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Evaluation of shear capacity of FRP reinforced concrete beams using artificial neural networks

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Abstract. To calculate the shear capacity of concrete beams reinforced with fibre-reinforced polymer (FRP), current shear design provisions use slightly modified versions of existing semi-empirical shear design equations that were primarily derived from experimental data generated on concrete beams having steel reinforcement. However, FRP materials have different mechanical properties and mode of failure than steel, and extending existing shear design equations for steel reinforced beams to cover concrete beams reinforced with FRP is questionable. This paper investigates the feasibility of using artificial neural networks (ANNs) to estimate the nominal shear capacity, V_n of concrete beams reinforced with FRP bars. Experimental data on 150 FRP-reinforced beams were retrieved from published literature. The resulting database was used to evaluate the validity of several existing shear design methods for FRP reinforced beams, namely the ACI 440-03, CSA S806-02, JSCE-97, and ISIS Canada-01. The database was also used to develop an ANN model to predict the shear capacity of FRP reinforced concrete beams. Results show that current guidelines are either inadequate or very conservative in estimating the shear strength of FRP reinforced concrete beams. Based on ANN predictions, modified equations are proposed for the shear design of FRP reinforced concrete beams and proved to be more accurate than existing equations.

Keywords: neural networks; fibre-reinforced polymer; shear strength; RC beams.

1. Introduction

To mitigate costly corrosion problems of reinforcing steel in concrete structures, there is a growing trend of using alternative materials, such as stainless, epoxy-coated, and galvanized steel bars. Fiber-reinforced polymer (FRP) has been proposed as an alternative material for reinforcing concrete structures due to its high resistance to corrosion and high strength to weight ratio (ACI 440R-96). The mechanical properties of FRP and its mode of failure differ from those of steel. Thus, a thorough investigation on its behavior as a reinforcement material is required prior to its safe implementation in demanding structural applications.

The tensile strength of FRP is around twice that of steel (Yost, *et al.* 2001), which allows FRP to replace longitudinal steel reinforcement in structural members subjected to flexural loading. Previous experimental and analytical studies concluded that the fundamental principals of flexural theories of concrete beams reinforced with steel bars are also valid for beams reinforced with FRP (Shehata 1999).

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However, due to its sudden and brittle mode of failure, a higher material reduction factor should be used for design purposes when FRP is considered for reinforcement (Shehata 1999). Furthermore, the longitudinal reinforcement contributes significantly to the nominal shear capacity, V_n of reinforced concrete (RC) beams. Such a contribution depends mostly on the axial rigidity and tensile strength of the reinforcing material. Since the axial rigidity of FRP is smaller than that of steel, the amount of shear resisted by concrete, V_{cf} is expected to be lower and needs to be evaluated.

Current shear design procedures and standards assume that the nominal shear capacity of conventionally reinforced concrete beams with shear reinforcement (stirrups), V_n is simply equal to the sum of the concrete contribution to shear resistance, V_c and that of the shear reinforcement, V_s:

$$V_s + V_c = V_n \tag{1}$$

V_c is considered as the shear resistance of a similar reinforced concrete beam but without shear reinforcement, and is usually calculated using a semi-empirical equation. V_s is determined based on the parallel truss model with 45° constant inclination diagonal shear cracks using the following equation:

$$V_s = \frac{A_{sv} f_{sv} d}{s} = \rho_{sv} f_{sv} b_w d \tag{2}$$

where A_{sv} and f_{sv} are the cross-sectional area and yield strength of steel stirrups; d is the beam's effective depth; b_w is the beam's width, s is the spacing between stirrups, and ρ_{sv} is the ratio of steel shear reinforcement.

Despite the substantial amount of experimental and analytical work on this subject, accurately predicting the shear capacity of conventionally reinforced concrete beams by itself remains a complex challenge that has not been fully resolved (Joint ACI-ASCE Committee 445 1998). Furthermore, the axial stiffness of FRP is lower than that of steel and a larger tensile strain in FRP bars is expected when it is used as longitudinal reinforcement, leading to a reduction in the depth of the neutral axis, wider shear cracks, and a reduction in the overall contribution to shear resistance mechanisms. Thus, when FRP is used as shear reinforcement, a reduction factor must be applied to its capacity. For instance, Shehata (1999) reported a reduction in the ultimate capacity of FRP stirrups by up to 79% of the guaranteed tensile strength parallel to the fibers, and observed that failure of FRP stirrups was more likely to occur at bends.

Due to the lack of detailed and reliable design standards and the fact that FRP materials do not offer the desirable plasticity provided by steel reinforcement (brittle sudden failure), current guidelines tend to highly underestimate the shear capacity of FRP reinforced concrete members. Consequently, the excessive amount of FRP needed to resist shear could be both costly and likely to create reinforcement congestion problems (Razaqpur, *et al.* 2004).

Studies (Yost, *et al.* 2001, Shehata 1999, Razaqpur, *et al.* 2004) were conducted to compare the ability of various shear design guidelines to calculate the contribution of concrete and/or FRP stirrups to the nominal shear capacity of concrete beams reinforced with FRP. It was concluded that current shear design guidelines are either inappropriate or highly conservative. Accordingly, the objective of this paper is to investigate the feasibility of using artificial intelligence to predict the shear capacity of concrete beams reinforced with FRP, compare such predictions to results obtained from different shear design guidelines namely, the provisions of the American Concrete Institute (ACI 440.1R-03), the

Canadian Standards Association (CSA S806-02), and the Japan Society of Civil Engineers (JSCE-97). The paper also evaluates the performance of shear design equations proposed by the Canadian Network of Centres of Excellence on Intelligent Sensing for Innovative Structures (ISIS Canada-01). A parametric study was also carried out to evaluate the ability of the proposed ANN model and current shear design guidelines to quantitatively account for the effects of basic shear design parameters on the shear strength of FRP-reinforced concrete beams. Finally, shear equations developed by Zsutty (1968) for steel reinforced concrete beams were modified based on ANN predictions to accommodate the axial rigidity of FRP longitudinal bars and the capacity of FRP stirrups, in order to propose new more accurate shear design equations for FRP-reinforced concrete beams.

2. Shear strength calculation methods

Shear provisions of current design codes and standards calculate the nominal shear capacity of conventionally reinforced concrete beams using Eqs. (1) and (2). They also calculate the shear capacity resisted by concrete, V_c using semi-empirical or statistically derived equations and ignore the influence of stirrups on the contribution of other shear mechanisms. Previous research (Russo and Puleri 1997) concluded that simply adding V_c and V_s to calculate V_n leads to underestimating the shear strength of steel-reinforced concrete beams. Furthermore, due to the brittle failure of FRP materials, current design guidelines tend to further underestimate the shear capacity of FRP-reinforced concrete members to achieve a more conservative design. The various shear design guidelines and shear prediction equations used in this study are briefly discussed below. Note that all strength reduction factors used in the following equations for design purposes are set to one for comparison.

2.1. ACI recommendations

Recommendations provided by ACI Committee 440 assume that Eq. (1) is also valid to calculate the nominal shear capacity, V_n of FRP-reinforced concrete beams as long as shear cracks are adequately controlled. Thus, taking into consideration the axial rigidity of the longitudinal reinforcement, ACI 440-03 evaluates the shear capacity of FRP-reinforced concrete beams without shear reinforcement using the following equation:

$$V_{cf} = \frac{\rho_{fl} E_{fl}}{90 \beta_1 f'_c} V_c \le V_c \qquad (3)$$

V_c is calculated according to shear provisions of ACI building code (ACI 318R-02) using one of the following equations:

$$V_c = (0.17\sqrt{f'_c})b_w d \le 0.3\sqrt{f'_c}\, b_w d \qquad \text{(ACI 11-3)} \quad (4)$$

$$V_c = \left(0.158\sqrt{f'_c} + 17.2\rho_{fl}\left(\frac{Vd}{M}\right)\right) b_w d \le 0.3\sqrt{f'_c}\, b_w d \qquad \text{(ACI 11-5)} \quad (5)$$

where f'_c is the compressive strength of concrete cylinder (MPa); ρ_{fl} is the longitudinal reinforcement ratio; V_c is the shear capacity of the reinforced concrete beam without shear reinforcement (N); E_{fl} is the modulus of elasticity of the longitudinal reinforcement (MPa); β_1 is a material reduction factor that depends on f'_c, $0.65 \leq \beta_1 \leq 0.85$; and M and V are the moment and shear force at a critical section, respectively.

ACI 440-03 limits the stress level in the FRP shear reinforcement to control the width of shear cracks and avoid failure at bends of stirrups. Accordingly, the shear capacity of FRP shear reinforcement is calculated as recommended by ACI 440-03 using Eq. (6):

$$V_f = \rho_{fv}(0.004E_{fv})b_w d \leq \rho_{fv} f_{bend} b_w d \tag{6}$$

where f_{bend}, E_{fv}, and ρ_{fv}, are the strength at the bend portions, modulus of elasticity, and ratio of FRP stirrups, respectively.

2.2. CSA recommendations

According to CSA S806-02, the nominal shear capacity of FRP-reinforced flexural members not subjected to significant axial tension can be calculated using:

$$V_n = V_{cf} + V_f \leq V_{cf} + 0.6\sqrt{f'_c}\, b_w d \tag{7}$$

For reasons similar to those explained above in ACI recommendations, the shear capacity provided by concrete, V_{cf} and by FRP stirrups, V_f in FRP-reinforced concrete beams with minimum shear reinforcement is calculated using Eqs. (8) and (9 and 10), respectively.

$$V_{cf} = 0.035\left(f'_c \rho_{fl} E_{fl} \frac{V_f d}{M_f}\right)^{1/3} b_w d \qquad a/d > 1.0 \tag{8a}$$

$$0.1\sqrt{f'_c}\, b_w d \leq V_{cf} \leq 0.2\sqrt{f'_c}\, b_w d \tag{8b}$$

$$V_f = 0.4\rho_{fv} f_{fuv} b_w d \leq 0.6\sqrt{f'_c}\, b_w d \tag{9}$$

f_{fuv} is the ultimate strength of FRP bars parallel to the direction of fibres.

For beams having an effective depth $d > 300$ mm and without shear reinforcement, or with shear reinforcement less than the minimum specified, V_{cf} is calculated using:

$$V_{cf} = \left(\frac{130}{1000 + d}\right)\sqrt{f'_c}\, b_w d \nless 0.08\sqrt{f'_c}\, b_w d \tag{10}$$

2.3. JSCE recommendations

Guidelines provided by JSCE-97 for the design and construction of FRP-reinforced concrete structures are a modified version of previous specifications for steel-reinforced members. For FRP-reinforced concrete flexural members, JSCE-97 recommends the following equations to calculate the shear capacity carried by concrete, V_{cf}:

$$V_{cf} = \frac{\beta_d \beta_p \beta_n f_{vcd}}{\gamma_b} b_w d \tag{11a}$$

$$\beta_p = \sqrt[3]{\frac{100\rho_{fl}E_{fl}}{E_s}} \le 1.5 \tag{11b}$$

$$\beta_d = \sqrt[4]{\frac{1000}{d}} \le 1.5 \tag{11c}$$

$$f_{vcd} = 0.2\sqrt[3]{f'_{cd}} \le 0.72 \quad \text{(MPa)} \tag{11d}$$

where $\beta_n = 1.0$ for members without axial force, γ_b is a strength reduction factor, $f'_{cd} = f'_c/\gamma_c$ is the design compressive strength, and γ_c is the compressive strength material factor = 1.3 for $f'_c < 50$ MPa and 1.5 otherwise.

The shear capacity carried by FRP shear reinforcement in non pre-stressed flexural members is given by:

$$V_f = \left[\frac{A_{fv}E_{fv}\varepsilon_{fv}(\sin\alpha_s + \cos\alpha_s)}{s}\right]\frac{jd}{\gamma_b} \tag{12a}$$

$$\varepsilon_{fv} = 0.0001\sqrt{\frac{f'_{mcd}\rho_{fl}E_{fl}}{\rho_{fv}E_{fv}}} \le \frac{f_{bend}}{E_{fv}} \tag{12b}$$

$$f_{bend} = \left(0.05\frac{r_b}{d_b} + 0.3\right)\frac{f_{fuv}}{\gamma_{mfb}} \tag{12c}$$

$$f'_{mcd} = f'_{cd}\left(\frac{h}{300}\right)^{-1/10} \tag{12d}$$

where ε_{fv} is the design value to limit the maximum strain in the FRP shear reinforcement, $jd = d/1.15$, α_s is the angle between the shear reinforcement and the member axis, h is the member's height, γ_{mfb}= 1.3 (safety factor for the bend portions of FRP bars), and r_b and d_b are the bend's radius and bar's diameter, respectively.

2.4. ISIS Canada

In its design manual for FRP-reinforced concrete structures, (ISIS Canada-01) adopted the same principals used by CSA A23.3-94 for shear design of steel-reinforced concrete. With slight modifications to existing equations, the ISIS Canada method calculates V_{cf} and V_f using the following equations:

$$V_{cf} = 0.2\sqrt{f'_c}b_w d\sqrt{\frac{E_{fl}}{E_s}} \qquad \text{for } d \le 300 \text{ mm} \tag{13}$$

$$V_{cf} = \left(\frac{260}{1000+d}\right)\sqrt{f'_c}b_w d\sqrt{\frac{E_{fl}}{E_s}} \ge 0.1\sqrt{f'_c}b_w d\sqrt{\frac{E_{fl}}{E_s}} \qquad \text{for } d > 300 \text{ mm} \tag{14}$$

$$V_f = \rho_{fv}\chi f_{fuv}b_w d \le 0.8b_w d\sqrt{\frac{f'_c E_{fv}}{E_s}} \qquad (\chi \text{ is assumed} = 0.4) \tag{15}$$

3. Artificial neural network approach

Multi-layer perceptron networks (MLP) are the most widely used ANNs in engineering applications due to their ability to implement non-linear transformations for functional approximation problems and to map a given input(s) into a desired output(s). They are highly adaptive data-driven trainable systems capable of capturing hidden and complex behavior through learning from training examples. MLP networks consist of multiple layers (an input layer, an output layer and one or more hidden layers). Each layer contains a number of processing elements (units) partially or fully connected to units in the consecutive layer. Connections between processing units are initially assigned random numerical values (weights) representing their strength. The main objective in building an artificial neural network-based model is to train specific network architecture to search for an optimum set of weights, for which the trained ANN can predict accurate values of outputs for a given set of inputs from within the range of the training data. Explaining the fundamental basis of ANNs is beyond the scope of this paper and details on how to build a neural network model can be found elsewhere (Haykin 1994, El-Chabib and Nehdi 2005).

3.1. Experimental database

In this study, shear strength results for 150 concrete beams (50 of which are without shear reinforcement) were collected from published literature (Yost, *et al*. 2001, Shehata 1999, Razaqpur, *et al.* 2004, El-Sayed, *et al*. 2005). Only rectangular, simply supported beams that exhibited shear failure were considered. Beams without shear reinforcement were reinforced with FRP bars as longitudinal reinforcement. All beams with shear reinforcement were reinforced with FRP stirrups, 72 of which had FRP bars and the others had steel bars for longitudinal reinforcement. The database was compiled in a patterned format. Each pattern consists of an input vector containing the geometrical and mechanical properties of RC beams, and an output vector containing the corresponding shear capacity of the beam. Table 1 provides the range and average values of all parameters used in the database.

3.2. ANN model

The network architecture adopted in this study to predict the shear strength of FRP reinforced concrete beams consists of an input layer, an output layer and one hidden layer. The input layer contains six variables representing the common shear design parameters of reinforced concrete beams (d, b_w, a/d,

Table 1 Range of shear design parameters and V_n for beams used in database

Parameters	Without shear reinf. (50 beams)			With shear reinf. (100 beams)		
	Minimum	Maximum	Average	Minimum	Maximum	Average
d (mm)	150.0	360.0	253.1	210.0	500.0	291.2
b_w (mm)	150.0	1000.0	360.5	135.0	300.0	200.8
a/d	1.8	6.5	3.7	1.2	4.3	2.7
$\rho_{fl} E_{fl}$ (GPa)	0.3	3.2	0.9	0.3	9.6	2.0
$\rho_{fv} f_{bend}$ (MPa)	0.0	0.0	0.0	0.3	13.4	3.2
f'_c (MPa)	22.7	49.0	38.5	22.5	84.2	39.5
V_n (kN)	31.5	190.0	87.8	56.2	375.5	164.1

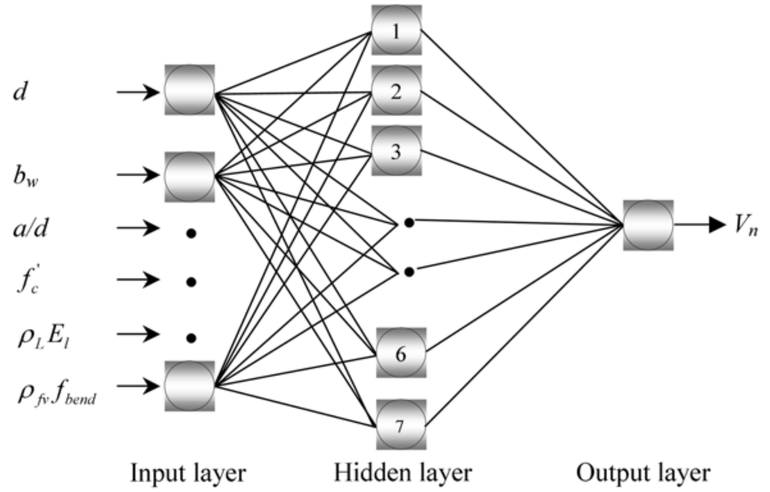

Fig. 1 Architecture of artificial neural network model

$\rho_{fv} f_{bend}$, and f'_c), in addition to the axial rigidity of the longitudinal reinforcement, $\rho_l E_l$. The output layer includes one unit representing the shear capacity, V_n and the hidden layer consists of seven processing units (Fig. 1). Full forward connection (between units of one layer and those of the subsequent layer) was adopted and variable learning rate and momentum were used to avoid lengthy training and ensure global convergence (Haykin 1994). The transfer function used for all processing units was a logarithm sigmoid function (El-Chabib and Nehdi 2005) with outputs varying between 0 and 1. Therefore, prior to the training process, variables in the generated database were scaled between 0 and 1 to speed up the training process and improve the network's generalization. The output values were scaled by simply dividing the shear capacity of each beam by the maximum shear capacity in the database, and input parameters were scaled using the following equation:

$$x_t = 0.1 + 0.8\frac{(x - x_{\min})}{(x_{\max} - x_{\min})} \tag{16}$$

Where x_t, $x_{\min}$ and $x_{\max}$ are the scaled, minimum, and maximum values of variable x in the database, respectively.

The primary factor that influences the performance of an ANN model is the completeness and accuracy of the database. Furthermore, it is recommended that the number of training patterns for a MLP network should be greater than 1.5 times the number of parameters (connections between neurons) to be optimized (El-Chabib and Nehdi 2005). Therefore, due to the relatively small number of beams without shear reinforcement, the ANN model thus developed was trained and validated on data for beams containing shear reinforcement only. The ANN model was trained on 80% of the data and tested on the remaining 20%.

4. Results and discussion

The performance of the ANN model thus developed and trained was evaluated using the testing database described earlier. The network was presented with the input vectors of the testing patterns and

Table 2 Performance of ANN model and various shear design methods in predicting the shear strength of FRP-reinforced concrete beams

Method	Without shear reinf. (50 beams)				With shear reinf. (100 beams)			
	AAE (%)	$V_{measured} / V_{predicted}$			*AAE* (%)	$V_{measured} / V_{predicted}$		
		Average	*STDV*	*COV*		Average	*STDV*	*COV*
ACI 440.1R-03	68.35	4.02	2.11	52.41	45.57	1.90	0.91	47.82
CSA S806-02	33.13	1.68	0.66	39.37	17.49	1.13	0.27	24.14
JSCE-97	33.16	1.69	0.71	41.84	50.05	2.22	0.73	32.67
ISIS Canada-01	34.57	1.69	0.81	48.07	29.52	1.06	0.33	31.61
ANN (training)	-	-	-	-	3.71	1.00	0.06	6.04
ANN (testing)	-	-	-	-	5.89	1.02	0.08	7.87
Proposed	26.01	1.45	0.49	33.85	19.75	1.25	0.33	26.23

asked to predict the corresponding shear capacities. The predicted shear capacities along with those calculated using ACI 440.1R-03, CSAS806-02, JSCE-97, and ISIS Canada-01 were subsequently compared to the experimentally measured values. The performance of each method was evaluated based on both the ratio of measured to predicted (or calculated) shear strength (V_m/V_p), and the average absolute error (*AAE*) calculated using Eq. (17):

$$AAE = \frac{1}{n}\sum\frac{|V_m - V_p|}{V_m} \times 100 \tag{17}$$

The average, standard deviation (*STDV*), and coefficient of variation (*COV*) for V_m/V_p, and the average absolute error (*AAE*) of the ANN model and other shear calculation methods investigated are listed in Table 2.

4.1. Beams without shear reinforcement

A comparative study was conducted to evaluate the performance of current shear design guidelines and recommendations for FRP-reinforced concrete beams without shear reinforcement. The shear strength of such beams calculated using current shear design provisions are plotted against the experimentally measured values in Fig. 2. It is shown that ACI shear design guidelines for FRP reinforced beams are highly conservative even without application of reduction factors (Fig. 2a). The guidelines assume (Eq. 3) that the shear strength of FRP-reinforced concrete beams increases linearly with the axial rigidity, $\rho_{fl}\ E_{fl}$ of the longitudinal reinforcement and decreases as the compressive strength of concrete increases. The first assumption leads to overestimating the shear capacity of highly reinforced concrete beams, especially beams reinforced with carbon fibre-reinforced polymer CFRP (high E_f), whereas the latter assumption yields very conservative shear strength for FRP-RC beams without stirrups.

Shear design equations proposed by CSA S806-02, JSCE-97, and ISIS Canada-01 generally provided conservative results again even before applying reduction factors (Figs. 2b-2d) with an average V_m/V_p of 1.69. However, the ISIS shear Eqs. (13) and (14) generated more scattered results

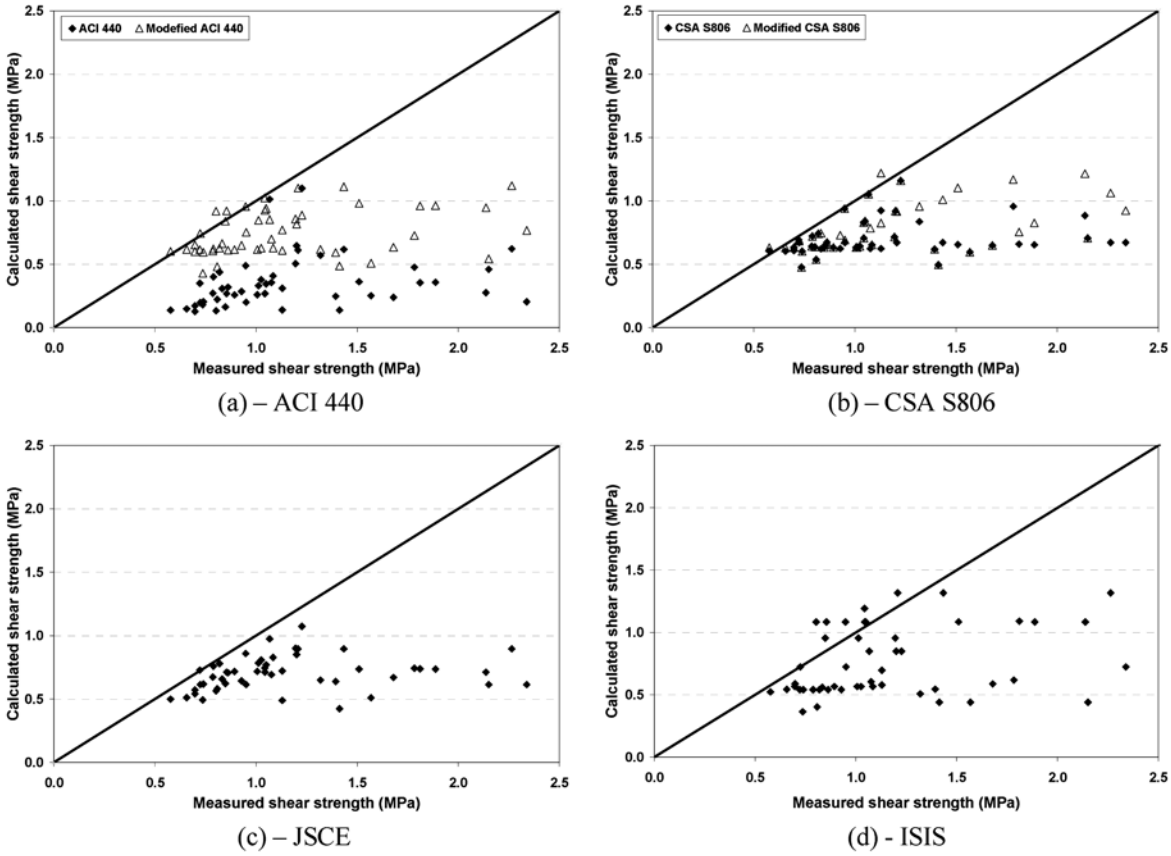

Fig. 2 Measured versus calculated shear capacity of FRP-reinforced concrete beams without shear reinforcement

than those of CSA and JSCE with a *COV* of 48% compared to 39% and 41% for CSA and JSCE, respectively. Current guidelines and recommendations simply assume that existing shear equations, initially derived for steel-reinforced concrete beams, are applicable to concrete beams reinforced with FRP bars provided that they are modified to account for the axial rigidity of the longitudinal reinforcement. However, these methods differ in accounting for the magnitude of the effect of axial rigidity of FRP bars on the total shear resistance, V_{cf} of concrete beams. CSA S806 and JSCE-97 assume that such an effect is to the magnitude of $(E_f/E_s)^{1/3}$, whereas the ISIS method considers the effect to be $(E_f/E_s)^{1/2}$. Furthermore, the ISIS method does not consider the contribution of other shear design parameters on V_{cf}, such as the longitudinal steel ratio, ρ_l, and the shear span to depth ratio, *a/d*. This could explain the very conservative values of shear strength calculated using shear equations of ISIS in the case of slightly reinforced slender concrete beams ($\rho_l < 0.5\%$ and $a/d > 2.5$).

4.2. Beams with shear reinforcement

ANN model predictions of shear strength for concrete beams reinforced with FRP stirrups and those calculated using the design guidelines discussed earlier are plotted against the experimentally measured

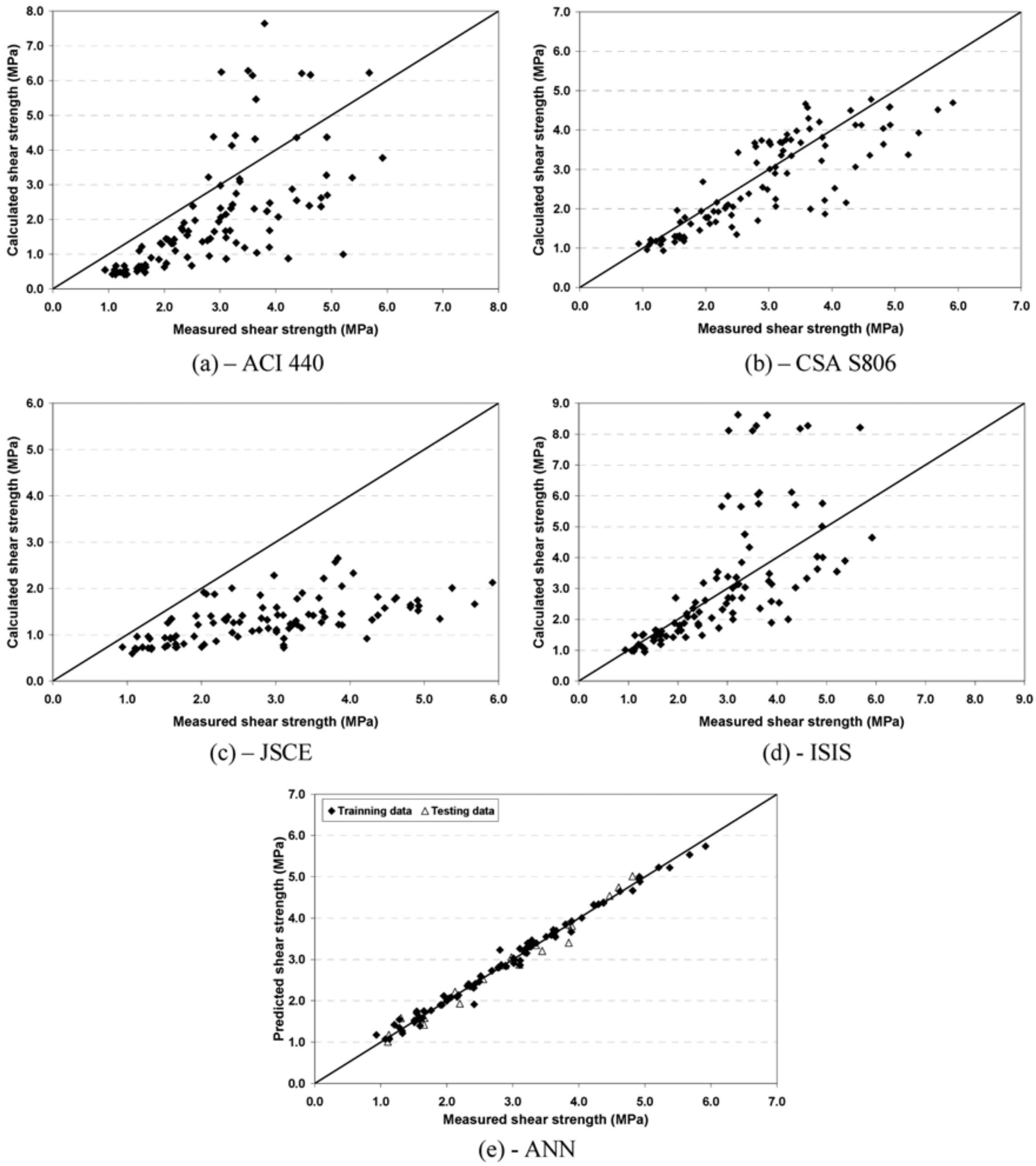

Fig. 3 Measured versus calculated shear capacity of reinforced concrete beams with FRP shear reinforcement

values in Fig. 3. The figure shows that the data points predicted by the ANN model (Fig. 3e) are located either on or slightly over/under, the equity line for both training and testing data, and therefore provided more accurate prediction for the shear strength of FRP-reinforced concrete beams compared to that of all other methods considered in this study. Despite the highly conservative nature of shear equations proposed by ACI 440 in calculating V_{cf} (Fig. 2a), these equations, in some cases, offered unsafe predictions of the nominal shear strength $V_n = V_{cf} + V_f$ (Fig. 3a). This indicates that ACI 440 tends to overestimate the shear capacity carried by FRP shear reinforcement. Analysis of the database indicated that most of the beams for which ACI 440 overestimated the nominal shear strength had either a high shear reinforcement ratio, ρ_{fv} or were reinforced with stirrups having a high modulus of elasticity (E_f). Similar observations are valid for shear equations proposed by ISIS (Fig. 3d), and to a lower extent for equations proposed by CSA S806 (Fig. 3b).

Shear provisions of ACI 440, CSA S806, and ISIS adopted the principal of Eq. (1) in calculating the nominal shear capacity, V_n. However, such provisions assume that stirrups yield at failure and the capacity of FRP shear reinforcement varies linearly with $\rho_{fv} E_{fv}$ (in the case of ACI) or with $\rho_{fv} f_{fuv}$ (for CSA and ISIS). Such assumptions lead to overestimating the shear capacity of concrete beams reinforced with FRP materials of high ultimate strength or high modulus of elasticity. Furthermore, CSA S806 (Eq. 9) and ISIS (Eq. 15) consider that the capacity of shear reinforcement depends on the strength of FRP bars at the bend portion, which is assumed to be 40% of the ultimate tensile strength parallel to the direction of FRP fibres. However, an experimental study conducted by Duranovic, *et al.* (1997) to investigate the shear strength and mode of failure of concrete beams reinforced with FRP stirrups lead to a conclusion that stresses in FRP stirrups at failure never exceeded 65% of their bend's capacity. Another study by Zhao, *et al*. (1995) also lead to the conclusion that the strain distribution in stirrups along a diagonal shear crack is not constant and can be expressed as a cubic root function. This could explain the relatively high nominal shear strength of FRP-reinforced concrete beams calculated using shear provisions of CSA S806 and ISIS Canada despite their conservative calculation of the shear capacity carried by concrete alone, V_{cf}.

4.3. Parametric study on effect of basic shear design parameters

4.3.1. Effect of FRP stirrups on shear strength

Current shear design provisions for FRP-reinforced concrete members generally assume that adding shear reinforcement to a RC beam will only enhance its shear strength by the shear capacity of stirrups.

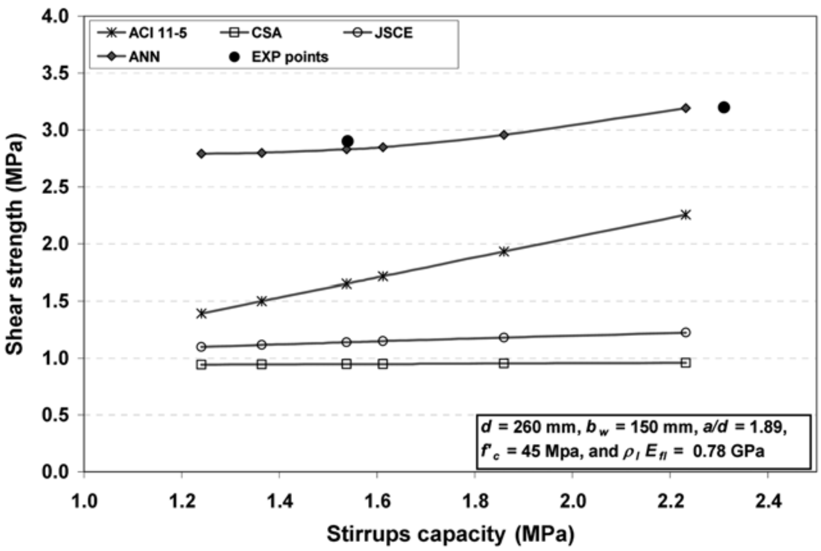

Fig. 4 Effect of FRP shear reinforcement, $\rho_{fv} f_{bend}$ on shear strength of RC beams

Such a practice presumes a linear relationship between the amount of shear reinforcement and the nominal shear strength and ignores the influence of stirrups on other shear resisting mechanisms. A sensitivity analysis was conducted in this study using the ANN model to investigate the effect of FRP stirrups on the shear strength of RC beams. The shear strength of a set of beams having geometrical and mechanical properties similar to those of a beam randomly selected from the database were also calculated for different amounts of shear reinforcement using the shear methods discussed above, and results are plotted in Fig. 4.

The figure shows the effect of $\rho_{fv} f_{bend}$ on the nominal shear strength of FRP-reinforced concrete beams. It can be observed that the ANN response is the closest to the experimental shear strength of beams measured by Vijay, *et al*. (1996). According to ACI 440, and for the range of FRP shear reinforcement considered in Fig. 4, increasing the amount of shear reinforcement by 80% increased the shear strength by more than 60%. The provisions of CSA S806 showed a slight increase of about 2%, whilst ANN and JSCE illustrated an increase of 14 and 11%, respectively. The large increase in shear strength due to the increase in ρ_{fv} as calculated by ACI 440 is due to the linear relationship assumed for the effect of ρ_{fv} on V_n by such provisions.

4.3.2. Effect of longitudinal reinforcement ratio on shear strength

A similar analysis to that used in investigating the effect of FRP stirrups was carried out to determine the effect of the longitudinal reinforcement ratio on the shear strength of FRP-reinforced concrete beams. Fig. 5 shows the relationship between $\rho_{fl} E_{fl}$ and the shear strength of a set of beams calculated using the ANN model and shear design methods considered herein. The figure also includes the experimental shear strength of two similar beams measured by Tottori and Wakui (1993). All beams share similar characteristics except the longitudinal reinforcement ratio, ρ_l. It is apparent that all methods, including the ANN model consider similar influence for the effect of ρ_l on shear strength. However, ACI 440 presumes a linear relationship for such an effect, as opposed to a non-linear effect for the other methods. The figure also shows that the amount of FRP flexural reinforcement exerted a

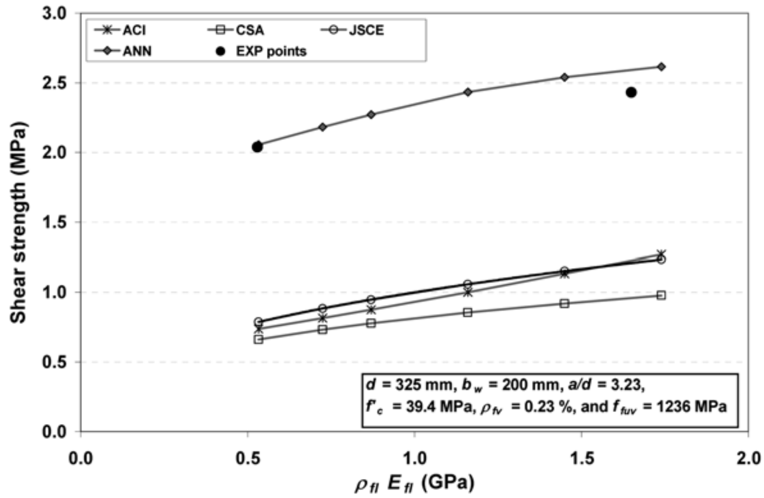

Fig. 5 Effect of the longitudinal reinforcement ratio, ρ_{fl} on shear strength of RC beams with FRP shear reinforcement

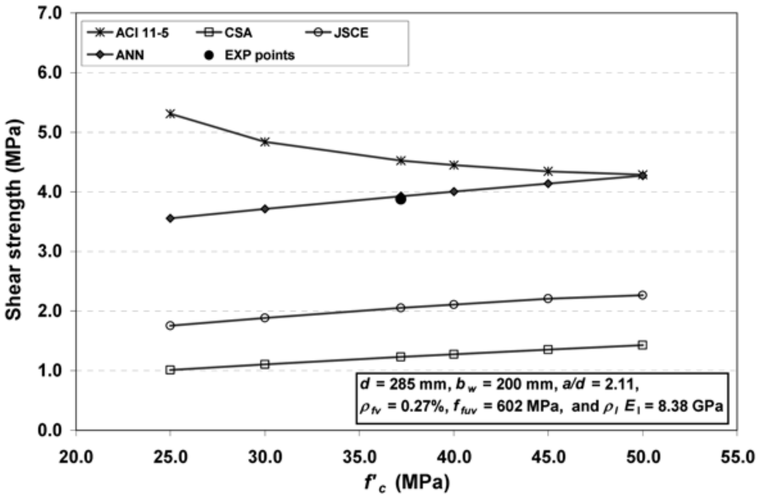

Fig. 6 Effect of concrete compressive strength, f'_c on shear strength of RC beams with FRP shear reinforcement

moderate to slight effect on the shear strength of RC beams with FRP shear reinforcement. Moreover, the ANN model response shows that the rate of such an effect decreased as ρ_l increased, which is in agreement with an experimental investigation conducted by Zhao, *et al*. (1995).

4.3.3. Effect of concrete compressive strength on shear strength

It is generally accepted that an increase in the compressive strength of concrete increases the shear strength of RC beams. In conventionally reinforced concrete beams with shear reinforcement, stirrups restrain shear cracks from widening and provide confinement to concrete in the compression zone, leading to a significant increase in the shear capacity of concrete. Due to the high tensile strain and low modulus of elasticity of FRP, such a benefit needs to be examined and validated. To investigate the ability of shear design guidelines considered in this study to quantitatively account for the effect of f'_c on shear strength of FRP-reinforced concrete beams, a set of six beams similar to a beam randomly selected from the database and tested by Tottori and Wakui were considered. The shear strength of all beams calculated using the various methods and those predicted by the ANN model are plotted against f'_c in Fig. 6. It is shown that all shear design methods account for the effect of f'_c, yet they vary in the magnitude of such an effect. While ISIS assumes that the effect of f'_c on shear strength of FRP-reinforced beams is similar to that in steel-reinforced beams, ACI 440 assumes that the shear strength of FRP-reinforced concrete beams decreases as f'_c increases (Fig. 6). All other methods, including the ANN model captured similar effects of f'_c on the shear strength of FRP-reinforced concrete beams as shown in Fig. 6.

4.3.4. Effect of shear span to depth ratio on shear strength

Fig. 7 shows the variation in shear strength of FRP-reinforced concrete beams with variable shear span to depth ratio (*a/d*). The figure illustrates the effect of *a/d* as estimated by the various shear provisions and calculation methods considered in this study. While ACI 440 and JSCE provisions disregard the effect of *a/d* on the shear strength of RC beams, CSA S806 incorporate a slight to negligible effect regardless of the value of *a/d*. The ANN response exhibited a slight influence of *a/d* on the shear strength of FRP-reinforced concrete beams when $a/d > 2.6$, but showed a larger effect for

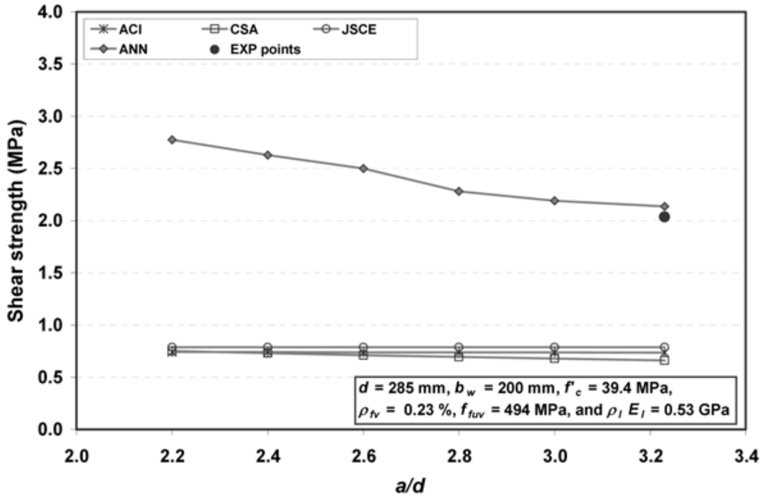

Fig. 7 Effect of shear span to depth ratio, *a/d* on shear strength of RC beams with FRP shear reinforcement

smaller values of *a/d*. Such a behavior is valid for steel-reinforced concrete beams due to the arch action mechanism of deep beams and it is reasonable to assume a similar effect for FRP-reinforced beams. However, due to the limited experimental data on deep beams reinforced with FRP, further investigation is needed for such a conclusion to be confirmed.

5. Proposed shear design equations

Despite the empirical nature of shear equations developed by Zsutty (1968, 1971), they are considered to be amongst the simplest and most accurate in predicting the shear capacity of steel-reinforced concrete beams without shear reinforcement (MacGregor and Bartlett 2000). Modifying Zsutty's method to account for the axial rigidity of the longitudinal reinforcement, the shear capacity of FRP reinforced concrete beams without stirrups can be calculated as follows:

$$V_{cf} = 2.2\left(\frac{E_{fl}}{E_s}f'_c\rho_{fl}\frac{V_f d}{M_f}\right)^{1/3} b_w d \qquad \text{for } a/d \geq 2.5 \quad (18)$$

If $a/d < 2.5$, V_{cf} above is multiplied by $(2.5d/a)$.

The performance of the proposed modification to Zsutty's Eq. (18) in calculating the shear strength of FRP-reinforced concrete beams without stirrups is presented in Fig. 8. It is shown that Eq. (18) provided the most accurate results for the shear strength of FRP-reinforced concrete beams without stirrups with an average value of V_m/V_p of 1.45 compared to 4.02 for the ACI and 1.68 for CSA, JSCE, and ISIS methods. Eq. (18) also provided the lowest *COV* and *AAE* values of 33 and 26%, respectively compared to 52 and 68% for ACI, 39 and 33% for CSA, 41 and 33% for JSCE, and 48 and 34% for ISIS, respectively.

Similar to the modified Zsutty's equation, CSA S806 accounts for the effect of most shear design

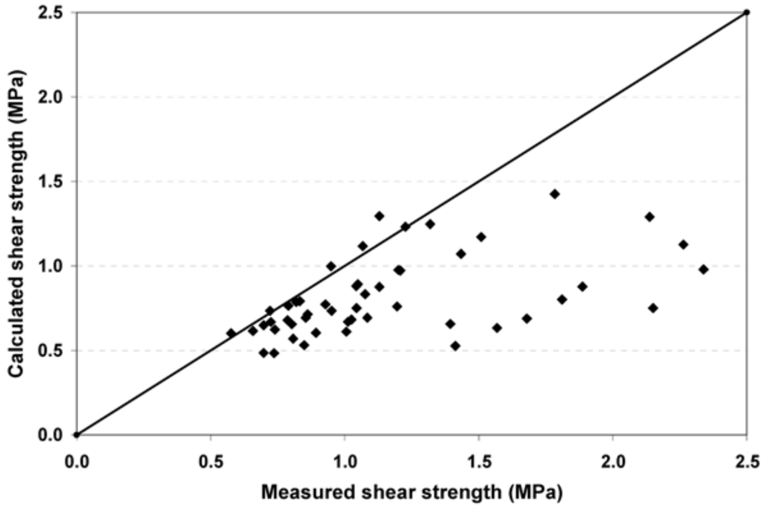

Fig. 8 Performance of the proposed equation in calculating v_{cf} of FRP-RC beams without stirrups

parameters on the shear strength of FRP-reinforced concrete beams. However, it does not consider Zsutty's recommendation to multiply the shear strength by 2.5 d/a when $a/d < 2.5$ for lack of experimental data on the behaviour of FRP-reinforced concrete deep beams. The shear strength of FRP-reinforced concrete beams calculated using the shear equations provided by CSA S806 and Zsutty's recommendation when $a/d < 2.5$ (modified CSA) are also plotted in Fig. 2b. It is shown that implementing Zsutty's recommendation yields more accurate results for the CSA method and reduces the average V_m/V_p from 1.68 to 1.52, and the *AAE* from 33% to 28%. To investigate the possibility of improving conservative results obtained using shear equations proposed by ACI 440, the shear strength of FRP-reinforced concrete beams were calculated using Eq. (5) multiplied by $(E_f/E_s)^{1/3}$ and results are plotted in Fig. 2a. It is shown that the shear strength of FRP-reinforced concrete beams calculated using the modified ACI 440 became reasonably accurate and comparable to those calculated by CSA, JSCE, and the modified Zsutty's equation. This suggests that the shear strength of FRP-reinforced concrete beams is a function of the cubic root of E_f/E_s. However, the less accurate results for beams having shear strength higher than 1.5 MPa (beams having large sections or $a/d > 2.5$) in Fig. 2 is due to the fact that current shear provisions for FRP-reinforced concrete beams do not account for the effect of a/d on the shear strength of deep beams and more research is needed in this area.

Furthermore, analysis on a logarithmic scale of the ANN response for the effect of FRP stirrups on the shear strength of FRP-reinforced concrete beams revealed that the capacity of FRP stirrups is a function of the square root of $\rho_{fv} f_{fuv}$ as opposed to a linear relationship adopted by most current shear design guidelines. Thus, the shear capacity of FRP stirrups can be calculated using Eq. (19):

$$V_f = \frac{\sqrt{\rho_{fv} E_{fv}}}{2} b_w d \tag{19}$$

Shear strength values calculated using Eqs. (18) and (19) versus the experimentally measured shear

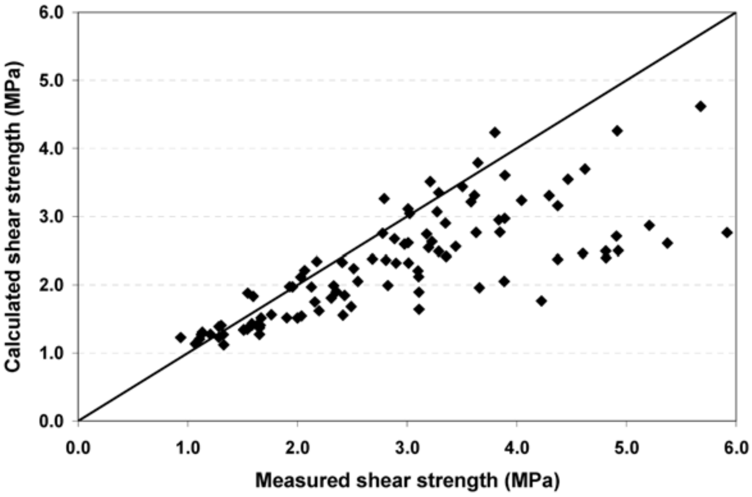

Fig. 9 Performance of the proposed equations in calculating v_n of FRP-RC beams with stirrups

strength of FRP-reinforced concrete beams are shown in Fig. 9. The figure shows that the proposed equations provided better, yet conservative, results than that of all current shear design guidelines considered in this study. Shear equations of CSA S806 offered a lower *AAE* than that of the proposed equations (Table 2). However, Fig. 3(b) shows that CSA S806 overestimated the shear strength of more than 30% of beams in the database, and in some cases the calculated shear strength was 40% higher than the measured value. Conversely, the proposed equations offered safe predictions for more than 95% of the beams with the highest overestimated shear strength only about 10% higher than the measured value. Due to the brittle and sudden failure of FRP materials, a safer design is required. Eqs. (18) and (19) generally offer a safer yet more accurate design compared to existing guidelines.

6. Performance of existing shear design provisions with present recommendations

As stated earlier, most shear design provisions for FRP-reinforced concrete beams are based on modified versions of shear equations initially derived for steel reinforced concrete beams. These provisions vary greatly in the number of parameters they consider and the intensity of the effect of such parameters. Implementing the proposed effects of the axial rigidity of longitudinal reinforcement and the capacity of FRP stirrups on the shear strength of FRP-reinforced concrete beams into equations provided by ACI 440 and CSA S806 are shown in Eqs. (20a) and (20b) and (21a, 21b, and 21c), respectively. The ability of these equations to safely estimate the shear strength of FRP-reinforced concrete beams compared to that of the original equations is shown in Figs. 10 and 11, respectively.

$$V_{cf} = \left(0.158\sqrt{f'_c} + 17.2\rho_{fl}\left(\frac{Vd}{M}\right)\right)\left(\frac{E_f}{E_s}\right)^{1/3} b_w d \le 0.3\sqrt{f'_c} b_w d\left(\frac{E_f}{E_s}\right)^{1/3} \tag{20a}$$

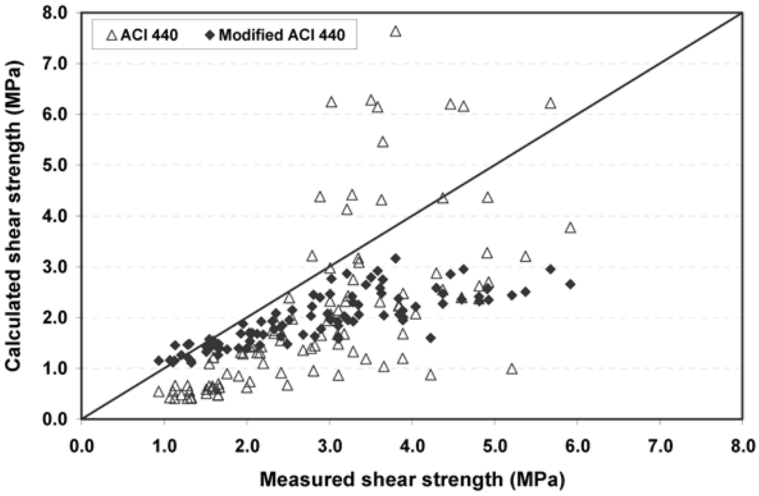

Fig. 10 Performance of the modified version of ACI 440 in calculating v_n of FRP-RC beams

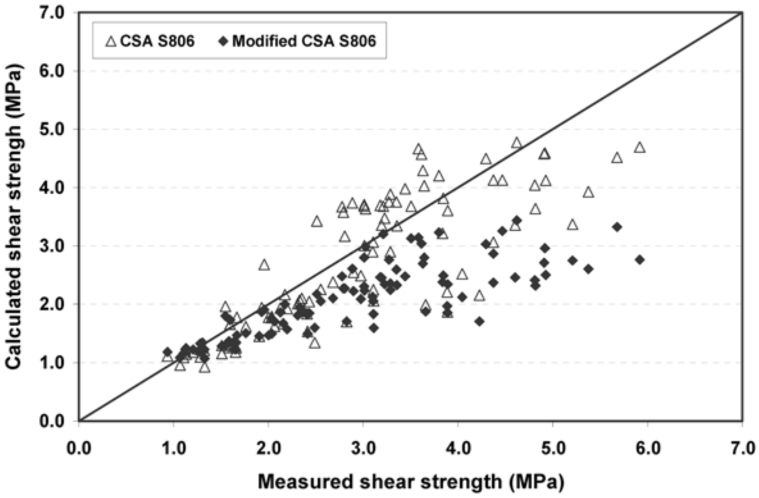

Fig. 11 Performance of the modified version of CSA S806 in calculating v_n of FRP-RC beams

$$V_f = \frac{\sqrt{\rho_{fv} f_{fuv}}}{2} b_w d \tag{20b}$$

$$V_{cf} = 0.035\left(f'_c \rho_{fl} E_{fl} \frac{V_f d}{M_f}\right)^{1/3} b_w d \qquad \text{for } a/d > 2.5 \tag{21a}$$

for $a/d < 2.5$ V_{cf} is multiplied by $2.5d/a$

$$0.1\sqrt{f'_c} b_w d \le V_{cf} \le 0.2\sqrt{f'_c} b_w d \tag{21b}$$

$$V_f = \frac{\sqrt{\rho_{fv} f_{fuv}}}{2} b_w d \tag{21c}$$

Fig. 10 shows the shear strength of RC beams calculated using current and suggested modified equations of ACI 440. It is observed that Eqs. (20a) and (20b) (modified equations of ACI 440) yield safer and less scattered results than that of current Eqs. (5) and (6). Incorporating the suggested modifications into current equations of ACI 440 lowered the average V_m/V_p from 1.90 to 1.43, and the *STDV*, *COV*, and the *AAE* from 0.91, 47.82, and 45.57 to 0.37, 25.70, and 27.61, respectively. The performance of the equations adopted by CSA S806 and the suggested modified version of such Eqs. (21a) and (21b) is illustrated in Fig. 11. It is shown that current equations of CSA S806 yield more accurate results than that of the modified ones. However, these equations overestimated the shear strength of a large number of concrete beams reinforced with FRP stirrups, and in some cases the overestimation was more than 35%. Bearing in mind that the shear failure of FRP-reinforced concrete beams is brittle and sudden, shear equations for FRP-reinforced concrete beams should be accurate yet conservative. Such requirements are better satisfied by the modified version of CSA S806 equations as shown in Fig. 11.

7. Conclusions

This study investigated using artificial neural networks for predicting the nominal shear strength of FRP-reinforced concrete beams with shear reinforcement, and compared such predictions with those of several existing shear design and calculation methods. Furthermore, a parametric study was carried out to evaluate the effect of basic shear design parameters on the shear strength of FRP-RC beams and the accuracy of current shear design provisions in capturing the effect of such parameters on shear strength. The following conclusions can be drawn:

- A successfully trained ANN model can be used as an effective tool for predicting the shear strength of FRP-RC beams and evaluating the effect of basic shear design parameters on the shear behaviour of such beams. The ANN approach outperformed all other shear design and calculation methods considered in this study.
- Shear provisions of ACI 440 are extremely conservative in estimating the shear strength of FRP-RC beams without shear reinforcement. All other shear provisions considered in this study also provided conservative results for such beams.
- The equation proposed by ACI 440 to calculate the capacity of FRP stirrups, V_f is adequate for FRP materials having low modulus of elasticity. However, it overestimates the capacity of FRP stirrups with high modulus of elasticity to a degree where the nominal shear capacity of FRP concrete beams reinforced with such stirrups becomes very unconservative despite its highly conservative estimation of V_{cf}.
- Shear provisions of JSCE-97 provided relatively accurate estimation of the shear capacity supplied by FRP shear reinforcement for low shear capacity beams. However, such provisions are very conservative for beams having high shear capacity.
- The CSA S806 method provided reasonably accurate estimations for the shear strength of concrete beams having relatively low shear reinforcement ratio, but overestimated the shear capacity of highly reinforced beams with FRP stirrups of high tensile strength. Due to the brittle failure of FRP materials, such equations need to be re-evaluated for the case of CFRP reinforcement.

- Shear design equations proposed in this study incorporated the axial rigidity of FRP longitudinal bars as a cubic root function of E_f/E_s and assumed that the contribution of FRP stirrups to shear strength is a square root function of $\rho_{fv} f_{fuv}$ rather than a linear function as proposed by CSA S806 and ACI-440. It provided the most accurate (but still conservative) results in calculating both V_{cf} and V_f amongst existing shear evaluation methods considered in this study.
- The effect of the axial rigidity of FRP longitudinal reinforcement on the shear capacity of FRP-reinforced concrete beams is better represented by a cubic root function rather than linear or square root relationships as proposed by ACI 440 and ISIS-01, respectively. CSA S806, JSCE, and the modified version of Zsutty's equation adopted such a relationship, while incorporating it into ACI 440 yielded better estimations of shear strength.

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