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Bi-spectrum for identifying crack and misalignment in shaft of a rotating machine

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Abstract. Bi-spectrum is a tool in the signal processing for identification of non-linear dynamic behvaiour in systems, and well-known for stationary system where components are non-linearly interacting. Breathing of a crack during shaft rotation is also exhibits a non-linear behaviour. The crack is known to generate 2X (twice the machine RPM) and higher harmonics in addition to 1X component in the shaft response during its rotation. Misaligned shaft also shows similar such feature as a crack in a shaft. The bi-spectrum method has now been applied on a small rotating rig to observe its features. The bi-spectrum results are found to be encouraging to distinguish these faults based on few experiments conducted on a small rig. The results are presented here.

Keywords: rotating machine; non-linear dynamics; cracked shaft; misaligned shaft; vibration experiments; bi-spectrum.

1. Introduction

Faults keep generating in rotating machines. Identification of faults and their rectification is important from safety and productivity consideration. Vibration based condition monitoring (Sinha 2002) is generally used for such requirement. In general, each fault can be identified by some kind of characteristic responses of shaft either during steady state machine operation or during transient (runup or rundown) operation. For example, both crack in a shaft or misaligned shaft generate 2X (twice the machine RPM) and higher harmonics in addition to 1X component in the shaft response during its rotation.

In case of a crack in shaft, the stiffness of the shaft will vary from high-to-low-to-high in a complete revolution of the shaft due to the breathing (opening and closing) of the crack due to the rotor self weight (Mayes and Davies 1984). This behaviour of the crack shaft also generates a 2X component (twice the machine RPM) in the dynamic response of shaft during its rotation. It is also known that both amplitude and phase of the 1X and 2X components change with the time of machine operation due to the propagation of the transverse crack. This can be observed in either the polar plots or the amplitude-phase versus time plots (Bently 1982). The orbit plot consisting of 1X and 2X components may change from a single loop to double loops like a '*figure eight*'. However during transient operation (run-up or run-down) of the machine, the shaft vibration may be very high when

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the machine speed passing through nearly half of the machine critical speed. At this particular moment the shape of the orbit plot changes from a *figure eight* to *a loop containing a small loop* inside indicating a significant change in the phase and amplitude of the 2X vibration which is close to the system natural frequency (Bently 1986, 1990, Adewusi and Al-Bedoor 2002). Analytically such an observation has also been shown by Jun, *et al.* (1992), and Yang, *et al.* (2002). Wauer (1990) and Gasch (1993) gave the comprehensive review on the dynamics of cracked rotors. However the explanation of the existence of 2X component in case of a misaligned shaft is not very straightforward like the breathing of a crack. Few earlier studies (Sekhar and Prabhu 1995, Xu, and Marangoni 1994) have considered some kind of forces and moments acting at the coupling due to misalignment in the shaft to explain the generation of 2X component. Jordan (1993) confirms that the misalignment initially affects the 1X response resulting in an elliptical orbit but in the case of severe misalignment the orbit plot may look like a *figure eight* due to the appearance of a high 2X component in the response. Hence both the faults (crack and misalignment) can be identified and distinguished but require transient operation of machine and the measurements in both horizontal and vertical directions in most cases.

In the recent years the higher order spectrum (HOS)-Bi-spectrum and Tri-spectrum is receiving attention for detection of a non-linear behaviour in structural dynamic response (Fackrell, *et al.* 1995a, 1995b, Collis, *et al.* 1998). Kocur and Stanko (2000) have demonstrated the overall condition of a good reciprocating machine can be distinguished from the bad one by the order bi-spectrum analysis of the vibration or sound signal based on their experiments on a car engine. Here also, the bi-spectrum was also computed for the shaft vibration responses for both the faults (crack shaft and misaligned shaft) simulated separately in the shaft of a small experimental rig. It has been observed that the computed bi-spectrum of the shaft response is a most promising tool to distinguish these faults. In fact, only one direction (either vertical or horizontal) shaft response measurement at a location may be capable to identify these faults. A simple analytical simulation of the experiment with a crack shaft was also carried out. The paper presents the vibration measurements, signal analysis, observations and the analytical simulation.

2. The bi-spectrum

Bi-spectrum (Fackrell, *et al.* 1995a, 1995b, Collis, *et al.* 1998) is a tool in the signal processing for identification of non-linear dynamic behaviour in system, and is known as the higher order spectrum (HOS). The conventional power spectrum (PS) provides information on the second-order properties (i.e. energy) of a signal whereas the bi-spectrum can provide information on the signal's third-order properties. In a physical sense, the bi-spectrum provides insight into non-linear coupling between frequencies (as it involves both amplitudes and phases) of a signal compared to the traditional PS that gives only the content of different frequencies and their amplitudes in a signal. The definition and concepts about the bi-spectrum can be found in the literatures (Fackrell, *et al*. 1995a, 1995b, Nikias and Petropulu 1993), and hence not discussed in details. However the computational approach used in the study is discussed here. As it is known that the power spectrum density (psd) of a time series $\mathbf{x}(t)$ is computed as

$$\mathbf{S}_{xx}(f_k) = \mathbf{X}(f_k)\mathbf{X}^*(f_k) \tag{1}$$

where $\mathbf{S}_{xx}(f_k)$ is the psd, $\mathbf{X}(f_k)$ and $\mathbf{X}^*(f_k)$ is the discrete Fourier Transformation (DFT) and its complex

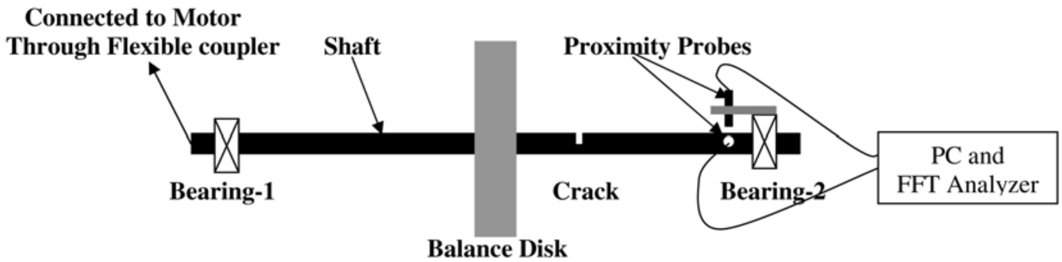

Fig. 1 Schematic of the experimental rig

conjugate at frequency f_k for the time series $\mathbf{x}(t)$. Here the psd has been computed for the several segments of the signal $\mathbf{x}(t)$, and then averaged. Similarly the bi-spectrum is computed here as (Fackrell, *et al.* 1995).

$$\mathbf{B}_{xxx}(f) = \mathbf{X}(f_k)\mathbf{X}(f_l)\mathbf{X}^*(f_{k+1}) \tag{2}$$

The bi-spectrum is also averaged over a large number of computed bi-spectrum for the different segments of the signal $\mathbf{x}(t)$ similar to the psd.

3. The experimental rig

The rig consists of a 10 mm OD steel shaft of 550 mm long through two bush bearings, which are directly mounted on a rigid massive base plate. The left end of the shaft is coupled with the motor shaft through a flexible coupler. The bearings are placed at 20 mm and at 510 mm from the left end of the shaft. The shaft also carries one balance disk made of steel and placed at mid-span of the two bearings. Balance disk dimensions are 75 mm OD, 10 mm ID and 25 mm thickness. The schematic of the rig is shown in Fig. 1. The first natural frequency of the shaft (with no crack and with an open crack) in the rig is identified at 27.50 Hz (both horizontal and vertical directions) by modal tests (Ewins 2000).

Considering the fact that the most of the rotating machines, e.g., Turbogenerator (TG) sets are now equipped with the proximity probes near the bearings to monitor the shaft relative displacements during normal operation. Here also proximity probes both in vertical and horizontal directions were mounted on the bearing-2 housing of the rig to measure the shaft displacement so that the observations made here can be useful for the real life machines. These arrangements are also shown in Fig. 1. The data were recorded in the computer with the sampling rate of 2560 samples/s with anti-aliasing filter of 2 kHz.

4. Experiments

Experiments were carried out on the rig for both the faults separately -one with a crack in the shaft and other with misaligned shaft. The experimental observations in both cases are discussed here.

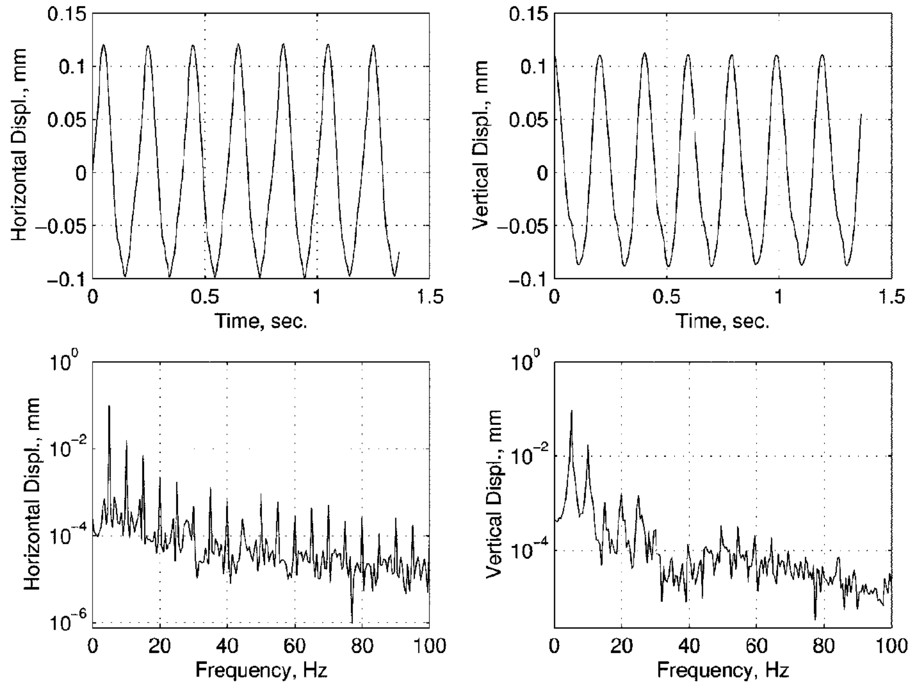

Fig. 2 Dynamic behaviour of a crack shaft at 300 RPM

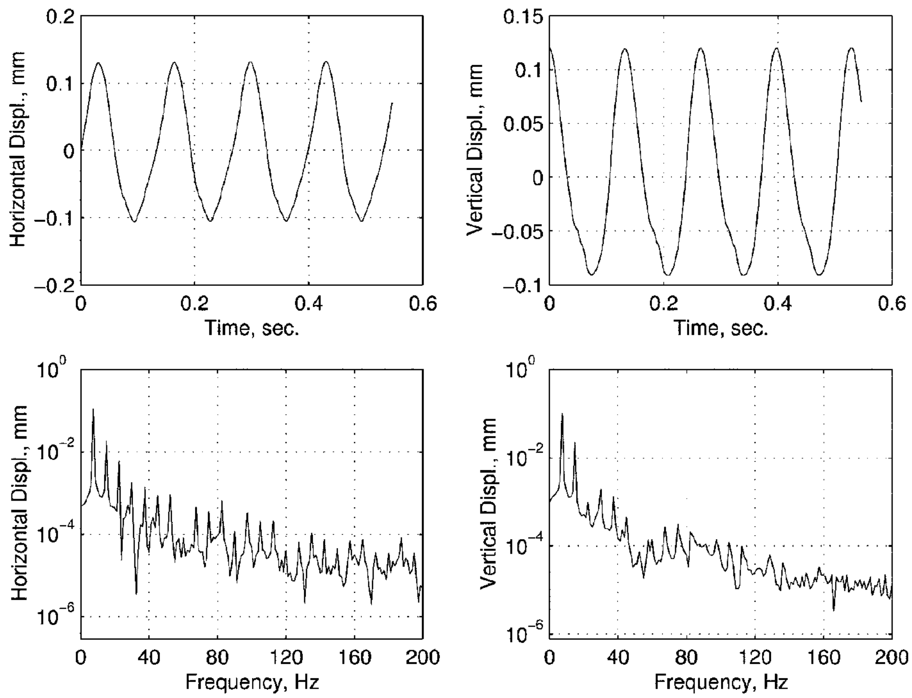

Fig. 3 Dynamic behaviour of a crack shaft at 450 RPM

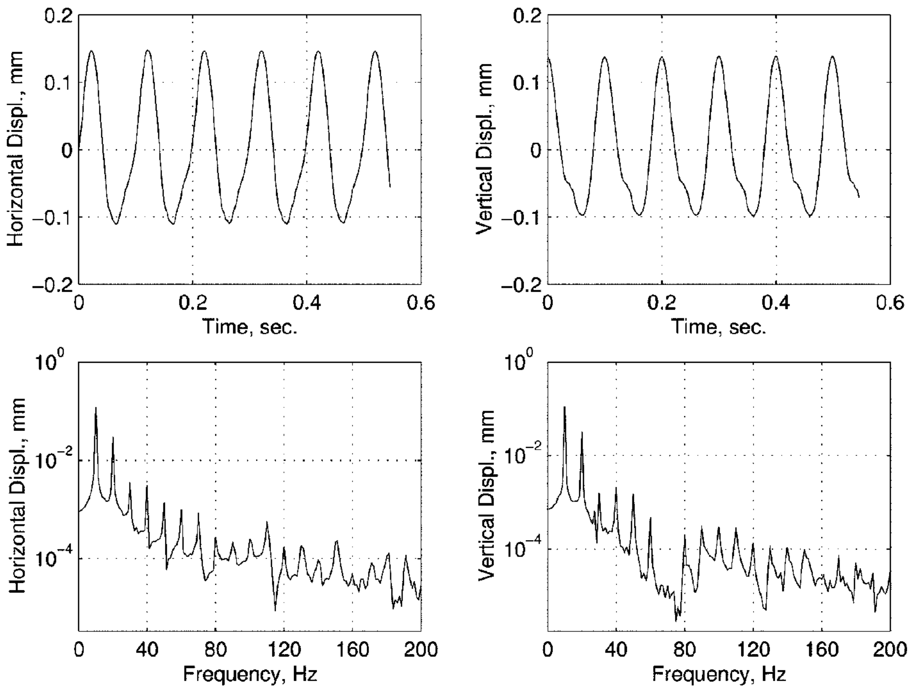

Fig. 4 Dynamic behaviour of a crack shaft at 600 RPM

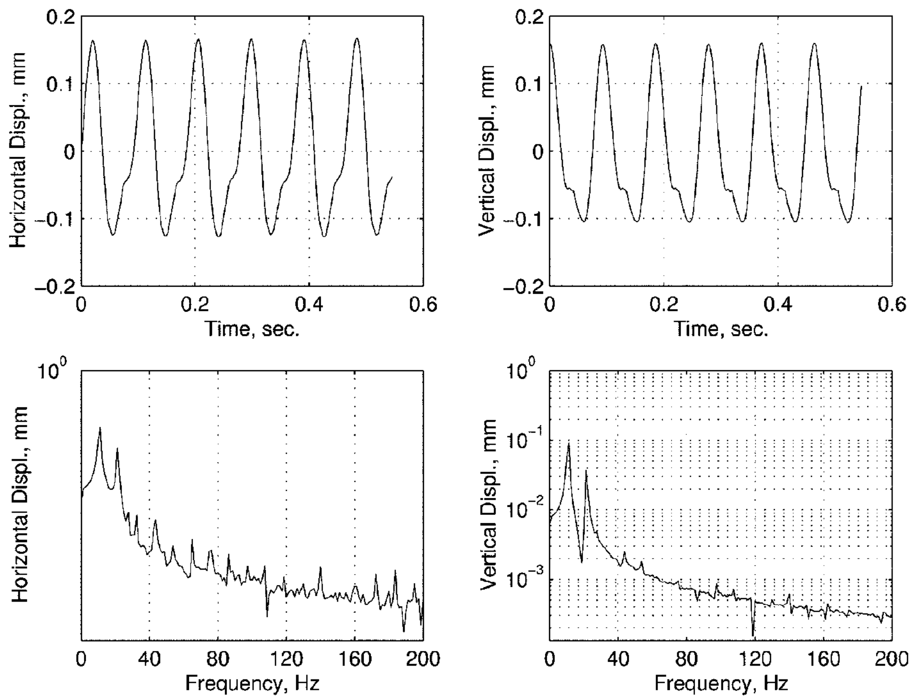

Fig. 5 Dynamic behaviour of a crack shaft at 650 RPM

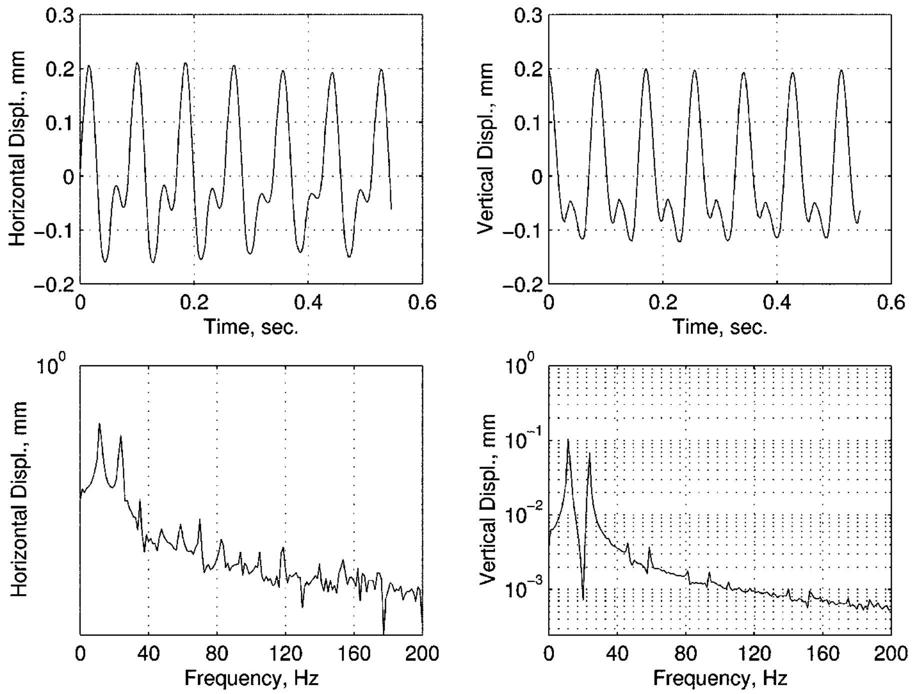

Fig. 6 Dynamic behaviour of a crack shaft at 700 RPM

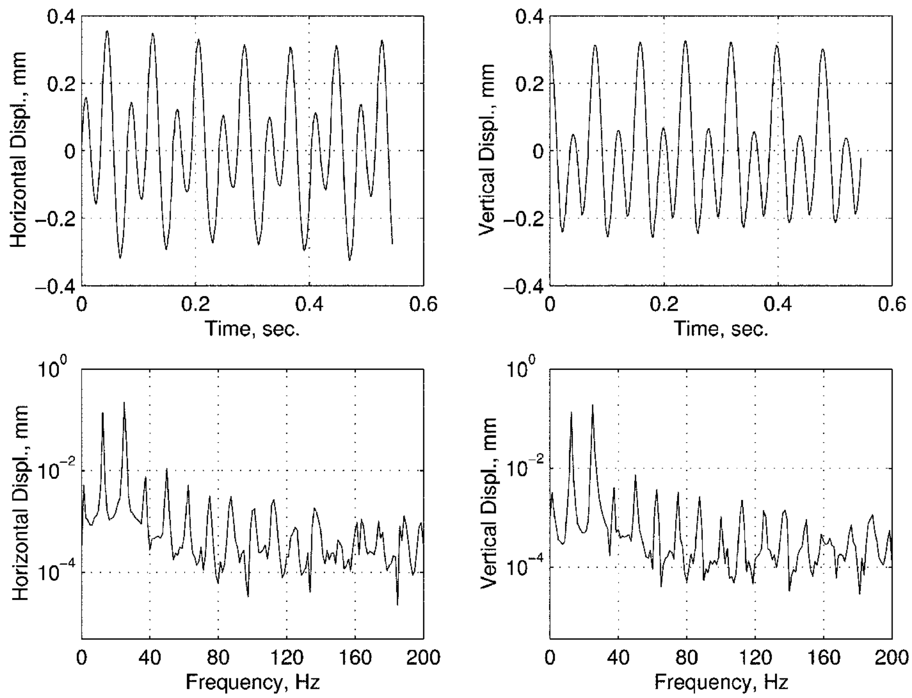

Fig. 7 Dynamic behaviour of a crack shaft at 750 RPM

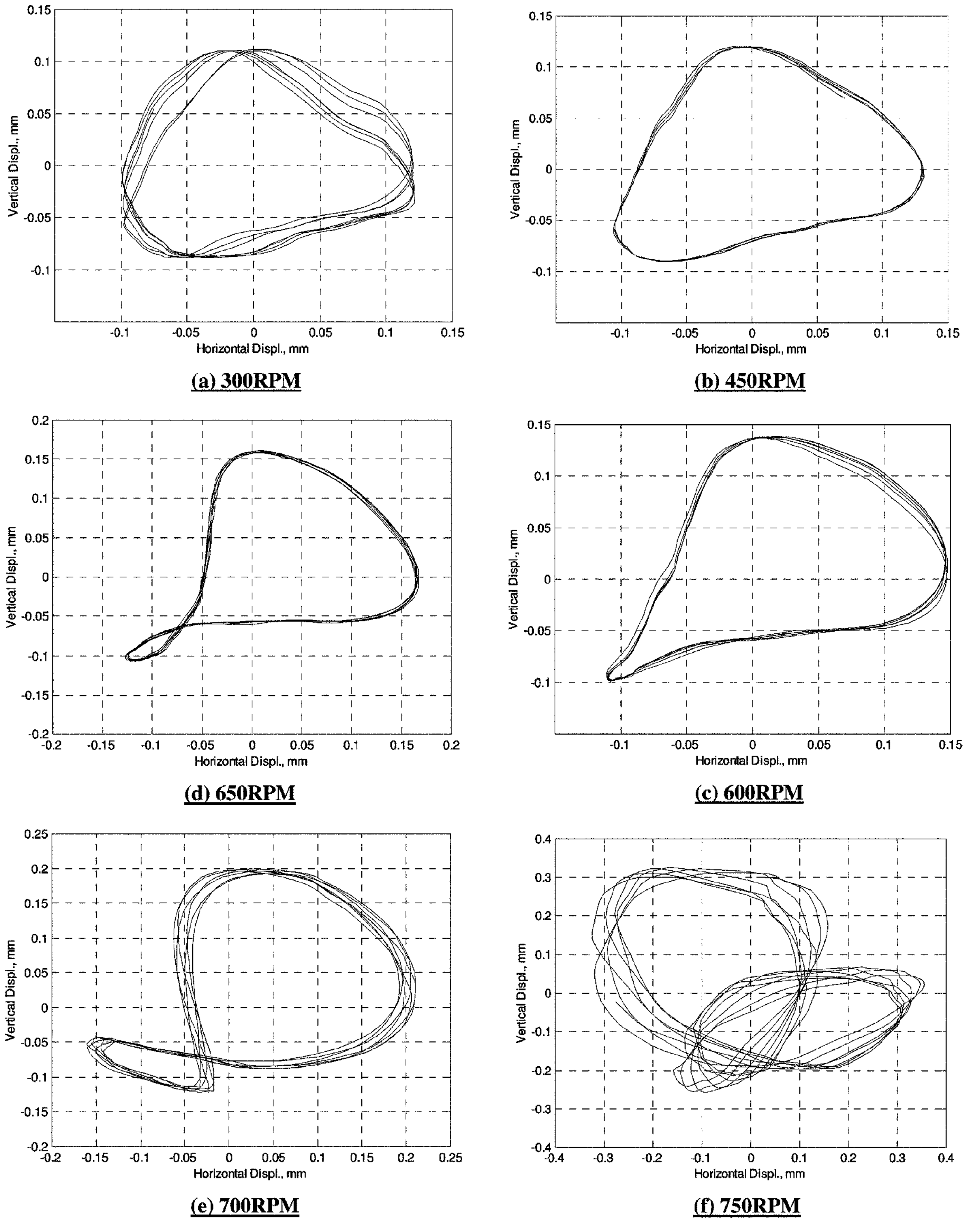

Fig. 8 Orbit plots of a cracked shaft responses at different rotor speed

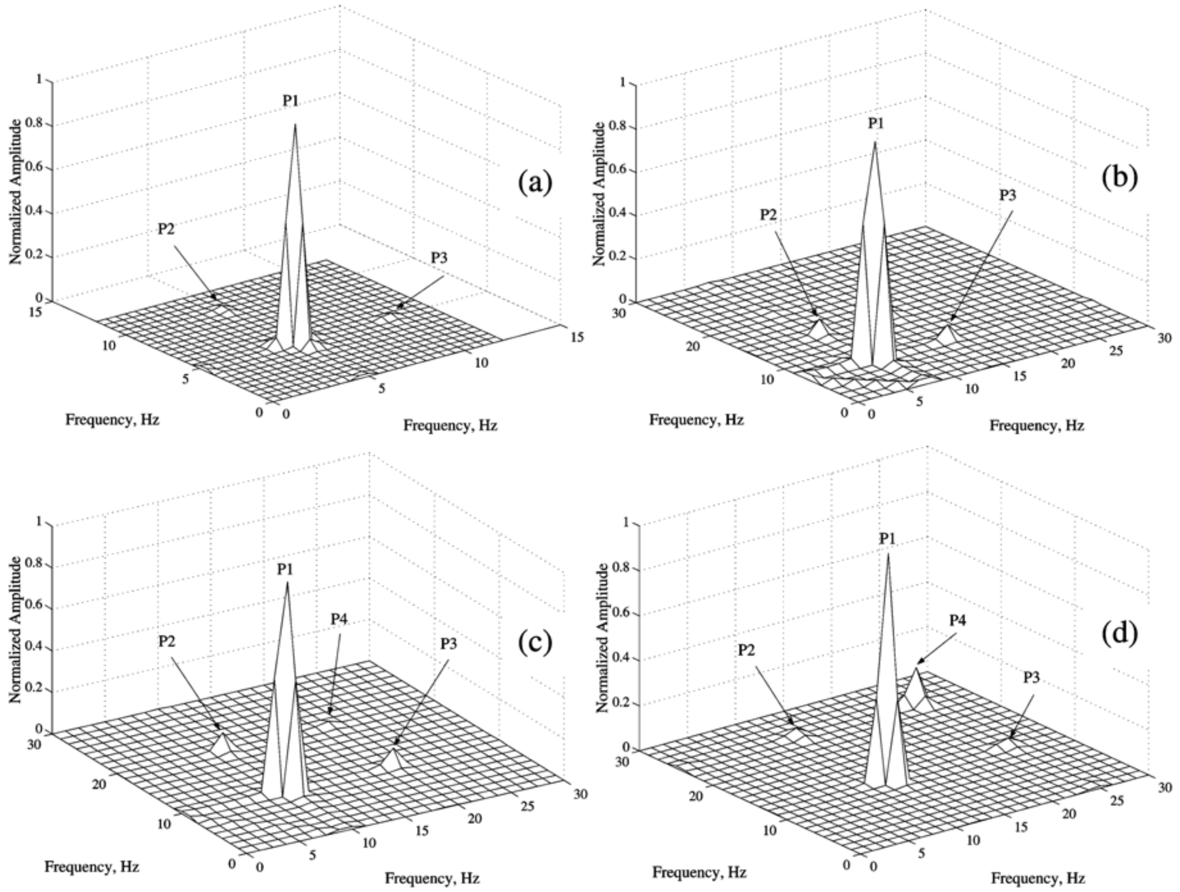

Fig. 9 Bi-spectra of a cracked shaft response in vertical direction
((a) 300 RPM, (b) 450 RPM, (c) 600 RPM, (d) 750 RPM)

4.1. Fault 1: Cracked shaft

The crack in the shaft was developed such that the breathing of the crack can be possible during the shaft rotation. The location of the crack was at 315 mm from left and depth equals to the half of the shaft diameter. The first natural frequency of the cracked shaft is identified at 26.25 Hz in vertical direction when the crack was fully open by modal tests (Ewins 2000).

Experiments were conducted at different rotating speeds during machine run-up to explore the behaviour of crack breathing during the shaft rotation. The measured vibration behaviour of the shaft both in vertical and horizontal directions in time and frequency (the amplitude spectrum only) domains are shown in Figs. 2-7 for the rotating speeds of 300 RPM, 450 RPM, 600 RPM, 650 RPM, 700 RPM and 750 RPM (close to haft the natural frequency of the cracked shaft). The orbit plots for these data are shown in Fig. 8. As seen from the figures the distortion in the vertical and horizontal displacements is very small at 300 RPM, and is progressive increasing with increase in speed. The formation of small loop in the orbit plot at higher speed and the shift of the loop location clearly indicate the phase shift from one speed to another speed. The orbit plots of '*eight figure*' in Figs. 8(d) and (e) indicate that the

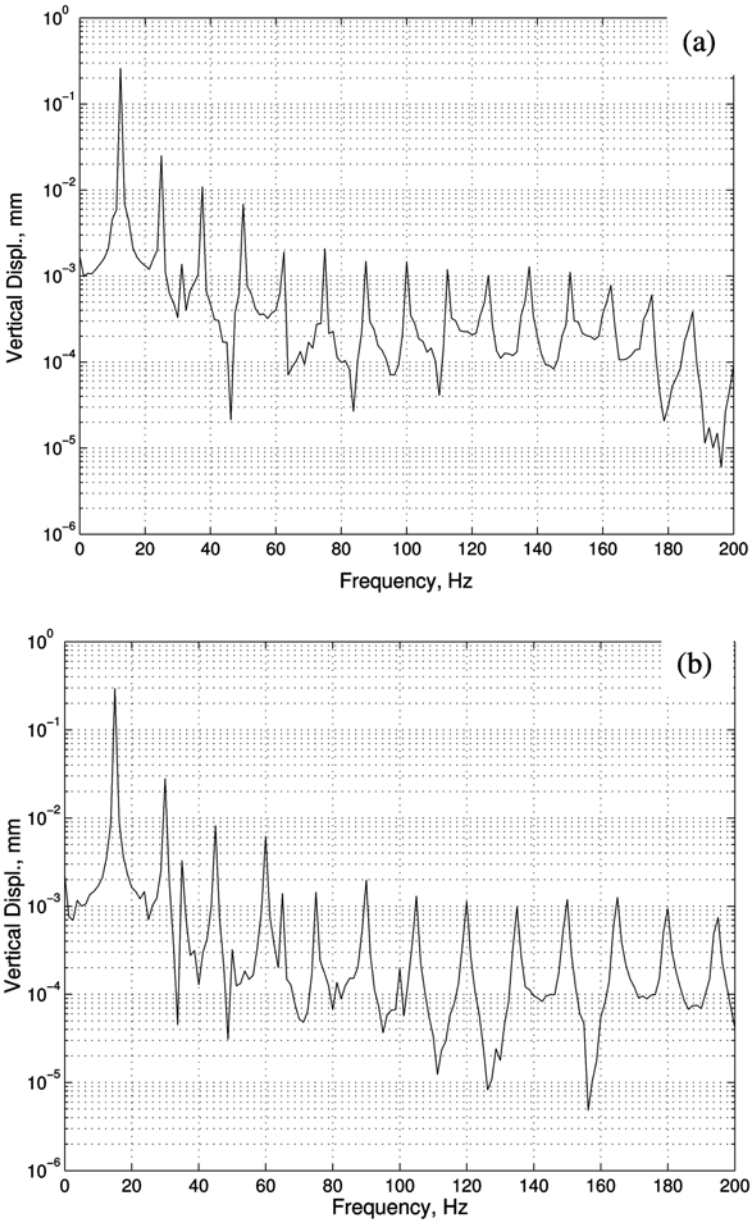

Fig. 10 Spectra of a misaligned shaft response in vertical direction ((a) 750 RPM, (b) 900 RPM)

dynamic behaviour is more like a severe misaligned shaft. However the orbit plot of *a loop containing a small loop* at 750 RPM (nearly half the natural frequency of crack shaft) indicates a significant change in the phase and amplitude of the 2X component which is close to the natural frequency of the crack shaft. These observations are well known and generally used in the vibration based condition monitoring.

The bi-spectra were also estimated at these rotating speeds. The 50 segments using 50% overlap in

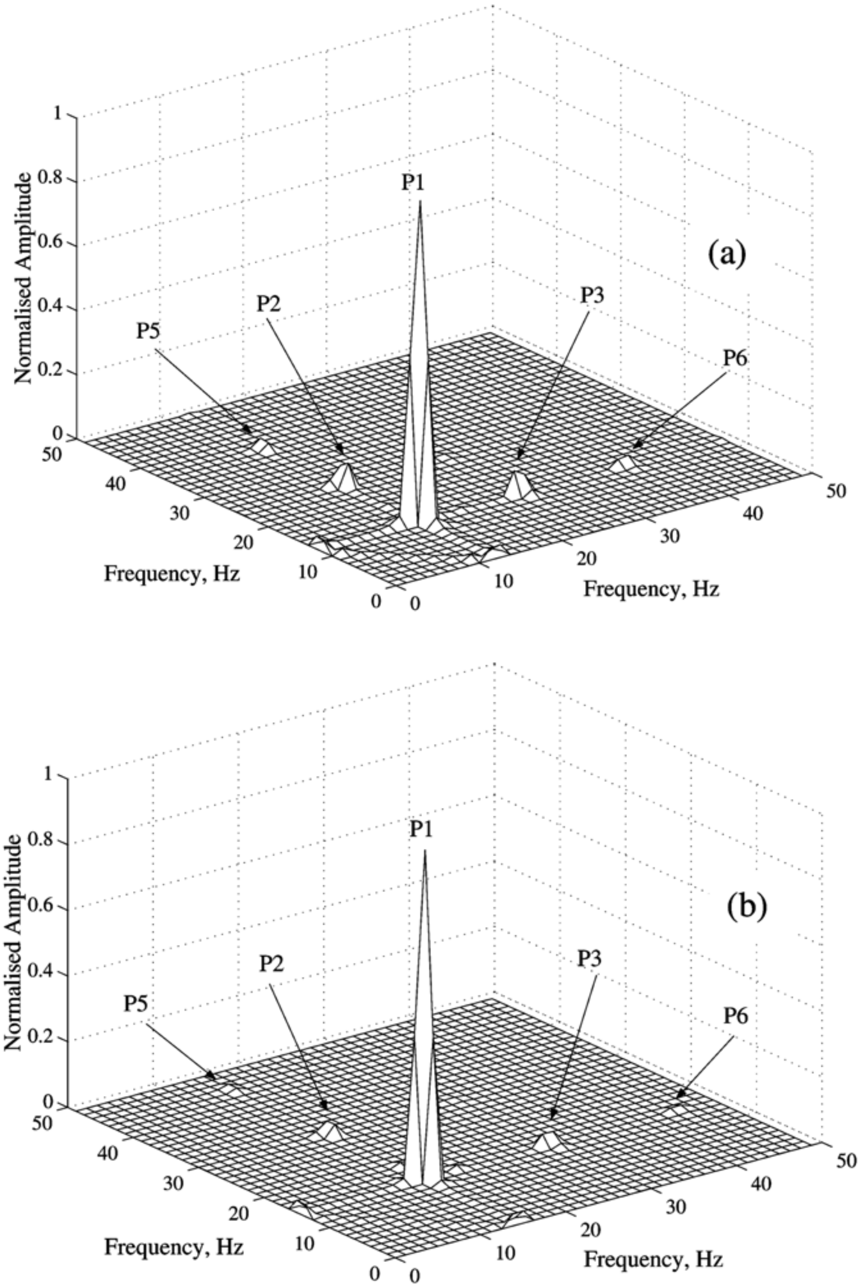

Fig. 11 Bi-spectra of a misaligned shaft response in vertical direction ((a) 750 RPM, (b) 900 RPM)

the time data with the frequency resolution of 1.25 Hz were used for the averaging of the computed bi-spectra using Eq. (2). Few bi-spectra at rotating speeds of 300 RPM, 450 RPM, 600 RPM and 750 RPM in vertical direction are shown in Fig. 9. Similar feature is seen in horizontal direction also.

4.2. Fault 2: Misaligned shaft

In the same rig, a misalignment was introduced in the shaft without crack by disturbing the alignment between two bearing pedestals. The bearing pedestal near the flexible coupling to the motor shaft has been misaligned by 1 mm and 0.5 mm in the vertical and horizontal directions respectively. Then the measurements were carried out at the RPM of 600, 750 and 900 to bring out the difference of behaviour in the bi-spectrum. Two typical amplitude spectra and bi-spectra of misaligned shaft responses in the vertical direction are shown in Figs. 10-11.

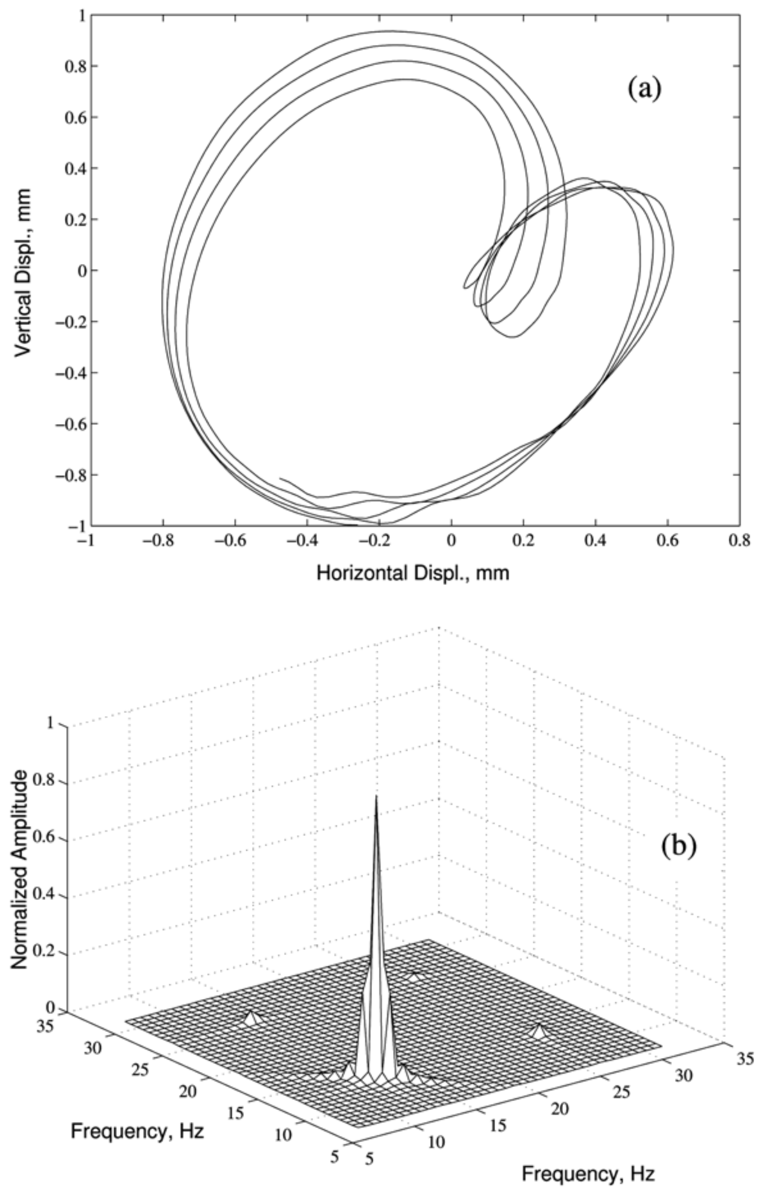

Fig. 12 Simulated dynamic behaviour of a crack shaft ((a) Orbit plot, (b) Bi-spectrum)

5. Discussion on computed bi-spectra

Altogether total six peaks (P1 to P6) are seen in the bi-spectra shown in Figs. 9 and 11. The diagonal peaks in a bi-spectrum are indicative of relation between two consecutive harmonics of a spectrum. So, the peak P1 relates 1X & 2X components and the P4 relates 2X & 4X components of a spectrum. The off-diagonal peaks seen at P2 and P3 relate responses at the three harmonic frequencies - 1X, 2X, and 3X whereas off-diagonal peak P5 & P6 give relation between 1X, 3X, and 4X components of a spectrum.

In case of a cracked shaft, three peaks (P1, P2 & P3) are only seen at the speeds of 300 RPM and 450 RPM. These speeds are well away from haft of the first natural frequency. However four peaks (P1 to P4) are seen at 600 RPM and 750 RPM which are close to half haft the natural frequency. The non-appearance of the peaks at P5 and P6 simply indicates the phase relation between 1X, 3X and 4X components is playing an important role. Hence the features (from three peaks to four peaks) observed in the bi-spectra during machine transient operation and/or the non-existence of peaks P5 & P6 during normal machine operation may be utilized as an additional parameter for detection of crack in the shaft. Moreover in case of a misaligned shaft the non-existence of peak at P4 and the similar feature at all the speed indicates the misalignment is not a speed dependent phenomena and the phase relation between 2X and 4X components is certainly much different then the cracked shaft.

6. Analytical simulation

A simple finite element simulation of the experiment was also carried out. A finite element model was created for the rotor using two-noded Euler-Bernoulli beam elements, each with two translational and two rotational degrees of freedom. The calculated first two natural frequencies are 26.53 Hz and 228.62 Hz in both the vertical and horizontal directions. Then the crack was introduced in the FE model using the method suggested earlier (Sinha, *et al*. 2002). The calculated natural frequencies in the vertical (fully open crack) and horizontal directions are 25.75 Hz & 225.68 Hz, and 26.10 Hz & 226.98 Hz respectively. The calculated frequencies are close to the experimentally measured values. The stiffness proportional damping matrix was also included in the model using experimentally measured modal damping of 0.3% at the first mode. To this model, the unbalance responses in vertical and horizontal directions were estimated assuming small unbalance at the disk. The responses were estimated when the shaft was assumed to be rotating at the speed of half the first natural frequency. The Newmark-β method was used for the numerical solution of the system dynamic equations. A very simple model for the breathing of crack (completely open during vertically lower position and completely closed for vertically top position for the crack in a rotation) was assumed for the response estimation. Fig. 12 gives the orbit plot and the bi-spectrum for the estimated responses at the measured locations which are close to the experimental observation seen in Figs. 8(f) & 9(d). The use of a more accurate damping model during the crack breathing and the exact nature of crack breathing during a shaft rotation would further improve accuracy of the simulation, however the present requirement is demonstrated adequately with this simple simulation. Similar simulation for the misaligned shaft could not be done, as the force function due to the shaft misalignment is not well stood.

7. Conclusions

In machine condition monitoring the crack and the misalignment in the shaft can be detected and distinguished using orbit plots during machine transient operation (run-up or run-down) though both the faults generate higher harmonics in the shaft responses. But they require measurements in both the vertical and horizontal directions. The computation of the bi-spectrum for such faults has shown encouraging results to identify and distinguish these two faults based on initial experiments on a small rig. In fact, it seems that the shaft response measurement in a direction (either vertical or horizontal) during normal machine operation is enough to identify these faults. But it requires few more experiments on relatively big rigs to confirm the same. At present only two faults - crack and misalignment are simulated in the experiments, similar experiments can also be explored for other types of faults so that the usefulness of the bi-spectrum and the response in one direction only in machine condition monitoring can be completely brought out. Which is underway.

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