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Shear wave in a fiber-reinforced anisotropic layer overlying a pre-stressed porous half space with self-weight

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Abstract. The main purpose of this paper is to study the effects of initial stress, gravity, anisotropy and porosity on the propagation of shear wave (SH-waves) in a fiber-reinforced layer placed over a porous media. The frequency equations in a closed form have been derived for SH-waves by applying suitable boundary conditions. The frequency equations have been expanded and approximated up to 2[nd] order of Whittaker's function. It has been observed that the SH-wave velocity decreases as width of fiber-reinforced layer increases. However, with the increase of initial stress, gravity parameter and porosity, the phase velocity increases. The results obtained are in perfect agreement with the standard results investigated by other relevant researchers.

Keywords: fiber-reinforced; fluid-saturated porous layer; initial stress; gravity parameter; phase velocity

1. Introduction

The surface wave propagation in an anisotropic layer is basically unalike from the propagation of same waves in an isotropic media. Therefore the study of surface waves in anisotropic layered media is of great interest for seismologists to understand the geophysical prospecting and the earth quake engineering. Also, the complex behaviour of layered structures can be understood with the propagation of shear waves in these different layers. Some important, useful and sincere attempts have been initiated to study the propagation of surface waves in layered complex structures coupling with magnetic, electrical and mechanical fields. The investigation of the mechanical response of a fiber-reinforced material is of sizeable significance in geotechnical engineering and geomechanics. Fiber-reinforced materials continue to give a distinctive interdisciplinary outlook and a logical perspective to understanding the latest developments in the geotechnical field. A continuum model is used to explain fiber-reinforced composites as they are widely used in different engineering applications including aviation, automotive and engineering structures due to their high stiffness, lightweight, strength and damping properties. Reinforced materials are superior to the structural materials in applications because reinforced composite has characteristic property where its components act together as a single anisotropic unit till they remain in the

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elastic condition. Also, fiber-reinforced composite concrete structures are significant due to their low weight and high strength. Earth can be chosen as a composite material with horizontally preferred direction perpendicular to the propagation of wave with different properties. During an earthquake, the spurious structures on the outside of the earth are excited which gives rise to violent vibrations in some cases. They act like a single unit in the elastic condition such that relative displacement can be absent between them. Belfield *et al.* (1983) presented the method of introducing a continuous self-reinforcement in elastic solid. Hashin and Rosen (1964) investigated the elastic moduli for fiber-reinforced materials. Boukhari *et al.* (2016) studied time-varying physical parameter identification of shear type structures based on discrete wavelet transform. Bose and Mal (1974) studied the propagation of time-harmonic elastic waves in a fiber-reinforced composite. Sengupta and Nath (2001) considered the surface waves in fiber-reinforced anisotropic elastic media. Singh (2007) obtained the reflection coefficients from free surface of an incompressible transversely isotropic fiber-reinforced elastic half-space for the case when outer slowness section is re-entrant. Singh and Yadav (2013) studied fiber-reinforced elastic solid half-space with magnetic field by taking the concept of reflection of plane waves. Singh (2002) and Sing and Singh (2004) presented many papers on fiber-reinforced materials. Sahu *et al*. (2014) investigated shear waves in a heterogeneous fiber-reinforced and porous media. Gupta *et al.* (2013) discussed propagation of Love waves in a porous layer under the influence of directional rigidities. Gupta (2014) studied surface waves in fiber-reinforced medium. Gupta and Gupta (2013) analyzed wave propagation in an anisotropic fiber-reinforced medium under temperature and stress. Wang *et al.* (2014) studied time-varying physical parameter identification of shear type structures based on discrete wavelet transform. Boukhari *et al.* (2016) proposed efficient shear deformation theory for wave propagation of functionally graded material plates.

The investigations of surface wave propagation in porous medium have acquired great interest in recent times. The SH-wave propagation in a fiber-reinforced anisotropic medium overlying a semi-infinite pre-stressed gravitational porous medium has prime importance in seismology and earthquake engineering on account of existence of inhomogeneity and porosity of the Earth. Earth is a porous media consists of different layers of uniform pores. Many researchers have discussed the elastic properties of porous media. The propagation of Love type waves with irregular boundary in a porous layer has been discussed by Chattopadhyay and De (1983). Dey and Gupta (1987) investigated wave propagation in void medium. Chattaraj *et al*. (2013a, b) studied Love wave propagation pre-stressed porous layer lying between two isotropic half-spaces and studied the effect of anisotropy and porosity on Love wave phase velocity. Gupta *et al*. (2013) presented a technical note on the propagation of Love wave in porous layer. Kundu *et al*. (2014a) showed the influence of various parameters such as porosity, rigidity, and anisotropy on Love wave propagation. Kundu *et al*. (2014b) discussed SH-waves in three layered media. The Earth's gravitational force affects the seismic wave propagation. The hydrostatic stresses in the gravitational half-space play an important role to analyze the static and dynamic problems of the Earth. Ghorai *et al*. (2010) discussed Love wave propagation in a porous layer overlying a gravitational half-space. Abd-Alla *et al*. (2013) investigated the effect of various parameters such as fibre-reinforcement, anisotropy and gravity of the elastic media on surface waves.

In this paper, the dispersion of SH-waves in a fiber-reinforced anisotropic layer overlying a pre-stressed gravitational porous half space has been briefly investigated. The influence of reinforce parameter, porosity and gravity parameter on the SH-wave propagation have discussed graphically. The obtained dispersion equation is in perfect agreement with the standard results investigated by other relevant researchers in the absence of reinforcement, porosity and stress-gravity parameters.

2. Formulation of the problem

Let H be the thickness of the steel fiber reinforced silica fume concrete layer placed over porous half-space. We consider x-axis along the direction of wave propagation and z-axis vertically downwards (Fig. 1).

3. Boundary conditions

The displacement components and stress components are continuous at $z = -H$, and at $z = 0$, therefore the geometry of the problem leads to the following conditions

At $z = -H$, the stress component $\tau_{23} = 0$.

At $z = 0$, the stress component of the layer and half space is continuous, i.e., $\tau_{23} = \sigma_{23}$.

At $z = 0$, the velocity component of both the layer is continuous, i.e., $u_2 = u_2'$.

4. Solution of the problem

4.1 Solution for the upper layer

The constitutive equations for a fiber reinforced linearly elastic anisotropic medium with respect to a preferred direction $\vec{a}$ (Belfield *et al.* 1983) are:

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\right) + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta\left(a_k a_m e_{km} a_i a_j\right) \quad (2)$$

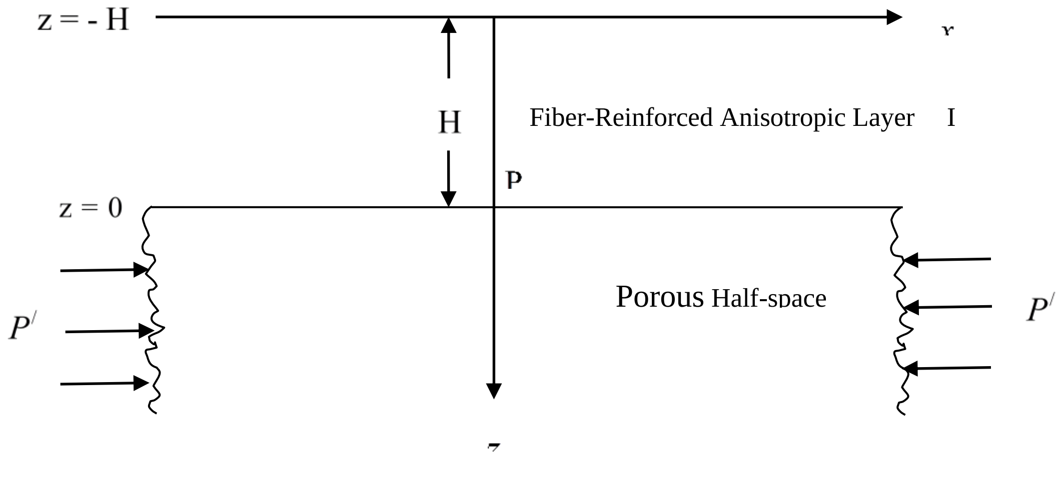

Fig. 1 Geometry of the problem

where , $e_{ij} = \frac{1}{2}(\mu_{i,j} + \mu_{j,i})$ are components of strain; $\alpha, \beta ?\mu_L - \mu_T$ are reinforced anisotropic elastic parameters; λ, μ_T are elastic parameters. Preferred direction of fibers are given by $\vec{a} = (a_1, a_2, a_3), a_1^2 + a_2^2 + a_3^2 = 1.$ If $\vec{a}$ has components that are (1, 0, 0) so that the preferred direction is the z-axis normal to direction of propagation. Relation (2) in the presence of initial compression simplifies as given below

$$\left.\begin{aligned}
\tau_{11} &= (\lambda + 2\alpha + 4\mu_L + \beta - 2\mu_T) e_{11} + (\lambda + \alpha) e_{22} + (\lambda + \alpha) e_{33} \\
\tau_{33} &= (\lambda + \alpha) e_{11} + \lambda e_{22} + (\lambda + 2\mu_T) e_{33} \\
\tau_{22} &= (\lambda + \alpha) e_{11} + (\lambda + 2\mu_T) e_{22} + \lambda e_{33} \\
\tau_{23} &= 2\mu_T e_{23} \\
\tau_{13} &= 2\mu_T e_{13} \\
\tau_{12} &= 2\mu_T e_{12}
\end{aligned}\right\} \tag{3}$$

The equations of motion in upper half are

$$\left.\begin{aligned}
\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} &= \rho \frac{\partial^2 u_1}{\partial t^2} \\
\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} &= \rho \frac{\partial^2 u_2}{\partial t^2} \\
\frac{\partial \tau_{31}}{\partial x} + \frac{\partial \tau_{32}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} &= \rho \frac{\partial^2 u_3}{\partial t^2}
\end{aligned}\right\} \tag{4}$$

$$u_1 = 0, \qquad u_3 = 0, \qquad u_2 = u_2(x, z, t) \tag{5}$$

Taking transversely isotropic and setting $a_2 = 0$ we get from Eqs. (3) as

$$\left.\begin{aligned}
\tau_{12} &= \mu_T \left(1 + \left(\frac{\mu_L}{\mu_T} - 1\right) a_1^2\right) \frac{\partial u_2}{\partial x} + \mu_T \left(\frac{\mu_L}{\mu_T} - 1\right) a_1 a_3 \frac{\partial u_2}{\partial z} \\
\tau_{23} &= \mu_T \left(1 + \left(\frac{\mu_L}{\mu_T} - 1\right) a_3^2\right) \frac{\partial u_2}{\partial z} + \mu_T \left(\frac{\mu_L}{\mu_T} - 1\right) a_1 a_3 \frac{\partial u_2}{\partial x} \\
\tau_{11} &= \tau_{22} = \tau_{33} = \tau_{23} = \tau_{13} = 0
\end{aligned}\right\} \tag{6}$$

Substituting Eq. (6) in Eq. (4), we get

$$\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\frac{\partial^2 u_2}{\partial x^2}+2\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3\frac{\partial^2 u_2}{\partial x\partial z}+\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right)\frac{\partial^2 u_2}{\partial z^2}=\frac{\rho}{\mu_T}\frac{\partial^2 u_2}{\partial t^2} \tag{7}$$

In order to solve Eq. (7), we take

$$u_2(x,z,t)=\xi(z)\,\mathrm{e}^{ik(x-ct)} \tag{8}$$

Here, k is wave number; c is the phase velocity of simple harmonic waves of wave length $2\pi/k$.

From Eqs. (7) and (8), we get

$$\left\{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right\}\frac{\partial^2\xi(z)}{\partial z^2}+\left\{2\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3ik\right\}\frac{\partial\xi(z)}{\partial z}+\left\{\frac{\rho}{\mu_T}\omega^2-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)k^2\right\}\xi(z)=0 \tag{9}$$

where, $\omega=kc$ is the angular frequency, k the wave number and c is the phase velocity.

Let the solution of Eq. (9) is

$$\xi(z)=\mathrm{M}e^{-ikXz}+\mathrm{Q}e^{-ikYz} \tag{10}$$

where, X and Y are arbitrary constants given by

$$X=\frac{\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3+\sqrt{\left(\frac{\mu_L}{\mu_T}-1\right)^2a_1^2a_3^2+\left\{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right\}\left\{\frac{c^2}{c_\rho^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right\}}}{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2} \tag{11}$$

$$Y=\frac{\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-\sqrt{\left(\frac{\mu_L}{\mu_T}-1\right)^2a_1^2a_3^2+\left\{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right\}\left\{\frac{c^2}{c_\rho^2}-\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right)\right\}}}{1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2} \tag{12}$$

and $c_\rho=\sqrt{\frac{\mu_T}{\rho}}$ is the shear velocity.

Therefore, the equation of displacement of the upper reinforced medium is the solution of Eq. (7) and is given by

$$u_2(x,z,t)=\left(\mathrm{M}e^{-ikXz}+\mathrm{Q}e^{-ikYz}\right)\mathrm{e}^{ik(x-ct)} \tag{13}$$

4.2 Solution for the porous half-space

We assume an anisotropic initially stressed half space. Neglecting the viscosity of water, the dynamic equations of motion in the porous half-space under the compressive initial stress P', in the absence of body forces (Biot 1965) are

$$\left.\begin{aligned}
&\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}+\frac{\partial\sigma_{13}}{\partial z}-P'\left(\frac{\partial\omega'_z}{\partial y}-\frac{\partial\omega'_y}{\partial z}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u'_1+\rho_{12}U'_x\right)\\
&\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}+\frac{\partial\sigma_{23}}{\partial z}-P'\left(\frac{\partial\omega'_z}{\partial x}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}v'_1+\rho_{12}V'_y\right)\\
&\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}+\frac{\partial\sigma_{13}}{\partial z}-P'\left(\frac{\partial\omega'_y}{\partial x}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}w'_1+\rho_{12}W'_z\right)\\
&\frac{\partial\sigma}{\partial x}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u'_1+\rho_{12}U'_x\right)\\
&\frac{\partial\sigma}{\partial y}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}v'_1+\rho_{12}V'_y\right)\\
&\frac{\partial\sigma}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}w'_1+\rho_{12}W'_z\right)
\end{aligned}\right\} \tag{14a}$$

where, $\sigma_{ij}(i, j=1,2,3)$ are the incremental stress components, (u'_1, v'_1, w'_1) are the components of the displacement vector of the solid, (U'_x, V'_y, W'_z) are the components of the displacement vector of the liquid and σ is the stress vector due to the liquid. This stress vector σ is related to the fluid pressure p by the relation $\sigma=-fp$, where f is porosity of the layer. The angular components $\omega'_x, \omega'_y, \omega'_z$ are given by

$$\omega'_x=\frac{1}{2}\left(\frac{\partial w'_1}{\partial y}-\frac{\partial v'_1}{\partial z}\right), \omega'_y=\frac{1}{2}\left(\frac{\partial u'_1}{\partial z}-\frac{\partial w'_1}{\partial x}\right), \omega'_x=\frac{1}{2}\left(\frac{\partial v'_1}{\partial x}-\frac{\partial u'_1}{\partial y}\right) \tag{14b}$$

The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are related to the densities ρ, ρ_s and ρ_w of the layer, solid and water, respectively, given by

$$\rho_{11}+\rho_{12}=(1-f)\rho_s, \quad \rho_{12}+\rho_{22}=f\rho_w \tag{14c}$$

So that the mass density of the aggregate

$$\rho=\rho_{11}+2\rho_{12}+\rho_{22}=\rho_s+f(\rho_w-\rho_s) \tag{14d}$$

These mass coefficients also obey the following inequalities

$$\rho_{11}>0, \quad \rho_{12}\le 0, \quad \rho_{22}>0, \quad \rho_{11}\rho_{22}-\rho_{12}{}^2>0 \tag{14e}$$

For SH- wave propagation, the stress-strain relations for the water saturated initially stressed anisotropic porous layer

$$\left.\begin{aligned}
&\sigma_{11} = (I+P')e_{xx} + (I-2N+P')e_{yy} + (J+P')e_{zz} + K\Delta \\
&\sigma_{22} = (I-2N)e_{xx} + Ie_{yy} + Je_{zz} + K\Delta \\
&\sigma_{33} = Je_{xx} + Je_{yy} + De_{zz} + K\Delta \\
&\sigma_{12} = 2Ne_{xy}, \sigma_{23} = 2Le_{yz}, \sigma_{13} = 2Le_{zx}
\end{aligned}\right\} \tag{14f}$$

where $e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$, $\Delta = div\,\vec{u}$ is the dilation, *I,F,C,N* and *G* are elastic constants for the medium; *N* and *L* are, in particular, shear moduli of the anisotropic layer in the *x* and *z* direction respectively. Further, *K* being the measure of coupling between the volume change of the solid and the liquid is a positive quantity.

The hydrostatic stresses in the self weight half-space are given by

$$\sigma_{11} = \sigma_{33} = -dgz \tag{14g}$$

where d is the density of the lower half-space.

The components of body force are due to gravity g and are

$$x = 0,\ y = 0,\ z = g \tag{14h}$$

For the SH- waves propagating along the *x*-direction, having the displacement of particles along the *y*-direction, we have

$$\left.\begin{aligned}
&U'_x = 0,\ U'_y = U'_y(x,z,t)\ \text{and}\ U'_z = 0 \\
&u'_1 = 0,\ u'_2 = u'_2(x,z,t)\ \text{and}\ u'_3 = 0
\end{aligned}\right\} \tag{14i}$$

These displacements will produce only the e_{xy} eand e_{yz} strain components and the other strain components will be zero. Hence, the stress-strain relations useful in the problem are

$$\left.\begin{aligned}
&\sigma_{12} = 2Ne_{xy} = N\frac{\partial u'_2}{\partial x} \\
&\sigma_{23} = 2Le_{yz} = L\frac{\partial u'_2}{\partial z} \\
&\text{Also, } \frac{\partial}{\partial y} = 0
\end{aligned}\right\} \tag{14j}$$

Therefore Eq. (14(a)) with the help of Eqs. (14(g)-14(j)) can be written as

$$\left.\begin{aligned}
&\frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{23}}{\partial z} - P'\left(\frac{\partial \omega'_z}{\partial x}\right) - dg\omega'_{23} - dgz\frac{\partial \omega'_{23}}{\partial z} + dgz\frac{\partial \omega'_{12}}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{11}u'_2 + \rho_{12}V'_y) \\
&\frac{\partial \sigma}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{12}u'_2 + \rho_{22}V'_y) = 0
\end{aligned}\right\} \tag{15}$$

where ω'_{ij} are the angular components, which are defined as

$$\left.\begin{aligned} \omega'_{12} &= -\frac{1}{2}\frac{\partial u'_2}{\partial x} \\ \omega'_{23} &= +\frac{1}{2}\frac{\partial u'_2}{\partial z} \end{aligned}\right\} \tag{16}$$

Eliminating U'_x (displacement of liquid part) from Eq. (15), we get

$$\frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{23}}{\partial z} - P'\left(\frac{\partial \omega'_z}{\partial x}\right) - dg\,\omega'_{23} - dgz\frac{\partial \omega'_{23}}{\partial z} + dgz\frac{\partial \omega'_{12}}{\partial x} = d\,\frac{\partial^2 u'_2}{\partial t^2} \tag{17}$$

where $d = \rho_{11} - \dfrac{\rho_{12}^2}{\rho_{22}}$.

Using Eq. (14(j)), Eq. (16) in Eq. (17), we have

$$\left(N - \frac{P'}{2} - \frac{dgz}{2}\right)\frac{\partial^2 u'_2}{\partial x^2} + \left(L - \frac{dgz}{2}\right)\frac{\partial^2 u'_2}{\partial z^2} - \left(\frac{dg}{2}\right)\frac{\partial u'_2}{\partial z} = d\,\frac{\partial^2 u'_2}{\partial t^2} \tag{18}$$

In order to solve Eq. (18), we take

$$u'_2(x,z,t) = \zeta(z)\,\mathrm{e}^{ik(x-ct)} \tag{19}$$

From Eqs. (18) and (19), we get

$$\frac{\partial^2 \zeta(z)}{\partial z^2} - \frac{G\delta k}{2\left(1-\dfrac{G\delta kz}{2}\right)}\frac{\partial \zeta(z)}{\partial z} - k^2\left(\frac{\left(\dfrac{N}{L} - \xi - \dfrac{G\delta kz}{2}\right) - \delta\dfrac{c^2}{c_2^2}}{\left(1-\dfrac{G\delta kz}{2}\right)}\right)\zeta(z) = 0 \tag{20}$$

where $c_2 = [L/\rho]^{\frac{1}{2}}$ is the shear velocity in the lower half-space, $G = dg/Lk$ known as gravity parameter, $\xi = \dfrac{P'}{2L}$ is called stress parameter, $\delta = \gamma_{11} - \gamma_{12}^2/\gamma_{22}$, $\gamma_{11} = \rho_{11}/\rho$, $\gamma_{12} = \rho_{12}/\rho$, $\gamma_{22} = \rho_{22}/\rho$, are the non-dimensional parameters for the material of the porous half-space, k is wave number.

Now substituting $\zeta(z) = \chi(z)\left(1-\dfrac{G\delta kz}{2}\right)^{-\frac{1}{2}}$ in Eq. (20) to eliminating term $\dfrac{\partial \zeta(z)}{\partial z}$, we obtain

$$\chi''(z) + k^2\left[\frac{G^2\delta^2}{16}\left(1-\frac{G\delta kz}{2}\right)^{-2}\left\{\left(\frac{N}{L} - \xi - \frac{G'\delta kz}{2}\right) - \delta\frac{c^2}{c_2^2}\right\}\left(1-\frac{G\delta k}{2}\right)^{-1}\right]\chi(z) = 0 \tag{21}$$

Introducing the dimensionless quantities $\eta=\frac{4}{G\delta}\left(\frac{N}{L}-\xi-\frac{G\delta kz}{2}\right)$ and $\chi(z)=g(\eta)$ in Eq. (21), we get,

$$\frac{d^2g(\eta)}{dz^2}+\left(-\frac{1}{4}+\frac{R}{\eta}+-\frac{1}{4\eta^2}\right)g(\eta)=0 \tag{22}$$

where, $R=\frac{1}{G}\left[\left(1-\frac{N}{L}-\xi\right)\frac{1}{\delta}+\frac{c^2}{c_2^2}\right]$

Eq. (22) is the well known Whittaker's equation (Whittaker and Watson 1990).

The solution of Whittaker's Eq. (21) is given by

$$g(\eta)=\mathrm{A}\mathrm{W}_{\mathrm{R},0}(\eta)+\mathrm{B}\mathrm{W}_{-\mathrm{R},0}(-\eta) \tag{23}$$

where A and B are arbitrary constants and $\mathrm{W}_{\mathrm{R},0}(\eta),\mathrm{W}_{-\mathrm{R},0}(-\eta)$ are the Whittaker function.

Now Eq. (23) satisfying the condition $\lim_{z\to\infty}u_2'\to 0$ i.e., $\lim_{z\to\infty}g(\eta)\to 0$ as $\eta\to 0$ may be taken as

$$g(\eta)=\mathrm{B}\mathrm{W}_{-\mathrm{R},0}(-\eta) \tag{24}$$

Hence the displacement for the SH-wave in the lower layer is

$$u_2'(x,z,t)=\mathrm{B}\left(1-\frac{G\delta kz}{2}\right)^{-\frac{1}{2}}\mathrm{W}_{-\mathrm{R},0}\left[-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right]e^{ik(\mathrm{x}-\mathrm{ct})} \tag{25}$$

5. Dispersion relation

The dispersion relation for SH-waves can be obtained by using boundary conditions given in section 3. Therefore, the displacement for the SH-waves in the in-homogeneous half-space using boundary conditions (1), (2) and (3) in Eq. (6), Eq. (13), Eqs. (14(j)) and (25)

$$\left\{\mathrm{M}\left[\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)e^{iXkH}+\mathrm{Q}\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)e^{iYkH}\right]\right\}=0 \tag{26}$$

$$\mu_T ik\left\{\mathrm{M}\left[\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)+\mathrm{Q}\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)\right]\right\}$$
$$=\ \mathrm{AL}\left\{\frac{\partial}{\partial z}\left[\left(1-\frac{G\delta kz}{2}\right)^{-\frac{1}{2}}\mathrm{W}_{-\mathrm{R},0}\left\{-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right\}\right]\right\}_{z=0} \tag{27}$$

$$\mathrm{M}+\mathrm{Q}=\mathrm{A}\left[\mathrm{W}_{-\mathrm{R},0}\left\{-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right\}\right]_{z=0} \tag{28}$$

Now eliminating M, Q and A from the Eq. (26), Eqs. (27) and (28), we obtain

$$\begin{vmatrix} \left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right)e^{iXkH} & \left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right)e^{iYkH} & 0 \\ ik\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2X\right) & ik\left(\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2Y\right) & -\frac{L}{\mu_T}\left\{\frac{\partial}{\partial z}\left[\left(1-\frac{G\delta kz}{2}\right)^{-\frac{1}{2}}\mathrm{W}_{-\mathrm{R},0}\left\{-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right\}\right]\right\}_{z=0} \\ 1 & 1 & -\left[\mathrm{W}_{-\mathrm{R},0}\left\{-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right\}\right]_{z=0} \end{vmatrix}=0 \tag{29}$$

On simplifying Eq. (29), we get

$$\tan\left[\frac{kH}{\varepsilon}\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\right]=\frac{L}{\mu_T}\frac{\left\{\frac{\partial}{\partial z}\left[\left(1-\frac{G\delta kz}{2}\right)^{-\frac{1}{2}}\mathrm{W}_{-\mathrm{R},0}\left\{-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right\}\right]\right\}_{z=0}}{\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\left\{\left[\mathrm{W}_{-\mathrm{R},0}\left\{-\frac{4}{G\delta}\left(1-\frac{G\delta kz}{2}\right)\right\}\right]\right\}_{z=0}} \tag{30}$$

On solving Eq. (30) (taking Whittaker's function up to second degree term), we get

$$\tan\left[\frac{kH}{\varepsilon}\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\right]=\frac{L}{\mu_T}\frac{\left(1-\frac{G\delta}{4}-\frac{\Psi}{2}\right)\left[16G\delta+(2\Psi+G\delta)^2\right]-\frac{G\delta}{2}(2\Psi+G\delta)^2}{\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\left[16G\delta+(2\Psi+G\delta)^2\right]} \tag{31}$$

where

$$p=\left(1+\left(\frac{\mu_L}{\mu_T}-1\right)a_1^2\right),\ q=\left[\left(\frac{\mu_L}{\mu_T}-1\right)a_1a_3-1\right],\ \varepsilon=\left[1+\left(\frac{\mu_L}{\mu_T}-1\right)a_3^2\right],\ \Psi=\frac{c^2}{c_2^2}+\frac{1}{\delta}\left(1-\frac{N}{L}-\frac{P'}{2L}\right)$$

Eq. (31) is the dispersion equation of SH-wave propagation in a fiber-reinforced anisotropic layer over a porous half-space under the effect of initial stress and gravity.

Particular Cases

If the layer is non-porous then $f\to 0$ and $\rho_s\to\rho$ which leads to $\gamma_{11}+\gamma_{12}\to 1$ and $\gamma_{12}+\gamma_{22}\to 0$, which leads to $\gamma_{11}-\frac{\gamma_{12}^2}{\gamma_{22}}\to 1$ or $\delta\to 1$. If the layer is porous then $f\to 1$, then $\rho_w\to\rho$, the liquid becomes fluid $\gamma_{11}-\frac{\gamma_{12}^2}{\gamma_{22}}\to 0$ or $\delta\to 0$, which means shear waves do not exit. Hence, for porous layer $0<f<1$ corresponds to $0<\delta<1$.

Case-1

For homogeneous reinforced medium over porous half space, we take $a_1 = 1, a_2 = a_3 = 0$ then $\rho \to \frac{\mu_L}{\mu_T}$ and $\mu_L \to \mu_T \to \mu_1$, $L = N = \mu_2$ therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{\left(1-\frac{G\delta}{4}-\frac{\Psi}{2}\right)\left[16G\delta+(2\Psi+G\delta)^2\right]-\frac{G\delta}{2}(2\Psi+G\delta)^2}{\sqrt{\frac{c^2}{c_1^2}-1}\left[16G\delta+(2\Psi+G\delta)^2\right]} \tag{32}$$

Case-2

For SH-wave propagation in a fiber-reinforced anisotropic layer over a porous half-space free from initial stress $\xi \to 0$, therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{\left(1-\frac{G\delta}{4}-\frac{\Psi'}{2}\right)\left[16G\delta+(2\Psi'+G'\delta)^2\right]-\frac{G\delta}{2}(2\Psi'+G\delta)^2}{\sqrt{\frac{c^2}{c_1^2}-1}\left[16G\delta+(2\Psi'+G\delta)^2\right]} \tag{33}$$

where, $\Psi' = \frac{c^2}{c_2^2} + \frac{1}{\delta}\left(1-\frac{N}{L}\right)$

Case-3

For SH-wave propagation in a fiber-reinforced anisotropic layer over a porous half-space free from gravity $G \to 0$, therefore, Eq. (31) reduces to

$$\tan\left[\frac{kH}{\varepsilon}\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\right] = \frac{L}{\mu_T}\frac{\left(1-\frac{c^2}{2c_2^2}-\frac{1}{2\delta}\left(1-\frac{N}{L}-\xi\right)\right)}{\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}} \tag{34}$$

Case-4

If the layer is non-porous then $\delta \to 1$, therefore, Eq. (31) reduces to

$$\tan\left[\frac{kH}{\varepsilon}\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\right] = \frac{L}{\mu_T}\frac{\left(1-\frac{G}{4}-\frac{\Psi''}{2}\right)\left[16G+(2\Psi''+G)^2\right]-\frac{G}{2}(2\Psi''+G)^2}{\sqrt{q^2+\varepsilon\left[\frac{c^2}{c_1^2}-p\right]}\left[16G+(2\Psi''+G)^2\right]} \tag{35}$$

where $\Psi'' = \frac{c^2}{c_2^2} + \left(1 - \frac{N}{L} - \frac{P'}{2L}\right)$

Case-4

For homogeneous reinforced medium over non-porous half space, we take $a_1 = 1, a_2 = a_3 = 0$ then $\rho \to \frac{\mu_L}{\mu_T}$ and $\mu_L \to \mu_T \to \mu_1$, $\delta \to 1$, $L = N = \mu_2$ therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{\left(1-\frac{G}{4}-\frac{1}{2}\left\{\frac{c^2}{c_2^2}+\left(1-\frac{N}{L}-\xi\right)\right\}\right)\left[16G+\left(\frac{1}{2}\left\{\frac{c^2}{c_2^2}+\left(1-\frac{N}{L}-\xi\right)\right\}+G\right)^2\right]-\frac{G}{2}\left(\frac{1}{2}\left\{\frac{c^2}{c_2^2}+\left(1-\frac{N}{L}-\xi\right)\right\}+G\right)^2}{\sqrt{\frac{c^2}{c_1^2}-1}\left[16G+\left(\frac{1}{2}\left\{\frac{c^2}{c_2^2}+\left(1-\frac{N}{L}-\xi\right)\right\}+G\right)^2\right]} \tag{36}$$

Case-5

For homogeneous reinforced medium over an homogeneous non-porous half space free from gravity $G \to 0$, stress parameter $\xi \to 0$ $a_1 = 1, a_2 = a_3 = 0$ then $\rho \to \frac{\mu_L}{\mu_T}$ and $\mu_L \to \mu_T \to \mu_1$, $\delta \to 1$, $L = N = \mu_2$ therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{1-\frac{1}{2}\frac{c^2}{c_2^2}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{37}$$

On approximation Eq. (37) gives

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right] = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{38}$$

Eq. (38) is the classical dispersion equation of SH-waves given by Love (1911) and Ewing *et al.* (1957).

6. Numerical analysis

To show the combined effects of stress, gravity, porosity and steel reinforced parameters on SH-wave propagation, the data is taken from Table 1, used by Chattaraj and Samal (2013a). Values of parameters for figures are given in Table 2. MATLAB software is used to explain the graphically effects of technical constants on SH-wave propagation in a fiber-reinforced anisotropic layer overlying a pre-stressed gravitational porous half space.

Table 1 Data for fiber-reinforced anisotropic layer and porous medium

Symbol	Numerical Value	Units
μ_T	5.65×10^9	N/m^2
μ_L	2.46×10^9	N/m^2
λ	5.65×10^9	N/m^2
α	-1.28×10^{10}.	N/m^2
β	220.09×10^9	N/m^2
ρ	7800	kg/m^3
L	0.1167×10^{10}	N/m^2
a_3^2	0.75	----
a_1^2	0.25	----
ρ_{11}	1.7567×10^3	Kg/m^3
ρ_{12}	-0.001567×10^3	Kg/m^3
ρ_{22}	0.19867×10^3	Kg/m^3
f	0.34	----

Table 2 Values of parameters for figures

	Figure 2	Figure 3	Figure 4	Figure 5	Figure 6	Figure 7	Figure 8
a_1^2, a_3^2	0.25, 0.75	0.25, 0.75	0.25, 0.75	0.25, 0.75	0.25, 0.75	0.25, 0.75	–
δ	–	0.4	0.4	0.4	0.4	0.4	0.4
G	0.5	0.5	0.5	–	0.5	0.5	0.5
N/L	0.5	0.5	–	0.5	0.5	0.5	0.5
μ_L/μ_T	2.5	2.5	2.5	2.5	–	2.5	2.5
$\bar{\xi}$	0.5	0.5	0.5	0.5	0.5	–	0.5
kH	4	–	4	4	4	4	4

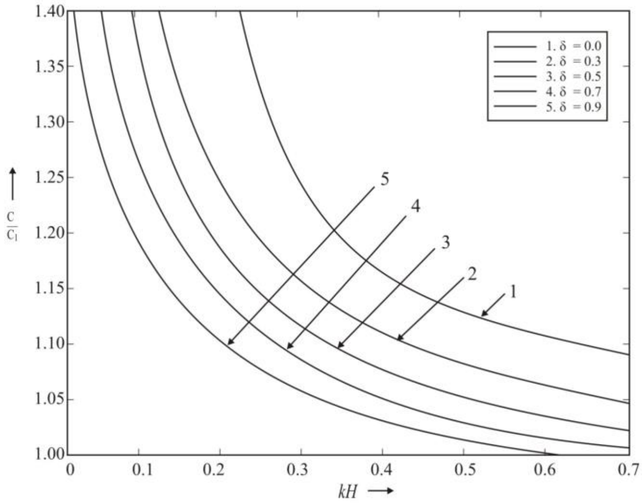

Fig. 2 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of porosity δ

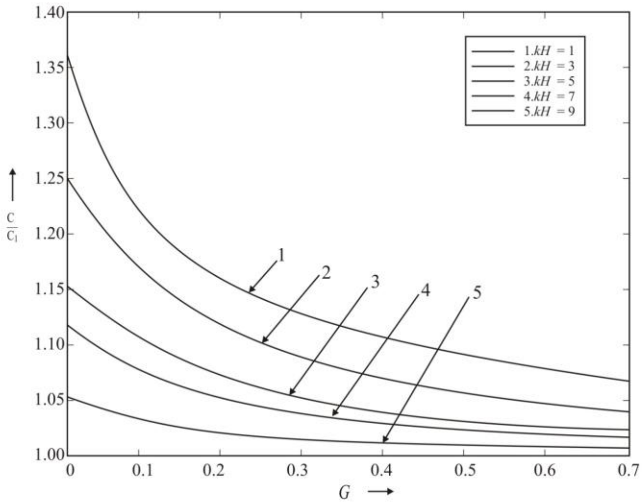

Fig. 3 Dimensionless phase velocity $\frac{c}{c_1}$ against G for different values kH

Fig. 2 discribes the variations of dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of porosity δ in the presence of reinforced parameters. It has been observed that with the increment of wave number there is decrement in the phase velocity for a particular value of δ. Further it is seen as δ increases the phase velocity decreases for a particular value of kH at it becomes constant after a certain value of kH. Fig. 3 depicts the variation of dimensionless phase velocity against dimensionless gravity parameter G for different values of kH. It is observed that the gravity parameter G has great influence on the phase velocity i.e., it decreases as gravity parameter G increases. Fig. 4 represents the variations of dimensionless phase velocity against dimensionless wave number for different values $\bar{\varsigma} = \frac{N}{L}$. From this figure it has been observed that as the dimensionless ratio $\frac{N}{L}$ increases, the phase velocity also increases at the same frequency. Fig. 5 is plotted between dimensionless phase velocity of SH-wave against dimensionless wave number for different values of Biot's parameter G. It shows that the phase velocity decreases as wave number increases for different values G.

Fig. 6 demonstrate the variation of phase velocity against wave number for different values $\mu' = \frac{\mu_L}{\mu_T}$, it is noted that as μ' increases, the phase velocity decreases and the curves are getting closer after some value of kH. Fig. 7 represents the variation of dimensionless phase velocity against dimensionless wave number for different values of stress parameter ξ. From this figure it has been observed that as the initial stress of the lower half-space increases the phase velocity also increases at the same frequency. Figs. 8 and 9 illustrates the variations of dimensionless phase velocity against wave number kH for different values reinforce parameters a_1^2 and a_3^2. It is seen from the diagram that as a_1^2 increases or decreases as well as a_3^2 decreases or increases, the velocity of SH-wave decreases.

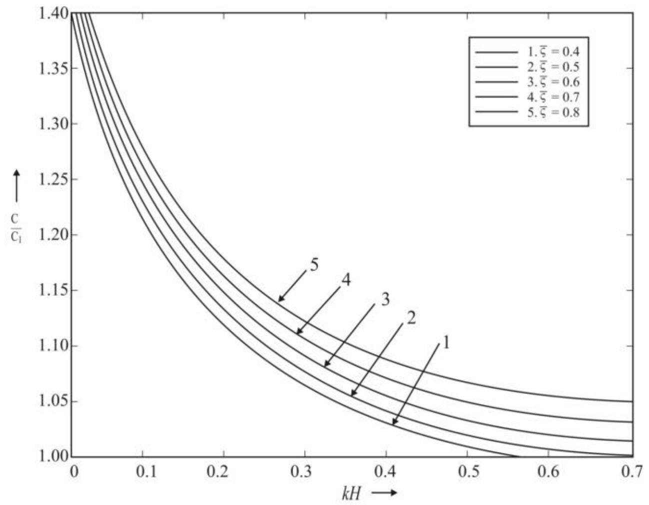

Fig. 4 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of $\bar{\varsigma} = N / L$

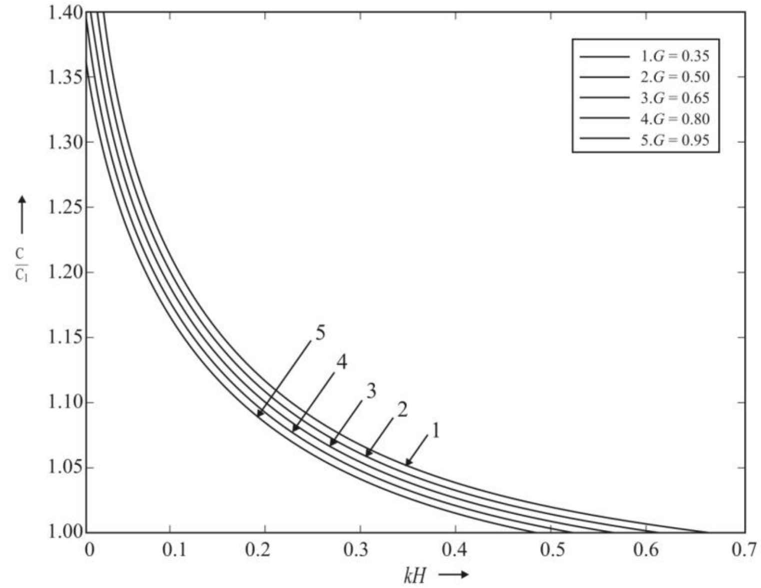

Fig. 5 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values Biot's parameter G

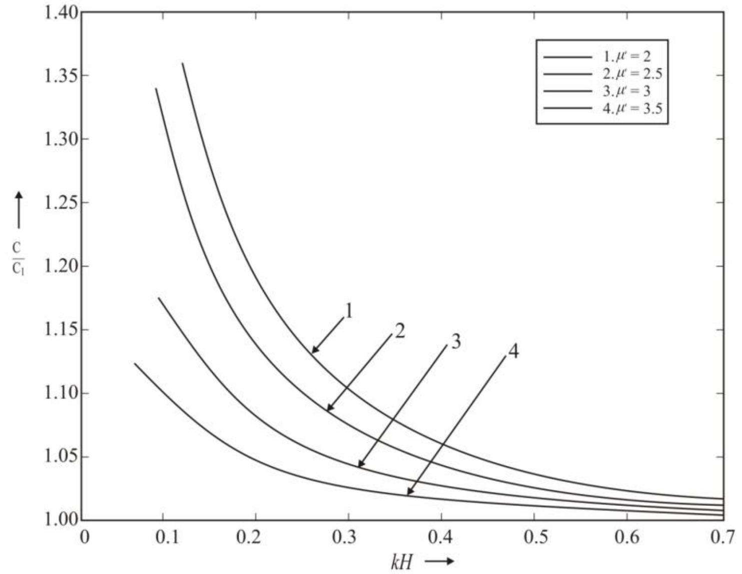

Fig. 6 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values μ'

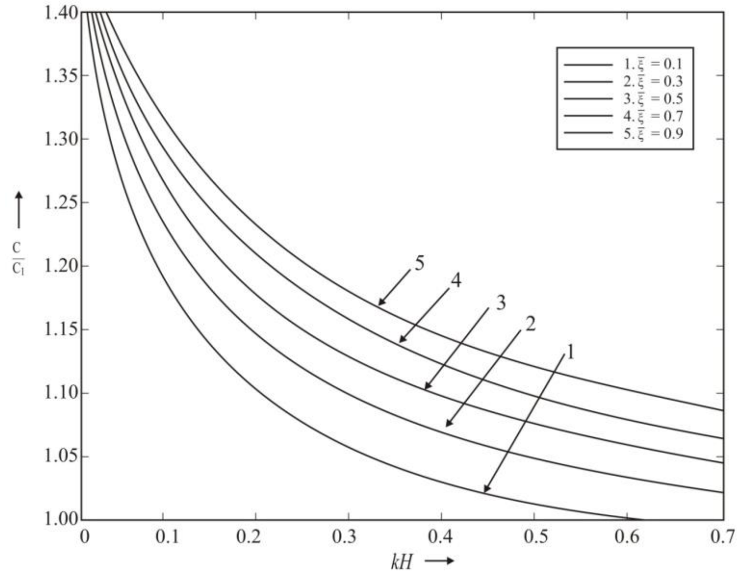

Fig. 7 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of stress parameter $\bar{\xi}$

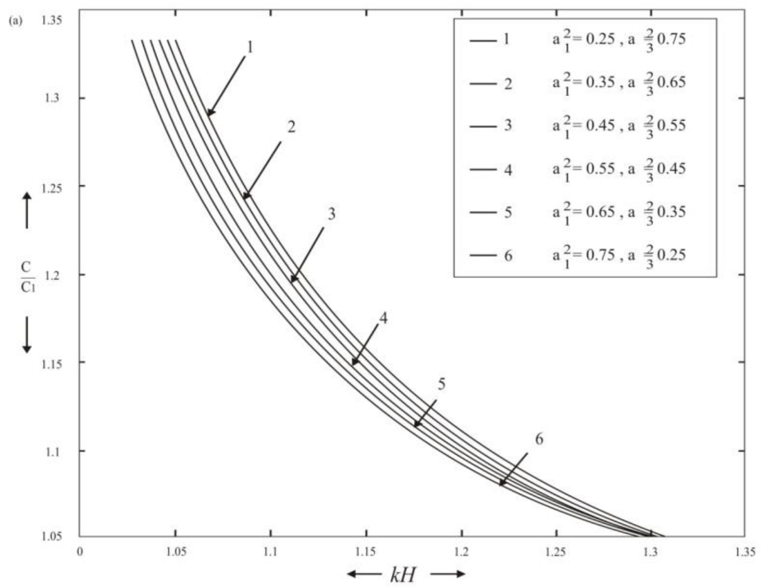

Fig. 8 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of reinforce parameters a_1^2 , a_3^2

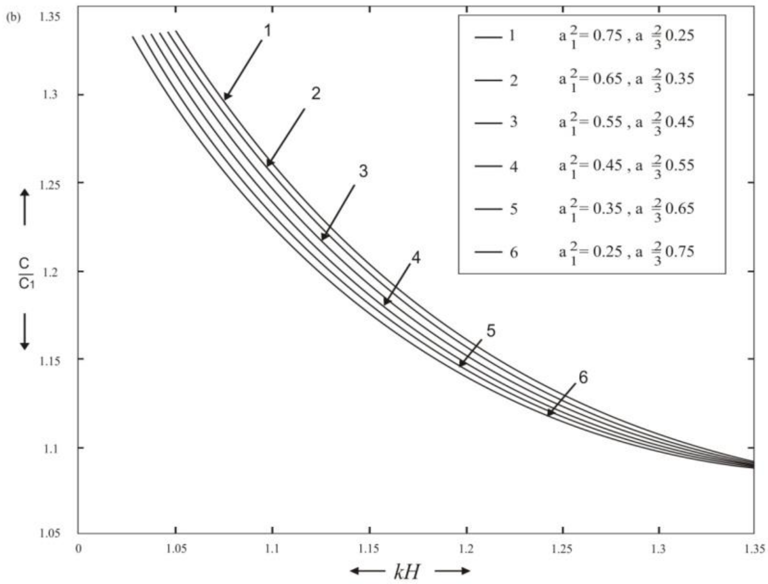

Fig. 9 Dimensionless phase velocity $\frac{c}{c_1}$ against dimensionless wave number kH for different values of reinforce parameters a_1^2, a_3^2

7. Conclusions

In this work, the dispersion of SH-waves in a fiber-reinforced anisotropic layer overlying a pre-stressed gravitational porous half space has been investigated analytically. It has been observed that the non-dimensional phase velocity is larger for a porous initially stressed gravitational elastic half-space as compared to a non-porous elastic half-space ($\delta \to 1$). The frequency equation for the propagation of SH-waves in a fiber-reinforced anisotropic layer overlying a pre-stressed gravitational porous half space has been derived in terms of second degree Whittaker. The dispersion relation shows that the reinforce parameters, stress, gravity and porosity plays a significant role on the propagation of SH-waves. It has been observed that for a fixed value of gravity parameter, the dimensionless phase velocity decreases as the width of the reinforced layer increases and increasing gravity parameter.

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