Smart Structures and Systems, Vol. 2, No. 4 (2006) 295-304
DOI: http://dx.doi.org/10.12989/sss.2006.2.4.295

Analysis of layered bases-foundations models under seismic actions

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(Received December 19, 2005, Accepted April 27, 2006)

Abstract. The paper considers the dynamic behaviour of the two-layered and multi-layered plate packets under dynamic (seismic) loading. These models correspond to the base-foundation packet structures. The analysis of the various models, including the models of contact between the layers, is derived on the base of the precise solutions of elasticity theory equations. It is shown that the application of the seismoisolator or, in general, less rigid materials between the base and the foundation brings to reduction of the natural frequencies of free vibrations of the packet base-foundation, as well as to the significant reduction of the negative seismic effect on the structures.

Keywords: free vibrations; seismoisolator; frequencies; layered plates.

1. Introduction

In the modern building practice the packet base-foundation is layered, in connection with layerity of the base (the local relief), and artificial, due to the application of the seismoisolators and other inclusions. The previously developed models result in the solution of the one-dimensional problems (Kelly 1997), yet such models do not always yield the true dynamic picture. A more detailed approach brings to the solution of the space boundary problems of elasticity and viscoelasticity theory for a layered beam-like or plate-like packet. An effective asymptotic method has been developed to solve these problems, (Aghalovyan 1997). A number of static and dynamic problems for layered structures modelling the behaviour of the seismoisolators and the base-foundation packages has been solved by this method (Aghalovyan and Gevorgyan 2000, Aghalovyan, *et al.* 2001, 2003, 2004).

In this paper, the classes of the space problems for the two-layered, three-layered and multilayered plates under full and incomplete (Coulomb) contact between the layers are considered. These models correspond to the packet "foundation-seismoisolator-base". The obtained exact solution proves the necessity of using seismoisolators for responsible structures like schools, hospitals and others.

2. The behaviour of a two-layered plate-like packet under a full contact between the layers

Let's consider forced vibrations of a two-layered packet $D=\{(x, y, z):((x, y) \in D_0, -h_2 \le z \le h_1)\}$ constructed from the orthotropic plates with of thickness h_1 and h_2, respectively. The parameters related

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to the first and second layer are denoted by the indices I and II, respectively.

The dynamic analysis requires the solution of the dynamic equations of elasticity theory for each layer. The equations take the form

$$\frac{\partial \sigma_{xx}^{k}}{\partial x} + \frac{\partial \sigma_{xy}^{k}}{\partial y} + \frac{\partial \sigma_{xz}^{k}}{\partial z} = \rho_k \frac{\partial^2 u^k}{\partial t^2} \qquad (x, y, z; y, v, w)$$

$$\frac{\partial u^k}{\partial x} = a_{11}^{k} \sigma_{xx}^{k} + a_{12}^{k} \sigma_{yy}^{k} + a_{13}^{k} \sigma_{zz}^{k}$$

$$\frac{\partial v^k}{\partial y} = a_{12}^{k} \sigma_{xx}^{k} + a_{22}^{k} \sigma_{yy}^{k} + a_{23}^{k} \sigma_{zz}^{k}$$

$$\frac{\partial w^k}{\partial z} = a_{13}^{k} \sigma_{xx}^{k} + a_{23}^{k} \sigma_{yy}^{k} + a_{33}^{k} \sigma_{zz}^{k}$$

$$\frac{\partial u^k}{\partial y} + \frac{\partial v^k}{\partial x} = a_{66}^{k} \sigma_{xy}^{k}$$

$$\frac{\partial w^k}{\partial x} + \frac{\partial u^k}{\partial z} = a_{55}^{k} \sigma_{xz}^{k}$$

$$\frac{\partial w^k}{\partial y} + \frac{\partial v^k}{\partial z} = a_{44}^{k} \sigma_{yz}^{k}, \qquad k = I, II \qquad (1)$$

where σ_{ij} are the components of the stress tensor, u, v, w are the components of the displacement vector, a_{ij} are the elasticity constants, ρ is the density, D_0 is the surface of the layers section with the characteristic tangential size $l >> h$, $h = \max(h_1, h_2)$. In earthquakes, vibrations of the structure arise due to vibration of the facial surface $z = -h_2$ of the lower layer. We suppose that vibration is harmonic, that is

$$u^{II}(-h_2) = u^{-}(\xi, \eta)\exp(i\Omega t), \quad \xi = x/l, \eta = y/l$$

$$v^{II}(-h_2) = v^{-}(\xi, \eta)\exp(i\Omega t)$$

$$w^{II}(-h_2) = w^{-}(\xi, \eta)\exp(i\Omega t) \qquad (2)$$

where Ω is the frequency of external forcing. Facial surface $z = h_1$ of the first layer is supposed fixed, that is

$$u^{I}(h_1) = v^{I}(h_1) = w^{I}(h_1) = 0 \qquad (3)$$

or free

$$\sigma_{xz}^{I}(h_1) = \sigma_{yz}^{I}(h_1) = \sigma_{zz}^{I}(h_1) = 0 \qquad (4)$$

We consider two types of the contact between the layers, the full contact

$$u^{I}(z = 0) = u^{II}(z = 0), v^{I}(z = 0) = v^{II}(z = 0), w^{I}(z = 0) = w^{II}(z = 0)$$

$$\sigma_{xz}^{I}(z = 0) = \sigma_{xz}^{II}(z = 0), \sigma_{yz}^{I}(z = 0) = \sigma_{yz}^{II}(z = 0), \sigma_{zz}^{I}(z = 0) = \sigma_{zz}^{II}(z = 0) \qquad (5)$$

and the incomplete contact (Coulomb friction)

$$w^I(z=0) = w^{II}(z=0),\ \sigma_{zz}^I(z=0) = \sigma_{zz}^{II}(z=0)$$

$$\sigma_{xz}^I(z=0) = \sigma_{xz}^{II}(z=0) = f_1\sigma_{zz}^I(z=0)$$

$$\sigma_{yz}^I(z=0) = \sigma_{yz}^{II}(z=0) = f_2\sigma_{zz}^I(z=0) \tag{6}$$

where f_1, f_2 are the coefficients of Coulomb friction. The conditions on the lateral area of the packet are uncertain. However, the effect of the lateral boundary conditions on the internal strain-stress state decreases rapidly and can be ignored in practice (Kelly 1997, Aghalovyan 1997).

The solution of the formulated dynamic problems will be sought in the form

$$(u^k, v^k, w^k) = [u_x^k(x,y,z), u_y^k(x,y,z), u_z^k(x,y,z)]\exp(i\Omega t)$$

$$\sigma_{\alpha\beta}^k(x,y,z,t) = \sigma_{jm}^k(x,y,z)\exp(i\Omega t),\ \alpha, \beta = x, y, z;\ j, m = 1, 2, 3 \tag{7}$$

Introducing the small parameter $\varepsilon = h/l$, considering the dimensionless coordinated and displacements

$$\xi = x/l,\ \eta = y/l,\ \zeta = z/h$$

$$U^k = u_x^k/l,\ V^k = u_y^k/l,\ W^k = u_z^k/l \tag{8}$$

and substituting (7) into the transformed equations (1), we obtain a singularly perturbed system. The asymptotic solution of this system takes the form (Aghalovyan 1997, Aghalovyan, *et al.* 2001)

$$\sigma_{ij}^k = \varepsilon^{-1+s}\sigma_{ij}^{(k,s)}(\xi, \eta, \zeta),\ U^k = \varepsilon^s U^{(k,s)}(\xi, \eta, \zeta),\ (U, V, W),\ s = \overline{0, N},\ k = I, II \tag{9}$$

After substituting (9) into the above mentioned system and setting equal the coefficients at the similar degrees of the parameter ε, we obtain the recurrent formulas for the coefficients $\sigma_{ij}^{(k,s)}$, $U^{(k,s)}$, $V^{(k,s)}$, $W^{(k,s)}$, and find $\sigma_{ij}^{(k,s)}$ as the functions of the displacements. If conditions (2) - (6) hold, the coefficients (9) can be determined.

If $u^-, v^-, w^- = const$, the exact solution can be found. In this case, under conditions (2), (3), (5), we have

$$U^I = \sqrt{\frac{\rho_{II}}{a_{55}^{II}}}\frac{u^-}{l}\frac{\Omega_*}{\Delta_1}\sin a_1^I\Omega_*(\zeta_1 - \zeta)$$

$$V^I = \sqrt{\frac{\rho_{II}}{a_{44}^{II}}}\frac{v^-}{l}\frac{\Omega_*}{\Delta_2}\sin a_2^I\Omega_*(\zeta_1 - \zeta)$$

$$W^I = \sqrt{A_{11}^{II}\rho_{II}}\frac{w^-}{l}\frac{\Omega_*}{\Delta_1}\sin a_3^I\Omega_*(\zeta_1 - \zeta) \tag{10}$$

if $0 \le \zeta \le \zeta_1$, $\zeta_1 = h_1/h$, and

$$U^{II} = \frac{u^-}{l}\frac{\Omega_*}{\Delta_1}\left(\sqrt{\frac{\rho_{II}}{a_{55}^{II}}}\sin a_1^I\Omega_*\zeta_1\cos a_1^{II}\Omega_*\zeta - \sqrt{\frac{\rho_I}{a_{55}^I}}\cos a_1^I\Omega_*\zeta_1\sin a_1^{II}\Omega_*\zeta\right)$$

$$V^{II} = \frac{v^-}{l}\frac{\Omega_*}{\Delta_2}\left(\sqrt{\frac{\rho_{II}}{a_{44}^{II}}}\sin a_2^I\Omega_*\zeta_1\cos a_2^{II}\Omega_*\zeta - \sqrt{\frac{\rho_I}{a_{44}^I}}\cos a_2^I\Omega_*\zeta_1\sin a_2^{II}\Omega_*\zeta\right)$$

$$W^{II} = \frac{w^-}{l}\frac{\Omega_*}{\Delta_3}\left(\sqrt{A_{44}^{II}\rho_{II}}\sin a_2^I\Omega_*\zeta_1\cos a_3^{II}\Omega_*\zeta - \sqrt{A_{11}^I\rho_I}\cos a_3^I\Omega_*\zeta_1\sin a_3^{II}\Omega_*\zeta\right) \tag{11}$$

if $-\zeta_2 \le \zeta \le 0$, $\zeta_2 = h_2/h$

The stress in the layers are calculated by the formulas

$$\sigma_{13}^k = \frac{1}{a_{55}^k}\frac{\partial U^k}{\partial\zeta}, \qquad \sigma_{23}^k = \frac{1}{a_{44}^k}\frac{\partial V^k}{\partial\zeta}, \qquad \sigma_{33}^k = A_{11}^k\frac{\partial W^k}{\partial\zeta}$$

$$\sigma_{12}^k = 0, \qquad \sigma_{11}^k = -A_{23}^k\frac{\partial W^k}{\partial\zeta}, \qquad \sigma_{22}^k = -A_{13}^k\frac{\partial W^k}{\partial\zeta}, \qquad k = I, II \tag{12}$$

where

$$\Omega_* = h\Omega, \quad a_{55} = \frac{1}{G_{13}}, \quad a_{44} = \frac{1}{G_{23}}$$

$$a_1^k = \sqrt{a_{55}^k\rho_k}, \quad a_2^k = \sqrt{a_{44}^k\rho_k}, \quad a_3^k = \sqrt{\frac{\rho_k}{A_{11}^k}}, \quad k = I, II$$

$$A_{11} = (a_{11}a_{22} - a_{12}^2)/\Delta, \quad A_{13} = (a_{11}a_{23} - a_{12}a_{13})/\Delta$$

$$A_{23} = (a_{22}a_{13} - a_{11}a_{23})/\Delta$$

$$\Delta = a_{11}a_{22}a_{33} + 2a_{12}a_{13}a_{23} - a_{11}a_{23}^2 - a_{22}a_{13}^2 - a_{33}a_{12}^2$$

$$\Delta_1 = \Omega_*\left(\sqrt{\frac{\rho_I}{a_{55}^I}}\cos a_1^I\Omega_*\zeta_1\sin a_1^{II}\Omega_*\zeta_2 + \sqrt{\frac{\rho_{II}}{a_{55}^{II}}}\sin a_1^I\Omega_*\zeta_1\cos a_1^{II}\Omega_*\zeta_2\right), \quad \zeta_2 = h_2/h$$

$$\Delta_2 = \Omega_*\left(\sqrt{\frac{\rho_I}{a_{44}^I}}\cos a_2^I\Omega_*\zeta_1\sin a_2^{II}\Omega_*\zeta_2 + \sqrt{\frac{\rho_{II}}{a_{44}^{II}}}\sin a_2^I\Omega_*\zeta_1\cos a_2^{II}\Omega_*\zeta_2\right)$$

$$\Delta_3 = \Omega_*\left(\sqrt{A_{11}^{II}\rho_I}\cos a_3^I\Omega_*\zeta_1\sin a_3^{II}\Omega_*\zeta_2 + \sqrt{A_{11}^{II}\rho_{II}}\sin a_3^I\Omega_*\zeta_1\cos a_3^{II}\Omega_*\zeta_2\right) \tag{13}$$

G_{13}, G_{23} are the shear modules. According to Eqs. (7), (9) the terminal solution is

$$(u^k, v^k, w^k) = l(U^k, V^k, W^k)\exp(i\Omega t), \quad k = I, II$$

$$\sigma_{\alpha\beta}^k = \frac{l}{h}\sigma_{ij}^k\exp(i\Omega t), \; \alpha, \beta = x, y, z; \quad i, j = 1, 2, 3 \tag{14}$$

The precise solution takes the form

$$U^I = \frac{b_1}{\delta_1}\frac{u^-}{l}\cos a_1^I \Omega_*(\zeta_1 - \zeta), \quad 0 \le \zeta \le \zeta_1$$

$$V^I = \frac{b_2}{\delta_2}\frac{v^-}{l}\cos a_2^I \Omega_*(\zeta_1 - \zeta)$$

$$W^I = \frac{b_3}{\delta_3}\frac{w^-}{l}\cos a_3^I \Omega_*(\zeta_1 - \zeta)$$

$$\sigma_{13}^I = \frac{a_1^{II}}{a_{55}^{II}}\frac{\Omega_*}{\delta_1}\frac{u^-}{l}\sin a_1^I \Omega_*(\zeta_1 - \zeta)$$

$$\sigma_{23}^I = \frac{a_2^{II}}{a_{44}^{II}}\frac{\Omega_*}{\delta_2}\frac{v^-}{l}\sin a_2^I \Omega_*(\zeta_1 - \zeta)$$

$$\sigma_{33}^I = \frac{A_1^{II} a_3^{II}}{\delta_3}\Omega_* \frac{w^-}{l}\sin a_3^I \Omega_*(\zeta_1 - \zeta)$$

$$\sigma_{12}^I = 0, \quad \sigma_{11}^I = -A_{23}^I \frac{\partial W^I}{\partial \zeta}, \quad \sigma_{22}^I = -A_{13}^I \frac{\partial W^I}{\partial \zeta}$$

$$b_1 = \frac{a_{55}^I a_1^{II}}{a_{55}^{II} a_1^I}, \quad b_2 = \frac{a_{44}^I a_2^{II}}{a_{44}^{II} a_2^I}, \quad b_3 = \frac{A_{11}^{II} a_3^{II}}{A_{11}^{II} a_3^I} \tag{15}$$

The parameters

$$\delta_1 = b_1 \cos a_1^I \Omega_* \zeta_1 \cos a_1^{II} \Omega_* \zeta_2 - \sin a_1^I \Omega_* \zeta_1 \sin a_1^{II} \Omega_* \zeta_2$$

$$\delta_2 = b_2 \cos a_2^I \Omega_* \zeta_1 \cos a_2^{II} \Omega_* \zeta_2 - \sin a_2^I \Omega_* \zeta_1 \sin a_2^{II} \Omega_* \zeta_2$$

$$\delta_3 = b_3 \cos a_3^I \Omega_* \zeta_1 \cos a_3^{II} \Omega_* \zeta_2 - \sin a_3^I \Omega_* \zeta_1 \sin a_3^{II} \Omega_* \zeta_2$$

correspond to conditions, Eqs. (2), (4), (5).

For the second layer ($-\zeta_2 \le \zeta \le 0$) we have

$$U^{II} = \frac{\bar{u}}{\delta_1 l}(b_1 \cos a_1^I \Omega_* \zeta_1 \cos a_1^{II} \Omega_* \zeta + \sin a_1^I \Omega_* \zeta_1 \sin a_1^{II} \Omega_* \zeta)$$

$$V^{II} = \frac{v^-}{\delta_2 l}(b_2 \cos a_2^I \Omega_* \zeta_1 \cos a_2^{II} \Omega_* \zeta + \sin a_2^I \Omega_* \zeta_1 \sin a_2^{II} \Omega_* \zeta)$$

$$W^{II} = \frac{\bar{w}}{\delta_3 l}(b_3 \cos a_3^I \Omega_* \zeta_1 \cos a_3^{II} \Omega_* \zeta + \sin a_3^I \Omega_* \zeta_1 \sin a_3^{II} \Omega_* \zeta) \tag{16}$$

The stress is calculated by formula (12) ($k = II$). The terminal solution is determined by formula (14). Solution (10), (11) takes place if $\Delta_1 \neq 0$, $\Delta_2 \neq 0$, $\Delta_3 \neq 0$. This implies $\delta_1 \neq 0$, $\delta_2 \neq 0$, $\delta_3 \neq 0$ for solution, Eqs. (15), (16). If at least one of the equalities

$$\Delta_1 = 0,\ \Delta_2 = 0,\ \Delta_3 = 0 \text{ or } \delta_1 = 0,\ \delta_2 = 0,\ \delta_3 = 0 \tag{17}$$

holds at a frequency Ω, then Ω coincides with the frequency of free vibrations of the packet (Aghalovyan, *et al.* 2001, Aghalovyan and Hovhannissyan 2005). In this case, the system is resonant. If the possible variations of the frequency Ω are known, then, using formulas, Eqs. (13) and (15), one can choose the layers parameters (elastic characteristics, density, thickness) in such a way that equalities (17) are violated. Thus, a proper choice of the layers parameters allows avoiding the resonance risk.

3. The behaviour of the two-layered plate-like packet under incomplete contact (Coulomb friction) between the layers

We obtain the solution of problem, Eqs. (1), (2), (3) under Coulomb friction, Eqs. (6) between the layers. First we take u^-, v^-, w^-. The solution is determined by formulas, Eqs. (7) and (9), in which $\sigma_{ij}^{(k,s)} = 0$, $U^{(k,s)} = 0$, $V^{(k,s)} = 0$, $W^{(k,s)} = 0$ if $s > 0$. Then, we have the following precise solution:

$$U^I = -\frac{f_1 w^- \Omega_* \sqrt{A_{11}^I A_{11}^{II} a_{55}^I \rho_{II}}}{l\Delta_3 \cos a_1^I \Omega_* \zeta_1} \cos a_3^I \Omega_* \zeta_1 \sin a_1^I \Omega_* (\zeta - \zeta_1)$$

$$V^I = -\frac{f_2 w^- \Omega_* \sqrt{A_{11}^I A_{11}^{II} a_{44}^I \rho_{II}}}{l\Delta_3 \cos a_2^I \Omega_* \zeta_1} \cos a_3^I \Omega_* \zeta_1 \sin a_2^I \Omega_* (\zeta - \zeta_1)$$

$$W^I = -\frac{w^- \Omega_* \sqrt{A_{11}^{II} \rho_{II}}}{l\Delta_3} \sin a_3^I \Omega_* (\zeta - \zeta_1) \tag{18}$$

for the first layer ($0 \le \zeta \le \zeta_1$), and

$$U^{II} = \frac{1}{\cos a_1^{II} \Omega_* \zeta_2} \left[-\frac{f_1 w^- \Omega_* \sqrt{A_{11}^I A_{11}^{II} a_{55}^I \rho_I}}{l\Delta_3} \cos a_3^I \Omega_* \zeta_1 \sin a_1^{II} \Omega_* (\zeta + \zeta_2) + \frac{u^-}{l} \cos a_1^{II} \Omega_* \zeta \right]$$

$$V^{II} = \frac{1}{\cos a_2^{II} \Omega_* \zeta_2} \left[-\frac{f_2 w^- \Omega_* \sqrt{A_{11}^I A_{11}^{II} a_{44}^I \rho_I}}{l\Delta_3} \cos a_3^I \Omega_* \zeta_1 \sin a_2^{II} \Omega_* (\zeta + \zeta_2) + \frac{v^-}{l} \cos a_2^{II} \Omega_* \zeta \right]$$

$$W^{II} = \frac{1}{\cos a_3^{II} \Omega_* \zeta_2} \left[-\frac{w^- \Omega_* \sqrt{A_{11}^I \rho_I}}{l\Delta_3} \cos a_3^I \Omega_* \zeta_1 \sin a_3^{II} \Omega_* (\zeta + \zeta_2) + \frac{w^-}{l} \cos a_3^{II} \Omega_* \zeta \right] \tag{19}$$

for the second layer ($-\zeta_2 \le \zeta \le 0$).

Using Eqs. (18), (19), we determine the stress in the layers by Eq. (12), and the terminal solution by

Eq. (14).

The system becomes resonant, if the external frequency Ω satisfies at least one of the equalities

$$\cos \overset{I}{a}_m \Omega_* \zeta_1 = 0, \quad \cos \overset{II}{a}_j \Omega_* \zeta_2 = 0, \quad \Delta_3 = 0, \; m = 1, 2; j = 1, 2, 3 \tag{20}$$

If $\Delta_3 \neq 0$, then, unlike the case of the full contact, the appearance of resonance depends on the physical, mechanical and geometrical characteristics of the layer.

The solution corresponding to conditions (2), (4), (6) is reduced to the form

$$\overset{I}{U} = \frac{f_1 w^- \Omega_* \sqrt{\overset{I}{A}_{11} \overset{II}{A}_{11} \overset{I}{a}_{55} \rho_{II}}}{l \Delta_4 \sin \overset{I}{a}_1 \Omega_* \zeta_1} \sin \overset{I}{a}_3 \Omega_* \zeta_1 \cos \overset{I}{a}_1 \Omega_* (\zeta - \zeta_1)$$

$$\overset{I}{V} = \frac{f_2 w^- \Omega_* \sqrt{\overset{I}{A}_{11} \overset{II}{A}_{11} \overset{I}{a}_{44} \rho_{II}}}{l \Delta_4 \sin \overset{I}{a}_2 \Omega_* \zeta_1} \sin \overset{I}{a}_3 \Omega_* \zeta_1 \cos \overset{I}{a}_2 \Omega_* (\zeta - \zeta_1)$$

$$\overset{I}{W} = \frac{w^- \Omega_* \sqrt{\overset{II}{A}_{11} \rho_{II}}}{l \Delta_4} \cos \overset{I}{a}_3 \Omega_* (\zeta - \zeta_1)$$

$$\Delta_4 = \Omega_* \left(\sqrt{\overset{II}{A}_{44} \rho_{II}} \cos \overset{I}{a}_3 \Omega_* \zeta_1 \cos \overset{II}{a}_3 \Omega_* \zeta_2 - \sqrt{\overset{II}{A}_{11} \rho_I} \sin \overset{I}{a}_3 \Omega_* \zeta_1 \sin \overset{II}{a}_3 \Omega_* \zeta_2 \right) \tag{21}$$

for the first layer ($0 \le \zeta \le \zeta_1$), and

$$\overset{II}{U} = \frac{1}{\cos \overset{II}{a}_1 \Omega_* \zeta_2} \left[\frac{f_1 w^- \Omega_* \sqrt{\overset{I}{A}_{11} \overset{II}{A}_{11} \overset{II}{a}_{55} \rho_I}}{l \Delta_4} \sin \overset{I}{a}_3 \Omega_* \zeta_1 \sin \overset{II}{a}_1 \Omega_* (\zeta + \zeta_2) + \frac{u^-}{l} \cos \overset{II}{a}_1 \Omega_* \zeta \right]$$

$$\overset{II}{V} = \frac{1}{\cos \overset{II}{a}_2 \Omega_* \zeta_2} \left[\frac{f_2 w^- \Omega_* \sqrt{\overset{I}{A}_{11} \overset{II}{A}_{11} \overset{II}{a}_{44} \rho_I}}{l \Delta_4} \sin \overset{I}{a}_3 \Omega_* \zeta_1 \sin \overset{II}{a}_2 \Omega_* (\zeta + \zeta_2) + \frac{v^-}{l} \cos \overset{II}{a}_2 \Omega_* \zeta \right]$$

$$\overset{II}{W} = \frac{w^-}{l} \frac{1}{\cos \overset{II}{a}_3 \Omega_* \zeta_2} \left[\frac{\Omega_* \sqrt{\overset{I}{A}_{11} \rho_I}}{\Delta_4} \sin \overset{I}{a}_3 \Omega_* \zeta_1 \sin \overset{II}{a}_3 \Omega_* (\zeta + \zeta_2) + \cos \overset{II}{a}_3 \Omega_* \zeta \right] \tag{22}$$

for the second layer ($-\zeta_2 \le \zeta \le 0$). Stress is calculated by formulae, Eq. (12). If the frequency Ω satisfies at least one of the equations

$$\Delta_4 = 0, \quad \sin \overset{I}{a}_m \Omega_* \zeta_1 = 0, \quad \cos \overset{II}{a}_j \Omega_* \zeta_2 = 0, \; m = 1, 2; j = 1, 2, 3 \tag{23}$$

then the system is resonant.

It can be shown that the approximation obtained is admissible for applications even if u^-, v^-, w^- are not constant

Compare Eqs. (10); (15), (16) with the Eqs. (18), (19); (21), (22). The comparison entails an important conclusion: in the case of the full (rigid) contact, both displacement and stress in all the layers depend on the values of the tangential boundary displacement u^{-}, v^{-}. In the case of the Coulomb friction between the layers, the tangential displacements u^{-}, v^{-} on the second layer don't affect the stress-strain state of the first layer.

Note that tangential displacement of the foundation are of the greatest danger under earthquakes. This makes the established property the fact of the primary importance. The application of the seismoisolators or the rubber-like materials between the foundation and the base makes their contact sensitive to the tangential displacement. Therefore, the use of the soft materials between the structural foundation and the base decreases the seismic danger.

This conclusion, based on the precise mathematical solution, has an evident applied significance in the seismo-stability problems. It demonstrates why the application of the seismoisolators decreases the danger of the seismic effect.

4. Specification of free and forced vibrations of three-layered and multilayered structures

The above developed method can be extended to the multilayered structures. In this case it is necessary to solve Eqs. (1) for every layer and satisfy the contact Eqs. (5) or (6) for the neighbouring layers. Eqs. (7) and (9) remain valid.

Now we pay attention to the qualitative features of the solutions for free and forced vibrations of the three-layered packet (Aghalovyan, *et al.* 2005).

There arise two kinds of the free shear vibrations and a longitudinal free vibration in the three-layered orthotropic plate. Consider the packet from two more rigid and one significantly elastic material under various interchanges of mutual location of the layers. We chose more rigid materials as glass-plastic STET with the parameters $E_1 = 35.22 \cdot 10^9$ Pa, $E_2 = 28.743 \cdot 10^9$ Pa, $G_{12} = 7.456 \cdot 10^9$ Pa, $\rho = 1950$ kg/m^3, $\nu_{12} = 0.177$, SVAM 10:1 ($E_1 = 38.26 \cdot 10^9$Pa, $E_2 = 17.64 \cdot 10^9$Pa, $G_{12} = 5.2 \cdot 10^9$ Pa, $\rho = 2000$ kg/m^3, $\nu_{12} = 0.22$), glass-plastic ASTT ($E_1 = 17.56 \cdot 10^9$ Pa, $E_2 = 12.85 \cdot 10^9$Pa, $G_{12} = 2.75 \cdot 10^9$ Pa, $\rho = 1900$ kg/m^3, $\nu_{12} = 0.15$), and iron ($E = 190 \cdot 10^9$Pa, $G = 75 \cdot 10^9$ Pa, $\rho = 7800$ kg/m^3, $\nu_{12} = 0.27$). As a flexible material, we chose

Table 1 Values of natural frequencies of layered plates

	ω_1	ω_2	ω_3	ω_4
I layer – SVAM	II layer – STET		III layer – rubber	
$h_1 = 0.3$ m, $h_2 = 0.5$ m, $h_3 = 2 \cdot 10^{-4}$ m	857.54	6623.11	12794	19577.3
I layer – SVAM	II layer – STET		III layer – iron	
$h_1 = 0.3$ m, $h_2 = 0.5$ m, $h_3 = 2 \cdot 10^{-4}$ m	3459.54	9456.14	16264.3	22472.2
I layer – SVAM	II layer – rubber		III layer – ASTT	
$h_1 = 0.3$ m, $h_2 = 5 \cdot 10^{-4}$ m, $h_3 = 0.2$ m	871.22	8879.22	14553.3	26262.7
I layer – SVAM	II layer – iron		III layer – ASTT	
$h_1 = 0.3$ m, $h_2 = 5 \cdot 10^{-4}$ m, $h_3 = 0.2$ m	3621.15	12161.1	19550.6	27816.0
I layer – SVAM	II layer – STET		III layer – ASTT	
$h_1 = 0.3$ m, $h_2 = 0.5$ m, $h_3 = 0.2$ m	2161.48	6865.08	11852.4	16676.1
I layer – iron	II layer – STET		III layer – ASTT	
$h_1 = 3 \cdot 10^{-4}$ m, $h_2 = 0.5$ m, $h_3 = 0.2$ m	2947.42	10747.7	17423.5	24042.2

rubber with the parameters $E = 7{\cdot}10^5$ Pa, $G = 2.4{\cdot}10^5$ Pa, $\rho = 1100$ kg/m^3, $\nu_{12} = 0.46$.

We obtain the next values of the frequencies of free shear vibrations (Table 1).

It follows from Table 1 that the packet frequencies diminish if the packet contains a significantly compliant inclusion between or under the layers. For example, the use of the strings between and under the layers results in the decrease of the natural frequencies of the packet. If the material between, under or on the layers is rigid (for example, iron), the frequencies increase. A similar effect takes place under free longitudinal vibrations. This effect remains valid if concrete, basalt, or other similar materials are used as the rigid layers.

In the case of the multi-layer packet, qualitative features of forced vibrations found for a two-layer packet remain valid. In particular, this implies that the application of the seismoisolators diminish the dangerous effect of earthquakes on constructions.

One can conclude that the negative effect of earthquakes on constructions can be diminished if there is a softer (rubber-like) material between the upper (foundation) and the rest of the layers (base). In other words, the contact between the layers should not be of the type of Coulomb friction. A proper choice of a compliant to shear material should yield cohesion (full contact).

5. Conclusions

The classes of problems of elasticity theory for layered plates under full and incomplete (Coulomb) contact between the layers have been solved. The analysis of the precise solutions has been performed. It has been shown that the implementation of the seismoisolators or the use of the compliant layer between the foundation and base diminishes the values of the free vibration frequencies for the packet “base-seismoisolator-foundation”. This allows diminishing the negative influence of seismic action on the foundation, and, as a result, on the structure as a whole.

Acknowledgements

The partial financial support of INTAS (grant N 03-51-5547), which made this investigation possible, is appreciated with gratitude.

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