} = 0 \tag{21}$$

$$f_{31}[\phi_{x,x} - 3c_1z^2(\phi_{x,x} + w_{0,xx})] + f_{31}[\phi_{y,y} - 3c_1z^2(\phi_{y,y} + w_{0,yy})] - g_{33}\frac{\partial^2\psi}{\partial z^2} - \mu_{33}\frac{\partial^2\varphi}{\partial z^2} = 0 \tag{22}$$

Thus, by adopting Crammer's rule, one may have

$$\begin{aligned} \frac{\partial^2\psi}{\partial z^2} &= \frac{(\mu_{33}e_{31} - g_{33}f_{31})\Delta}{h_{33}\mu_{33} - g_{33}^2} \\ \frac{\partial^2\varphi}{\partial z^2} &= \frac{(h_{33}f_{31} - g_{33}e_{31})\Delta}{h_{33}\mu_{33} - g_{33}^2} \end{aligned} \tag{23}$$

where

$$\Delta = \left[\phi_{x,x} - 3c_1 z^2\left(\phi_{x,x} + w_{0,xx}\right)\right] + \left[\phi_{y,y} - 3c_1 z^2\left(\phi_{y,y} + w_{0,yy}\right)\right] \tag{24}$$

The second integral of Eq. (23) is given by

$$\psi = \frac{(\mu_{33}e_{31} - g_{33}f_{31})\Delta}{h_{33}\mu_{33} - g_{33}^2}\left(\frac{z^2}{2} - \frac{h^2}{8}\right) + \frac{V_0}{h}z + \frac{V_0}{2}$$
$$\varphi = \frac{(h_{33}f_{31} - g_{33}e_{31})\Delta}{h_{33}\mu_{33} - g_{33}^2}\left(\frac{z^2}{2} - \frac{h^2}{8}\right) + \frac{\Omega_0}{h}z + \frac{\Omega_0}{2} \tag{25}$$

where the electric and magnetic boundary conditions are prescribed as $\psi(h/2)$=V0 and $\psi(-h/2)$=0 and $\varphi(h/2) = \Omega_0$ and $\varphi(-h/2) = 0$.

$$N_{xx} = c_{11}u_{0,x}h + f_{31}\Omega_0 + e_{31}V_0 + c_{12}v_{0,y}h \tag{26}$$

$$N_{yy} = c_{12}u_{0,x}h + f_{31}\Omega_0 + e_{31}V_0 + c_{22}v_{0,y}h \tag{27}$$

$$N_{xy} = c_{66}\left(u_{0,y} + v_{0,x}\right)h \tag{28}$$

$$M_{xx} = -\frac{h^3}{20}\left[\left(\frac{c_{11}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{x,x} + w_{0,xx}\right) + \left(\frac{c_{12}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{y,y} + w_{0,yy}\right)\right] + \frac{h^3}{12}\left[(c_{11} + f_{31}M_2 + e_{31}M_1)\phi_{x,x} + (c_{12} + f_{31}M_2 + e_{31}M_1)\phi_{y,y}\right] \tag{29}$$

$$M_{yy} = -\frac{h^3}{20}\left[\left(\frac{c_{12}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{x,x} + w_{0,xx}\right) + \left(\frac{c_{22}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{y,y} + w_{0,yy}\right)\right] + \frac{h^3}{12}\left[(c_{12} + f_{31}M_2 + e_{31}M_1)\phi_{x,x} + (c_{22} + f_{31}M_2 + e_{31}M_1)\phi_{y,y}\right] \tag{30}$$

$$M_{xy} = -\frac{h^3}{60}c_{66}\left(\phi_{x,y} + \phi_{y,x} + 2w_{0,xy}\right) + \frac{h^3}{12}c_{66}\left(\phi_{x,y} + \phi_{y,x}\right) \tag{31}$$

$$P_{xx} = -\frac{h^5}{112}\left[\left(\frac{c_{11}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{x,x} + w_{0,xx}\right) + \left(\frac{c_{12}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{y,y} + w_{0,yy}\right)\right] + \frac{h^5}{80}\left[(c_{11} + f_{31}M_2 + e_{31}M_1)\phi_{x,x} + (c_{12} + f_{31}M_2 + e_{31}M_1)\phi_{y,y}\right] \tag{32}$$

$$P_{yy} = -\frac{h^5}{112}\left[\left(\frac{c_{12}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{x,x} + w_{0,xx}\right) + \left(\frac{c_{22}}{3} + e_{31}M_1 + f_{31}M_2\right)\left(\phi_{y,y} + w_{0,yy}\right)\right] + \frac{h^5}{80}\left[(c_{12} + f_{31}M_2 + e_{31}M_1)\phi_{x,x} + (c_{22} + f_{31}M_2 + e_{31}M_1)\phi_{y,y}\right] \tag{33}$$

$$P_{xy} = -\frac{h^5}{168}c_{66}w_{0,xy} + \frac{h^5}{105}c_{66}\left(\phi_{x,y} + \phi_{y,x}\right) \tag{34}$$

$$Q_x = \frac{2h}{3}c_{44}\left(\phi_x + w_{0,x}\right) \tag{35}$$

$$Q_y = \frac{2h}{3}c_{44}\left(\phi_y + w_{0,y}\right) \tag{36}$$

$$R_x = \frac{h^3}{3}c_{44}\left(\phi_x + w_{0,x}\right) \tag{37}$$

$$R_y = \frac{h^3}{3}c_{44}\left(\phi_y + w_{0,y}\right) \tag{38}$$

with

$$M_1 = \frac{(\mu_{33}e_{31} - g_{33}f_{31})}{h_{33}\mu_{33} - g_{33}^2} \tag{39}$$

$$M_2 = \frac{(h_{33}f_{31} - g_{33}e_{31})}{h_{33}\mu_{33} - g_{33}^2} \tag{40}$$

By substituting Eqs. (26)-(38) to Eqs. (14)-(18), one can yield

$$\left[c_{11}u_{0,xx} + c_{12}v_{0,xy} + c_{66}\left(u_{0,yy} + v_{0,xy}\right)\right]h = 0 \tag{41}$$

$$\left[c_{12}u_{0,xy} + c_{22}v_{0,yy} + c_{66}\left(u_{0,xy} + v_{0,xx}\right)\right]h = 0 \tag{42}$$

$$\left[\left(\frac{\phi_{x,xxx}+\phi_{x,xyy}+\phi_{y,xxy}+\phi_{y,yyy}}{210} - \frac{w_{0,xxxx}+w_{0,yyyy}+2w_{0,xxyy}}{84}\right)(e_{31}M_1 + f_{31}M_2) + c_{66}\left(\frac{\phi_{x,xxy}+8\phi_{y,xxx}-5w_{0,xxxy}}{315}\right) + c_{12}\left(\frac{4\left(\phi_{x,xyy}+\phi_{y,xxy}\right)}{315} - \frac{w_{0,xxyy}}{126}\right) + c_{11}\left(\frac{4\phi_{x,xxx}}{315} - \frac{w_{0,xxxx}}{252}\right) + c_{22}\left(\frac{4\phi_{y,yyy}}{315} - \frac{w_{0,yyyy}}{252}\right)\right]h^3 + \left[\frac{8\left(\phi_{x,x}+\phi_{y,y}+w_{0,xx}+w_{0,yy}\right)}{15}\right]h + w_{0,xx}\left(N_{xm} + N_{xe} + N_{xa} - k_g\right) + w_{0,yy}\left(N_{ym} + N_{ye} + N_{ya} - k_g\right) + k_w w_0 = 0 \tag{43}$$

$$\begin{aligned}\Big((9\phi_{x,xx} + 9\phi_{y,xy} - 12w_{0,xxx} - 12w_{0,xyy})(e_{31}M_1 &\\ + f_{31}M_2) + c_{12}(17\phi_{y,xy} - 4w_{0,xyy}) &\\ + c_{11}(17\phi_{x,xx} - 4w_{0,xxx}) &\\ + c_{66}(17\phi_{x,yy} + 17\phi_{y,xy} &\\ - 8w_{0,xyy})\Big)h^3 - 168c_{44}(\phi_x + w_{0,x})h &\\ = 0\end{aligned} \tag{44}$$

$$\begin{aligned}\Big((9\phi_{x,xy} + 9\phi_{y,yy} - 12w_{0,xxy} - 12w_{0,yyy})(e_{31}M_1 &\\ + f_{31}M_2) + c_{12}(17\phi_{x,xy} - 4w_{0,xxy}) &\\ + c_{22}(17\phi_{y,yy} - 4w_{0,yyy}) &\\ + c_{66}(17\phi_{x,xy} + 17\phi_{y,xx} &\\ - 8w_{0,xxy})\Big)h^3 - 168c_{44}(\phi_y + w_{0,y})h &\\ = 0\end{aligned} \tag{45}$$

$$\bar{u}_0 = \frac{u_0}{l}\ ;\bar{v}_0 = \frac{v_0}{l}\ ;\bar{w}_0 = \frac{w_0}{l}\ ;\bar{x} = \frac{x}{l}\ ;$$
$$\bar{y} = \frac{y}{b}\ ;\delta = \frac{h}{l};\eta = \frac{l}{b};\theta = \frac{1}{\delta^2};\ \bar{c}_{ij} = \frac{c_{ij}}{c_{11}} \tag{46}$$

$$\bar{e}_{31}\bar{M}_1 = \frac{e_{31}M_1}{c_{11}}\ ;\bar{f}_{31}\bar{M}_2 = \frac{f_{31}M_2}{c_{11}}\ ;$$
$$\bar{k}_w = \frac{k_w l^4}{h^3 c_{11}}\ ;\bar{k}_g = \frac{k_g l^2}{h^3 c_{11}}\ ;\bar{N}_{ij} = \frac{N_{ij} l^2}{h^3 c_{11}} \tag{47}$$

$$[\ \bar{c}_{11}\bar{u}_{0,xx} + \bar{c}_{12}\eta\bar{v}_{0,xy} + \bar{c}_{66}(\eta^2\bar{u}_{0,yy} + \eta\bar{v}_{0,xy})]\theta = 0 \tag{48}$$

$$[\ \bar{c}_{12}\eta\bar{u}_{0,xy} + \bar{c}_{22}\eta^2\bar{v}_{0,yy} + \bar{c}_{66}(\eta\bar{u}_{0,xy} + \bar{v}_{0,xx})]\theta = 0 \tag{49}$$

$$\begin{aligned}&\left(\frac{\phi_{x,xxx} + \eta^2\phi_{x,xyy} + \eta\phi_{y,xxy} + \eta^3\phi_{y,yyy}}{210}\right.\\ &\left.- \frac{\bar{w}_{0,xxxx} + \eta^4\bar{w}_{0,yyyy} + 2\eta^2\bar{w}_{0,xxyy}}{84}\right)(\bar{e}_{31}\bar{M}_1 + \bar{f}_{31}\bar{M}_2)\\ &+ \bar{c}_{66}\left(\frac{8\eta\phi_{x,xxy} + 8\phi_{y,xxx} - 5\eta\bar{w}_{0,xxxy}}{315}\right)\\ &+ \bar{c}_{12}\left(\frac{4(\eta^2\phi_{x,xyy} + \eta\phi_{y,xxy})}{315} - \frac{\eta^2\bar{w}_{0,xxyy}}{126}\right)\\ &+ \bar{c}_{11}\left(\frac{4\phi_{x,xxx}}{315} + \frac{\bar{w}_{0,xxxx}}{252}\right) + \bar{c}_{22}\left(\frac{4\eta^3\phi_{y,yyy}}{315} - \frac{\eta^4\bar{w}_{0,yyyy}}{252}\right)\\ &+ \left[\frac{8\left(\phi_{x,x} + \eta\phi_{y,y} + \bar{w}_{0,xx} + \eta^2\bar{w}_{0,yy}\right)}{15}\right]\theta\\ &+ (\bar{N}_{xm} + \bar{N}_{xe} + \bar{N}_{xa} - \bar{k}_g)\bar{w}_{0,xx}\\ &+ (\bar{N}_{ym} + \bar{N}_{ye} + \bar{N}_{ya} - \bar{k}_g)\eta^2\bar{w}_{0,yy} + \bar{k}_w\bar{w}_0 = 0\end{aligned} \tag{50}$$

$$\begin{aligned}(9\phi_{x,xx} + 9\eta\phi_{y,xy} - 12\bar{w}_{0,xxx} - 12\eta^2\bar{w}_{0,xyy})(\bar{e}_{31}\bar{M}_1 &\\ + \bar{f}_{31}\bar{M}_2) &\\ + \bar{c}_{12}\left(17\eta\phi_{y,xy} - 4\eta^2\bar{w}_{0,xyy}\right) &\\ + \bar{c}_{11}(17\phi_{x,xx} - 4\bar{w}_{0,xxx}) &\\ + \bar{c}_{66}(17\eta^2\phi_{x,yy} + 17\eta\phi_{y,xy} &\\ - 8\eta^2\bar{w}_{0,xyy}) - 168\bar{c}_{44}\theta(\phi_x + \bar{w}_{0,x}) &\\ = 0\end{aligned} \tag{51}$$

$$\begin{aligned}\left(9\eta\phi_{x,xy} + 9\eta^2\phi_{y,yy} - 12\eta\bar{w}_{0,xxy} - 12\eta^3\bar{w}_{0,yyy}\right)(\bar{e}_{31}\bar{M}_1 &\\ + \bar{f}_{31}\bar{M}_2) + \bar{c}_{12}(17\eta\phi_{x,xy} - 4\eta\bar{w}_{0,xxy}) &\\ + \bar{c}_{22}(17\eta^2\phi_{y,yy} - 4\eta^3\bar{w}_{0,yyy}) &\\ + \bar{c}_{66}(17\eta\phi_{x,xy} + 17\phi_{y,xx} - 8\eta w_{0,xxy}) &\\ - 168\bar{c}_{44}\theta(\phi_y + \eta w_{0,y}) = 0\end{aligned} \tag{52}$$

For the simply supported magnetoelectroelastic plate we have following boundary conditions

$$\begin{aligned}\bar{u} = \bar{v} = \bar{w} = M_{xx} = 0\ \ , x = 0, a\\ \bar{u} = \bar{v} = \bar{w} = M_{yy} = 0\ \ , y = 0, b\end{aligned} \tag{53}$$

$$\begin{aligned}\bar{u}_0 &= U\cos(\alpha\bar{x})\sin(\beta\bar{y})\\ \bar{v}_0 &= V\sin(\alpha\bar{x})\cos(\beta\bar{y})\\ \bar{w}_0 &= W\sin(\alpha\bar{x})\sin(\beta\bar{y})\\ \phi_x &= A\cos(\alpha\bar{x})\sin(\beta\bar{y})\\ \phi_y &= B\sin(\alpha\bar{x})\cos(\beta\bar{y})\end{aligned} \tag{54}$$

In wich $\alpha$ and $\beta$ are defined as $\alpha = m\pi$ and $\beta = n\pi$, respectively, and m and n are the half wave numbers.

By substituting Eqs. (54) into Eqs. (48)-(52), one have

$$(-\bar{c}_{11}\alpha^2 - \bar{c}_{66}\eta^2\beta^2)\theta U + (-\bar{c}_{12}\eta\alpha\beta - \bar{c}_{66}\eta\alpha\beta)\theta V = 0 \tag{55}$$

$$(-\bar{c}_{12}\eta\alpha\beta - \bar{c}_{66}\eta\alpha\beta)\theta U + (-\bar{c}_{22}\eta^2\beta^2 - \bar{c}_{66}\alpha^2)\theta V = 0 \tag{56}$$

$$\begin{aligned}&\Big[-\tfrac{1}{84}(\alpha^4 + \eta^4\beta^4 + 2\eta^2\alpha^2\beta^2)(\bar{e}_{31}\bar{M}_1 + \bar{f}_{31}\bar{M}_2) + \\ &\tfrac{1}{252}(4\eta\alpha^3\beta\bar{c}_{66} + 2\eta^2\alpha^2\beta^2\bar{c}_{12} - \alpha^4\bar{c}_{11} - \eta^4\beta^4\bar{c}_{22}) + \\ &\tfrac{8}{15}(-\alpha^2 - \eta^2\beta^2)\bar{c}_{44}\theta - (\bar{N}_{xm} + \bar{N}_{xe} + \bar{N}_{xa} - \bar{k}_g)\alpha^2 + \\ &(\bar{N}_{ym} + \bar{N}_{ye} + \bar{N}_{ya} - \bar{k}_g)\eta^2\beta^2 + \bar{k}_w\Big]W + \Big[\tfrac{1}{210}(\alpha^3 + \\ &\eta^2\alpha\beta^2)(\bar{e}_{31}\bar{M}_1 + \bar{f}_{31}\bar{M}_2) + \tfrac{4}{315}(\eta^2\alpha\beta^2\bar{c}_{12} + \alpha^3\bar{c}_{11} - \\ &2\eta\alpha^2\beta\bar{c}_{66}) - \tfrac{8}{15}\alpha\bar{c}_{44}\theta\Big]A + \Big[\tfrac{1}{210}(\eta\alpha^2\beta + \eta^3\beta^3)(\bar{e}_{31}\bar{M}_1 + \\ &\bar{f}_{31}\bar{M}_2) + \tfrac{4}{315}(-2\alpha^3\bar{c}_{66} + \eta^3\beta^3\bar{c}_{22} + \eta\alpha^2\beta\bar{c}_{12}) - \\ &\tfrac{8}{15}\eta\beta\bar{c}_{44}\theta\Big]B = 0\end{aligned} \tag{57}$$

$$\Big[\tfrac{1}{315}(12\alpha^3 + 12\eta^2\alpha\beta^2)(\bar{e}_{31}\bar{M}_1 + \bar{f}_{31}\bar{M}_2) + \tag{58}$$

$$\frac{4}{315}\alpha^2\eta\beta^2\bar{c}_{12}+\frac{4}{315}\alpha^3\bar{c}_{11}+\frac{8}{315}\eta^2\alpha\beta^2\bar{c}_{66}-\frac{8}{15}\alpha\bar{c}_{44}\theta\Big]W+\Big[-\frac{1}{35}\alpha^2(\bar{e}_{31}\bar{M}_1+\bar{f}_{31}\bar{M}_2)-\frac{17}{315}\alpha^2\bar{c}_{11}-\frac{17}{315}\eta^2\beta^2\bar{c}_{66}-\frac{8}{15}\bar{c}_{44}\theta\Big]A+\Big[-\frac{1}{35}\eta\alpha\beta(\bar{e}_{31}\bar{M}_1+\bar{f}_{31}\bar{M}_2)-\frac{17}{315}\eta\alpha\beta\bar{c}_{12}-\frac{17}{315}\eta\alpha\beta\bar{c}_{66}\Big]B=0$$

$$\Big[\frac{1}{315}(12\alpha^2\beta+12\eta^2\beta^3)(\bar{e}_{31}\bar{M}_1+\bar{f}_{31}\bar{M}_2)+\frac{4}{315}\eta^2\alpha\beta\bar{c}_{12}+\frac{4}{315}\beta^3\bar{c}_{22}+\frac{8}{315}\eta^2\alpha^2\beta\bar{c}_{66}-\frac{8}{15}\beta\bar{c}_{44}\theta\Big]W+\Big[-\frac{1}{35}\alpha\beta(\bar{e}_{31}\bar{M}_1+\bar{f}_{31}\bar{M}_2)-\frac{17}{315}\eta^2\alpha\beta\bar{c}_{66}-\frac{17}{315}\eta\alpha\beta\bar{c}_{12}\Big]A+\Big[-\frac{1}{35}\eta\beta^2(\bar{e}_{31}\bar{M}_1+\bar{f}_{31}\bar{M}_2)-\frac{17}{315}\beta^2\bar{c}_{22}-\frac{17}{315}\eta\alpha^2\bar{c}_{66}-\frac{8}{15}\bar{c}_{44}\theta\Big]B=0 \quad (59)$$

Eqs. (55)-(59) can be written as

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix}\begin{Bmatrix} U \\ V \\ W \\ A \\ B \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (60)$$

where coefficients $L_{11}$ through $L_{55}$ are given in Appendix A.

## 3. Results and discussion

Numerical examples of buckling behaviors for magnetoelectroelastic plate are investigated. The magnetoelectroelastic composite made of the piezoelectric material $BaTiO_3$ as the inclusions and piezomagnetic material $CoFe_2O_4$ as the matrix is considered. The materials properties for the composite are (Wang and Han 2007): $c_{11}$=226x10$^9$ N/m$^2$, $c_{12}$= 124x10$^9$ N/m$^2$, $c_{22}$=216x10$^9$ N/m$^2$, $c_{44}$=44x10$^9$N/m$^2$, $e_{31}$=-2.2C/m$^2$, $f_{31}$=290.2N/Am, $h_{33}$=6.35x10$^{-9}$C$^2$/Nm$^2$,$g_{33}$=2737.5x10$^{-12}$Ns/VC, $\mu_{33}$=83.5x10$^{-6}$Ns$^2$/C$^2$. The length of the magnetoelectro-elastic plate is $l$=1m.

### 3.1 Comparisons

Firstly, in order to validate the accuracy of the present method, a comparison has been carried out with previously published results by Li (2014) for both thin and moderately thick square plates . Plates are subjected to various loads.

The buckling load parameters are listed in Table 1 with the available results by Li (2014), obtained by Mindlin plates theory for shear correction facto $\pi^2/12$. It can be observed an excellent agreement between the present results and those given by Li (2014).

Table 1 Comparison study of buckling load parameters, $P_{cr}$ for square magnetoelectroelastic plate ($V_0$ =$\Omega_0$ =0, $k_w$=$k_g$=0, m=n=1)

| | | δ=l/h | | | | | |
|---|---|---|---|---|---|---|---|
| | Theories | 0.001 | 0.01 | 0.05 | 0.1 | 0.15 | 0.2 |
| λ=0 | FSDT* ($ks$=$\pi^2$/12) | 3.2794 | 3.2760 | 3.1975 | 2.9747 | 2.6652 | 2.3264 |
| | Present | 2.9824 | 2.9794 | 2.9089 | 2.7087 | 2.4304 | 2.1254 |
| λ=0.5 | FSDT* ($ks$=$\pi^2$/12) | 2.1862 | 2.1840 | 2.1317 | 1.9831 | 1.7768 | 1.5509 |
| | Present | 1.9882 | 1.9863 | 1.9392 | 1.8058 | 1.6203 | 1.4169 |
| λ=-0.5 | FSDT* ($ks$=$\pi^2$/12) | 6.5587 | 6.5520 | 6.3950 | 5.9494 | 5.3304 | 4.6527 |
| | Present | 5.9647 | 5.9588 | 5.8177 | 5.4174 | 4.8608 | 4.2508 |

* Taken from Ref (Li. 2014)

#### 3.2.1 Effect of the lateral load

The effect of lateral load parameter on the buckling load is considered firstly. From Fig. 1, one can see that buckling load increases with increasing length-to-width ratio for a rectangular magnetoelectroelastic thinner plate. The responses are not following a monotonous trend and revert from the expected line with the thickness ratio. This is because of the fact that the thin structure may not follow a monotonous trend of results due to severity in geometrical distortion.

#### 3.2.2 Effect of the electric load

Fig. 2 displays the influence of electric potential on the normalized buckling load under different thickness-to-length ratio for a square magnetoelectroelastic plate. It is seen that the buckling load decreases linearly with an increase in the value of electric load.

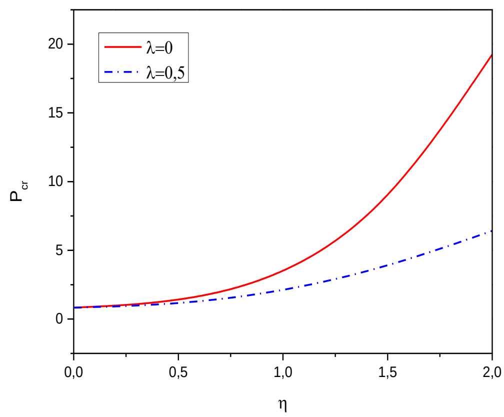


Fig. 1 Variations of buckling load, $P_{cr}$, versus $\eta$, for rectangular magnetoelectroelastic plate under different lateral load parameter ($\delta=0.001, V_0=\Omega_0=0, k_w=k_g=0, m=n=1$)

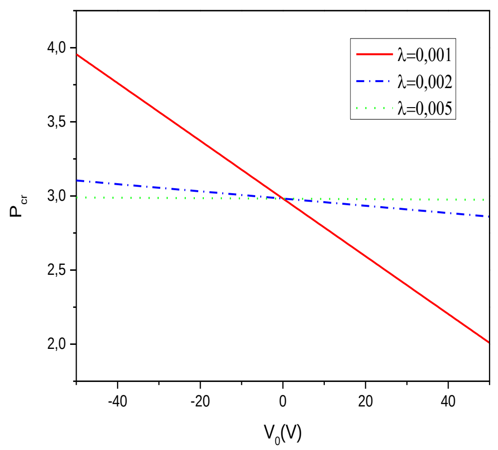


Fig. 2 Variations of buckling load, $P_{cr}$, versus electric potential, $V_0$, for square magnetoelectroelastic plate under different $\delta$ $(\Omega_0 = 0, k_w = k_g, = 0, \lambda = 0, m = n = 1)$

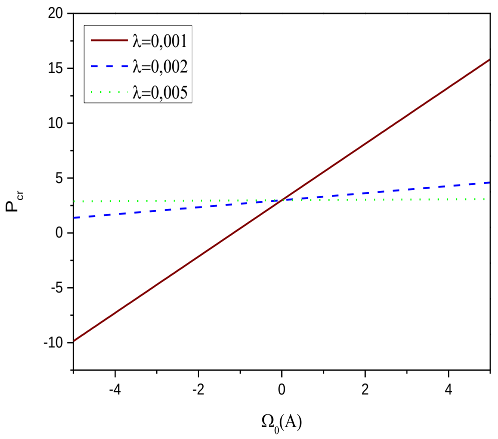


Fig. 4 Variations of buckling load, $P_{cr}$, versus magnetic potential, $\Omega_0$, for square magnetoelectroelastic plate under different $\delta$ $(V_0 = 0, k_w = k_g, = 0, \lambda = 0, m = n = 1)$

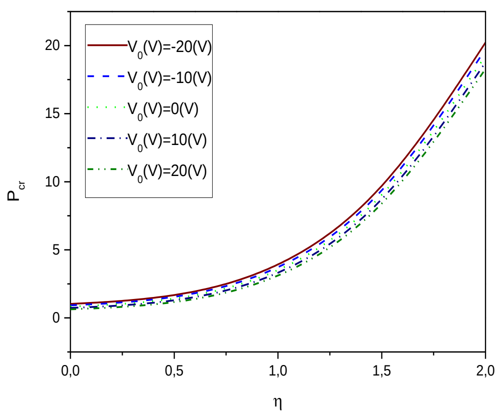


Fig. 3 Variations of buckling load, $P_{cr}$, versus, for rectangular magnetoelectroelastic plate under different electric potential $V_0\,(\delta = 0.001, \Omega_0 = 0,\ k_w = k_g = 0,\ \lambda = 0, m = n = 1)$

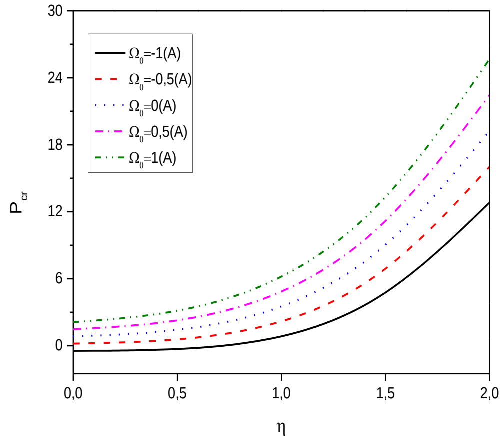


Fig. 5 Variations of buckling load, $P_{cr}$, versus $\eta$, for rectangular magnetoelectroelastic plate under different magnetic potential $\Omega_0$, $(\delta = 0.001, V_0 = 0, k_w = k_g = 0,\ \lambda = 0, m = n = 1)$

Fig. 3 illustrates the influence of the length to width ratio on the buckling load of a rectangular plate for varies electric load. It can be conclude that the buckling load increases with an increase in the value of length to width ratio and the trend becomes more apparent for a negative electric load. This is because of the fact that the thin flexible structures may not follow a specified trend of results due to the geometrical distortions are nonlinear in nature.

### *3.2.3 Effect of the magnetic load*

Fig. 4 shows the effect of magnetic load on the buckling load under different thickness to length ratio for a square plate. Contrary to the case of electric load shown in Fig. 2, the buckling load increases with the increase of magnetic load.

The effect of the length to width ratio on the normalized buckling load of a rectangular plate for varies magnetic potential is shown in Fig. 5. It is clear that the buckling load increases with an increase in the value of length to width ratio especially for a larger magnetic load.

## 4. Conclusions

In this paper, the buckling behavior of a magnetoelectroelastic plate resting on a Pasternak elastic foundation is investigated using the third-order shear deformation plate theory by taking the nonlinear equations according to Von Karman's theory sense to incorporate the true geometrical distortion in geometry.

The in plane electric and magnetic fields can be ignored for plates. According to Maxwell equation and magnetoelectric boundary condition, the variation of electric and magnetic potentials along the thickness direction of the plate is determined. From the numerical results, some conclusions can be drawn.

- Buckling does not mean failure of the structure, rather it is a state of geometrical instability due to excess thermal and/or mechanical distortion of structural geometry, and it may lead to catastrophic failure. This geometrical distortion may also be reckoned as the geometrical non-linearity, and it has been modelled through von-Karman type non-linear strain-displacement relation.
- For a rectangular magnetoelectroelastic plate, the buckling load increases with an increase in the value of length to width ratio.
- The buckling load decreases with an increase in the values of lateral load and thickness to length ratio for a square magnetoelectroelastic plate.
- For a rectangular magnetoelectroelastic plate, the buckling load decreases linearly with the increase of electric load, and increases with the increase of magnetic load.

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## Appendix A

$L_{11} = -\overline{c}_{11}\alpha^2\theta - \overline{c}_{66}\eta^2\beta^2\theta$

$L_{12} = -\overline{c}_{12}\eta\alpha\beta\theta - \overline{c}_{66}\eta\alpha\beta\theta$

$L_{13} = L_{14} = L_{15} = 0$

$L_{21} = -\overline{c}_{12}\eta\alpha\beta\theta - \overline{c}_{66}\eta\alpha\beta\theta$

$L_{22} = -\overline{c}_{22}\eta^2\beta^2\theta - \overline{c}_{66}\alpha^2\theta$

$L_{23} = L_{24} = L_{25} = 0$

$L_{31} = L_{32} = 0$

$L_{33} = -\frac{1}{84}(\alpha^4 + \eta^4\beta^4 + 2\eta^2\alpha^2\beta^2)(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) +$
$\frac{1}{252}(4\eta\alpha^3\beta\overline{c}_{66} + 2\eta^2\alpha^2\beta^2\overline{c}_{12} - \alpha^4\overline{c}_{11} - \eta^4\beta^4\overline{c}_{22}) +$
$\frac{8}{15}(-\alpha^2 - \eta^2\beta^2)\overline{c}_{44}\theta - (\overline{N}_{xm} + \overline{N}_{xe} + \overline{N}_{xa} - \overline{k}_g)\alpha^2 +$
$(\overline{N}_{ym} + \overline{N}_{ye} + \overline{N}_{ya} - \overline{k}_g)\eta^2\beta^2 + \overline{k}_w$

$L_{34} = \frac{1}{210}(\alpha^3 + \eta^2\alpha\beta^2)(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) +$
$\frac{4}{315}(\eta^2\alpha\beta^2\overline{c}_{12} + \alpha^3\overline{c}_{11} - 2\eta\alpha^2\beta\overline{c}_{66}) - \frac{8}{15}\alpha\overline{c}_{44}\theta$

$L_{35} = \frac{1}{210}(\eta\alpha^2\beta + \eta^3\beta^3)(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) +$
$\frac{4}{315}(-2\alpha^3\overline{c}_{66} + \eta^3\beta^3\overline{c}_{22} + \eta\alpha^2\beta\overline{c}_{12}) - \frac{8}{15}\eta\beta\overline{c}_{44}\theta$

$L_{41} = L_{42} = 0$

$L_{43} = \frac{1}{315}(12\alpha^3 + 12\eta^2\alpha\beta^2)(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) +$
$\frac{4}{315}\alpha^2\eta\beta^2\overline{c}_{12} + \frac{4}{315}\alpha^3\overline{c}_{11} + \frac{8}{315}\eta^2\alpha\beta^2\overline{c}_{66} - \frac{8}{15}\alpha\overline{c}_{44}\theta$

$L_{44} = -\frac{1}{35}\alpha^2(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) - \frac{17}{315}\alpha^2\overline{c}_{11} -$
$\frac{17}{315}\eta^2\beta^2\overline{c}_{66} - \frac{8}{15}\overline{c}_{44}\theta$

$L_{45} = -\frac{1}{35}\eta\alpha\beta(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) - \frac{17}{315}\eta\alpha\beta\overline{c}_{12} -$
$\frac{17}{315}\eta\alpha\beta\overline{c}_{66}$

$L_{51} = L_{52} = 0$

$L_{53} = \frac{1}{315}(12\alpha^2\beta + 12\eta^2\beta^3)(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) +$
$\frac{4}{315}\eta^2\alpha\beta\overline{c}_{12} + \frac{4}{315}\beta^3\overline{c}_{22} + \frac{8}{315}\eta^2\alpha^2\beta\overline{c}_{66} - \frac{8}{15}\beta\overline{c}_{44}\theta$

$L_{54} = -\frac{1}{35}\alpha\beta(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) - \frac{17}{315}\eta^2\alpha\beta\overline{c}_{66} -$
$\frac{17}{315}\eta\alpha\beta\overline{c}_{12}$

$L_{55} = -\frac{1}{35}\eta\beta^2(\overline{e}_{31}\overline{M}_1 + \overline{f}_{31}\overline{M}_2) - \frac{17}{315}\beta^2\overline{c}_{22} -$
$\frac{17}{315}\eta\alpha^2\overline{c}_{66} - \frac{8}{15}\overline{c}_{44}\theta$