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Sensor placement selection of SHM using tolerance domain and second order eigenvalue sensitivity

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Abstract. Monitoring large-scale civil engineering structures such as offshore platforms and high-large buildings requires a large number of sensors of different types. Innovative sensor data information technologies are very extremely important for data transmission, storage and retrieval of large volume sensor data generated from large sensor networks. How to obtain the optimal sensor set and placement is more and more concerned by researchers in vibration-based SHM. In this paper, a method of determining the sensor location which aims to extract the dynamic parameter effectively is presented. The method selects the number and place of sensor being installed on or in structure by through the tolerance domain statistical inference algorithm combined with second order sensitivity technology. The method proposal first finds and determines the sub-set sensors from the theoretic measure point derived from analytical model by the statistical tolerance domain procedure under the principle of modal effective independence. The second step is to judge whether the sorted out measured point set has sensitive to the dynamic change of structure by utilizing second order characteristic value sensitivity analysis. A 76-high-building benchmark mode and an offshore platform structure sensor optimal selection are demonstrated and result shows that the method is available and feasible.

Keywords: sensor optimal selection; tolerance domain; second order sensitivity.

1. Introduction

1.1. Background

Over the past decade, more and more attention has been paid to the work of determining the placement and number of sensor in the process of damage identification. Determining the best number and positions of sensors to be used for SHM is of crucial importance for costs and efficiency reasons. More generally, the issue of sensor positioning for investigating the properties of dynamical systems has been investigated from different points of view: average energy, observability, controllability, monitoring performance index (e.g. power of statistical test), entropy information content of measured or transformed

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data. As well known, one main advantage of optimizing sensor can obtain the maximum modal information by the minimum sensor. Another strongpoint comes from the demand of decreasing the dimension of gained data which always been regarded as the one of most difficulties of identifying the dynamic parameter and extracting the abnormal behavior of large structures when they are suffered from an incidental strong load or in the damage situation. In which how to select the location of sensor and the optimal number sensor is a key technology to obtain the parameter and situation of health of structure. That is precisely focus of this paper. Recent research results showed that the optimal positioning is also an important side to damage algorithm in success or not because the most algorithms depending upon the sensors' number will interferes strongly the speed of convergence of large equation of model correction when in the process of computation of real-time.

Different types of criteria have been used for quantifying the relevance of the number and the positions of sensors in a given set and they have been optimized for selecting the best possible sets. In particular, several scalar functions of matrices such as observability and controllability matrices, Fisher information matrix, modal assurance criterion matrix have been proposed. Shah and Udwadia (1978) at first presented the method of modal independence in the field of dynamic system identification. Udwadia (1985) brought the method of linear effective modal independence. Bayard and Tongco (1988, 1996) introduced the technique of sensor selection based on modal energy decomposition. Lim and Kamme (1991) extended Udwadia's technique to discuss the theory foundation of modal independence and presented the method which to describes the validity by Fisher information matrix and answered the question of eliminating the redundant sensors by using the theorem of information. Hemez (1994) used strain energy to optimize sensor. At same year, Larson and Zimmerman (1994) presented the method of modal orthogonal decomposition. Came (1995) selected the sensor through minimizing the off-diagonal element by modal assurance criterion based on the modal independence. Cobb and Liebst (1996) optimized the sensor by the minimizing the modal information matrix directly. Cherng (1999) utilized the singular value decomposition of modal Hankel matrix instead of Fisher matrix, to determine the spot of sensor. Lew (1999) suggested a new method to choose the sensor according to the assignment of vector to the sensitivity matrix. Yap and Zimmerman (2000) presented the technique which determines the location by the condition number decomposition of modal information matrix on the grounds of modal independence principle.

Above methods of measured point optimization can be divided into two kinds. One is the method based on the modal independence another based on sensitivity technique. Because method of modal independence can obtain high modal assurance factor and separate the close modal frequencies effectively in the interested scope of frequencies. So it can make eigenvector matrix independent linearly up to the extent of satisfaction. When SNR (signal noise ratio) is good it can obtain good performance of non-singular from the view of theory when the eigenvector matrix is used in structural updating reducing. So the modal independence method is not only to get appropriate sensor but also improve the speed of convergence of equation. But the flaw of modal independence is a few sensor scenarios must be offered firstly. Theses schemes can be given without difficulty in the experiment of identifying the modal parameters of undamaged structure but it maybe meet trouble when used in damage diagnosis to exist structure. The main cause lies the effect of damage to the structure is not considered and so this method would lose the important damage information if it is employed directly to find the location of sensor in the practice of the exit building test. Fortunately, the method of sensitivity can make the selected measured point reflect sensitively the change of structure. The short of this method is the selected sensor heavy correlates with the various parameters of being identified. With regard to the complex structure, for the gigantic number DOF, the burden of computation is terrible. In the same time the alteration quickly of parameter of structure, the error of result of the first order sensitivity is becoming greater and greater.

1.2. Scope

For improving the efficiency of sensor layout and taking into account the influence of damage to measured point, a statistical method named of tolerance domain is offered and combined the second order sensitivity technique who can competent for large parameter shift of structure when suffered damage or other causations are added into the modal independence to make decision to the position and number of sensor. It first determines the total measured point through the tolerance domain and second to select the candidate subset of sensor under modal independence principle and at last to check the validity of chosen sensor by second order sensitivity so as to pick the optimum location and quantity of sensor.

The layout of the paper is as following: section two will describe the theory of the second order sensitivity and section three will introduce tolerance domain technique. Section four presents the method of modal independence based on the determinant principle. The results of benchmark model and offshore platform when encounter strong noise are shown in section five and section six will provide the discussion of concerning work and some conclusions. The last part is the acknowledgement and reference.

2. Eigenvalue second order sensitivity

Utilizing the derivation and combining with the theory of modal linear combination, first calculate the first order sensitivity of eigenvalue and based on theses sensitivity values, we can obtain the second order sensitivity of eigenvalue. Typically, the viscous damping equation of free vibration can be written as:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = 0 \tag{1}$$

where $\mathbf{M}$,$\mathbf{C}$ and $\mathbf{K}$ are $n\times n$ matrices, $\ddot{\mathbf{X}}, \dot{\mathbf{X}}$ and $\mathbf{X}$ are $n\times 1$ acceleration, velocity, displacement, and force vector, respectively.

The first order eigenvalue sensitivity of structure from Eq. (1) has following form:

$$\frac{\partial \lambda_r}{\partial p_m} = -\left(\lambda_r^2 \Psi_r^T \frac{\partial \mathbf{M}}{\partial p_m}\Psi_r + \lambda_r \Psi_r^T \frac{\partial \mathbf{C}}{\partial p_m}\psi_r + \Psi_r^T \frac{\partial \mathbf{K}}{\partial p_m}\Psi_r\right) \tag{2}$$

In which, λ_r is the r order eigenvalue chosen, p_m is the mth concerned structural parameter. Ψ_r is the rth modal shape vector and equals to $\begin{bmatrix}\Psi_r \\ \lambda_r\Psi_r\end{bmatrix}$, where, $\Psi_r = [\varphi_1, \varphi_2,..., \varphi_N]$. ψ_r is upper part of conjugate matrix of Ψ_r. T denotes transposition. When p_m takes value from m_{ij} or k_{ij} or c_{ij}, the Eq. (2) is the first order sensitivity of eigenvalue corresponding to mass and stiffness or damping.

Grounded on the assumption of eigenvector linear combination (Fox and Kapoor 1968), the general formula of second order sensitivity as below:

$$\frac{\partial^2 \lambda_r}{\partial p_m^2} = -\frac{\partial \lambda_r}{\partial p_m}\left(\Psi_r^T \frac{\partial \mathbf{C}}{\partial p_m}\Psi_r + 2\lambda_r \Psi_r^T \frac{\partial \mathbf{M}}{\partial p_m}\Psi_r\right) - 2\left(\lambda_r^2 \Psi_r^T \frac{\partial \mathbf{M}}{\partial p_m} + \lambda_r \Psi_r^T \frac{\partial \mathbf{C}}{\partial p_m} + \Psi_r^T \frac{\partial \mathbf{K}}{\partial p_m}\right)\frac{\partial \Psi_r}{\partial p_m} \tag{3}$$

Replacing p_m into m_{ij} or k_{ij} or c_{ij}, the structural parameter's second order sensitivity is written as below respectively:

$$\frac{\partial^2 \lambda_r}{\partial m_{ij}^2} = \begin{cases} 8\lambda_r^3 \varphi_{jr}^2 \varphi_{ir}^2 - 2\lambda_r^2(\varphi_{jr}\phi_i + \varphi_{ir}\phi_j), & (s \neq r,\ i \neq j),\ \eta_s = \dfrac{\lambda_r^2}{\lambda_s - \lambda_r}(\varphi_{is}\varphi_{jr} + \varphi_{js}\varphi_{ir}) \\ 2\lambda_r^3 \varphi_{jr}^3 - 2\lambda_r^2 \varphi_{ir}\phi_j, & (s \neq r,\ i \neq j),\ \eta_s = \dfrac{\lambda_r^2}{\lambda_s - \lambda_r}\varphi_{is}\varphi_{jr} \\ 8\lambda_r^3 \varphi_{jr}^2 \varphi_{ir}^2 - 2\lambda_r^2(\varphi_{jr}\phi_i + \varphi_{ir}\phi_j), & (s \neq r,\ i \neq j),\ \eta_s = -2\lambda_r \varphi_{ir}\varphi_{jr} \\ 2\lambda_r^3 \varphi_{ir}^3 - 2\lambda_r^2 \varphi_{ir}\phi_j, & (s \neq r,\ i \neq j),\ \eta_s = -\lambda_r \varphi_{ir}^2 \end{cases} \tag{4}$$

$$\frac{\partial^2 \lambda_r}{\partial k_{ij}^2} = \begin{cases} -2(\varphi_{jr}\phi_i + \varphi_{ir}\phi_j), & (s \neq r,\ i \neq j),\ \eta_s = \dfrac{1}{\lambda_s - \lambda_r}(\psi_{is}\psi_{jr} + \psi_{ir}\psi_{js}) \\ 2\varphi_{ir}\phi_i, & (s \neq r,\ i \neq j),\ \eta_s = \dfrac{1}{\lambda_s - \lambda_r}\psi_{is}\psi_{ir} \\ 0, & (s \neq r,\ i \neq j) \\ 0, & (s \neq r,\ i \neq j) \end{cases} \tag{5}$$

To finish this step, the second order sensitivity of large complex structure can be acquired by using the Eqs. (4) and (5).

3. Measured point whole design based on tolerance domain

Tolerance domain is a statistic technique which uses the limit value to determine the initial sensor set which subjects to certain probability distribution. The tolerance domain limit value varies from the different sample population distribution. When total distribution is normal, the design limit value can be calculated using following formula:

$$K = \frac{1}{\sqrt{n}} t_{(n-1,\, u_p\sqrt{n})(1-\alpha)} \tag{6}$$

Where, p is the tolerance domain ratio value, $1-\alpha$ is the confidence level, n is total quantity of sample, K is subsample volume designed, t is Student distribution.

If the distribution of population is not known but satisfies continuous, the tolerance domain ratio value calculation can be obtained by probability integration. The limit value of tolerance domain can be replaced by the minimum subsample capability directly. So the tolerance domain ratio value can get as below:

$$1 - np^{n-1} + (n-1)p^n \geq 1 - \alpha \Rightarrow np^{n-1} - (n-1)p^n \leq 1 - (1 - \alpha) \tag{7}$$

Based on the theory of finite discontinuity point, the discontinuity point if exist can be substituted by the removable discontinuity point. Therefore, Eq. (7) can be used to the situation of discrete process such as measured point collection.

4. Sensor selection by modal independence

The main task of modal independence is to look for the right subset from the initial whole measured point set. According to the postulation of Fisher, the optimal information condition must be satisfied like this: one is test noise must not relevant with the statistical character of acquired input signal, another is the information in the test process has identical statistical feature. Under these hypothesises, there are several most commonly used optimum designs such as 1) maximize the minimum eigenvalue of Fisher information matrix; and 2) maximize the trace of Fisher matrix; and 3) maximize the determinant of information matrix. Because the results of determinant method equals to the minimum measured point when the measured point lied in the confidence area holds the modal independence. So determinant optimal method can be chosen to take out the candidate subset from original measured set came from tolerance domain.

Presumption modal matrix is **A**, according to Fisher information matrix, it has the form as below:

$$\mathbf{A}^{-1} = [\hat{\Psi}^{m^T}\hat{\Psi}^{m}]^{-1} \tag{8}$$

where, m and T express DOF number of structure in FEM and transposition of matrix, $\hat{\Psi}$ is complex modal matrix of structural system.

Set $m=c$, is the original measured point set. Implement the singular value decomposition to $\hat{\Psi}^{c}$, we can get the following equation:

$$\hat{\Psi}^{c} = \mathbf{U}_0\mathbf{S}_0\mathbf{V}^T \tag{9}$$

where, $\mathbf{U}_0$ is left eigenvalue matrix, $\mathbf{V}$ is right eigenvalue matrix, $\mathbf{S}_0$ is singular matrix, T denotes transposition.

Considering the Eq. (9), the structural modal reduced information matrix can be written as following:

$$\mathbf{A}_r = \mathbf{V}\mathbf{S}_0\mathbf{U}_0^T\mathbf{T}_m\mathbf{U}_0\mathbf{S}_0\mathbf{V}^T \tag{10}$$

where, $\mathbf{T}_m$ is an introduced state observation matrix, m is the measured DOF.

According to the determinant calculation of information matrix (Gantmacher 1971), standard style of $\mathbf{A}_r$ has the form as below:

$$\frac{Det(\mathbf{A}) - Det(\mathbf{A}_r)}{Det(\mathbf{A})} = \sum_{i=1}^{n} q_i \tag{11}$$

where, $q_i = \frac{1}{2}[tr({}_n\mathbf{B}^i) - q_1 tr({}_n\mathbf{B}^{i-1}) - \cdots - q_{i-1}q_i] = O(q_1^i)$, $o(\cdot)$ denotes high order infinitesimal, ${}_n\mathbf{B} = \mathbf{U}_{0d}^T\mathbf{U}_{0d}$, $\mathbf{U}_{0d}$ is an $nd\times d$ matrix constituted with row vectors of $\mathbf{U}_0$ corresponding to deleted DOF, nd is the number of deleted DOF. Using the trace of matrix and neglecting the high order of the q_i, it can be derived the equation:

$$\frac{Det(\mathbf{A}) - Det(\mathbf{A}_r)}{Det(\mathbf{A})} \cong q_1 = tr({}_n\mathbf{B}) = \sum_{k=1}^{nd} u_{0k}u_{0k}^T$$

$$= \sum_{k=1}^{nd} \hat{\psi}_{rk}^{c} (\hat{\Psi}^{c^T} \hat{\Psi}^{c})^{-1} \hat{\psi}_{rk}^{c^T} = \sum_{k=1}^{nd} \underset{k}{diag} (\hat{\Psi}^{c} (\hat{\Psi}^{c^T} \hat{\Psi}^{c})^{-1} \hat{\Psi}^{c^T}) \tag{12}$$

In which, u_{0k} is the kth row vector of $\mathbf{U}_0$, $\hat{\psi}_{rk}^{c}$ is the kth row vector of $\hat{\Psi}^{c}$.

From the Eq. (12), information loss can be expressed by the determinant of contributed factor matrix. If the contributed factor corresponding to the DOF is considerable it can be retained as valid measured point so as to void losing information, and vice versa the measured point will be eliminated when the contributed factor is less than a small quantity. So the principle of maximum modal information obtained from the minimum sensor can be translated into pursuing to maximize of minimum determinant matrix.

Let $\rho = \sum_{k=1}^{nd} \underset{k}{diag} (\hat{\Psi}^{c} (\hat{\Psi}^{c^T} \hat{\Psi}^{c})^{-1} \hat{\Psi}^{c^T})$ is a small quantity; the modal independence principle can be derived out as following (Kim and Park 1997):

$$\sum_{i=1}^{n} q_i \le \sum_{i=1}^{n} |q_i| \le \sum_{i=1}^{n} \rho^i \le \frac{\rho}{1-\rho} < 1 \tag{13}$$

Eq. (13) shows the DOF eliminating principle. From Eq. (13) it can be found that the upper bound of information loss is 0.5. Because the upper bound is a enlarging process gradually, in order to avoiding losing excess useful information, the upper bound is should not greater than 0.5. However, because the impact of noise, the upper bound of information does not always lie around of 0.5. For taking into account the effect of the noise and considering the value of noise is a small quantity corresponding to the main signal, so we add additional item of $\Delta\rho$ as a noise perturbation quantity to the back of ρ, thus the new upper bound ρ can be rewritten as following:

$$\rho = \sum_{k=1}^{nd} \underset{k}{diag} (\hat{\Psi}^{c} (\hat{\Psi}^{c^T} \hat{\Psi}^{c})^{-1} \hat{\Psi}^{c^T}) + \Delta\rho = \rho' + \Delta\rho \tag{14}$$

In which ρ' is the upper bound of information lose without noise environment. Taking the Eq. (14) into Eq. (13), we can obtain the new span of ρ considering the noise as below:

$$\frac{\rho' + \Delta\rho}{1 - (\rho' + \Delta\rho)} < 1 \Rightarrow \rho' < 0.5 - \Delta\rho \tag{15}$$

When in practice, the suitable value of $\Delta\rho$ should be decided under consideration with the scale of structure and request of real computing time and health monitoring system others factors.

Summarizing above, the sensor selection based on modal independence has following steps:

1. constitute the modal matrix, $\hat{\Psi}^{c}$;
2. establish the information matrix, $\mathbf{A}$;
3. determine the noise factor, $\Delta\rho$;
4. confirm the upper bound of information lose factor, ρ';
5. eliminate the measured point set whose contribution factor under the ρ';
6. obtain the selected sensor set from the original measured point set based on the tolerance domain;
7. check the chose sensor set whether lies in area of second sensitivity or not, and get the optimal sensor set finally.

5. 76-story building benchmark model sensor selection

The building considered, whose plan and elevation view show in Figs. 1 and 2, is a 76-story 306 meters office tower proposed for the city of Melbourne, Australia. This is a reinforced concrete building consisting of concrete core and concrete frame. The total mass of the building, including heavy machinery in the plant rooms, is 153,000 metric tons. The total volume of the building is 510,000 m^3, resulting in a mass density of 300 kg per cubic meter, which is typical of concrete structures. The building is slender with a height to width ratio (aspect ratio) of $306.1/42 = 7.3$. The 76-story tall building is modelled as a vertical cantilever beam. A finite element model is constructed using the Mass21 and Beam188 elements of ANSYS by considering the portion of the building between two adjacent floors as a classical beam element of uniform thickness, leading to 76 translational and 76 rotational degrees of freedom. Then, all the 76 rotational degrees of freedom have been removed by the static condensation. This results in 76 degrees of freedom, representing the displacement of each floor in the lateral direction. The first five natural frequencies are 0.16, 0.765, 1.992, 3.790 and 6.395 Hz and the first three modeshapes are shown in Fig. 3. The proportional (76×76) damping matrix for the building with 76 lateral degrees of freedom is calculated by assuming 1% damping ratio for the first five modes using Rayleigh's approach. This model, having (76×76) mass, damping and stiffness matrices, is referred to as the "76 DOF Model".

At first determine the initial measured point subset using the tolerance domain. The first modeshape of structure is used the population of tolerance domain sample as following:

$$10^{-2} \times [0.04, 0.08, 0.13, \ldots, 9.58, 9.95, 10.33, \ldots, 22.19, 22.71, 23.22]_{76}$$

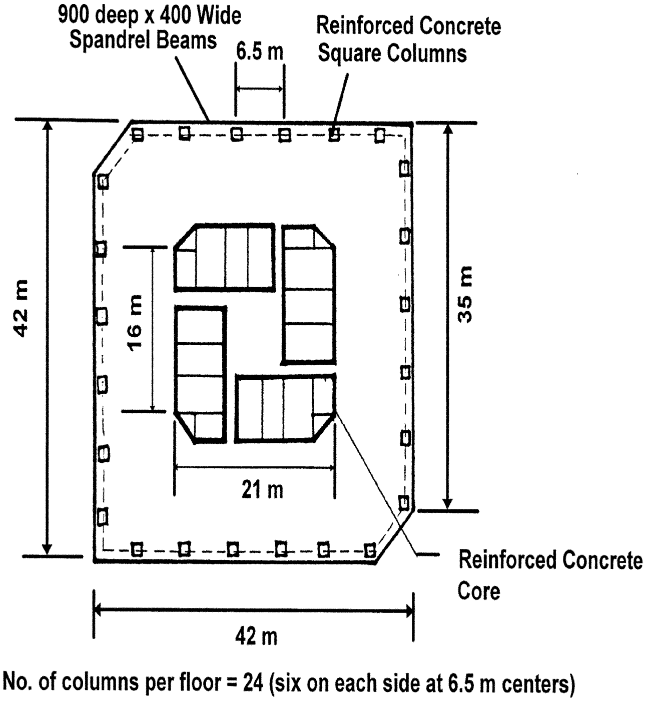


Fig. 1 Plan view of the 76-story building

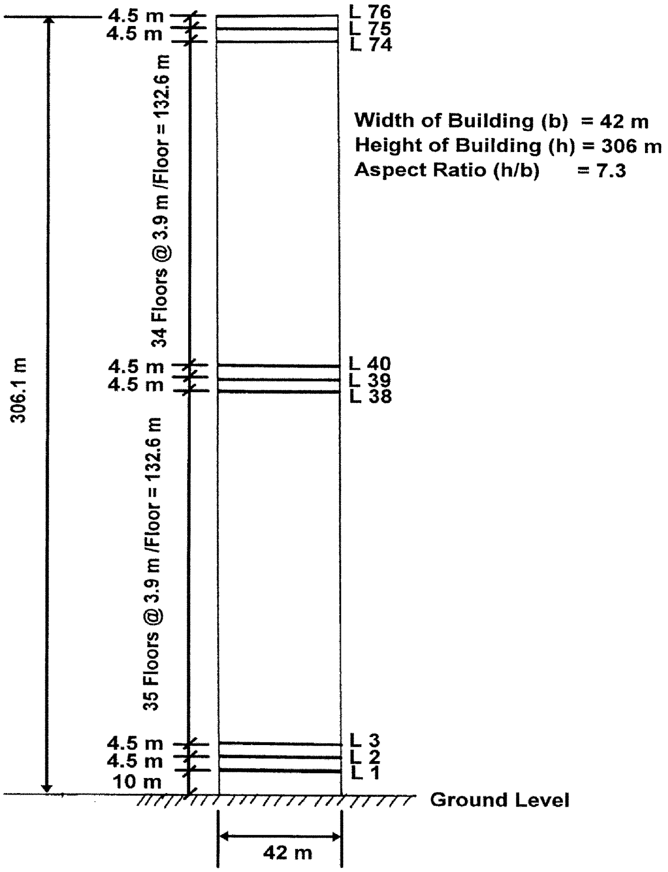
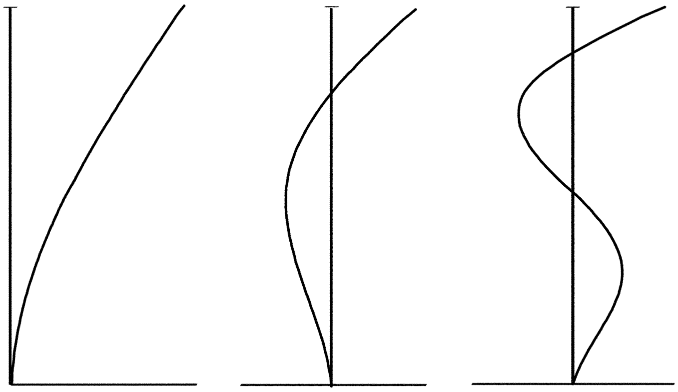


Fig. 2 Elevation view of the building



Mode1 **Mode2** **Mode3**

Fig. 2 Elevation view of the building

The estimated mean of sample and least likelihood variance of above modeshape sample are: 0.0897 and 0.0052. When set the probability ratio value of tolerance domain is 0.90 and the confidence level is 0.90, the designed limit value $K = 1.607$, tolerance upper and lower limit values are: 0.098,0.0814. The selected sensor subset volume according to the upper limit is 32 corresponding to DOS as below: $[45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76]_{32}$ and the lower limit is $[40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76]_{37}$. When the total distribution probability is not known the subsample volume of measured point is 48. So choose the lower limit 37 as the initial subset DOF.

For considering the noise effect in the processing of sensor selection, the noise scale depend on the output signal is added into the acquired response from the structure. The constitution of noise is defined as following:

$$E_n = E_s + Sqrt(x_n)*rand(1, N) \tag{16}$$

Where, E_n is noise signal, E_n is signal strength of noise, $Sqrt(x_n)$ is the noise variance, x_n is the output signal from structure, $rand(1, N)$ is the Gauss distribution signal which has N time acquisition point corresponding to the sample length of output signal of structure. By the Eq. (16),we can add the color noise into the structural response signal so as to the effect of noise to the sensor selection can be considered in the method of optimal measured point position based on modal independence.

Adding 50, 40 30, 20 per cent color noise into the output signal respectively is to find and compare the impaction of noise on the sensor selection. The calculated ρ' respected to with the different color noise can be computed to obtain with 0.48, 0.47, 0.45 and 0.43 respectively. The $\rho' = 0.40$ is corresponding to the situation of no noise influence. The selected the effective modal sensor number is 25, 23, 21, 19 and 11 respectively. Their locations corresponding to the DOF are list as below:

(1) When 50% *clolor noise* $\Rightarrow \rho' = 0.48$:
Sensor position:$\mathbf{[76,75,70,68,63,61,55,54,50,49,40,38,36,34,23,21,19,17,15,10,8,6,4,2,1]_{25}}$**;**
(2) When 40% *clolor noise* $\Rightarrow \rho' = 0.47$:
Sensor position:$\mathbf{[76,75,70,68,63,61,55,54,50,49,40,38,36,34,23,17,15,10,8,6,4,2,1]_{23}}$**;**
(3) When 30% *clolor noise* $\Rightarrow \rho' = 0.45$:
Sensor position:$\mathbf{[76,75,70,68,63,61,55,54,50,49,40,38,23,17,15,10,8,6,4,2,1]_{21}}$**;**
(4) When 20% *clolor noise* $\Rightarrow \rho' = 0.43$:
Sensor position:$\mathbf{[76,75,70,68,63,61,55,54,50,49,40,38,23,17,10,8,6,2,1]_{19}}$**.**
(5) When 0% *clolor noise* $\Rightarrow \rho' = 0.40$:
Sensor position: $\mathbf{[76,70,63,55,50,49,38,23,17,8,1]_{11}}$**.**

From above results, under the initial subset DOF of 37, the increased sensor per cent corresponding to the case (5) (no noise scenario) caused by the noise is 22%, the most increased sensor per cent is 38% corresponding to case (1) (50% color noise). Because the modal information statistical uncertainty respected with the analytical model, the error of 22% can be deemed better than the error of dynamic finite element method (DFEM) such as the modreshape and frequency and damp ratio. On the other hand, the noise of model and measurement can often be decreased below 20%. So in practice, we can get much better sensor optimal result than above theory calculated situation.

The last step is check if the selected sensor lies in the area of sensitivity. The second order sensitivity of the first eigenvalue corresponding to the mass and stiffness is shown in Figs. 4 and 5, respectively.

Viewing of the stiffness sensitivity of Fig. 5.5, when $\rho' = 0.40, 0.43, 0.45$, the picked sensors sets are all second order sensitive. The chose sensors meet the necessary of sensitivity of second order. This means that the optimal measured point set can be selected from three noise scenarios. Within the three

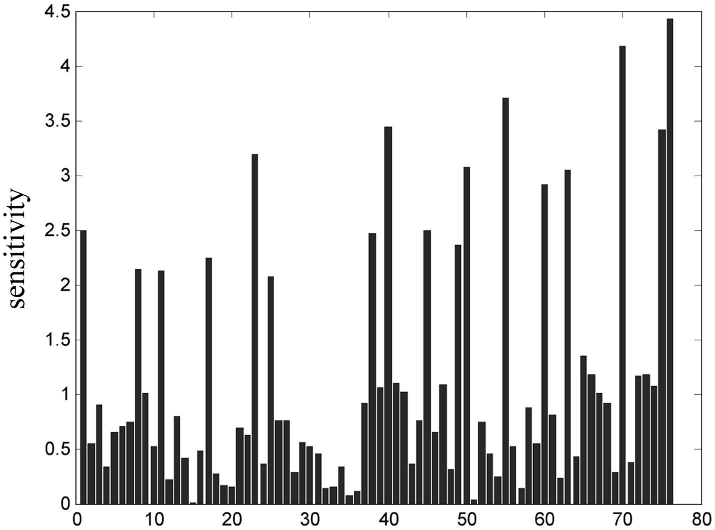


Fig. 4 Second order sensitivity of the first frequency corresponding to mass

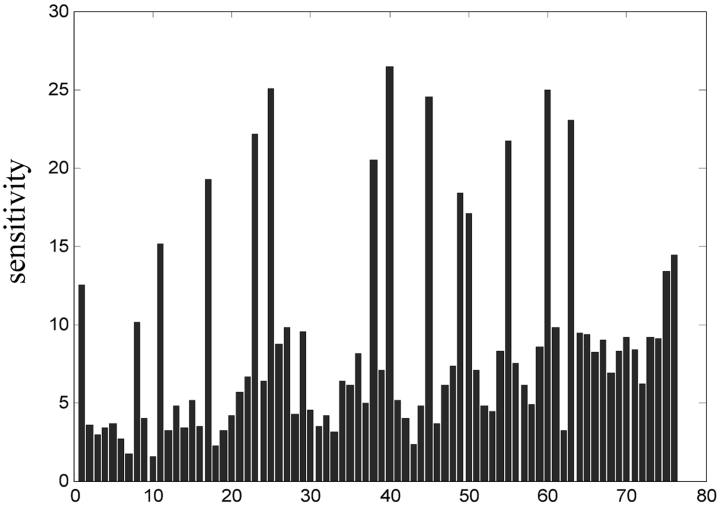


Fig. 5 Second order sensitivity of the first frequency corresponding to stiffness

situations, comparing Fig. 4 with Fig. 5, it can be seen that the sensors set satisfied with mass second order sensitivity are much more than stiffness second order sensitivity. It shows that the tolerance domain can remove the redundant measured point. Form the view of most independent modal information and minimum sensors principle, the finally selected optimal sensor set of came from the cases of (3),(4), and (5) can be determined as the constitution of (5) whose corresponding to DOF is **[76, 70, 63, 55, 50, 49, 38, 23, 17, 8, 1]$_{11}$.**

For further verifying the validity of selected sensor, the dynamic parameters are identified by using the time series model method based on the output information from the selected sensors set. Three signals among of selected sensors with the 76-story and 70-story and 63-story are used as the initial

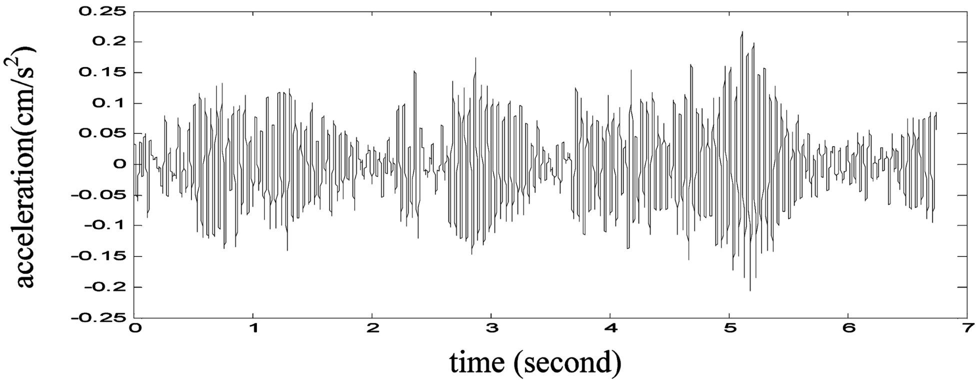


Fig. 6 Output acceleration from the 76th

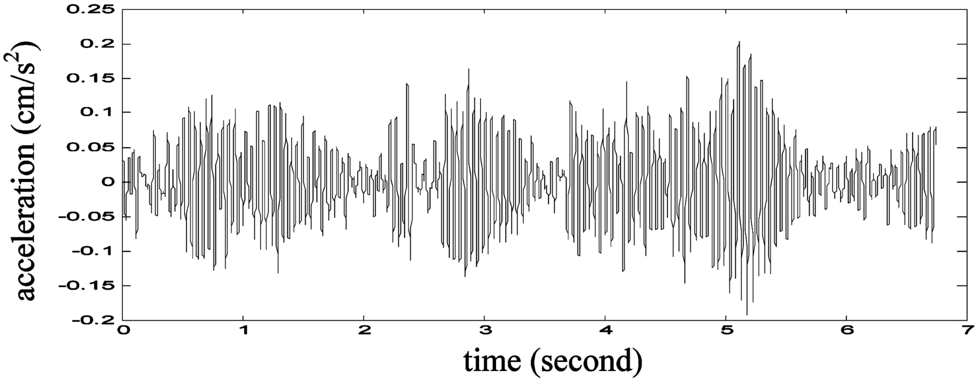


Fig. 7 Output acceleration from the 76th floor

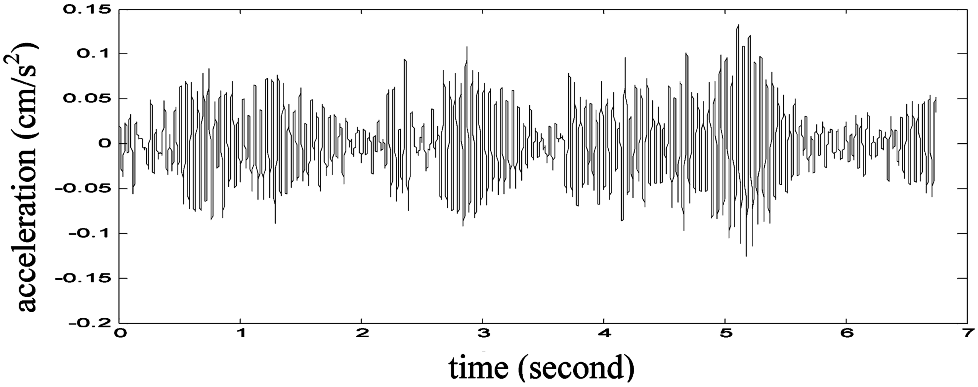


Fig. 8 Output acceleration from the 63th floor

output signal to form the simulated noise of environment and test. It is showed in Fig. 6, Fig. 7 and Fig. 8.

The identified of frequencies and damping ratios are listed in Table 1. Contrasting the identified value with Benchmark value listed in Table 1, the identified frequencies is very close to the Benchmark values and what's more, the higher close frequencies are separated clearly. This is demonstrated that the

Table 1 Comparing values of Benchmark with identified by selected sensor

modal	Frequency (Hz)			Damping ratio (%)		
	Benchmark identified	Identified by selected sensor	error	Benchmark identified	Identified by selected sensor	error
1	0.1600	0.1622	0.0022	2.0000	1.9834	0.0166
2	0.7651	0.7543	0.0108	2.0000	2.2035	0.2035
3	1.9921	1.9838	0.0083	2.0000	2.4536	0.4536
4	3.7899	3.7645	0.0254	2.0000	2.6428	0.6428
5	6.3945	6.4001	0.0056	2.0000	2.6901	0.6901
6	9.4577	9.4409	0.0168	2.2819	2.7003	0.0166
7	13.2481	13.2602	0.0121	2.7903	2.7421	0.0482
8	17.5136	17.5099	0.0037	3.4462	3.2826	0.1636
9	22.8172	22.8204	0.0032	4.3149	3.7019	0.6130
10	28.2235	28.1977	0.0258	5.2294	4.7384	0.4910
11	34.5331	34.5011	0.0320	6.3155	5.5792	0.7363
12	41.2564	41.0651	0.1913	7.8454	6.6074	1.2380
13	49.1601	49.2603	0.1002	8.8701	7.9015	0.9698
14	56.8448	56.7058	0.1390	10.2225	9.2663	0.9562
15	65.6324	65.5425	0.0899	11.7735	10.7035	1.0700

obtained modal space information computed from the selected sensor set is effective and the sensor location can get good modal characteristic to extract the dynamic parameters in the processing of SHM based on the global and local structural response.

Additional, if we don't use the tolerance domain, all the 76 DOF is select to used as initial measured sensor set directly, the result based on the modal independence is 14 whose DOF distributes as $[76, 75, 70, 63, 55, 50, 49, 38, 32, 23, 17, 8, 5, 1]_{14}$. Comparing this subset with the one under $\rho' = 0.40$, the method without tolerance domain has more only three points than using the original measured point came from the tolerance domain. The loss modal information using tolerance domain is 27 per cent but the computation efficient makes much more progress because the method of tolerance domain can eliminate many sensors once they do not lie in the sample population space. The more analytical nodes and more complex to the structure, the more efficiency for get the optimal sensor set.

6. JZ202-MUQ offshore platform sensor selection

JZ202-MUQ offshore platform lies in Liaodong gulf of Bo sea in east China. It is a fixed steel jack shallow sea platform. It has been served for fifteen years since it was built up in 1991. Its structure below in water is fifteen and half mater. The jack is made of four legs which is fabricated by steel pipe and reinforce concrete pile in the pipe. The each jack was been hammered in sea bed soil with a inclination layout. The design gradient of directions of two jack is 1:10 however that of fact is 1:7. The plan structure presents quadrilateral. Its design life is twenty years. Because the aging of material and the accumulating of fatigue caused by the environment corrosion and the periodical dynamic oceanic load, the SHM seems very important to the everyday product of oil and gas and obtaining the safe status. However, for the large size of structure, how to determine the most optimal sensor position is

Fig. 9 View of JZ202-MUQ offshore platform

being the first emphasis technique and kernel step. The reasonable appropriate and measurable sensors is our one of focus in the damage finding and evaluation. The picture of the offshore platform is showed in Fig. 9. JZ202-MUQ is made of living platform and producing platform. The left platform in the photo is the living platform. The sensor selection of SHM is carried out on this platform.

The MUQ platform has the distance of 54.9 meter from the top of platform to the bed river and its total weight is about 2000 tons. Harbin Institute of Technology made a scale mode to perform the vibration experiment to grasp the dynamic character and master its behave under ambient exciting. Because the size of vibrating table is 3×4 m the, geometrical similarity ratio of scale mode is 1:10 and the stiffness similarity ratio is 1:35 and the mass likelihood ratio is 1:450. The load ratio of similitude is 1:266. The deformation similarity ratio is been given 1:7.6 and admeasure the acceleration similarity ratio 1:8.4.

This experiment was designed to demonstrate the sensor selection method and illustrate the accuracy of estimating parameters and improving locating damage of SHM in the scenario of damage in which simulates to move the inclined brace which was been found to occur usually in offshore platform. The schematic of offshore platform is shown in Fig. 10. The dimensions and properties of the structure are summarized in Table 2.

In Fig. 10, there are six dissipater braces in the middle part of platform for structural vibration control and a bold black line is been simulated as damaged bar. A linear viscoelastic damped FEM of the offshore platform was constructed using BEAM188 element in conjunction with MASS21 element (ANSYS SoftWare). BEAM188 is a linear (2-node) beam element in 3-D. BEAM188 has six degrees of freedom at each node. A compressed model with 3854-DOF and 10020-DOF considering the rotated and space distribution damped FEMs were generated. The first sensor determination method based on empirical and FEM has 23-position in which been chosen to install accelerometer and displacement sensors. Among these measured point, besides 22-acceleromwter and displacement measured point, there is one rotation measured point among the 23-poision. The letter of ***a*** in Fig. 10 denotes accelerometer and the letter of ***d*** denotes the displacement. In Fig.10, the measured point of **6*a*** indicates the rotation measured position. In order to simulate the ambient excitation and to obtain the response from platform, Lanczos algorithm was used to calculate the modal response of structure. The meshed platform is

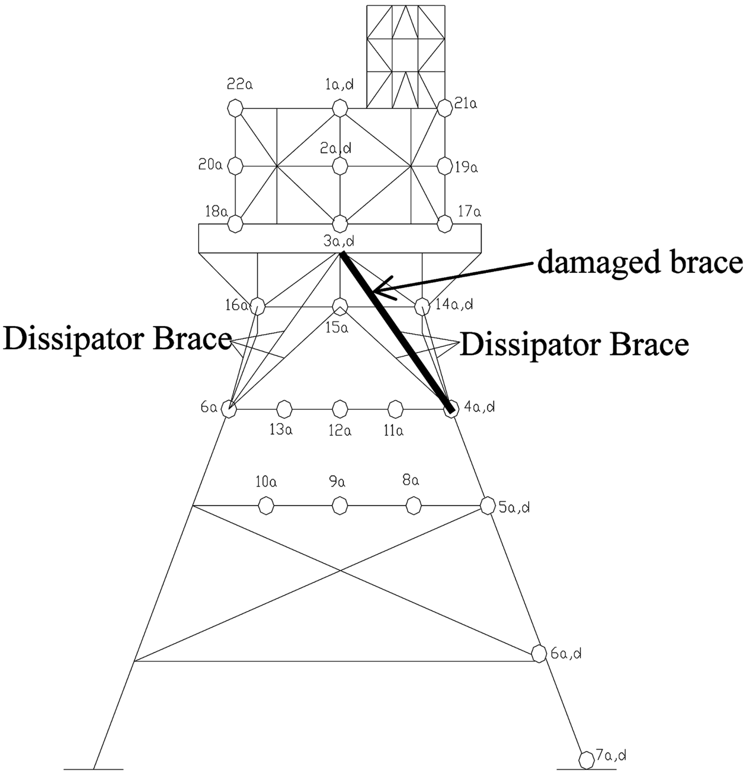


Fig. 10 Damaged strucutre platform

Table 2 Material properties

Type of Steel: A3 Circle Steel
Young's Modulus: 2.05E+2GP$_a$
Poisson's ratio:0.3
Density: 7.85E+3 kg/m^3

shown as Fig. 11. The first ten frequencies are listed in Table 3. It can be seen that the frequency is very close and dense.

Pipes and foundation and the transfer layer are the key components which are bearing the platform structural load. So their health situations are kernel of SHM of offshore platform. Usually, the platform structure of SHM is simplified into two parts. One is the steel deck which stays bearing heavy concentrated mass above the working layer the other is lower pipe jack bars which have the nodes of 1340 DOF in the total nodes distribution. The sensor selection processing is implemented on the simplified structure which includes the deck and the pipe jack.

The first step of sensor positioning is to calculate the measured point of tolerance domain. The modal information matrix is made from the first modeshape as [1.00, 0.99, ···, 1.73]1340. The estimated mean and variance of modal information are 0.51, 0.08, respectively. Given the p=0.95, $1-\alpha = 0.95$, then $k = 1.84$ when sample has the normal distribution. The chosen subsample according to upper limit is 47 and the number of DOF corresponding to lower limit is 124. When the distribution is not determined the

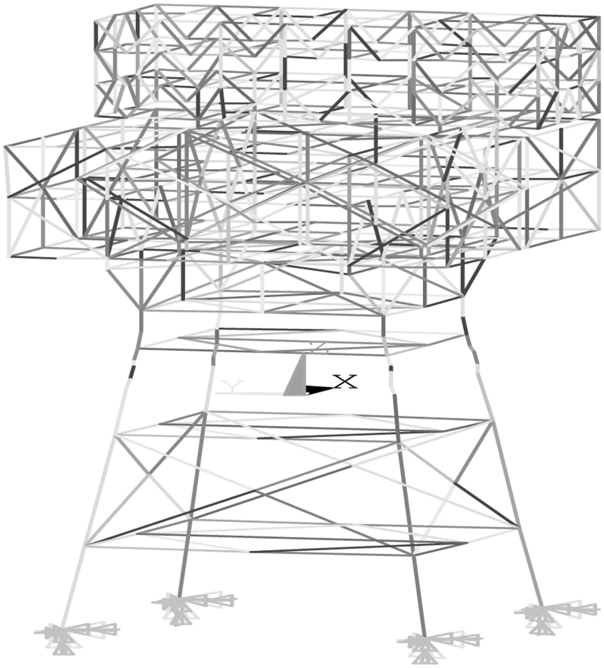


Fig. 11 Model of MUQ

Table 3 The first ten analytic frequencies of MUQ

Modal order	1	2	3	4	5	6	7	8	9	10
Frequency (Hz)	0.66	0.72	0.74	1.53	1.21	1.97	2.06	2.31	2.36	2.41

subsample volume is 123. So the largest subsmple corresponding to 124 is being taken as the tolerance domain subset.

The second step is to take out the subset DOF through the modal independence method. The color noise to the platform is treated as the 76-story building in part five above. The added noise is 30, 20, and 10 per cent according to the output signal intensity, respectively. The selected effective modal sensor set and their locations corresponding to the DOF are list as below:

(1) When 0% *clolor noise* $\Rightarrow \rho' = 0.40$:
Sensor position: [23, 28, 74, 103, 179, 205, 220, 242, 270, 330, 345, 355, 360, 400, 405, 410, 415]$_{17}$;

(2) When 15% *clolor noise* $\Rightarrow \rho' = 0.42$
Sensor position: [23, 28, 74, 103, 179, 205, 220, 242, 270, 330, 345, 355, 360, 400, 405, 410, 415; 416, 420;]$_{19}$;
Contrast with (1), the added DOF is:[416,420]$_2$

(3) When 20% *clolor noise* $\Rightarrow \rho' = 0.45$:
Sensor position: [23, 28, 74, 103, 179, 205, 220, 242, 270, 330, 345, 355, 360, 400, 405, 410, 415; 33, 38, 48, 62, 117, 200, 237]$_{24}$;
Contrast with (1), the added DOF is: [33, 38, 48, 62, 117, 200, 237]$_7$

(4) When 30% *clolor noise* $\Rightarrow \rho' = 0.47$:

Sensor position: [23, 28, 74, 103, 179, 205, 220, 242, 270, 330, 345, 355, 360, 400, 405, 410, 415; 5, 11, 17, 20, 25, 52, 57, 72, 79, 84, 122, 132, 210, 215,225, 230, 385, 390]$_{42}$;
Contrast with (1), the added DOF is: [5, 11, 17, 20, 25, 52, 57, 72, 79, 84, 122, 132, 210, 215,225, 230, 385, 390]$_{18}$

From above results by modal independence, it can be known that the need sensors will increase with the aggravating of noise effect. However, all the sensor sets chose from the noise ambient contain the sensor set computed through the weak noise or non-noise case. Additional, theses added DOFs have strong modal consistency because they almost distributed in the same bar or pipe. The case of (2) contaminated by 15 per cent noise has only more two points that the case of (1) non-noise situation and the added points response near node of **415** and **410. So** the result in case of (1), which has the modal information upper bound value of 0.40, is been selected as the optimal modal independence measured point set.

The selected measured point assigned in Figs. 12 and 13. The location corresponding to case (1) apportioned in Fig. 12. In Fig. 13, the sign of circle denotes sensor set corresponding to the cases (1) and (2). The two black circles among the circles means the added two points corresponding to case (2). The sign of square expresses case (3) and the sign of triangle to the case (4). From Fig. 13, we can found that the added points under cases (3) and (4) lie in the jack pipe leg for the noise impaction. Because the steel leg of platform has very large stiffness, the modal information under case (1) can embody all the needed cases (3) and (4). That means that the case (1) is enough to express the effective modal information. So the subset sensor under $\rho' = 0.40$ is decided to the picked out measured point and neglect the added two points showed by black circles in Fig. 13. The ultimate sensor subset obtained from the modal independence is indicated in Fig. 12.

Comparing with Fig. 10 with Fig. 12, the measured sensors are less that that by the common method just only based on the FEM analysis. The sensors selected are more concise and representative than before. For verifying the selected sensors, the second order sensitivity is been checked on theses sensors. The platform model mass second order sensitivity is shown in Figs. 14~16.

From Fig. 14, the selected measured point set of case (1) lies in the sensitive zone except the node of 74. The mass sensitivity indicates the sensor subset of case (1) is right. Noticing the Figs. 15 and 16, it shows that the higher order, the more sensitive to the sensor in the pipe leg. This is resulted by

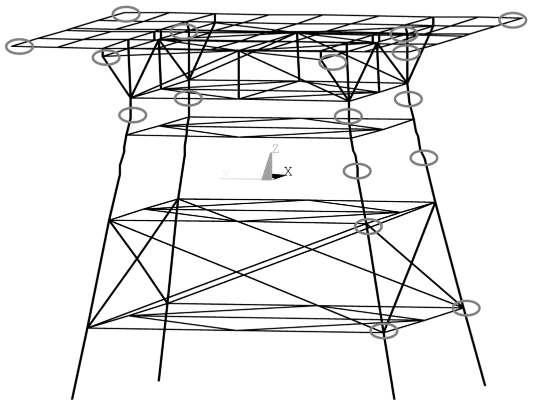


Fig. 12 Optimal measured point

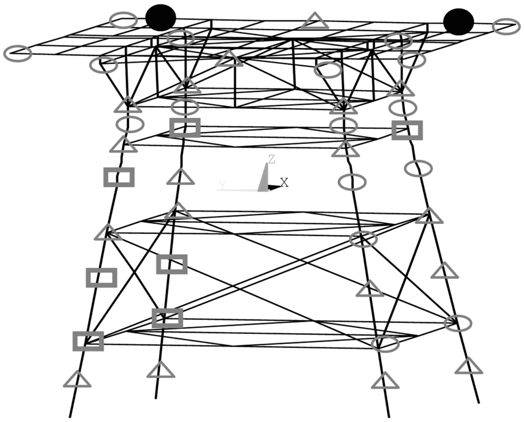


Fig. 13 Distribution of measured sensor

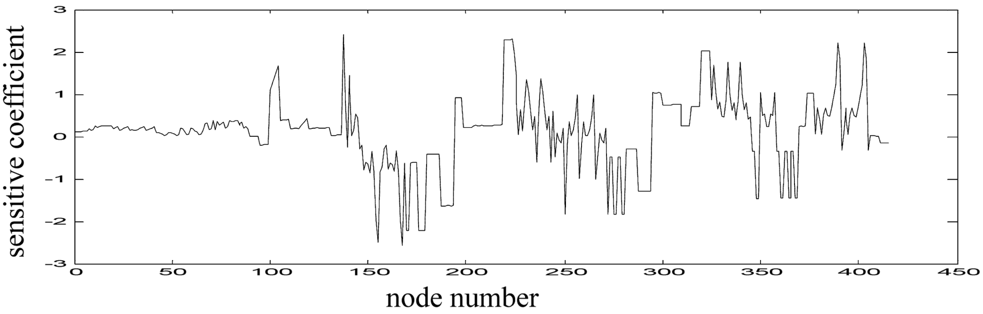


Fig. 14 Second order mass sensitivity to the first frequency

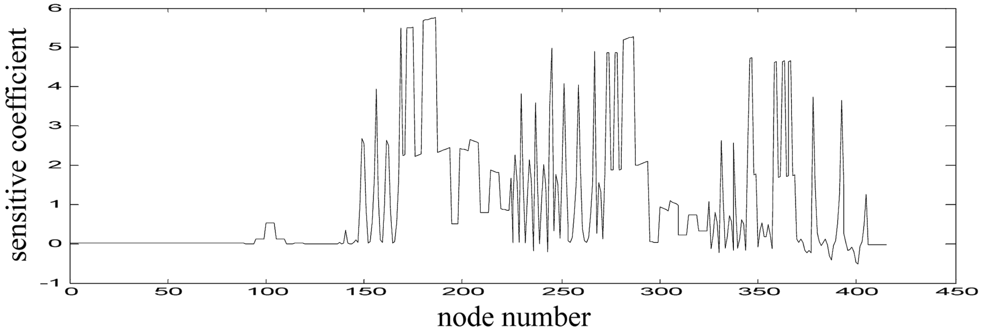


Fig. 15 Second order mass sensitivity to the second frequency

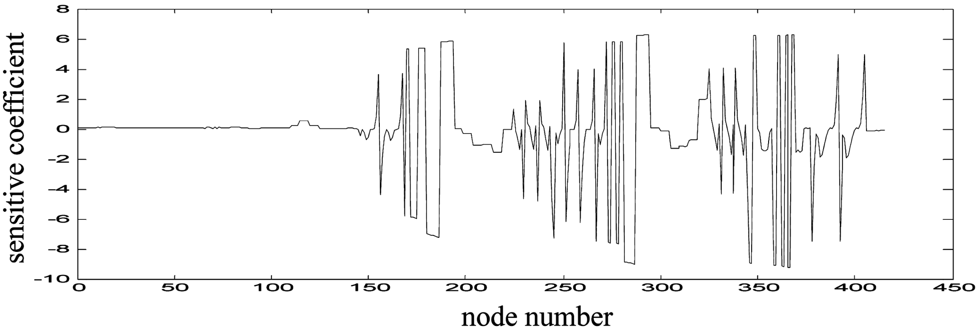


Fig. 16 Second order mass sensitivity to the third frequency

centralized mass of upper structure and therefore the selected sensor always in the mass sensitive area. The trend is also cab be reflected from the Fig. 14.

The first three second order stiffness of structure sensitivities are shown in Figs. 17~19. Analysing the Fig. 17, we can know clearly that the selected sensor set corresponding to $\rho' = 0.40$ are sensitive except the points of 23 and 28 and 105. The DOFs of 23 and 28 are not sensitive points in the Figs. 18~19. This appearance caused from the fact that the nodes bar attached those DOFs lie in dock plan and far away the big stiffness node in the mode FEM. The stiffness change in these nodes can not arouse obvious second order sensitivity. It can also discover that the sensitive point increases quickly when the order get higher. Stiffness second order sensitivity shows that the sensor subset corresponding to case (1) is validity.

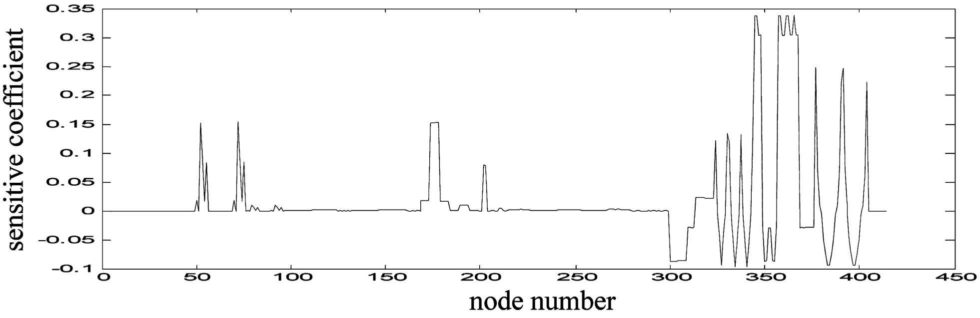


Fig. 17 Second order stiffness sensitivity to the first frequency

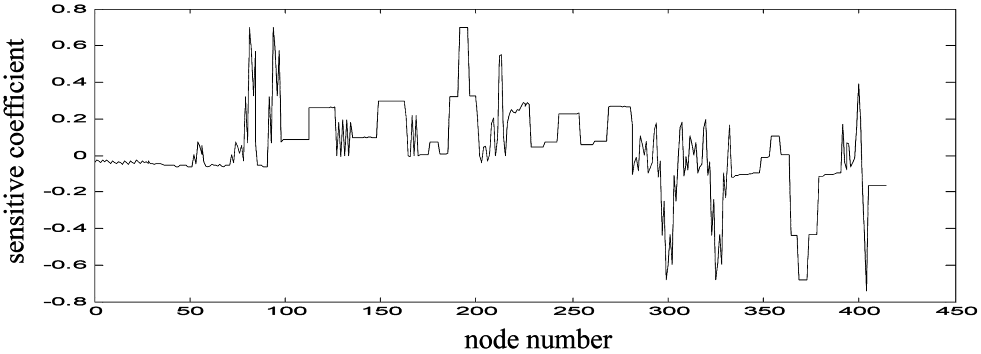


Fig. 18 Second order stiffness sensitivity to the second frequency

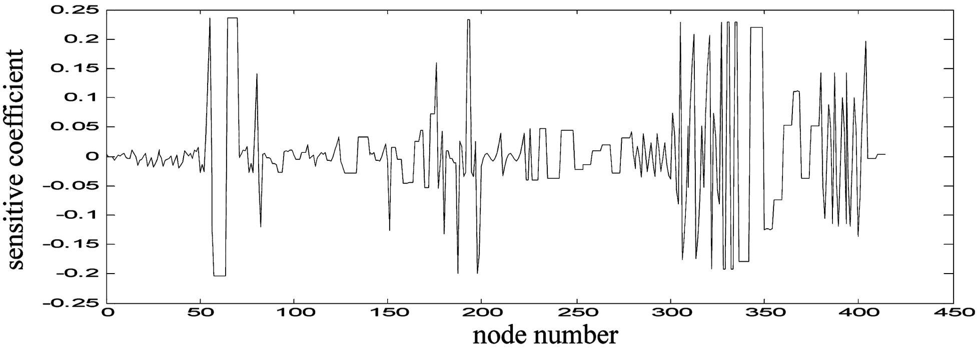


Fig. 19 Second order stiffness sensitivity to the third frequency

For demonstrating the ability of extracting the modal information from the selected optimal sensor set, The output signal from these sensor installed in the real structure of platform MUQ has been used to identify the frequency. In Fig. 20 we contrasted the frequency result corresponding to case (1) with the other cases and the computed frequency based on the sensor from common FEM show in Fig. 10. From the Fig. 20, it demonstrated clearly the modal information obtained from case (1) is satisfied through the lower order frequency to higher frequency. The selected position can be used to gain the offshore platform structural dynamic information and serve for the necessary of SHM.

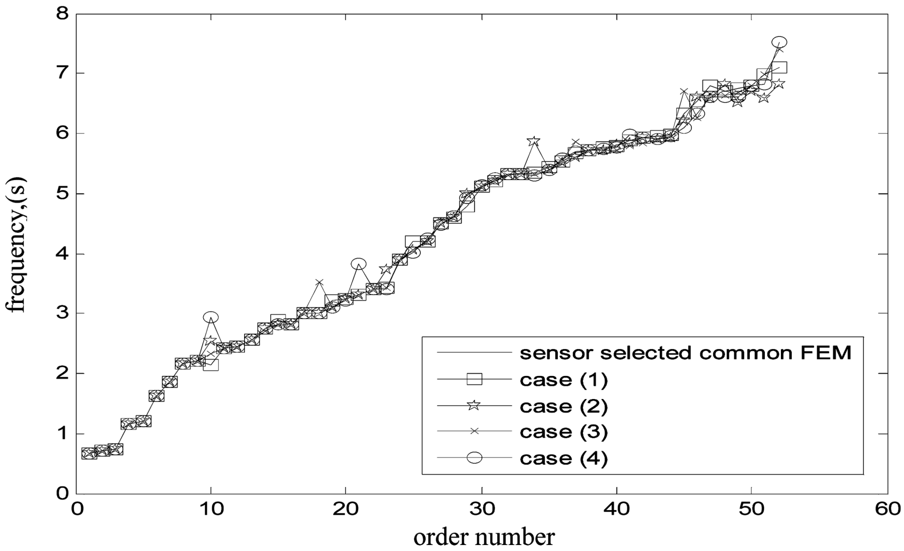


Fig. 20 Comparing identified frequency with all the cases

7. Conclusions

Using the minimum measured point set to obtain maximum modal information is essential principle of selecting the sensor subset. Tolerance domain method can offer efficient initial subset and retain the main modal information. The modal independence combining with tolerance domain make the original measured set hold measured point meet observable and make the modal loss within reasonable range. Sensor set passed the checking of structural second order sensitivity can more reflect the change of structure when their performance is becoming degradable. The method becomes more efficient when structure is more complex and gigantic. The results came from numeric model simulation and the real offshore platform sensor selection processing showed that the selected sensor can present the modal information well when upper bound of modal contribution loss factor is less than 0.5. On the other hand, for the selected sensor must be checked by the second order sensitivity, the picked out measured point set can reflect the change of structure more sensitive in the profile of SHM. It makes these points obtain more useful information which is very advantage to make tracks for damage of structure. So the sensor subset chosen technique is valid for large structure dynamic parameters identification and damage or fault location and valuation in SHM.

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