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Optimum LCVA for suppressing harmonic vibration of damped structures

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Abstract. Explicit design formulae of liquid column vibration absorber (LCVA) for suppressing harmonic vibration of structures with small inherent structural damping are developed in this study. The developed design formulae are also applicable to the design of a tuned mass damper (TMD) and a tuned liquid column damper (TLCD) for damped structures under harmonic force excitation. The optimum parameters of LCVA for suppressing harmonic vibration of undamped structures are first derived. Numerical searching of the optimum parameters of tuned vibration absorber system for suppressing harmonic vibration of damped structure is conducted. Explicit formulae for these optimum parameters are then obtained by a series of curve fitting techniques. The analytical result shows that the control performance of TLCD for reducing harmonic vibration of undamped structure is always better than that of non-uniform LCVA for same mass and length ratios. As for the effects of structural damping on the optimum parameters, it is found that the optimum tuning ratio decreases and the optimum damping ratio increases as the structural damping is increased. Furthermore, the optimum head loss coefficient is inversely proportional to the amplitude of excitation force and increases as the structural damping is increased. Numerical verification of the developed explicit design expressions is also conducted and the developed expressions are demonstrated to be reasonably accurate for design purposes.

Keywords: damped structure; liquid column vibration absorber; harmonic excitation; curve fitting

1. Introduction

The protection of buildings against the effects of wind is essential for a wind-sensitive tall building. Liquid Column Vibration Absorber (LCVA) or Tuned Liquid Column Damper (TLCD), a special class of LCVA with uniform sectional area, is shown to be effective for suppressing vibration by a number of studies if the damper parameters are properly selected (Gao *et al*. 1997, Lee *et al*. 2012, Min *et al.* 2015, Bigdeli and Kim 2016). Due to its simple way of adjusting damper parameters, LCVA is considered to be one of the cost effective solutions for suppressing the vibration of structures. It has attracted great attention from researchers to improve the performance of LCVA or apply LCVA to different structures. To facilitate the application of LCVA, Wu *et al.* (2012) presented a set of empirical formulae for the determination of head loss coefficient by the opening ratio of orifice. Lee and Juang (2012) studied the feasibility of an integrated system of floating platform with an underwater TLCD. The underwater TLCD could provide buoyancy to the system and would not occupy additional space for installation. Diana *et al*. (2013) developed a new methodology which allows direct linking of TLCD with the increased damping acting on the structure, facilitating the preliminary design of these devices. Cheng *et al.* (2015) studied the application of MR-TLCD to structures by a series of experimental test with focus on the effects of MR-fluid viscosity. More recently, Di Matteo *et al.* (2015) and Di Matteo *et al.* (2016) developed a new formulation of TLCD based on fractional calculus and experimentally demonstrated that the proposed formulation can accurately capture structural responses and liquid displacements of TLCD. Bigdeli and Kim (2017) proposed a new Neuro-Wavelet control algorithm based on a cost function to actively control the vibrations of structures under earthquake loads.

To maximize the performance of LCVA, a large number of studies have been conducted to optimize the performance of vibration absorbers under different type of loadings. Chang (1999) first derived the optimum parameters of LCVA for suppressing vibration of structure under white noise excitation. Yalla and Kareem (2000) also derived the optimum parameters of TLCD and developed an explicit formula for the optimum head loss coefficient in terms of the optimum parameters derived from linearized equation of motion for a TLCD-structure system under white noise excitation. Wu et al. (2005) proposed some guidelines and presented some numerical results in tabular form for designing TLCD for damped structures under a white noise type of wind excitation. Konar and Ghosh (2010) determined the optimum combination of the design parameters of LCVA for the most efficient control performance. Di Matteo *et al*. (2014) and

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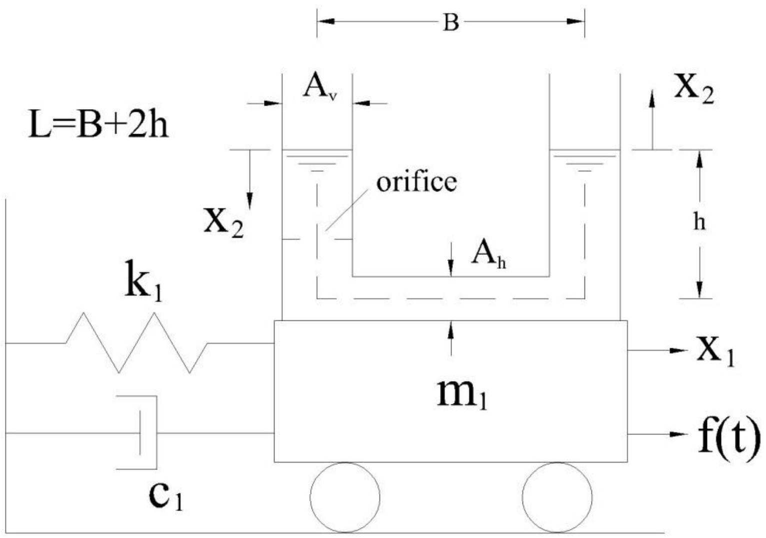


Fig. 1 Liquid column vibration absorber

Di Matteo *et al.* (2015) developed a pre-design simplified formula for choosing the optimal parameters of TLCD on lightly damped structures under random ground acceleration. Min *et al.* (2015) also studied the performance of passive TLCD with an optimally designed blocking ratio under wind excitation and derived some analytical formulae for design purposes. The effects of structural damping on the optimum parameters of LCVA were also studied by Ghosh and Basu (2007a), Shum *et al.* (2011) and Shum (2015). Although all of these previous studies were significant developments of the optimum parameters of LCVA or TLCD, each of the aforementioned studies are limited to the case of random type excitation, which might not well represent harmonic type crosswind excitation due to vortex shedding or galloping. Following the perturbation approach presented by Liu and Liu (2005), Shum (2009) derived the optimum parameters of TLCD for suppressing harmonic vibration of undamped structure and presented some numerically searched optimum values of TLCD parameters for suppressing harmonic vibration of damped structure. Nevertheless, the optimum damping of TLCD for undamped structures derived by perturbation approach is relatively cumbersome and the numerical results presented in his studies are only applicable to the case of TLCD for few number of structural damping. Later, Wu *et al.* (2009) also studied the optimum parameters of LCVA for suppressing harmonic vibration of damped structures by numerical method and presented some design tables for LCVA. However, only few LCVA configurations were considered in their studies.

In order to facilitate the design process of LCVA, explicit design formulas of LCVA for suppressing harmonic vibration of structures with small inherent structural damping are developed in present study. The optimum damping and tuning frequency ratio of a tuned vibration absorber system for damped structure under harmonic force excitation are obtained through a numerical searching approach. Explicit formulae for these optimum parameters are then obtained by a sequence of curve fitting techniques. Similar curve fitting approach was used by Tsai and Lin (1993) to determine explicit expressions for optimum parameters of TMD under harmonic support excitation. The effects of structural damping on the optimum parameters of LCVA are also investigated in this study.

2. Problem formulation

The problem being considered is shown in Fig. 1. The system consists of a structural mass m_1, supported by a spring with stiffness k_1 and a viscous damper with damping coefficient c_1. The motion of structural mass relative to ground is denoted as $x_1(t)$. A LCVA is installed on the structural mass. The motion of liquid inside the vertical column relative to the container is denoted as $x_2(t)$. The system is subjected to a harmonic force, $f(t)$. By considering the dynamic equilibrium condition and the interaction between the structural mass and the liquid in LCVA, the equation of motion of a structural mass equipped with a LCVA for lateral vibration control are (Hitchcock *et al.* 1997, Taflanidis *et al.* 2007)

$$\begin{bmatrix} m_1 + m_d & \rho A_v B \\ \rho A_v B & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & \frac{1}{2}\rho A_v \delta |\dot{x}_2| \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & 2\rho A_v g \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [F] \tag{1}$$

under the condition

$$x_2 \leq \frac{L-B}{2} - \frac{s}{2} \tag{2}$$

where $m_d = \rho A_v[(A_h/A_v)B + (L-B)]$ is the total mass of liquid inside container; $m_2 = \rho A_v[(L-B) + B(A_v/A_h)]$ is defined as the equivalent mass of liquid inside LCVA; ρ is the density of the LCVA liquid; B is the horizontal length of liquid column; L is the total length of liquid column; s is the thickness of horizontal tube of container; A_v and A_h denote the vertical and horizontal cross-sectional areas of liquid column, respectively; g is the acceleration due to gravity; δ is the head loss coefficient due to an orifice inside the vertical part of liquid column; [F] is the force vector. The mass m_2 can be regarded as the mass of a uniform liquid column with cross-sectional area A_v that possesses the same kinetic energy. The inherent nonlinear damping force of liquid motion could be replaced by a linear equivalent damping force $c_{eq}\dot{x}_2$ (Gao *et al*. 1997) using equivalent linearization technique. The equivalent damping ratio of liquid motion ξ_2 is determined by $\xi_2 = c_{eq}/(2m_2\omega_2)$. ω_2 is the circular natural frequency of the liquid inside LCVA. The frequency tuning ratio λ and the mass ratio of LCVA to structure μ are defined as

$$\lambda = \frac{\omega_2}{\omega_1}; \qquad \mu = \frac{m_d}{m_1} \tag{3}$$

After replacing nonlinear damping force of liquid motion by linear equivalent damping force, Eq. (1) can be simplified into the following form

$$\begin{bmatrix} (1+\mu)m_1 & \alpha\chi_1\mu m_1 \\ \alpha\chi_2 m_2 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2m_1\omega_1\xi_1 & 0 \\ 0 & 2m_2\omega_2\xi_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} m_1\omega_1^2 & 0 \\ 0 & m_2\omega_2^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [F] \tag{4}$$

in which ω_1 is the natural frequency of structure; ξ_1 is the damping ratio of structure; $\alpha = \frac{B}{L}$; $r_A = \frac{A_v}{A_h}$; $\chi_1 = \frac{r_A}{\alpha + r_A - \alpha r_A}$; $\chi_2 = \frac{1}{1-\alpha+\alpha r_A}$; $m_d = \frac{\rho A_v L}{\chi_1}$; $m_2 = \frac{\rho A_v L}{\chi_2}$ and $\omega_2 = \sqrt{\frac{2g\chi_2}{L}}$. Eq. (4) can be re-written into the following matrix form.

$$\begin{bmatrix} 1+\mu_2 & \mu_3 \\ \mu_3 & \mu_1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2\omega_1\xi_1 & 0 \\ 0 & 2\mu_1\omega_2\xi_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \mu_1\omega_2^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{m_1}[F] \tag{5}$$

The displacement responses of structure and liquid motion of LCVA determined from the linearized Eq. (5) would depend on a value of equivalent damping ξ_2. For a given excitation amplitude $\bar{f}$, the equivalent damping ξ_2 could be achieved by adjusting the value of head loss coefficient. The three mass ratios μ_1, μ_2, and μ_3 in the above equation depend on the type of vibration absorber being considered. For the case of LCVA-structure system (α≠1, r_A≠1 for LCVA)

$$\mu_1 = \frac{m_2}{m_1} \qquad \mu_2 = \frac{m_d}{m_1} = \mu \tag{6}$$

$$\mu_3 = \chi_2\alpha\mu_1 \ or\ \chi_1\alpha\mu_2$$

For the case of TMD-structure system (α=1, r_A=1 for TMD)

$$\mu_1 = \mu_2 = \mu_3 = \mu = \frac{m_2}{m_1} \tag{7}$$

where m_2 is the mass of TMD attached to the primary structure. Therefore, the results derived based on Eq. (5) are applicable to TMD and LCVA. Eq. (5) can be considered to be a unified analytical model for a structure equipped with a tuned vibration absorber and can be converted to the equation of motion of TMD-structure system when substituting Eq. (7) into Eq. (5). In this study, results derived from unified analytical model for structure-tuned vibration absorber system under harmonic force excitation are generalized in terms of efficiency index. An efficiency index of tuned vibration absorber is defined as (Chang 1999)

$$\gamma = \frac{\mu_3^2}{\mu_1} \tag{8}$$

The efficiency index for TMD and LCVA is determined as follows

$$\gamma_{TMD} = \mu \tag{9}$$

$$\gamma_{LCVA} = \mu\alpha^2\chi \tag{10}$$

in which $\chi = \chi_1\chi_2$. The efficiency index for TLCD is determined by substituting r_A=1 into Eq. (6).

$$\gamma_{TLCD} = \mu\alpha^2 \tag{11}$$

3. Optimization of LCVA

In this section, the closed-form optimum parameters of a LCVA are first determined for the case of an undamped primary structure subjected to a harmonic excitation. The steady-state responses of structure and liquid motion of LCVA are first obtained by solving the linearized Eq. (5). The frequency and damping parameters of LCVA for minimizing the resonant vibration amplitudes are then optimized based on fixed-point theory. The closed-form optimum parameters of a LCVA for suppressing harmonic vibration of damped structure are then determined through a numerical searching technique. After the optimum LCVA parameters for damped structures are found through numerical searching, a series of curve fitting technique is applied to find the explicit expressions that can best represent the numerically searched optimum values.

3.1 Undamped structure

In case of forced motion, there is only one external harmonic force f(t) acting on the structural mass. The corresponding force vector is

$$[F] = \begin{bmatrix} f(t) \\ 0 \end{bmatrix} \tag{12}$$

where $f(t) = \bar{f}e^{i\tilde{\omega}t}$; $\bar{f}$ is the amplitude of harmonic force and $\tilde{\omega}$ is the frequency of external applied force.

Considering the feasible range of head loss coefficient in practice, the liquid motion due to quadratic damping induced by orifice can be assumed to be weakly nonlinear. The steady-state responses of the structure and liquid motion of LCVA under harmonic force excitation are also harmonic (Gao *et al*. 1997) and are given by

$$[x_1 \quad x_2]^T = [\bar{X}_1 \quad \bar{X}_2]^T e^{i\tilde{\omega}t} \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (5) with some manipulation, it yields the steady-state response amplitudes of structure and liquid.

$$\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|^2 = \frac{[(\lambda^2-\beta^2)]^2+(2\lambda\beta\xi_2)^2}{\{[(1+\mu_2)-\gamma]\beta^4-[1+(1+\mu_2)\lambda^2]\beta^2+\lambda^2\}^2+\{2\lambda\beta\xi_2[1-(1+\mu_2)\beta^2]\}^2} \tag{14}$$

$$\left|\frac{\bar{X}_2}{\bar{f}/k_1}\right|^2 = \frac{\left(\frac{\mu_3}{\mu_1}\beta^2\right)^2}{\{[(1+\mu_2)-\gamma]\beta^4-[1+(1+\mu_2)\lambda^2]\beta^2+\lambda^2\}^2+\{2\lambda\beta\xi_2[1-(1+\mu_2)\beta^2]\}^2} \tag{15}$$

in which $\beta = \tilde{\omega}/\omega_1$. Fig. 2 shows the variation of the steady-state response amplitude of structure with frequency ratio under different values of equivalent damping of LCVA. It can be observed from Fig. 2 that all four curves intersect at the two points irrespective of the values of equivalent damping. Similar findings were first reported by the classical work on tuned mass dampers by Ormondroyd and Den Hartog (1928). The optimum parameters can be obtained by having zero gradients and equal amplitudes at these two fixed points. As the ordinates of the two fixed points are independent of the equivalent damping ξ_2, their ordinates can be determined by considering two extreme conditions of the equivalent damping, i.e., when the equivalent damping is zero and infinitely large. Therefore, the locations of the fixed-points can be determined by the following condition.

$$\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|_{\xi_2=0} = \left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|_{\xi_2=\infty} \tag{16}$$

corresponding to

$$\frac{\lambda^2-\beta^2}{[(1+\mu_2)-\gamma]\beta^4-[1+(1+\mu_2)\lambda^2]\beta^2+\lambda^2} = \pm\frac{1}{1-(1+\mu_2)\beta^2} \tag{17}$$

Use of the plus sign leads to the root $\beta = 0$, which is considered to be the static case. Use of the minus sign leads to the following quadratic equation in β^2.

$$[2(1+\mu_2)-\gamma]\beta^4 - 2[1+(1+\mu_2)\lambda^2]\beta^2 - 2\lambda^2 = 0 \tag{18}$$

The roots of this quadratic equation correspond to the ordinates of the fixed points, namely β_P and β_Q, in Fig. 2.

Their sum is determined by

$$\beta_P^2 + \beta_Q^2 = \frac{2[1+(1+\mu_2)\lambda^2]}{[2(1+\mu_2)-\gamma]} \tag{19}$$

The optimum value of the ordinates of points P and Q can be determined by letting $\xi_2 \to \infty$ in Eq. (14) and then setting $\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|_P = \left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|_Q$. Therefore

$$\frac{1}{1-(1+\mu_2)\beta_P^2} = \frac{-1}{1-(1+\mu_2)\beta_Q^2} \tag{20}$$

which yields

$$\beta_P^2 + \beta_Q^2 = \frac{2}{1+\mu_2} \tag{21}$$

Substituting Eq. (21) into Eq. (19), the optimum frequency tuning ratio is determined by

$$\lambda = \frac{\sqrt{1+\mu_2-\gamma}}{1+\mu_2} \tag{22}$$

The location of the two invariant points can be determined by solving the quadratic equation in Eq. (18) with the substitution of Eq. (22).

$$\beta_P^2 = \frac{1}{1+\mu_2}\left[1 - \sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] \tag{23}$$

$$\beta_Q^2 = \frac{1}{1+\mu_2}\left[1 + \sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right]$$

Substituting (λ, β_P) or (λ, β_Q) into Eq. (14) and letting $\xi_2 \to \infty$, one would obtain the common ordinate at the frequency ratios of β_P and β_Q.

$$\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|_{P,Q} = \sqrt{\frac{2(1+\mu_2)-\gamma}{\gamma}} \tag{24}$$

The optimum damping of LCVA can be determined by having zero gradients at points P and Q.

$$\frac{d}{d\beta^2}\left(\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|^2\right) = 0 \tag{25}$$

or

$$\frac{dp}{d\beta^2}q - \frac{dq}{d\beta^2}p = 0 \tag{26}$$

where p is the numerator of $\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|^2$, $p = (\lambda^2-\beta^2)^2 + (2\lambda\beta\xi_2)^2$ and q is the denominator of $\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|^2$, $q = \{[(1+\mu_2)-\gamma]\beta^4 - [1+(1+\mu_2)\lambda^2]\beta^2 + \lambda^2\}^2 + \{2\lambda\beta\xi_2[1-(1+\mu_2)\beta^2]\}^2$.

Differentiating p and q with respect to β^2, we have

$$\frac{\partial p}{\partial \beta^2} = 2(\beta^2-\lambda^2) + 4\lambda^2\xi_2^2 \tag{27}$$

$$\begin{aligned}\frac{\partial q}{\partial \beta^2} = 2[(1+\mu_2-\gamma)\beta^4 - (1+\lambda^2+\mu_2\lambda^2)\beta^2 \\ + \lambda^2][2(1+\mu_2-\gamma)\beta^2 \\ - (1+\lambda^2+\mu_2\lambda^2)] \\ + 4\lambda^2\xi_2^2[1-(1+\mu_2)\beta^2][1 \\ - 3(1+\mu_2)\beta^2]\end{aligned} \tag{28}$$

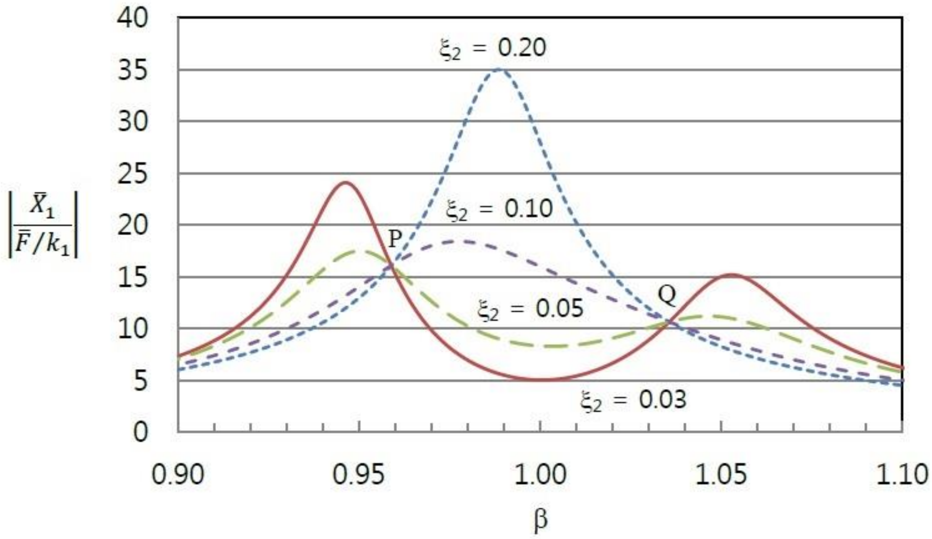


Fig. 2 Steady state response of structure with LCVA installed under harmonic forced motion (μ=0.02, λ=1, r_A =0.8, r_A=0.5)

Eq. (26) can be re-written as follows.

$$\frac{\partial p}{\partial \beta^2} = \frac{p}{q}\frac{\partial q}{\partial \beta^2} \tag{29}$$

Under the optimum condition of LCVA

$$\frac{p}{q} = \left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|^2_{P,Q} = \frac{2(1+\mu_2)-\gamma}{\gamma} \tag{30}$$

Substituting Eqs. (27), (28) and (30) into Eq. (29), Eq. (29) becomes

$$\begin{aligned} 2(\beta^2-\lambda^2)+4\lambda^2\xi_2^2 = \frac{2(1+\mu_2)-\gamma}{\gamma} & \{2[(1+\mu_2-\gamma)\beta^4 \\ & -(1+\lambda^2+\mu_2\lambda^2)\beta^2+\lambda^2][2(1+\mu_2-\gamma)\beta^2 \\ & -(1+\lambda^2+\mu_2\lambda^2)] \\ & +4\lambda^2\xi_2^2[1-(1+\mu_2)\beta^2][1-3(1+\mu_2)\beta^2]\} \end{aligned} \tag{31}$$

Rearranging Eq. (31), the damping of LCVA is given by

$$\xi_2^2 = \frac{[2(1+\mu_2)-\gamma][(1+\mu_2-\gamma)\beta^4-(1+\lambda^2+\mu_2\lambda^2)\beta^2+\lambda^2][2(1+\mu_2-\gamma)\beta^2-(1+\lambda^2+\mu_2\lambda^2)]-\gamma(\beta^2-\lambda^2)}{2\gamma\lambda^2-2\lambda^2[2(1+\mu_2)-\gamma][1-(1+\mu_2)\beta^2][1-3(1+\mu_2)\beta^2]} \tag{32}$$

Substituting Eqs. (22) and (23) into Eq. (32), the optimum damping of LCVA at points P and Q is given by

$$\begin{aligned} \xi_P^2 &= \frac{\gamma}{8(1+\mu_2)}\left[3-\sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] \\ \xi_Q^2 &= \frac{\gamma}{8(1+\mu_2)}\left[3+\sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] \end{aligned} \tag{33}$$

The optimum damping ratio of LCVA can be taken as the average value of ξ_P^2 and ξ_Q^2, which is given as

$$\xi_2^2 = \frac{3\gamma}{8(1+\mu_2)} \tag{34}$$

Substituting (λ, β_P, ξ_P) or (λ, β_Q, ξ_Q) into Eq. (15), one would obtain the common ordinate of the displacement response of tuned vibration absorber at the frequency ratios of β_P and β_Q.

$$\left|\frac{\bar{X}_2}{\bar{f}/k_1}\right|_{P,Q} = \frac{\mu_3}{\mu_1}\frac{1+\mu_2}{\gamma} \tag{35}$$

It should be noted that the approach with substitution of Eq. (30) into Eq. (29) saves an undue amount of labor work. The determination of stationary point by a direct solving of Eq. (25) could lead to a prohibitive complexity as indicated by Liu and Liu (2005). Eq. (24) shows that the displacement response of structure decreases as the efficiency index γ increases. This suggests that the performance of LCVA is enhanced if increasing the value of the efficiency index. If the same amount of mass is used for TMD, TLCD and LCVA and the same length ratio is adopted for TLCD and LCVA, the following inequality always holds regardless of the area ratio r_A of LCVA (Wu and Shen 2004, Taflanidis *et al*. 2007).

$$\gamma_{LCVA} < \gamma_{TLCD} < \gamma_{TMD} \tag{36}$$

As indicated by the above inequality relation, if the same mass and length ratios are used for TLCD and LCVA, the control performance of TLCD is always better than non-uniform LCVA. On the other hand, the above inequality relation also indicates that TMD always performs better than TLCD under the same mass ratio.

By substituting the efficiency index of TMD γ_{TMD} into Eqs. (22), (33) and (34), the optimum frequency tuning ratio and the optimum damping ratio of TMD can be determined by the following expressions.

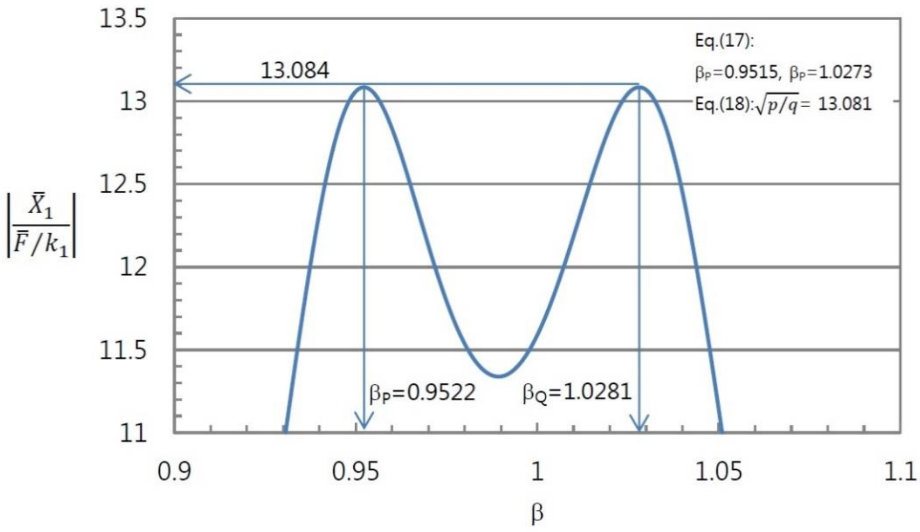


Fig. 3 Steady state response of structure with optimum LCVA installed (μ=0.02, r_A =0.8, r_A=0.5, λ=0.9844, ξ_2=0.0660)

Table 1 Optimal parameters of LCVA for various mass ratios, liquid column length ratios, and area ratios

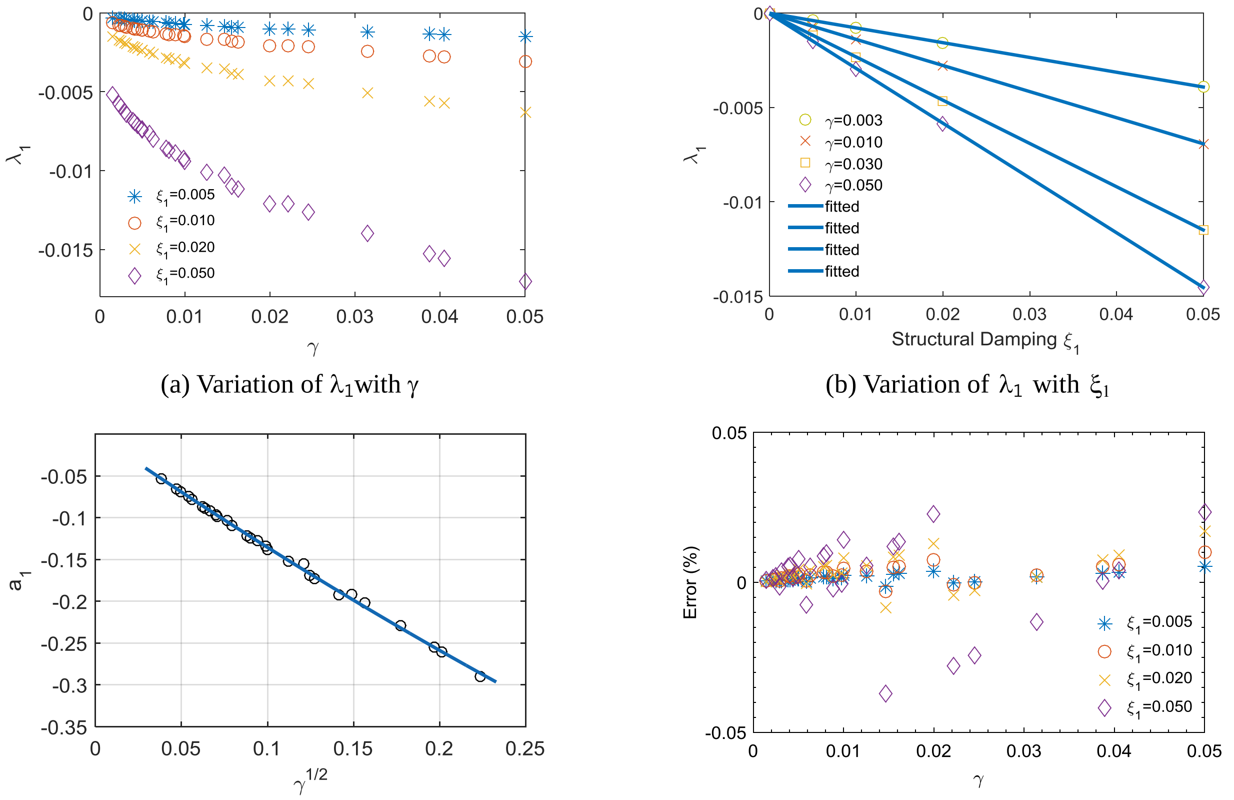
μ	α	r_A	Explicit Formulas		Numerical Method		Maximum Difference (%)
			λ	ξ_2	λ	ξ_2	
0.02	0.7	0.5	0.9858	0.0571	0.9858	0.0571	0.00%
0.02	0.7	1.0	0.9854	0.0600	0.9854	0.0600	0.01%
0.02	0.9	0.5	0.9826	0.0755	0.9826	0.0755	0.01%
0.02	0.9	1.0	0.9823	0.0772	0.9823	0.0772	0.01%
0.05	0.7	0.5	0.9655	0.0890	0.9655	0.0890	0.00%
0.05	0.7	1.0	0.9644	0.0935	0.9644	0.0935	0.00%
0.05	0.9	0.5	0.9577	0.1176	0.9577	0.1177	0.01%
0.05	0.9	1.0	0.9569	0.1203	0.9569	0.1203	0.01%

$$\lambda = \frac{1}{1+\mu} \tag{37}$$

$$\xi_{P,Q}^2 = \frac{\mu}{8(1+\mu)}\left[3 \pm \sqrt{\frac{\mu}{2+\mu}}\right] \qquad \xi_2^2 = \frac{3\mu}{8(1+\mu)} \tag{38}$$

which are same as the classic results obtained by Brock (1946).

The derived formulae are verified through a comparison with results obtained from numerical method. The optimum tuning ratio λ and damping ratio ξ_2 of LCVA are numerically determined for specified mass ratio μ, length ratio α, area ratio r_A, and structural damping ratio ξ_1. The response curves as a function of excitation frequency are first computed by substituting a set of λ and ξ_2 into Eq. (14). For a fixed value of λ, the maximum amplitudes of response curves for different values of ξ_2 are found, and the minimum value of these maximum amplitudes is selected, which is the minimax amplitude for that value of λ. Then, the above process is repeated for different value of λ to find the minimax of each λ. The set of λ and ξ_2 corresponding to the smallest value of the minimaxes is the optimum parameters of LCVA. The optimum parameters of LCVA for various mass ratios, liquid column and length ratios are tabulated in Table 1. It is found that the difference between the results derived in this study and the results from numerical method is less than 0.01%. This indicates that the formulae derived in this study are of the same order of accuracy as those from numerical method. Verifications are carried out further by investigating the frequency response curve of an undamped structure equipped with LCVA under harmonic forced motion. The parameters of LCVA are taken as μ = 0.02, α = 0.8, r_A = 0.5. The frequency tuning and the damping of LCVA are adjusted to the optimum values as given by Eqs. (22) and (34) for forced motion. It can be seen in Fig. 3 that the frequency response curve of undamped structure equipped with LCVA under harmonic forced motion has two equal peaks, whose amplitude match very well with the results given by Eq.



(a) Variation of λ_1 with γ (b) Variation of λ_1 with ξ_1

(c) Fitting curve of a_1 (d) Error of explicit formula

Fig. 4 Curve fitting of optimal tuning ratio

(24). The locations of the two peaks (β_P, β_Q) also match very well with the results given by Eq. (23).

3.2 Damped structure

For the case of damped primary structure, invariant points no longer exist. The closed-form solution of the optimal parameters for damped structures cannot be obtained using fixed-point theory as used for undamped structures. Nevertheless, the optimal condition of a vibration absorber for the case of a damped primary structure can still be achieved numerically when the two resonant peaks in the frequency response curve of structural displacement are of equal amplitude. Numerical searching can be conducted to determine the optimum parameters of tuned vibration absorber. Explicit formulae for these optimum parameters are then obtained by a series of curve fitting techniques. In this study, the optimal parameters of tuned vibration absorber are numerically determined for the case of ξ_1 = 0, 0.005, 0.010, 0.020 and 0.050 and μ_2 = 0.005, 0.010, 0.020 and 0.050 and α = 0.7, 0.9, and 1.0. For $\alpha \neq 1$, r_A= 0.2, 0.5 and 1.0. Hence, numerical search of the optimal parameters of tuned vibration absorber are conducted for a total of 140 cases and the range of the corresponding efficiency index γ is from 0.0015 to 0.0500. These set of parameters cover three types of tuned vibration absorbers, TMD, TLCD and LCVA with various mass ratio or U-shaped container geometry considered.

For a damped primary structure without tuned vibration absorber subjected to force excitation, the frequency ratio at resonant is $\sqrt{1-\xi_1^2}$. The optimal tuning ratio should be very close to this value when the mass ratio of a tuned vibration absorber-structure system is very small. When the damping of primary structure is equal to zero, the optimal tuning ratio should be given by Eq. (22). Therefore, the optimal tuning ratio of tuned vibration absorber for damped primary structure can be approximately determined by the following expression.

$$\lambda = \frac{\sqrt{1+\mu_2-\gamma}}{1+\mu_2} + \sqrt{1-2\xi_1^2} - 1 + \lambda_1 \tag{39}$$

where λ_1 is the differences between the optimal tuning ratio of LCVA for undamped structure and damped structure and is function of structural damping and efficiency index. The variation of λ_1 with the efficiency index is plotted in Fig. 4(a) and the variation of λ_1 with structural damping ξ_1 for some fixed value of efficiency index is plotted in Fig. 4(b). It is found that λ_1 has a linear relation with structural damping ξ_1 for same efficiency index and therefore the following equation holds for tuned vibration absorber with same efficiency index.

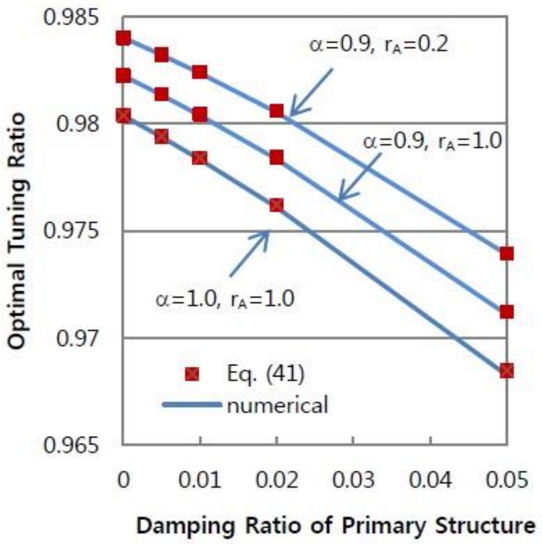
$$\lambda_1 = a_o + a_1\xi_1 \tag{40}$$

As shown in Fig. 4(b), regression analysis of λ_1 on

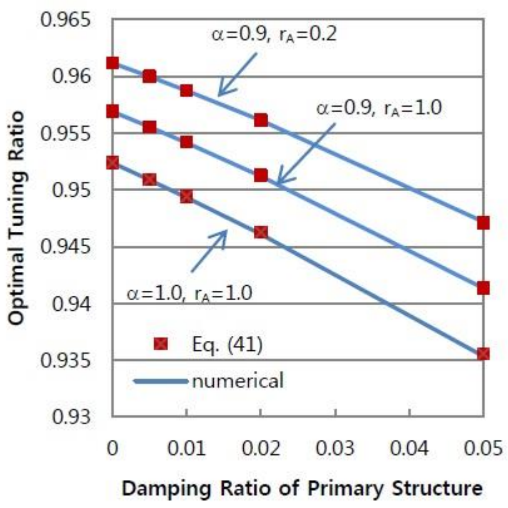
ξ_1 indicate that values of a_o are negligible, generally having an order of 10^{-5}. The value of a_1 is the slope of the linear relation of λ_1 with structural damping ξ_1 for same value of γ. This process is repeated for a range of γ. The variation of a_1 with ξ_1 is then fitted by a polynomial of $\gamma^{\frac{1}{2}}$, as shown in Fig. 4(c). It is found that the parameter a_1 is function of efficiency index γ and the relation can be determined by a curve fitting technique. Combining Eqs. (39) and (40) together with the fitted relation of a_1 with $\gamma^{\frac{1}{2}}$, the optimal tuning ratio of LCVA for damped structure is given by

$$\lambda = \frac{\sqrt{1+\mu_2-\gamma}}{1+\mu_2} + \sqrt{1-2\xi_1^2} - 1 + (0.6213\gamma - 1.4184\gamma^{0.5})\xi_1 \quad (41)$$

The differences between Eq. (41) and the numerical searched values are shown in Fig. 4(d). It can be seen that the maximum error is less than 0.04%. Eq. (41) is also applicable to the case of TMD, TLCD and LCVA for different values of absorber parameters with efficiency index γ up to a value of 0.05.



(a) $\mu = 0.02$



(b) $\mu= 0.05$

Fig. 5 Structural damping effects on optimal tuning ratio of tuned vibration absorber

The accuracy of Eq. (41) would gradually decrease when the efficiency index γ is increased beyond value of 0.05. The structural damping effect on the optimal tuning ratio of a tuned vibration absorber is shown in Fig. 5 for mass ratios of 0.02 and 0.05.

The optimal tuning ratio decreases as the structural damping ratio is increased from 0 to 0.05. In reality, the inherent structural damping is unlikely to exceed a value of 0.02. To better demonstrate the accuracy of Eq. (41), the upper bound of structural damping is extended to 0.05 and results from Eq. (41) are in good agreement with the searched values from the numerical method. The same numerical optimization method described in section 3 was used to determine the optimum parameters of LCVA for damped structures. A comparison of results from Wu et al. (2009) is presented in Table 2. It is found that the results from Eq. (41) are in good agreement with those from Wu *et al.* (2009).

Table 3 compares the optimal tuning ratio of a TMD for suppressing displacement induced by harmonic excitation. The comparison includes the results from the numerical method and Eq. (41) as well as the results found in the literature. As there is no previous study on the explicit optimum damping of LCVA for the case of a damped primary structure, the results obtained from Eq. (41) could only be compared with the results from the previous studies on tuned mass damper (TMD). By substituting $\gamma = \mu$ and $\mu_2 = \mu$ into Eq. (41), the optimum tuning ratio of TMD is obtained. It can be seen from Table 3 that the deviations between the results from Eq. (41) and those from the numerical method are very small for the range of mass ratios and damping ratios considered. The number in the bracket represents the relative difference between the result from the method of that column and the result from numerical method. The maximum deviation between the results from Eq. (41) and those from the numerical method is found to be about 0.02% when the mass ratio is 0.05 and the damping ratio of primary structure is 0.05. The maximum deviation for the empirical formula proposed by Ioi and Ikeda (1978) is about 0.41% when the mass ratio is 0.02 and the damping ratio of primary structure is 0.05. The discrepancy between the results from numerical method and the results from the empirical formula proposed by Ioi and Ikeda (1978) apparently decreases with the increasing mass ratio. The maximum deviation of the explicit formula derived by Ghosh and Basu (2007b) is about 1.32% when the mass ratio is 0.05 and the damping ratio of primary structure is 0.05. The discrepancy between the results from numerical method and the results from the explicit formula derived by Ghosh and Basu (2007b) apparently increases with the increasing mass ratio. Table 3 also shows that the optimum tuning ratio of TMD decreases as the structural damping is increased. Eq. (41) provides a better estimation of optimum tuning ratio of TMD when comparing with the formulas available in the existing literatures.

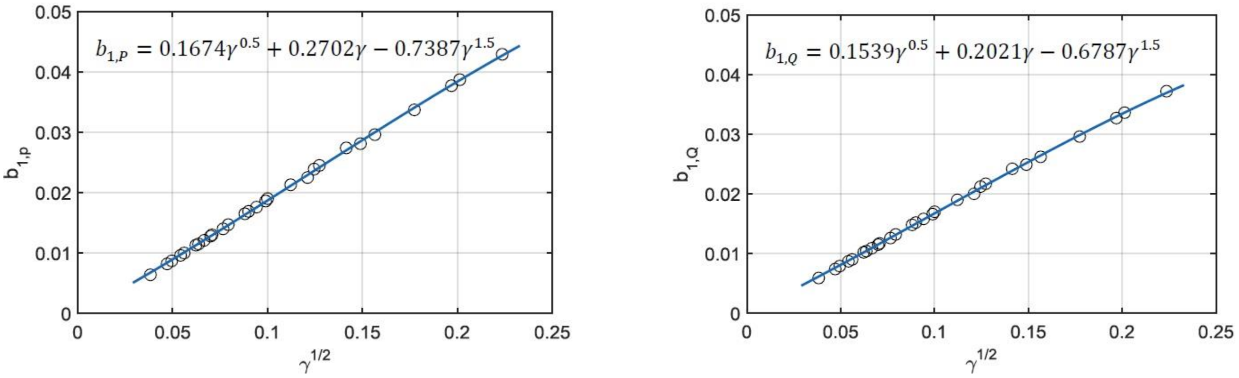
The optimal damping ratio of LCVA for damped primary structure at each optimized resonant peak can be approximated by the following expression.

Table 2 Comparison of optimum tuning ratio of a LCVA under harmonic excitation

μ	α	r_A	ξ_1	Wu *et al.* (2009)	Eq. (41)
0.02	0.7	1	0.02	0.9823	0.9825 (0.02%)
0.02	0.9	1	0.02	0.9783	0.9786 (0.04%)
0.05	0.7	1	0.02	0.9599	0.9601 (0.02%)
0.05	0.9	1	0.02	0.9512	0.9515 (0.03%)
0.02	0.7	2	0.02	0.9829	0.9831 (0.02%)
0.02	0.9	2	0.02	0.9788	0.9791 (0.03%)
0.05	0.7	2	0.02	0.9612	0.9614 (0.02%)
0.05	0.9	2	0.02	0.9521	0.9524 (0.03%)

Table 3 Comparison of optimum tuning ratio of a TMD under harmonic excitation

μ	ξ_1	numerical	Eq. (41)	Ioi *et al.* (1978)	Basu *et al.* (2007b)
0.02	0.000	0.9804	0.9804 (0.00%)	0.9804 (0.00%)	0.9804 (0.00%)
0.02	0.005	0.9794	0.9794 (0.00%)	0.9790 (-0.04%)	0.9803 (0.10%)
0.02	0.010	0.9783	0.9784 (0.01%)	0.9776 (-0.08%)	0.9802 (0.19%)
0.02	0.020	0.9761	0.9762 (0.01%)	0.9745 (-0.16%)	0.9796 (0.36%)
0.02	0.050	0.9683	0.9685 (0.02%)	0.9643 (-0.41%)	0.9755 (0.75%)
0.05	0.000	0.9524	0.9524 (0.00%)	0.9524 (0.00%)	0.9524 (0.00%)
0.05	0.005	0.9509	0.9509 (0.01%)	0.9508 (-0.01%)	0.9523 (0.15%)
0.05	0.010	0.9493	0.9494 (0.01%)	0.9491 (-0.02%)	0.9522 (0.30%)
0.05	0.020	0.9461	0.9463 (0.02%)	0.9456 (-0.05%)	0.9516 (0.59%)
0.05	0.050	0.9354	0.9356 (0.02%)	0.9341 (-0.13%)	0.9477 (1.32%)



(a) Fitting curve of a_1 for ε_P (b) Fitting curve of a_1 for ε_Q

Fig. 6 Curve fitting of optimal damping ratio

$$\xi_{P,Q}^2 = \frac{\gamma}{8(1+\mu_2)}\left[3 \pm \sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] + \varepsilon_{P,Q} \quad (42)$$

where $\varepsilon_{P,Q}$ are the differences between the optimal damping ratio of LCVA for undamped structure and damped structure and are functions of structural damping and efficiency index. It is found that the functions $\varepsilon_{P,Q}$ also have a linear relation with the structural damping for tuned vibration absorber with same efficiency index and therefore the following equation holds.

$$\varepsilon_P = b_{o,P} + b_{1,P}\xi_1 \qquad \varepsilon_Q = b_{o,Q} + b_{1,Q}\xi_1 \quad (43)$$

Regression analysis indicated that the values of $b_{o,P}$ and $b_{o,Q}$ are negligible, with an order of 10^{-5}. The variation of $b_{1,P}$ and $b_{1,Q}$ with $\gamma^{\frac{1}{2}}$ is shown in Fig. 6. Combining Eq. (42) and Eq. (43) together with the relation of $b_{1,P}$ and $b_{1,Q}$ with $\gamma^{\frac{1}{2}}$ determined by curve fitting technique, the optimal damping ratio of LCVA for damped structure is given by

$$\xi_P^2 = \frac{\gamma}{8(1+\mu_2)}\left[3 - \sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] + (0.1674\gamma^{0.5} + 0.2702\gamma - 0.7387\gamma^{1.5})\xi_1 \quad (44a)$$

$$\xi_Q^2 = \frac{\gamma}{8(1+\mu_2)}\left[3+\sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] + (0.1539\gamma^{0.5}+0.2021\gamma - 0.6787\gamma^{1.5})\xi_1 \quad (44b)$$

When applying Eq. (44), it should be noted that Eq. (44) is applicable to the case of TMD, TLCD and LCVA for different values of absorber parameters with efficiency index γ up to a value of 0.05. The accuracy of Eq. (44) would gradually decrease when the efficiency index γ is beyond value of 0.05.

Fig. 7 compares the optimal damping ratio of a TMD for suppressing displacement induced by harmonic excitation. The comparison includes the results from the numerical method and Eq. (44) as well as the results from the empirical formula proposed by Ioi and Ikeda (1978). By substituting $\gamma = \mu$ and $\mu_2 = \mu$ into Eq. (44), the optimum damping ratio of TMD is obtained by taking the average of ξ_P and ξ_Q. Fig. 7 shows that the optimal damping ratios of a TMD can be predicted quite well by either Eq. (44) or by the empirical formula proposed by Ioi and Ikeda (1978). It can also be seen from Fig. 7 that the discrepancy between the results from numerical method and the results from Eq. (44) is very small. For the range of mass ratios and damping ratios being considered, the maximum deviation between the results from Eq. (44) and those from the numerical method is found to be about 0.21% when the mass ratio is 0.02 and the damping ratio of primary structure is 0.02. The maximum discrepancy of the empirical formula proposed by Ioi and Ikeda (1978) is about 0.73%. Fig. 7 also shows that the optimum damping ratio of TMD increases as the structural damping is increased.

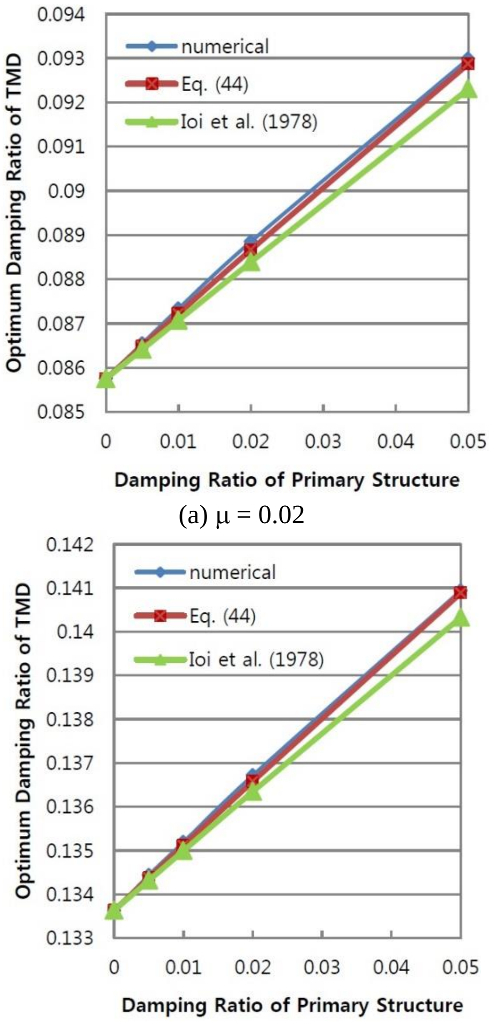
The explicit formulas for the ordinates of two resonant peaks in the frequency response curve and the corresponding maximum response at two resonant peaks are determined by the same curve fitting technique described above and are given by the following expressions.

$$\beta_P^2 = \frac{1}{1+\mu_2}\left[1-\sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] - (0.395+3.119\gamma^{0.5}-6.1957\gamma)\xi_1 \quad (45a)$$

$$\beta_Q^2 = \frac{1}{1+\mu_2}\left[1+\sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}\right] + (0.4952-0.7812\gamma^{0.5}-3.057\gamma)\xi_1 - (6.0086-23.467\gamma^{0.5}+49.346\gamma)\xi_1^2 \quad (45b)$$

$$\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right|_{P,Q} = \frac{1}{2\xi_1+\sqrt{\frac{\gamma}{2(1+\mu_2)-\gamma}}} - \left(9.7136-\frac{4.6648}{\gamma^{0.5}}+\frac{0.0168}{\gamma}\right)\xi_1 + \left(250.32-\frac{79.91}{\gamma^{0.5}}-\frac{0.0153}{\gamma}\right)\xi_1^2 \quad (46)$$

Regression analysis indicated that the ordinate of the second resonant peak β_Q and the maximum response at two resonant peaks have a quadratic relation with the structural damping. For the range of efficiency index being included in this study, the maximum error of Eq. (45(a)) or Eq. (45(b)) is about 0.06% while the maximum error of Eq. (46) is about 2.4%.



(a) μ = 0.02

(b) μ= 0.05

Fig. 7 Comparison of optimal damping of a TMD under harmonic excitation

3.3 Optimal head loss coefficient

Energy dissipation due to quadratic damping of liquid motion is governed by head loss coefficient induced by an orifice placed in the container of LCVA. In order to maximize the energy dissipation, it is essential to estimate the optimal head loss coefficient under a given loading condition. However, as the damping of LCVA is nonlinear, the optimal damping derived from linearized equation of LCVA-structure system could not be applied directly. An equivalent damping coefficient is then introduced by using equivalent linearization method and the equivalent damping coefficient c_{eq} could be determined by equating energy dissipated by nonlinear damping force with energy dissipated by linear equivalent damping force in a cycle and it is given by (Gao *et al*. 1997)

$$c_{eq} = \frac{4}{3\pi}\rho A_v \delta \tilde{\omega} |\bar{X}_2| \quad (47)$$

where $|\bar{X}_2|$ is the steady state response amplitude of liquid displacement; $\tilde{\omega}$ is the frequency of external applied force

Table 4 Optimal head loss coefficient of LCVA for various mass ratios, liquid column length ratios, and area ratios

ξ_1	μ	α	r_A	C	Explicit Formulas		Numerical Approach		Maximum Difference
					λ	δ	λ	δ	
0.00	0.02	0.7	0.5	0.00025	0.9858	17.83	0.9859	17.83	0.03%
0.00	0.02	0.7	0.5	0.00050	0.9858	8.92	0.9859	8.93	-0.20%
0.00	0.02	0.9	0.5	0.00025	0.9826	27.24	0.9828	27.27	-0.10%
0.00	0.02	0.9	0.5	0.00050	0.9826	13.62	0.9825	13.64	-0.17%
0.00	0.02	0.7	1.0	0.00025	0.9854	31.88	0.9855	31.97	-0.28%
0.00	0.02	0.7	1.0	0.00050	0.9854	15.94	0.9855	15.95	-0.06%
0.00	0.02	0.9	1.0	0.00025	0.9823	52.93	0.9826	52.96	-0.05%
0.00	0.02	0.9	1.0	0.00050	0.9823	26.47	0.9825	26.51	-0.18%
0.02	0.02	0.7	0.5	0.00025	0.9829	30.79	0.9830	31.19	-1.27%
0.02	0.02	0.7	0.5	0.00050	0.9829	15.40	0.9830	15.58	-1.16%
0.02	0.02	0.9	0.5	0.00025	0.9789	41.90	0.9791	42.48	-1.37%
0.02	0.02	0.9	0.5	0.00050	0.9789	20.95	0.9790	21.20	-1.17%
0.02	0.02	0.7	1.0	0.00025	0.9823	53.84	0.9824	54.55	-1.31%
0.02	0.02	0.7	1.0	0.00050	0.9823	26.92	0.9824	27.31	-1.42%
0.02	0.02	0.9	1.0	0.00025	0.9784	80.76	0.9787	81.60	-1.03%
0.02	0.02	0.9	1.0	0.00050	0.9784	40.38	0.9787	40.82	-1.08%

or support motion. Eq. (47) indicates that when the level of external excitation is decreased, a smaller liquid displacement response is induced and a larger head loss coefficient is required to achieve the optimal condition. Re-arranging Eq. (47) with some manipulation, the head loss coefficient at the two fixed points is given by

$$\delta_{P,Q} = \frac{3\pi g}{\lambda\omega_1^2}\frac{\xi_{P,Q}}{\beta_{P,Q}|\bar{X}_2|_{P,Q}} \tag{48}$$

The optimal head loss coefficient can be taken as the average value of δ_P^2 and δ_Q^2, which is given as

$$\delta^2 = \frac{1}{2\left|\frac{\bar{X}_2}{\bar{f}/k_1}\right|^2_{\beta_{P,Q}}}\left(\frac{3\pi}{\lambda C}\right)^2\left(\frac{\xi_P^2}{\beta_P^2}+\frac{\xi_Q^2}{\beta_Q^2}\right) \tag{49}$$

where the excitation force amplitude is expressed in term of fraction of the weight of structure, $\bar{F} = Cm_1 g$ for the case of forced motion. For an undamped structure, the optimum head loss coefficient of LCVA can be determined by the following expression with the substitution of Eqs. (23), (33) and (35) into Eq. (49).

$$\delta = \frac{3\pi}{2\sqrt{2}C}\frac{\mu\alpha\chi_1\sqrt{[3(1+\mu)-2\mu\alpha^2\chi]\mu\alpha^2\chi}}{(1+\mu-\mu\alpha^2\chi)} \tag{50}$$

Solving the linearized equation of motion of liquid displacement, the amplitude of liquid displacements $|\bar{X}_2|$ is determined by the following equation.

$$\left|\frac{\bar{X}_2}{\bar{f}/k_1}\right| = \frac{\chi_2\alpha\beta^2}{\sqrt{(\lambda^2-\beta^2)^2+4\lambda^2\beta^2\xi_2^2}}\left|\frac{\bar{X}_1}{\bar{f}/k_1}\right| \tag{51}$$

The expression for estimating the optimal head loss coefficient of LCVA is verified with the results obtained from numerical method. The response curves as function of excitation frequency were numerically determined by solving the original nonlinear equation in time domain using Wilson-θ method. The time step and the total time duration used in time domain analysis were 0.005 sec and 500 sec. The optimal parameters of λ and δ were then determined by searching the corresponding smallest minimaxes. Table 4 summarizes the optimal head loss coefficient of LCVA for various mass ratios, liquid column length ratios, and area ratios. It is found that the difference between the optimal head loss coefficient determined by the explicit formulas and by numerical method is less than 1.5% when damping ratio is 0.02. For the case of undamped structure, the results from explicit expressions are in good agreement with those from numerical optimization method, with the maximum difference less than 0.3%. This indicates that the developed explicit formulae are accurate for designing LCVA for suppressing harmonic vibration of damped structures. Table 4 also shows that the optimum head loss coefficient increases as the structural damping is increased.

4. Conclusions

Explicit formulae for determining the optimum absorber parameters for a linear damped primary structure under harmonic force excitation were developed in this study using a series of curve fitting technique. The formulae are applicable to the design of tuned mass damper (TMD), tuned liquid column damper (TLCD), and liquid column vibration absorber (LCVA) for abating the vibration of damped structure under harmonic excitation. It was demonstrated the control performance of TLCD for reducing harmonic vibration of undamped structure is always better than that of non-uniform LCVA for same mass and length ratios. As for the effects of structural damping on the optimum parameters, it was found that the optimum tuning ratio decreases and the optimal damping ratio increases as the structural damping is increased. Furthermore, the optimum head loss coefficient is inversely proportional to the amplitude of excitation force and increases as the structural damping is increased. Numerical verification of the developed explicit design expressions was also conducted and the developed expressions were found to be reasonably accurate for design purposes.

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