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Adaptive fuzzy sliding mode control of seismically excited structures

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Abstract. In this paper, an adaptive fuzzy sliding mode controller (AFSMC) is designed to reduce dynamic responses of seismically excited structures. In the conventional sliding mode control (SMC), direct implementation of switching-type control law leads to chattering phenomenon which may excite unmodeled high frequency dynamics and may cause vibration in control force. Attenuation of chattering and its harmful effects are done by using fuzzy controller to approximate discontinuous part of the sliding mode control law. In order to prevent time-consuming obtaining of membership functions and reduce complexity of the fuzzy rule bases, adaptive law based on Lyapunov function is designed. To demonstrate the performance of AFSMC method and to compare with that of SMC and fuzzy control, a linear three-story scaled building is investigated for numerical simulation based on the proposed method. The results indicate satisfactory performance of the proposed method superior to those of SMC and fuzzy control.

Keywords: sliding mode control; adaptive fuzzy control; chattering-free

1. Introduction

Natural hazards mitigation in a smart way has drawn the attention of many researchers over decades. Various control systems have been developed as smart methods to alleviate the dynamic quantities of structures subjected to earthquake load and other dynamic loads (He *et al*. 2003, Hechmi and Kahla 2014, Kerber *et al.* 2007, Loh and Chang 2008).

Control schemes like active, semi-active and hybrid contain control algorithms as a processor makes them able to adapt to structural changes and changing environment. However, the selected algorithm could strongly affect the performance of these control systems. Since the earthquake is not known a prior and uncertainty in parameters is possible, linear feedback theories are not suitable to control the response of structures against earthquakes. Alternate nonlinear control algorithms including sliding mode control (Yang and Wu 1995), adaptive control (Wada and Das 1991) and fuzzy sliding (Alli and Yakut 2005) can be considered.

Since SMC method is insensitive against changes and external excitations, it has become a superior choice among the other control methods. The ability of switching between different control laws has made SMC a flexible method capable to change its structure which is also known as variable structure method. Moreover, since 1980, developments in sliding mode control have made this method more attractive in the field of active control (Hung *et al*. 1993). Previous studies in structural vibration control which have been done by Yang and his co-authors showed the advantages of SMC (Yang *et al*. 1995, Agrawal *et al.* 1998, Hsu 1996). Beside the advantages of SMC, switching between two values may cause chattering phenomenon where too many switches happen in the control bounds and time history of control force becomes incapable for real control actuators. To improve the control performance of conventional SMC, discontinuous part generating switching is approximated by a fuzzy system to attenuate chattering.

Over past two decades researchers have focused on real modeled systems with probable uncertainties and parameter changes. In spite of Conventional formulated control methods requiring precise mathematical model, smart control methods have been utilized in complex or ill-modeled control systems (Hsu 1996, Mamdani 1974). Fuzzy controllers, founded on human knowledge, are based on conditional linguistic statements and approximation reasoning. The expert control rules of conditional linguistic statements are applied on the relationship of system variables in fuzzy controllers (Van der Wal 1995, Procyk and Mamdani 1979). Guclu and Yazici (2008) applied fuzzy logic controller to a 15-story building equipped with ATMD. Li *et al*. (2010) developed FLC algorithm to control a nonlinear high-rise structure under earthquake. However, in some situations, it is not easy to acquire the knowledge from a skilled operator. Suitable parameters determination demands time-consuming trial and error, so properly regulation of fuzzy rules and membership functions become the key point. Integrating SMC and Fuzzy Logic controller (FLC) by applying sliding mode concept on fuzzy control and fuzzifying the sliding surface lead to fuzzy sliding mode control, i.e., FSMC benefits the advantages of both SMC and FLC by avoiding chattering and the mentioned problems in FLC. Many control efforts have been made in various fields of engineering by FSMC. Alli and Yakut (2005) applied FSMC for isolation of earthquake-excited

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structures.Wang and Lee (2002) designed a FSMC based on generic algorithms for a building structure. In order to reduce the vibration of large structures, Kim and Yun (2000) verified the FSMC algorithm on a benchmark structure initiated by ASCE. A fuzzy-sliding mode controller is presented to control the dynamics of semi-active suspension systems of vehicles using magneto-rheological fluid dampers by Zheng and Li (2009). Control algorithm for seismic protection of building structures based on sliding mode control is presented by Nikos *et al.* (2009) focusing on sliding surface design by pole assignment algorithm where the poles of the system in the sliding surface are obtained. Results showed that by using sliding mode control the response of a structure is reduced significantly compared to the response of the uncontrolled structure. Byeongil *et al*. (2013) investigate application of a control algorithm called model predictive sliding mode control to active vibration suppression of a cantilevered aluminum beam. The results showed significant reduction of vibration when model predictive sliding mode control is used.

Generally, in the systems with uncertainties where the upper bound of uncertain parameters are known, sliding mode control may be useful to guarantee the stability and consistent performance. However, the upper bound increase of uncertainty raises the control effort. Obtaining an optimal value of uncertainty and hence minimizing the control cost is possible by proposing an adaptive law to the FSMC Algorithm directing us to adaptive fuzzy sliding mode control. The adaptive law is used to tune the centers of membership functions in the consequent part of fuzzy rules and simplifies the establishment of fuzzy rule bases.

The purpose of this paper is to develop the AFSMC method to prevent the conventional drawbacks in SMC and FLC methods. Free-chattering control law is achieved by smoothing the control discontinuity in a thin boundary layer neighboring the sliding surface using fuzzy inference mechanism to approximate discontinuity. The effectiveness of the method is illustrated by applying on a linear scaled building studied by Kobori and Kamagata (1992) which is excited by El Centro earthquake scaled to a maximum acceleration of 0.112 g.

2. Conventional SMC

The equation of motion in classical form for a dynamic system subjected to active control force, $u(t)$ and system disturbance, $d(x(t),t)$ is presented by

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) \\ \cdots \\ \dot{x}_{n-1}(t) = x_n(t) \\ \dot{x}_n(t) = a_1x_1(t) + a_2x_2(t) + \ldots + a_nx_n(t) + u(t) + d(x(t),t) \end{cases} \tag{1}$$

Where, $x_1(t), x_2(t), \ldots, x_n(t)$ are state variables. The upper bound for system disturbance is considered as $d(x(t),t) \le \delta(x(t),t)$. The control task is to find a suitable control law $u(t)$ to force the system move toward the equilibrium point. Based on sliding mode theory, switching control law drives the nonlinear system-state trajectory on a specified and user-chosen surface in the state-space. The system state trajectory is forced by controller to remain on the surface for all subsequent time, and slides towards the origin. The sliding surface is chosen to be a linear function of the system-states

$$S(x) = cx = c_1x_1(t) + c_2x_2(t) + \ldots + c_nx_n(t) \tag{2}$$

By choosing a suitable sliding surface equation S(x), system reaches the surface in a limited time and slides on it towards the x=0. We suppose the sliding surface as

$$S(x) = 0 \qquad and \qquad \dot{S}(x) = 0 \tag{3}$$

These conditions force the trajectory to reach the sliding surface and stays on it. By differentiation of sliding surface with respect, t and substituting from Eq. (1) an equivalent control force, $u_{eq}(t)$ can be obtained while neglecting d(x(t),t).

$$\dot{S}(x)|_{u=u_{eq}} = a_1x_1(t) + (a_2+c_1)x_2(t) + \ldots + (a_n+c_{n-1})x_n(t) + u_{eq}(t) = 0 \tag{4}$$

$$u_{eq}(t) = -a_1x_1(t) - (a_2+c_1)x_2(t) - \ldots - (a_n+c_{n-1})x_n(t) \tag{5}$$

Further, it is shown that while state trajectory slides on the surface S(x)=0, the following (n-1)th- order equation is obtained.

$$c_n\dot{x}_{n-1}(t) + c_{n-1}x_{n-1}(t) + \ldots + c_2x_2(t) + c_1x_1(t) = 0 \tag{6}$$

The coefficients c_i, i=1,2,…,n can be chosen properly by designer such that all the roots in the Eq. (6) are in the open left-half of the complex plane. However, in order to account for the presence of disturbances, the control law has to be discontinuous (Slotine and Li 1991). In this case, the control law can be obtained satisfying

$$S(x)\dot{S}(x) < \quad \sigma\|S\| \qquad \text{when} \qquad S(x) \neq 0 \tag{7}$$

Where σ is a strictly positive constant. Substituting $u(t)$ as

$$u(t) = -a_1x_1(t) - (a_2+c_1)x_2(t) - \ldots - (a_n+c_{n-1})x_n(t) - [\delta(x(t),t)+\sigma]sign(S) \tag{8}$$

$$u(t) = u_{eq}(t) - [\delta(x(t),t)+\sigma]sign(S) \tag{9}$$

The Eq. (4) can be re-written as

$$\dot{S}(x) = -[\delta(x(t),t)+\sigma]sign(S) + d(x(t),t) \tag{10}$$

Multiplying both sides of Eq. (10) to S(x), it becomes

$$\begin{aligned} S(x)\dot{S}(x) &= -\sigma|S(x)| - \delta(x(t),t)|S(x)| - d(x(t),t)|S(x)| \\ &= -\sigma|S(x)| - \delta(x(t),t)|S(x)|[1 - d(x(t),t)/\delta(x(t),t)] \le -\sigma|S(x)| \end{aligned} \tag{11}$$

Therefore, Eq. (7) to be satisfied by Eq. (8). The term sign(S) in Eqs. (8) and (9) causes high frequency switches which is unable to be modeled in real systems. This phenomenon is named chattering. Free-chattering control law can be achieved by smoothing the control discontinuity in a thin boundary layer neighboring the sliding surface. The mathematical form of this solution is to replace the “sign” term with unit step function (Kobori and Kamagata 1992) and saturation function (Slotine and Li 1991) inside the boundary layer to alleviate chattering. In the present paper, fuzzy inference mechanism is suggested to approximate the discontinuous part of Eqs. (8) and (9).

3. Fuzzy sliding mode control

In this section the structure of a fuzzy logic controller will be drawn. A FLC system consists of a fuzzifier, a knowledge base, an inference system and a defuzzifier. The basic structure of a FLC is illustrated in Fig. 1. Various units of a fuzzy logic controller are defined as follows (Aldawod *et al.* 2001):

Fuzzifier: The fuzzifier maps the crisp inputs into fuzzy linguistic values using fuzzy reasoning mechanism.

Knowledge base: This unit is a collection of IF-THEN rules created by an expert to achieve the control goals. A fuzzy IF-THEN rule based on sliding mode theory is as follows

jth rule: IF $S(x)$ is A_1^j and $\dot{S}(x)$ is A_2^j THEN u_f is B^j.

Where the switching variable $S(x)$ and the variation of $S(x)$ as $\dot{S}(x)$ represent the input variables. The control variable u_f represents the fuzzy output variable. A_i^j and B^j are the fuzzy variables and j is the number of control rules.

Inference engine: This unit performs various fuzzy logic operations to infer the control action from a given fuzzy input.

Difuzzifier: The defizzifier maps fuzzy control action to a crisp control value.

The magnitude of the control output not only depends on $S(x)$ but also is influenced by $\dot{S}(x)$, hence fuzzy rule base can be set according to the condition of the trajectory in the neighborhood of sliding surface, e.g., if the trajectory in the phase plane leaves the sliding surface with a large angle ($\dot{S}(x)$ is big) and $S(x)$ is large, then the control value will be large in value with opposite direction to drive the trajectory toward the sliding surface. Specifically, a fuzzy system as an estimator is achieved using the singleton fuzzification, product inference and center-average defuzzification. The corresponding output of the fuzzy system is obtained as (Hsiao *et al*. 2005),

$$u_f = \frac{\sum_{j=1}^{m} w_j c_j}{\sum_{j=1}^{m} w_j} = v^T \psi \tag{12}$$

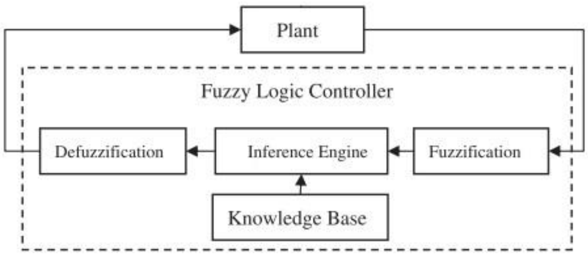

Fig. 1 Basic structure of a FLC

$$w_j = \prod_{i=1}^{n} \mu_{F_i^j}(x_i) \tag{13}$$

Where m is the number of fuzzy rules; c_j represents the center of the membership function in the consequent part of the jth rule. $v = [c_1, \ldots, c_m]^T$ denotes vector of adjustable parameter. w_j represents the firing strength of the jth rule and then ψ, named as firing strength vector, is used for the remaining part of equation as

$$\psi = \frac{[w_1 \quad \ldots. \quad w_m]^T}{\sum_{j=1}^{m} w_j} \tag{14}$$

Following, a specific fuzzy inference system is used to approximate the discontinuous part of Eqs. (8) and (9). Ultimately, the sign function is replaced with the fuzzy control output of Eq. (12)

4. Adaptive law design

The main task of this section is to derive an adaptive law to adjust the centers of the membership functions such that the estimated fuzzy output can be optimally approximated to achieve the minimum control cost under the situation of uncertainties in the external excitation. From Eq. (9), it is obvious that the upper bound of the uncertainty is required. Moreover, the increase in control cost due to the rise of the upper bound makes it necessary to set an adaptive law to obtain an optimum output with minimum control cost.

Let $\hat{u}_f$ be a reasonable approximation by fuzzy logic system which minimizes the control cost and allows attenuating the discontinuous control law in conventional sliding mode. From Eq. (12), $\hat{u}_f$ can be written as follows

$$\hat{u}_f = \hat{v}^T \psi \tag{15}$$

Where, $\hat{v}^T$ is an optimal vector which minimizes the control cost. Define the approximation error vector as

$$\tilde{v} = v - \hat{v} \tag{16}$$

We consider the Lyapunov’s function candidate for the problem as

$$V = \frac{1}{2}(S^2 + \frac{1}{\alpha}\tilde{v}^T\tilde{v}) \quad (17)$$

The time derivative of V is

$$\begin{aligned}\dot{V} &= S\dot{S} + \frac{1}{\alpha}\tilde{v}^T\dot{v} = S(u_f + d(x(t),t)) + \frac{1}{\alpha}\tilde{v}^T\dot{v} \\ &= S(u_f - \hat{u}_f) + S\hat{u}_f + Sd(x(t),t) + \frac{1}{\alpha}\tilde{v}^T\dot{v} \\ &= S(v^T - \hat{v}^T)\psi + S\hat{u}_f + Sd(x(t),t) + \frac{1}{\alpha}\tilde{v}^T\dot{v} \quad (18) \\ &= S\tilde{v}^T\psi + S\hat{u}_f + Sd(x(t),t) + \frac{1}{\alpha}\tilde{v}^T\dot{v} \\ &= \frac{1}{\alpha}\tilde{v}^T(\dot{v} + \alpha S\psi) + S\hat{u}_f + Sd(x(t),t)\end{aligned}$$

in which $\dot{\tilde{v}} = \dot{v}$.

The adaptive law is designed as follows

$$\dot{v} = -\alpha S\psi \quad (19)$$

Where α is a positive constant. Substituting Eq. (19) into Eq. (18) leads to

$$\dot{V} = S\hat{u}_f + Sd(x(t),t) < -\sigma|S| \quad (20)$$

The adaptive law is derived from Lyapunov's theory and $\dot{V} < 0$ is obtained. This means that V converges to zero. Thus, $\tilde{V}$ converges to zero and the optimal control output $\hat{u}_f$ leading to minimum control cost. This adaptive law adjusts the centers of the membership functions in THEN-part.

5. AFSMC formulation of control problem

The equation of motion for a structural system subjected to ground acceleration, $\ddot{x}_g(t)$ and active control force, $u(t)$ is described in the matrix form as the following

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = MR\ddot{x}_g(t) + Bu(t) \quad (21)$$

Where M, C and K are (N×N) dimensional mass, damping and stiffness matrices, respectively. $\ddot{x}(t)$, $\dot{x}(t)$ and $x(t)$ are (N×1) dimensional relative acceleration, velocity and displacement vectors, respectively. Further, R and B are earthquake influence (with terms of equal to one) and control location vectors of size (N×1), respectively.

The Eq. (21) can also be expressed in the state space form as follows

$$\dot{z}(t) = Az(t) + B_1u(t) + B_2\ddot{x}_g(t) \quad (22)$$

Where

$$A = \begin{bmatrix} o & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad B_1 = \begin{bmatrix} o \\ M^{-1}B \end{bmatrix} \qquad B_2 = \begin{bmatrix} o \\ R \end{bmatrix} \qquad z(t) = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix}$$

The SMC law with sliding surface equation as S=PZ while satisfying the condition in Eq. (7) can be written as

$$u(t) = u_{eq}(t) - [\delta(x(t),t) + \sigma]sign(S^TPB) \quad (23)$$

In which

$$u_{eq}(t) = -(PB)^{-1}PAZ \quad (24)$$

and P is the coefficient vector of sliding surface. The theory of sliding mode control requires designing a sliding surface, whereas the motion on it is stable. The method of optimal sliding mode described in (Yang *et al*. 1995) is used to design the sliding surface, where the integral of the quadratic function of the state vector is minimized to derive the sliding surface.

$$J = \int_0^\infty Z^T(t)QZ(t)dt \quad (25)$$

Where, Q denotes a 2n×2n positive definite weighting matrix.

As mentioned above, direct application of Eq. (9) generates chattering. Fuzzy inference system is applied to reduce chattering. The triangular membership function is used for each fuzzy member in this paper and the fuzzy rule base to obtain u_f is presented in Table 1. The definitions of the fuzzy variables of input and output membership function are as follows: NB = Negative Big, NM = Negative Medium, NS = Negative Small, Z =Zero, PS = Positive Small, PM = Positive Medium and PB = Positive Big, and $\overline{S}$ represents S^TPB.

The initial value of v used in the adaptive law presented in the Eq. (12) is a 49×1 vector which is specified according to the initial fuzzy control rules in Table 1. According to section 3, the values of v are the points at which the membership functions of the corresponding part of the rules achieve their maximum value. For example, for the first fuzzy rule in Table 1: IF S is NB and $\dot{S}$ is PB THEN u_f is PM. So the value of $v(1)$ is the point at which the membership function of PM achieves its maximum value. Therefore, the control law becomes

Table 1 Initial fuzzy control rules of u_f

$\dot{\overline{S}} \backslash \overline{S}$	NB	NM	NS	Z	PS	PM	PB
PB	PM	PS	NS	NM	NM	NM	NB
PM	PB	PS	Z	NS	NS	NM	NB
PS	PB	PM	PS	NS	NS	NM	NB
Z	PB	PM	PS	Z	NS	NM	NB
NS	PB	PM	PS	PS	NS	NM	NB
NM	PB	PM	PS	PS	Z	NS	NB
NB	PB	PM	PM	PM	PS	NS	NM

$$u(t) = u_{eq}(t) + u_f \tag{26}$$

and the block diagram of AFSMC is shown in Fig. 2.

6. Numerical study

To demonstrate the application of AFSMC scheme and its effectiveness in relation to control of structural response, a 3-story scaled building equipped with active bracing system in the first story is studied. The model is firstly studied by Kobori and Kamagata (1992). The performance of the model structure is evaluated and compared with conventional SMC and fuzzy control results. Fig. 3 illustrates the schematic view of the structural model chosen for this study. Every story unit is identically constructed and the mass, stiffness and damping coefficients have been shown in Table 2.

In order to time history analysis of the structure, the NS component of El Centro earthquake is selected and scaled to a maximum acceleration of 0.112 g as the input excitation. The time history of the scaled acceleration of the earthquake was drawn in Fig. 4.

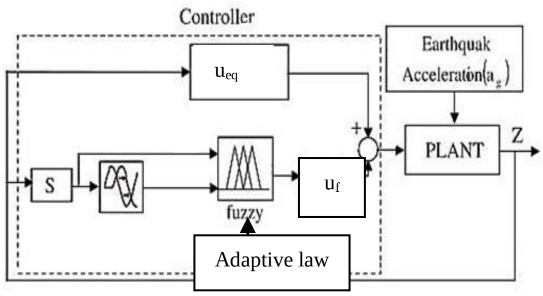

Fig. 2 Block diagram of AFSMC

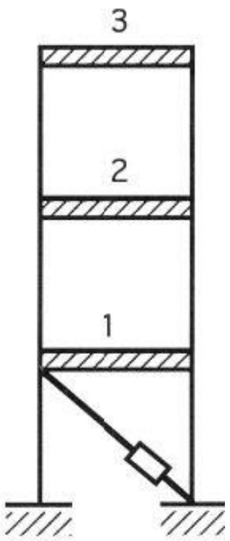

Fig. 3 Building model with active bracing system

Table 2 Mass, stiffness and damping values of structure

story	Mass (kg)	Stiffness (N/m)	Damping (N.s/m)
1	1000	980000	1407
2	1000	980000	1407
3	1000	980000	1407

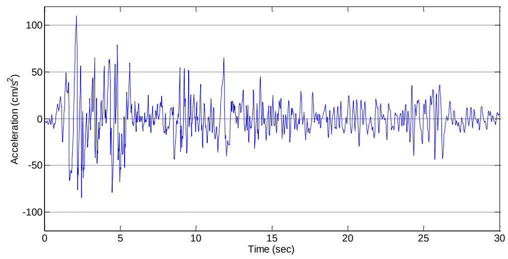

Fig. 4 Time history of El Centro earthquake scaled to 0.112 g

Since a controller is installed in the first story as shown in Fig. 3, there is one sliding surface. For full-state feedback, while using LQR method to design the sliding surface, the diagonal weighting matrix, Q in Eq. (25), is considered as $Q = diag(10^5, 10^4, 10^3, 1, 1, 1)$ and this results the sliding surface equation as

$$S(x) = 223.6x_1(t) - 17.32x_2(t) + 6.01x_3(t) + 3.68\dot{x}_1(t) + 2.68\dot{x}_2(t) + 1.01\dot{x}_3(t) \tag{27}$$

Fig. 5 shows uncontrolled and controlled displacements of the first floor using discontinuous SMC. Displacement and acceleration time histories of the second and third floors are illustrated in Figs. 6 and 7, respectively. Applied control force for SMC is illustrated in Fig. 8. It can be seen that high-frequency chattering in the time history of control force is occurred due to using discontinuous sliding mode controller.

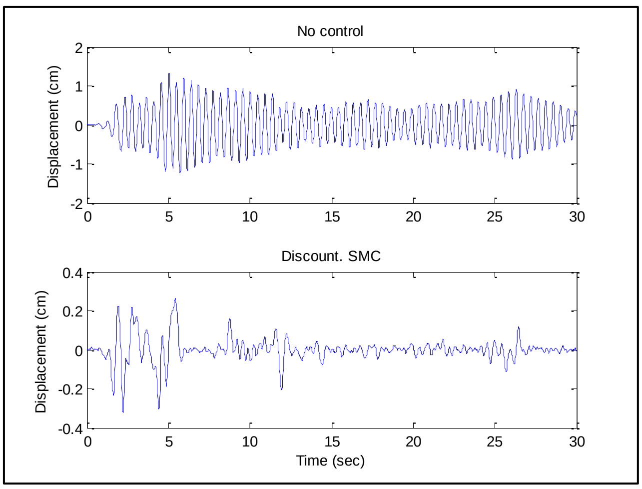

Fig. 5 Time histories of displacement

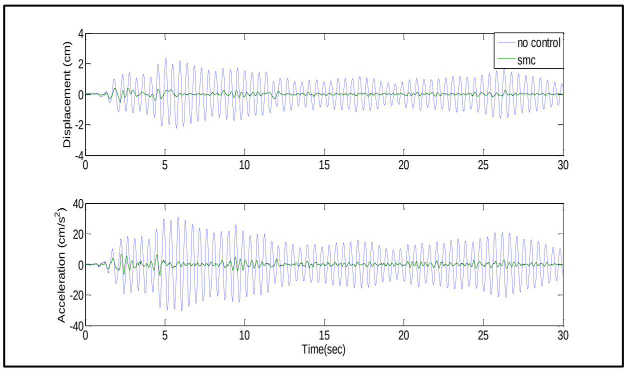

Fig. 6 Second floor time histories of response

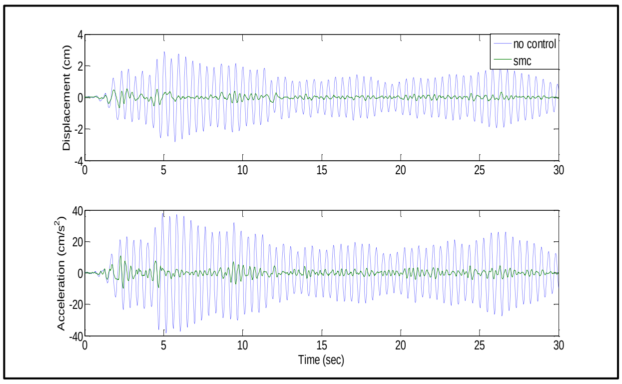

Fig. 7 Third floor time histories of response

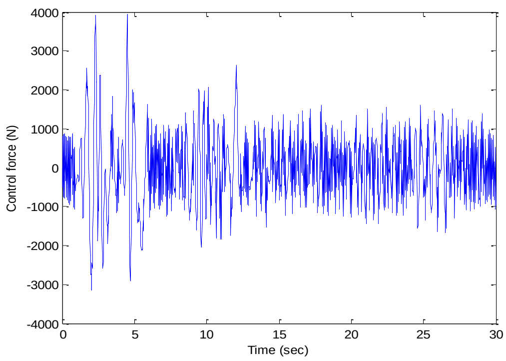

Fig. 8 Time history of sliding mode control force

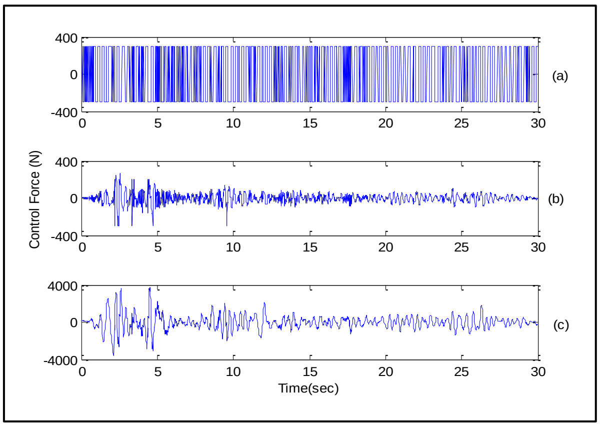

Fig. 9 Time history of control force: (a) Discontinuous part of SMC, (b) Fuzzy approximation and (c) AFSMC

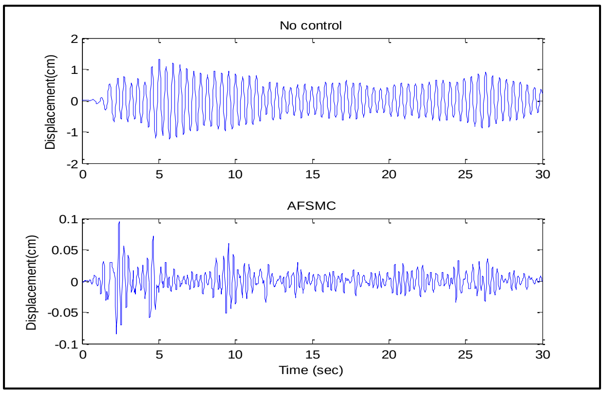

Fig. 10 Time histories of displacement at first floor

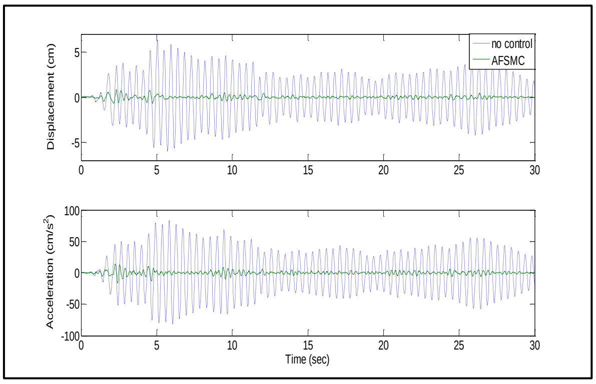

Fig. 11 Second floor time histories of response

As explained in previous section, the effectiveness of using a fuzzy approximator is to reduce chattering. The results of implementation of Fuzzy rules in controller are compared in Fig. 9. Fig. 9(a) represents the time history of the control force resulted from the discontinuous part of the Eq. (9) containing chattering. As it is shown in Fig. 9(b), the chattering has been removed when the Fuzzy approximation is implemented. Ultimately, in Fig. 9(c) the time history of AFSMC control force is presented. Fig. 10 indicates the time history of uncontrolled and controlled displacements of the first floor using AFSMC. Moreover, dynamic responses of the second and third floors regarding the application of AFSMC are shown in Figs. 11 and 12.

For the comparison purpose, a FLC method is performed. Fig. 13 represents the block diagram of a fuzzy controller. As it is obvious, fuzzy block has 2 inputs as displacement and velocity of the first floor and an output as the control force. S_e, dS_e and S_u are factors used to scale the inputs and output which are obtained by trial and error. Five membership functions for displacement, three membership functions for velocity and seven membership functions for control output are suggested in triangular shapes. Table 3 indicates the fuzzy rule base, where X, $\dot{X}$, N, Z and P represent displacement, velocity, Negative, Zero and Positive, respectively.

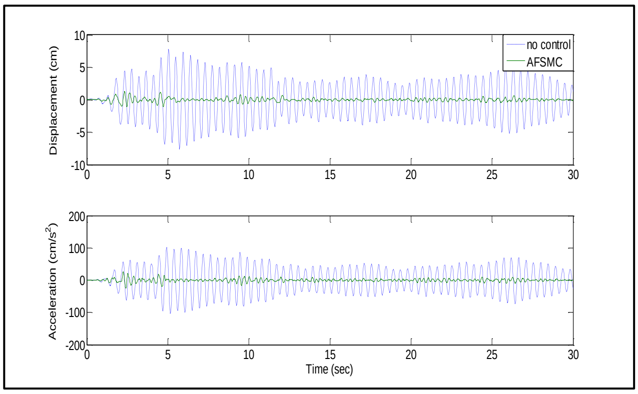

Fig. 12 Third floor time histories of response

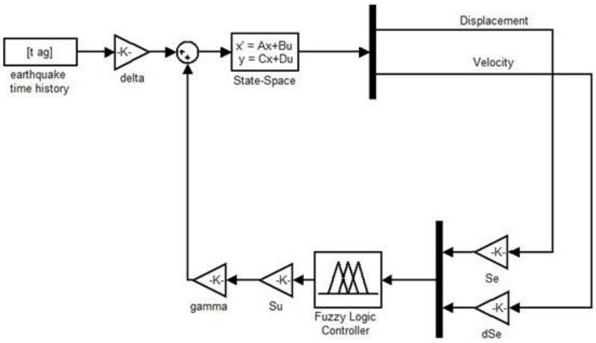

Fig. 13 Block diagram of FLC

Table 3 Fuzzy rule basis For FLC

$X \setminus \dot{X}$	N	Z	P
NB	NB	NM	NS
NS	NM	NS	Z
Z	NS	Z	PS
PS	Z	PS	PM
PB	PS	PM	PB

Choosing S_e=42, dS_e=1 and S_u=10^4 leads to Fig. 14 representing the controlled displacement and control force time history of the first floor for El Centro earthquake. Displacement and acceleration time histories of the second and third floors while applying FLC are illustrated in Figs. 15 and 16.

Furthermore, to assess the performance of control systems, five evaluation criteria are proposed. The first criterion evaluates the ability of the proposed method to reduce inter-story drift. The second and third criteria relate to the ability of the method to reduce the maximum displacement and acceleration. The forth one assesses the ability to reduce the maximum force of the floor and the last one relates to the control devices.

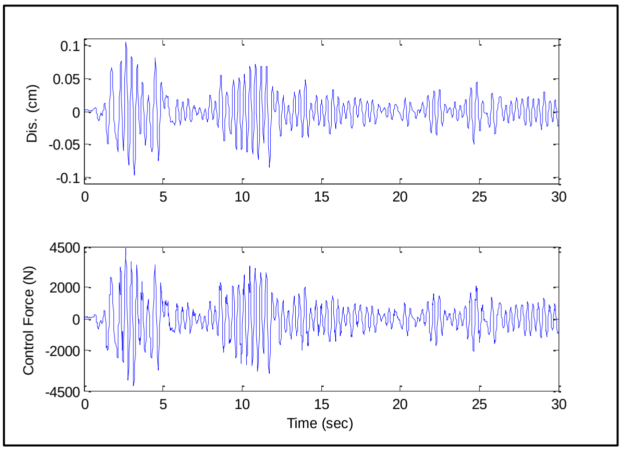

Fig. 14 Time histories of displacement and control force using FLC

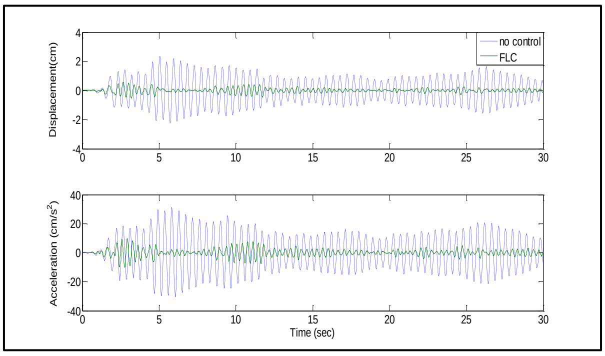

Fig. 15 Second floor time histories of response

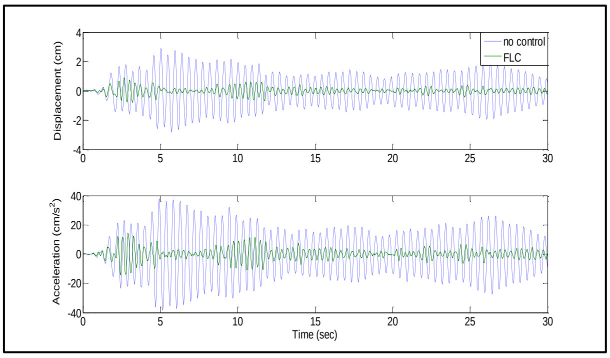

Fig. 16 Third floor time histories of response

$$J_1 = \frac{\max\limits_{t,i} \dfrac{|d_i^c(t)|}{h_i}}{\max\limits_{t,i} \dfrac{|d_i^u(t)|}{h_i}} \tag{28}$$

$$J_2 = \frac{\max\limits_{t,i} \dfrac{|x_i^c(t)|}{h_i}}{\max\limits_{t,i} \dfrac{|x_i^u(t)|}{h_i}} \tag{29}$$

Table 4 Evaluation criteria of El Centro earthquake

Evaluation criteria	AFSMC	FLC	SMC
J1	0.1549	0.3461	0.2195
J2	0.1670	0.2971	0.2265
J3	0.4737	0.5588	0.3158
J4	0.3651	0.4298	0.3219
J5	0.1244	0.1452	0.1311

$$J_3 = \frac{\max\limits_{t,i} |\ddot{x}_{ai}^{c}(t)|}{\max\limits_{t,i} |\ddot{x}_{ai}^{u}(t)|} \tag{30}$$

$$J_4 = \frac{\max\limits_{t,i} |\sum_i m_i \ddot{x}_{ai}^{c}(t)|}{\max\limits_{t,i} |\sum_i m_i \ddot{x}_{ai}^{u}(t)|} \tag{31}$$

$$J_5 = \frac{\max |f_L(t)|}{W} \tag{32}$$

Where, d_i, x_i and $\ddot{x}_i$ are the drift, displacement and acceleration of the ith story, respectively. F_L is the control force produced by Lth device. m_i and h_i is mass and height of the ith story. W is seismic weight of building. The superscript terms ‘c’ and ‘u’ refer to the controlled and uncontrolled system.

Table 4 indicates a competent control performance based on evaluation criteria. By comparison between the results of evaluation criteria for El Centro ground motion, it can be concluded that the proposed control method leads to satisfactory level of performance.

7. Conclusions

In this study, dynamic responses of a linear structure are evaluated and controlled by Sliding mode control (SMC), Fuzzy logic control (FLC) and adaptive fuzzy sliding mode control (AFSMC) algorithms. The robustness of SMC-based algorithm against uncertainties and disturbances has made us to propose AFSMC method. The simulation results indicate chattering-free time history of outputs which are gained by relating the algorithms of adaptive fuzzy control and SMC. Hence, the resultant method takes advantages of both methods by using S and $\dot{S}$ as fuzzy inputs which represent displacements and velocities of all stories. It removes chattering by replacing the discontinuous part with the fuzzy inference system and adapting its membership functions to minimize control costs due to earthquake uncertainty. Also, the trial and error process of obtaining fuzzy rule basis in the fuzzy logic control is avoided by applying the condition of the trajectory in the neighborhood of the sliding surface to construct the fuzzy rules. Moreover, the results reveal reasonable performance of the AFSMC method in dissipating the responses as satisfactorily as two other methods. Consequently, the results indicate that AFSMC is an effective method for seismic dissipation of structures.

References

Agrawal, A.K., Yang, J.N. and Wu, J.C. (1998), “Non-linear control strategies for duffing systems”, *Int. J. Non-Linear Mech.*, **33**(5), 829-841.

Aldawod, M., Samali, B. and Naghdy, F. (2001), “Active control of along wind response of tall building using a fuzzy controller”, *Eng. Struct.*, **23**(11), 1512-1522.

Alli, H. and Yakut, O. (2005), “Fuzzy sliding-mode control of structures”, *Eng. Struct.*, **27**(2), 277-284.

Byeongil, K., Gregory, N.W. and Hwan-Sik, Y. (2013), “Active vibration suppression of a 1D piezoelectric bimorph structure using model predictive sliding mode control”, *Smart Struct. Syst.*, **11**(6), 623-635.

Guclu, R. and Yazici, H. (2008), “Vibration control of a structure with ATMD against earthquake using fuzzy logic controllers”, *J. Sound Vib*., **318**, 36-49.

He, W.L., Agrawal, A.K. and Yang, J.N. (2003), “A novel semi-active friction controller for linear structures against earthquakes”, *J. Struct. Eng. - ASCE*, **129**(7), 941-950.

Hsiao, F.H., Chen, C.W., Liang, Y.W., Xu, S.D. and Chiang, W.L. (2005), “T-S fuzzy controllers for nonlinear interconnected systems with multiple time delays”, *T. Circuits Syst. I: Regular Papers*, **25**(9), 1883-1893.

Hsu, S.Y. (1996), *Seismic response control using continuous sliding mode control with compensator*, Ph.D. Thesis, University of California, Irvine.

Hung, J.Y., Gao, W.G. and Hung, J.C. (1993), “Variable structure control: A survey”, *IEEE T. Ind. Electron.*, **40**(1), 2-22.

Kerber, F., Hurlebaus, S., Beadle, B.M. and Stobener, U. (2007), “Control concepts for an active vibration isolation system”, *Mech. Syst. Signal Pr.*, **21**, 3042-3059.

Kim, S.B. and Yun, C.B. (2000), “Sliding mode fuzzy control: theory and verification on a benchmark structure”, *Earthq. Eng. Struct. D.*, **29**, 1587-608.

Kobori, T. and Kamagata, S. (1992), “Dynamic intelligent buildings-active seismic response control”, *Intelligent Struct.*, **2**, 279-282.

Li, L., Song, G. and Ou, J, (2010), “Hybrid active mass damper (AMD) vibration suppression of nonlinear high-rise structure using fuzzy logic control algorithm under earthquake excitation”, *Struct. Control Health Monit*., **18,** 698-709.

Loh, C.H. and Chang, C.M. (2008), “Application of centralized and decentralized control to building structure: analytical study”, *J. Eng. Mech. - ASCE*, **134**(11), 970-982.

Mamdani, E.M. (1974), “Application of fuzzy algorithms for control of simple dynamic plant”, *Proc. IEEE*, **121**, 1585-1588.

Ouni, M.H.E. and Kahla, N.B. (2014), “Active tendon control of a Geiger dome”, *J. Vib. Control*, **20**, 241-255.

Pnevmatikos, N.G. and Gantes, C.J. (2009), “Sliding mode control for structures based on the frequency content of the earthquake loading”, *Smart Struct. Syst.*, **5**(3), 209-221.

Procyk, T.J. and Mamdani. E.M. (1979), “A linguistic self-organizing process controller”, *Automatica*, **15**, 15-30.

Slotine, J.J.E. and Li, W. (1991), *Applied nonlinear control*, Prentice Hall. Englewood Cliffs, New Jersey.

Van der Wal, A.J. (1995), "Application of fuzzy control in industry", *Fuzzy Set. Syst.*, **74**, 33-41.

Wada, B.K. and Das, S. (1991), "Application of adaptive structures concepts to civil structures", *Intell. Struct.-Monit. Control*, **1**, 195-217.

Wang, A.P. and Lee, C.D. (2002), "Fuzzy sliding mode control for a building structure based on genetic algorithms", *Earthq. Eng. Struct. D.*, **31**, 881-895.

Yang, J.N., Wu, J.C. and Agrawal, A.K. (1995), "Sliding mode control of nonlinear and hysteretic structures", *J. Eng. Mech. - ASCE*, **121**(12), 1386-1390.

Zheng, L. and Li, Y.N. (2009), "Fuzzy-sliding mode control of a full car semi-active suspension systems with MR dampers", *Smart Struct. Syst.*, **5**(3), 261-277.

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