Smart Structures and Systems, *Vol. 18, No. 5 (2016) 839-864*
DOI: http://dx.doi.org/10.12989/sss.2016.18.5.839

Modified gradient methods hybridized with Tikhonov regularization for damage identification of spatial structure

S.S. Naseralavi[1], S. Shojaee[*2] and M. Ahmadi[2]

[1]*Department of Civil Engineering, Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran*
[2]*Department of Civil Engineering, Shahid Bahonar University of Kerman, Kerman, Iran*

(Received October 12, 2015, Revised April 2, 2016, Accepted April 6, 2016)

Abstract. This paper presents an efficient method for updating the structural finite element model. Model updating is performed through minimizing the difference between the recorded acceleration of a real damaged structure and a hypothetical damaged one. This is performed by updating physical parameters (module of elasticity in this study) in each step using iterative process of modified nonlinear conjugate gradient (M-NCG) and modified Broyden–Fletcher–Goldfarb–Shanno algorithm (M-BFGS) separately. These algorithms are based on sensitivity analysis and provide a solution for nonlinear damage detection problem. Three illustrative test examples are considered to assess the performance of the proposed method. Finally, it is demonstrated that the proposed method is satisfactory for detecting the location and ratio of structural damage in presence of noise.

Keywords: damage identification; gradient methods; Tikhonov regularization; truss structure

1. Introduction

Quantitative and objective condition assessment for infrastructure protection has been the subject of many researchers conducted within the engineering community. To achieve this, methodologies of the routine inspections either with fixed intervals or the continuous monitoring, which provide information on safety reliability or remaining life of the structure, have been under development in recent years. Inspecting structural components for damage diagnosis is vital to take decisions about their repairs.

Structural health monitoring is divided into four stages: determining the existence, location of damages, and estimating the amount of damage and the remaining life of structure (Rytter 1993). Among the tissues, detection and localization of damage are currently of growing interest among researchers (Basseville *et al.* 2004). The detection methods generally can be divided into two categories, the methods which are based upon static responses (Chou and Ghaboussi 2001, Wang *et al*. 2011) and the other methods which use dynamic data (Yan *et al.* 2007, Salawu 1997, Pandey *et al.* 1991, Fan and Qiao 2011). The prevailing interest among researchers is focused on damage detection using vibration data.

Since the damage is a nonlinear characteristic, the direct solution of the resulted system of

∗Corresponding author, Dr., E-mail: saeed.shojaee@uk.ac.ir

http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

equations is formidable or maybe impossible. This particular set of equations should be solved through a numerically iterative process. Current methods of damage detection are mostly composed of updating finite element model through minimizing the difference between responses of real damaged and hypothetically damaged structures. Some of the mentioned methods apply the explorer optimization algorithms to update the structural model during an iterative process. The other methods based on sensitivity analysis, utilize various algorithms to update the finite element model by minimizing the objective function.

Many effective sensitivity-based damage detection methods have been suggested in the literature. Esfandiari *et al*. (2009) updated the structural model using a least square algorithm using an appropriate stabilization method. Their method can detect the ratio and location of damage in trusses, where the noise exists in the frequency response function. Teughels and Roeck (2005) presented two sensitivity-based algorithms, modified Gauss-Newton and coupled local minimizers (CLM), for updating the finite element model. They detected the location and ratio of damages in bridges by model updating through minimization of the difference between frequencies and mode shapes. Bakir *et al.* (2007) implemented another sensitivity-based method called trust region algorithm, for updating the finite element model. They obtained the location and ratio of damage in reinforced concrete frames by minimizing the frequency and mode shape residuals. Their algorithm was also efficient in the case of noisy data. Jaishi and Ren (2006) generated a sensitivity-based finite element model updating for damage diagnosis. They utilized a modal flexibility residual as the objective function and trust region Newton algorithm for minimizing the objective function. Numerical results indicate that the proposed method can provide a reliable tool to accurately identify the multiple structural damages. Lee (2009) introduced a method for identifying multiple cracks in a beam using the Newton-Raphson method, sensitivity analysis and natural frequencies.

Naseralavi *et al.* (2012) proposed a new damage detection algorithm for space structures under static loads. To make the method stable to noise, in their method, first a set of damage candidates is obtained using a discrete version of ant colony optimization. In the second stage by employing continuous ant colony optimization the damage extents are evaluated. Naseralavi *et al.* (2012) identified the damages by searching exhaustively within the sensitivity vector of elements to obtain the associated subspace in which response change vector lies in the best. Torkzadeh *et al*. (2013) used modal strain energy changes to restrict the damage candidates. The damage severity of damage candidates are computed by minimizing the norm of difference between the response of damaged structure and that of the mathematical model of structure as the objective function. The minimization is performed by using the heuristic particle swarm optimization. Kaveh and Maniat (2014) employed the novel charged system search as a heuristic optimization tool for detecting multi damages using changes in natural frequencies and incomplete mode shapes. They verified their method for beams, frames and trusses in various noise levels. They obtained satisfactory results in all cases.

Sarvi *et al*. (2014) proposed a method for identifying damage based on sensitivity analysis using the updating finite element model method. They used Levenberg-Marquardt algorithm for solving damage equation and acceleration responses as input data for this method. Their proposed method was effective in the presence of noise and performed satisfactorily in detecting damages.

Although various methods have been developed for model updating, still the novel ideas are being presented. The main objective of this paper is to develop an efficient method for updating the structural finite element model for solving system of nonlinear equations using gradient based methods such as nonlinear conjugate gradient (NCG) and BFGS. These algorithms are based on sensitivity analysis and provide a linear solution for nonlinear damage detection problem.

This paper is organized as follows. First the fundamentals of damage detection are reviewed. Then in Section 3, the formulation of NCG and BFGS algorithms for model updating and damage detection method are presented. In Section 4 the proposed algorithm is given. Finally, the results of the numerical simulations of three spatial structures are discussed and the efficiency of the proposed approach is investigated.

2. Fundamentals of damage detection

The main idea of updating methods for damage detection is based on the fact that changes in structural response are due to changes in physical properties. In other words, through a reversed model updating the detailed changes of physical model can be found using the known responses. The considered responses of damaged structure are functions of the structural damage. It means that a specific damage causes a unique response. This point can be used to find the damage. The damage can be simulated by inflicting changes on structural parameters such as Young's modulus or cross sectional area of members. The damage detection equation can be stated as

$$\mathbf{R}_{\mathrm{d}} = \mathrm{R}(\mathbf{X}) \Rightarrow \mathbf{X} = ? \tag{1}$$

$$\mathbf{r} = \mathrm{R}(\mathbf{X}) - \mathbf{R}_{\mathrm{d}} \tag{2}$$

$$\mathbf{X} = \{x_1, x_2, \ldots x_n\},\ 0 \le x_i \le 1 \tag{3}$$

where $\mathbf{r}$ is the residual function, $\mathrm{R}(\mathbf{X})$ and $\mathbf{R}_{\mathrm{d}}$ are the response vectors of hypothetically damaged structure and the existing damaged structure respectively. $\mathbf{X}$ represents the damage vector which consists of all structural members' damage (x_i) and n is the number of members. The goal of damage detection is to find the damage vector $\mathbf{X}$, using the response vector of the damaged structure.

Eq. (1) can be expressed as follows using the Taylor series expansion

$$\mathbf{R}_{\mathrm{d}} = \mathrm{R}(\mathbf{X}) = \mathbf{R}_{\mathrm{h}} + \frac{\partial \mathrm{R}}{\partial \mathbf{X}} \Delta\mathbf{X} + \ldots \tag{4}$$

where $\mathbf{R}_{\mathrm{h}}$ is the response of healthy structure and $\Delta\mathbf{X}$ is change of damage vector. If first-order approximation (linearization) is applied, regardless of higher order terms, Eq. (4) can be rewritten as

$$\mathbf{R}_{\mathrm{d}} = \mathbf{R}_{\mathrm{h}} + \frac{\partial \mathbf{R}}{\partial \mathbf{X}} \Delta\mathbf{X} \ \Rightarrow\ \mathbf{R}_{\mathrm{d}} - \mathbf{R}_{\mathrm{h}} = \Delta\mathbf{R} \cong \mathbf{S}\Delta\mathbf{X} \quad , \qquad \mathbf{S} = \frac{\partial \mathbf{R}}{\partial \mathbf{X}} \tag{5}$$

where $\mathbf{S}$ is sensitivity matrix, $\mathbf{S} \in \mathrm{R}^{m\times n}$ and $m \ge n$ that m and n are number of rows and columns of matrix S respectively. Finally damage detecting problem leads to the solution of nonlinear equation system, then residual function is rewritten as follows

$$\mathbf{r} = \Delta\mathbf{R} - \mathbf{S}\Delta\mathbf{X} \tag{6}$$

By minimizing residual vector using Newton-Raphson method during update process, the damage solution is resulted.

2.1 Updating parameter

The updating parameter is the unknown physical features of the model. In this paper the ratio of updated modulus of elasticity ($E^e - E_0^e$) to its initial value (E_0^e) is considered as updating parameter. The dimensionless updating parameter is defined as follows

$$x^e = -\frac{E^e - E_0^e}{E_0^e} \rightarrow E^e = E_0^e(1 - x^e) \tag{7}$$

2.2 Objective function

The damage detection procedure using model updating is similar to the identification of unknown parameters in an optimization problem using model updating. By spotting the error between responses of healthy structure and updated structure and minimizing the objective function in each step of iterative process, $\mathbf{X}^e$ is updated. Minimizing the objective function is defined as a nonlinear least square minimization problem (Teughels *et al*. 2002)

$$\mathrm{f}(\mathbf{X}) = \frac{1}{2}\left(\mathrm{R}(\mathbf{X}) - \mathbf{R}_\mathrm{d}\right)^\mathrm{T}\left(\mathrm{R}(\mathbf{X}) - \mathbf{R}_\mathrm{d}\right) = \frac{1}{2}\sum_{j=1}^{m}\mathbf{r}_j^2(\mathbf{X}) = \left\|\mathrm{r}(\mathbf{X})\right\|^2 \tag{8}$$

where $\mathbf{r}$ is the residual value of damage function, and $\| \;\|$ is Euclidean norm. The structural model updating, as employed in this work, is represented by minimizing the difference between the acceleration response of actual damaged structure and the hypothetically damaged one.

3. Algorithm of solving damage equation

3.1 Methodology

In a system of equations when the coefficient matrix is non-square and the number of equations is more than the number of unknowns, the system will come up with no consistent solution. However, by utilizing optimization methods it would be possible to reach the best answer with the minimum residual value (**r**). In case of damage identification issue based on sensitivity analysis, we may come across such equation systems for which a practical method is needed. Optimization could be defined as finding out the best damage solution among all the possible ones. Among various optimization methods we herein consider the line search methods. These methods start from a specified point in solution space and using rules that are based on mathematics and geometry, and achieve to a specific result, the coming points could be identified as more optimized answers. These methods are the basis of most multivariable optimization methods. If $\mathbf{X_n}$ is the identified solution in the *n*th step, the solution in the next step will be as follows

$$\mathbf{X_{n+1}} = \mathbf{X_n} + \alpha_n \mathbf{S_n} \tag{9}$$

where α_n is the step size and S_n is a vector and similar to X_n in dimension which determines the direction of the movement. Finding the proper α_n and S_n is the major goal in line search process. Among the developed line search techniques which have been introduced in the literature, the conjugate gradient method and BFGS is considered in this research.

3.2 Nonlinear conjugate gradient algorithm

In numerical optimization, the nonlinear conjugate gradient method generalizes the conjugate gradient method to the nonlinear optimization. For a quadratic function f(**X**)

$$\mathrm{f}(\mathbf{X}) = \|\mathbf{AX} - \mathbf{b}\|^2 \tag{10}$$

The minimum of ***f*** is obtained when the gradient is equal to zero

$$\nabla_x \mathrm{f} = 2\mathbf{A}^\mathrm{T}\left(\mathbf{AX} - \mathbf{b}\right) = 0 \tag{11}$$

whereas linear conjugate gradient seeks a solution to the linear equation $\mathbf{A}^\mathrm{T}\mathbf{Ax}=\mathbf{A}^\mathrm{T}\mathbf{b}$, the nonlinear conjugate gradient method is generally used to find the local minimum of a nonlinear function using its gradient $\nabla_\mathbf{x}$f alone. It works when the function is approximately quadratic near the minimum, which is the case when the function is twice differentiable at the minimum.

3.3 BFGS algorithm

BFGS is a Quasi-Newton second-derivative line search family method, which is one of the most powerful methods to solve unconstrained optimization problem. Consider the unconstrained optimization problem

$$\min \mathrm{f}(\mathbf{X}),\ \ \mathbf{X} \in \mathrm{R}^\mathrm{n} \tag{12}$$

$$\mathrm{f}(\mathbf{X}) = \|\mathbf{AX} - \mathbf{b}\|^2 \tag{13}$$

where f is a general non-convex function having continuous second derivatives. The quasi-Newton method is one of the most well-known and efficient methods for solving (13) and has the general form

$$\mathbf{X}_{\mathrm{k+1}} = \mathbf{X}_\mathrm{k} - \alpha_\mathrm{k}\mathbf{H}_\mathrm{k}\mathbf{X}_\mathrm{k} \tag{14}$$

where $\mathbf{H}_\mathrm{k}$ is an approximation to the inverse Hessian and α_k is the step-size. BFGS technique identified to be the most efficient quasi-Newton method as declared by Broyden, Fletcher, Goldfarb, and Shanno, independently. In Quasi–Newton methods, the idea is to construct matrices approximating the Hessian matrix and/or its inverse, instead of exact computing the Hessian matrix as in Newton-type methods. The matrices are adjusted on each iteration and can be produced in many different ways ranging from simple techniques to advanced ones. For more details about these algorithms, see Luenberger and Ye (2007).

3.4 Tikhonov regularization

The responses of damaged structures are measured by sensors in a laboratory or from in situ structures. Always there is a noise in responses measured by sensors. The noise is artificially added to responses coming from numerical simulation. This error is called measurement error and is applied to the responses by the following equation (Li and Law 2010)

$$\mathbf{a}_{\mathrm{measured}} = \mathbf{a}_{\mathrm{calculated}} + \mathrm{E}_{\mathrm{p}} \times \mathbf{N}_{\mathrm{noise}} \times \mathbf{a}_{\mathrm{calculated}} \tag{15}$$

where $\mathbf{a}_{\mathrm{measured}}$ is the noisy acceleration response vector and $\mathbf{a}_{\mathrm{calculated}}$ is the acceleration response vector calculated from the damaged structure and E_p is the noise level (e.g., 1 to 5 percent,…). $\mathbf{N}_{\mathrm{noise}}$ is the normally distributed vector with zero mean and unit standard deviation.

Most methods which are used to identify damage cannot withstand the influence of measurement errors resulting to wrong damage solutions. Moreover, in such problems, small measurement errors can lead to large variations in model parameters. Therefore to resolve these problems, the effect of measurement errors should be reduced. This is done by means of regularization technique.

The main obstacle in ill-posed problems is large condition number. In other words, the spectrum of the eigenvalues of matrix coefficients is wide. Therefore, it is necessary to consider the specification of real damage cases for stabilizing the problem. “Tikhonove method” is the most common method in stabilization of ill-posed problems, especially in solving inverse problems.

The idea of this method were presented by Phillips (1962) and Tikhonov (1963) almost simultaneously but independently. From a statistical point of view, this method was categorized as Bayesian methods for solving inverse problems. These problems were used when the information or the basic premise of unknowns were existed. In Tikhonov method such as least square scheme, we assume that the observation errors are random with zero mean probability distribution function. Therefore, in this method as well as the least squares method, we look for a damage solution with the lowest residual. However, achieving such a solution with the lowest residual in ill-posed system of equation is not easily possible. Hence in Tikhonov method, the norm of residual vector was minimized and at the same time, the feature of unknowns can be reduced to prevent immensity response. In general the Tikhonov method can be defined as a minimization of Tikhonov function given by

$$\mathrm{f}_{\mathrm{Tikhonov}}(\mathbf{X},\lambda) = \left\|\mathbf{AX} - \mathbf{b}\right\|_2^2 + \lambda\Omega(\mathbf{X}) \tag{16}$$

In above relation, the function $\Omega(\mathbf{X})$ is determined according to the basic assumptions and information of the unknowns. It should be noted that the Tikhonov method with the definition above, is also called "generalized Tikhonov". In regularization of ill-posed problems with generalized Tikhonov method, we can often use the semi-norm of problem solution by defining function Ω as follow

$$\Omega(\mathbf{X}) = \left\|\mathbf{LX}\right\|_2^2 \quad , \mathbf{L} \in \mathrm{R}^{m?} \tag{17}$$

Putting the function Ω in equation (16), differentiating from unknowns and setting derivatives equal to zero, we achieve the generalized Tikhonov regularization strategy. In this study, the main diagonal in matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is considered to be the main diagonal of the matrix **L.**

$$\mathbf{L}=\mathrm{diag}(\mathbf{A}^{\mathrm{T}}\mathbf{A}) \tag{18}$$

By selecting matrix **L** as above, the condition number will be further reduced so that the system of equations will be more stable and thus will reach to a more accurate solutions of the equations. For applying NCG and BFGS methods to damage detection process, we need to use regularizer, because these algorithms cannot converge without regularization method.

4. Modified algorithms based on damage identification

In the procedure of damage identification based on sensitivity analysis, we face damage system of equations given in Eq. (5). In order to solve such a system, the objective function is defined as shown in Eq. (8). Optimizing the objective function, the best solution will be obtained for the damage system of equations. As the system of equations is inconsistent, the objective function has to be stabilized by adding penalty function as follows

$$\mathrm{f}(\mathbf{X}) = \left\|\mathrm{r}(\mathbf{X})\right\|^2 + \lambda\left\|\mathbf{L}\mathbf{X}\right\|^2 = \left\|\mathbf{S}\mathbf{X} - \mathbf{R}_{\mathrm{d}}\right\|^2 + \lambda\left\|\mathbf{L}\mathbf{X}\right\|^2 \tag{19}$$

where **S** represents sensitivity matrix, **X** is the damage solution in each updating step, $\mathbf{R}_{\mathrm{d}}$ is the response of the damaged structure, and **L** is a diagonal matrix whose diagonal elements are equal to those of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$. λ is a factor which is selected in a way that the value of objective function finds a descending process. In order to be applied to conjugate gradient algorithm and BFGS, the first and second order gradient of objective function is calculated as follows

$$\nabla\ \mathrm{f} = 2\mathbf{S}^{\mathrm{T}}\left(\mathbf{S}\mathbf{X} - \mathbf{R}_{\mathrm{d}}\right) + 2\lambda(\mathbf{L}^{\mathrm{T}}\mathbf{L})\mathbf{X} \tag{20}$$

$$\nabla_2\, \mathrm{f} = 2\mathbf{S}^{\mathrm{T}}\mathbf{S} + 2\lambda(\mathbf{L}^{\mathrm{T}}\mathbf{L}) \tag{21}$$

To execute conjugate gradient and BFGS, the parameter α_n should be found in a line search process. To this aim, the first and second order gradient of the objective function is computed as follows

$$\alpha_{NCG} = \frac{\mathbf{s}_n{}^{\mathrm{T}}\Delta\mathbf{x}_n}{\mathbf{s}_n{}^{\mathrm{T}}\nabla_2\,\mathrm{f}\ \ \mathbf{s}_n} \tag{22}$$

$$\alpha_{BFGS} = \frac{\mathbf{r}_n{}^{\mathrm{T}}\mathbf{H}_n\ \mathbf{r}_n}{(\mathbf{H}_n\ \mathbf{r}_n)^{\mathrm{T}}\nabla_2\,\mathrm{f}\ \ (\mathbf{H}_n\ \mathbf{r}_n)} \tag{23}$$

The steps of algorithm with an effective method for structural damage detection can be described in a step-by-step procedure as follows:

1. Choice the starting point properly and the control parameters of the algorithm.
2. Select the appropriate factor λ, it is important and it differs in different cases.
3. Update the sensitivity matrix.
4. Update the damage index using the suggested M-NCG and M-BFGS.

To stabilize the updating process, Tikhonov regularization is used as the following steps:

1. Update the objective function.
2. If the value of objective function is reduced, it is accepted.
3. If the objective function value is more than the previous one, a new value is ascribed to the λ and the $\mathbf{X}$ is calculated again.
4. Use the above process and new damage index, the objective function is again calculated with updated parameters.
5. If the value of the objective function decreases, the updated damage index is feasible.
6. If else the updated damage index does not change the objective function, updating process will continue with the failure index of previous stage.

5. Numerical examples

To demonstrate the efficiency of the M-NCG and M-BFGS algorithms in solving complex discrete structures, in three space-structures damage are detected. Using the acceleration response recorded in some points, structure damage detection is done by extension of NCG and BFGS algorithms in updating the structure models. Modulus of elasticity and the weight per unit volume are respectively 210000 Mpa and 7850 Kg/m^3. The Riley damping has been used to model the structural damping. The response of structure is obtained through a linear time-history analysis. Triangular impulsive load in time step 0.005 sec is induced vertically on the structure nodes. The sensors are considered to be tri-axial, getting the accelerations in all three dimensions, having the sampling frequency of 200 Hz. The losing weight and temperature are ignored in all examples during the analysis.

Table 1 Different condition for 52-element dome

Condition	Scenario	Sensor Pattern	Noise Level %
A	2	a	0
B	2	b	0
C	4	a	3
D	4	b	3
E	3	a	1
F	1	b	1

5.1 Space dome with 52 members

A 52-member dome as shown in Fig. 3 is considered here to confirm the proposed methods. Finite element model of the structure is composed of 21 nodes with 39 active degrees of freedom.

In this example, some sensor patterns have been considered as shown in Fig. 4. Also, the label of the nodes and members are shown in Fig. 1. Various conditions and patterns for damages are given in Tables 1 and 2 respectively. In this example we use Levenberg-Marquardt results from reference (Sarvi *et al.* 2014) to be compared with M-NCG and M-BFGS methods.

Table 2 Different damage scenarios 52-element dome

Scenario	Number of element	Damage ratio %
1	27	10
	34	10
2	22	30
	44	30
3	2	30
	10	30
	30	30
4	9	40
	13	30
	40	30

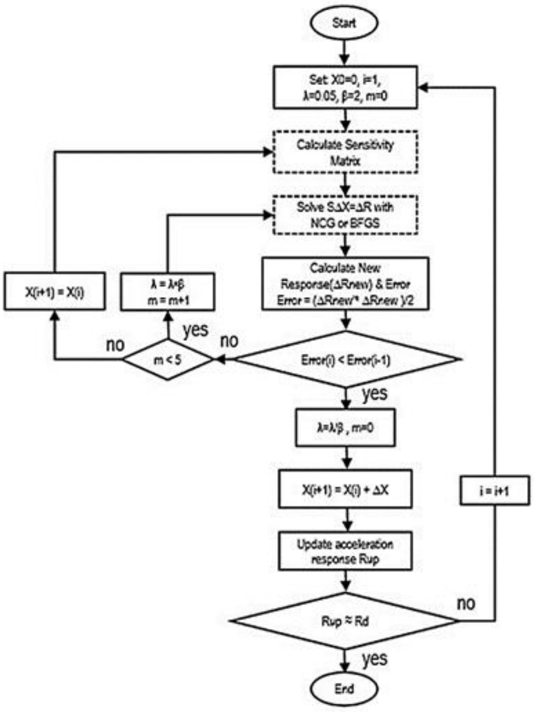

Fig. 1 The flowchart of the proposed method

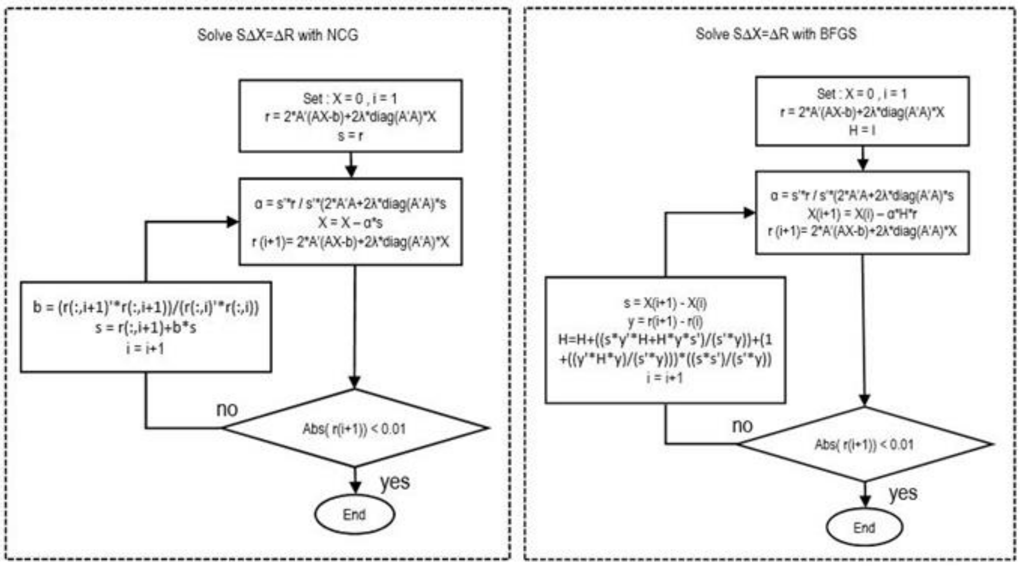

Fig. 2 The Flowchart of M-BFGS and M-NCG in the proposed method

The norm of residual, the difference between real damage and identified damage, is shown in Table 3.

Table 3 Result summary for 52-member dome

Scenario	Method	Norm of residual (actual-detected)	Max iteration	Time (s)
A	M-NCG	0.0002	82	160
	M-BFGS	0.0002	41	156
	Lev-Mar	0.0000	-	153
B	M-NCG	0.0004	73	157
	M-BFGS	0.0004	41	150
	Lev-Mar	0.0000	-	153
C	M-NCG	0.0628	101	201
	M-BFGS	0.0625	42	166
	Lev-Mar	0.0581	-	155
D	M-NCG	0.0714	89	177
	M-BFGS	0.0625	43	159
	Lev-Mar	0.0717	-	154
E	M-NCG	0.0203	105	182
	M-BFGS	0.0207	44	162
	Lev-Mar	0.0202	-	156
F	M-NCG	0.0187	66	166
	M-BFGS	0.0191	37	160
	Lev-Mar	0.0204	-	156

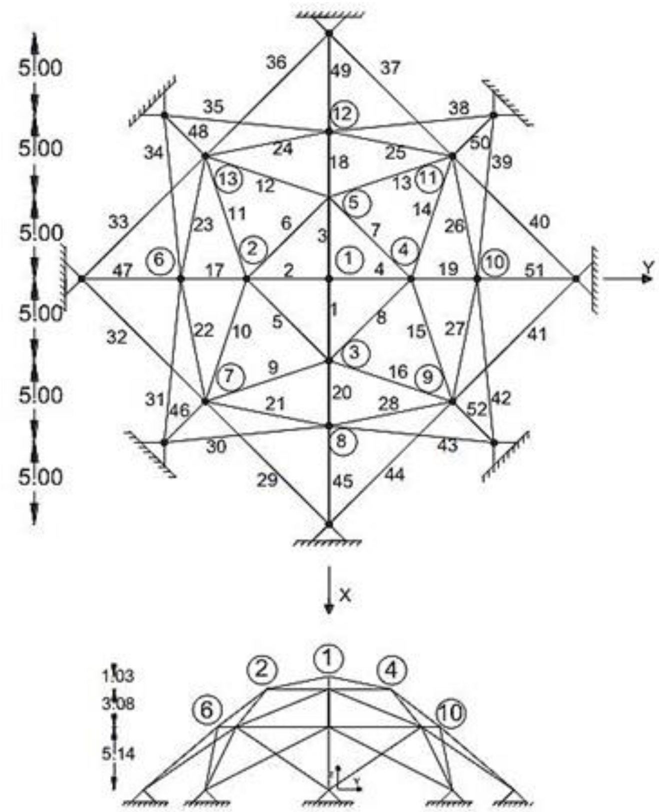

Fig. 3 52-element dome structure

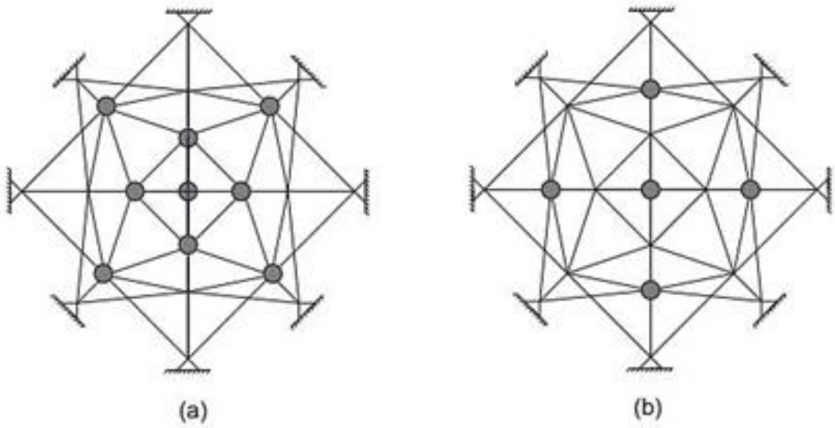

Fig. 4 Two sensor patterns in 52-element dome structure

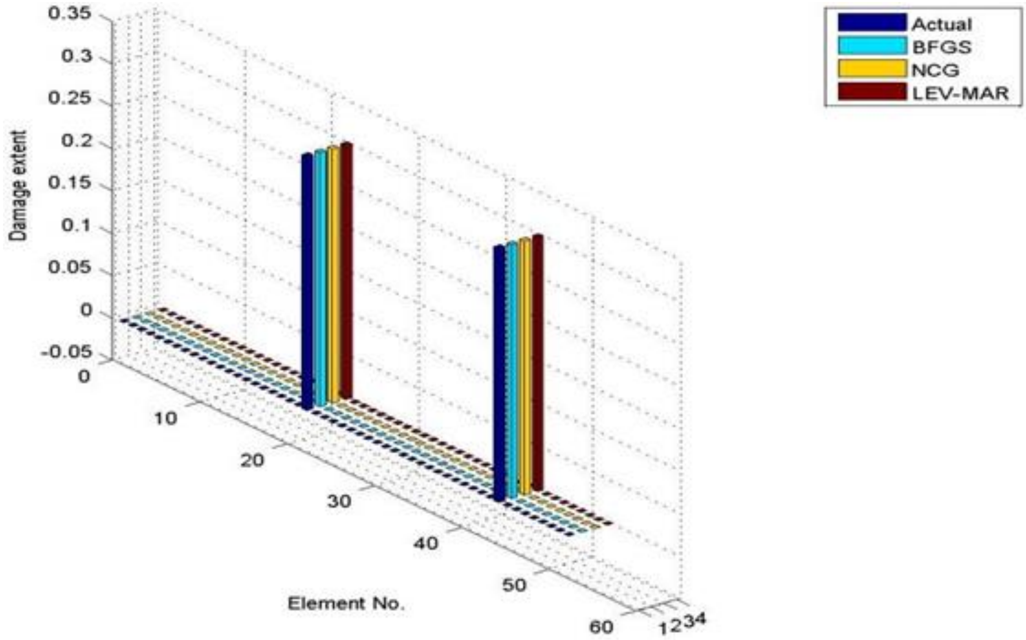

Fig. 5 Damage detection result for 52-element dome structure for Conditions A and B

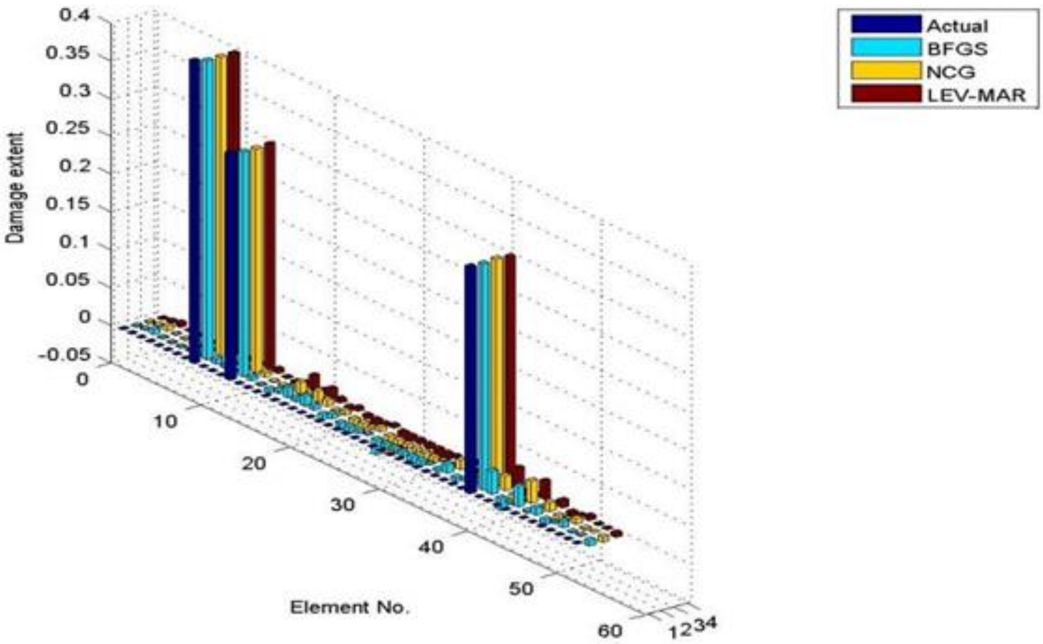

Fig. 6 Damage detection result for 52-element dome structure for Conditions C and D

Figs. 5 and 6 show equal accuracy for the three methods of damage detection using M-BFGS, M-NCG and LEV-MAR algorithms. According to these figures, in the absence of noise in data, accuracy of three methods is same, but in the presence of noise M-BFGS method works a little better.

5.2 Space dome with 120 members

A 120-member dome as shown in Fig. 9 is considered here to confirm the proposed method. The section area of members of the 120-member dome is optimized under static loading (Kaveh and Talatahari 2009). Physical model of the structure consists of 49 nodes with 117 active degrees of freedom. The label of the joints and members are shown in Fig. 9. Various conditions and damage pattern are given in Tables 4 and 5 respectively. In this example, the results from reference (Sarvi *et al*. 2014) is used to be compared with M-NCG and M-BFGS methods.

Table 4 Different condition for 120-element dome

Condition	Scenario	Sensor Pattern	Noise Level %
A	2	a	0
B	2	b	0
C	4	a	0
D	4	b	0
E	3	b	5
F	1	a	2

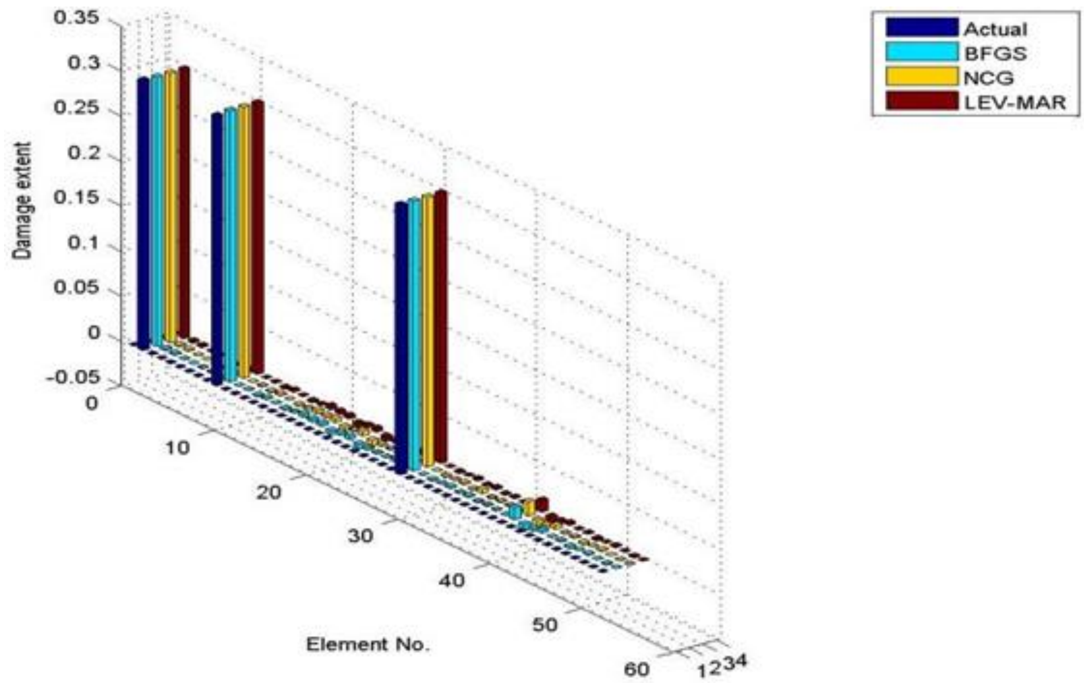

Fig. 7 Damage detection result for 52-element dome structure for Condition E

Table 5 Different damage scenarios in 120-element dome

Scenario	Number of element	Damage ratio %
1	18	10
	51	40
	52	30
	111	25
	118	5
2	16	30
	27	12
	41	32
	49	5
	62	40
	96	25
	117	10
3	1	25
	34	30
	42	40
	56	5
	58	10
	65	35
	67	25
	77	17
	97	9
4	6	25
	22	25
	32	40
	34	5
	35	40
	40	30
	91	17
	92	12
	99	10
	112	11
	119	9

Table 6 Results summary for 120-member dome

Scenario	Method	Norm of residual (actual-detected)	Max iteration	Time (s)
A	M-NCG	0.0005	312	721
	M-BFGS	0.0004	95	552
	Lev-Mar	0.0004	-	448
B	M-NCG	0.0000	175	754
	M-BFGS	0.0000	74	514
	Lev-Mar	0.0000	-	623
C	M-NCG	0.0017	294	656
	M-BFGS	0.0017	91	539
	Lev-Mar	0.0017	-	453
D	M-NCG	0.0003	180	615
	M-BFGS	0.0003	76	539
	Lev-Mar	0.0003	-	446
E	M-NCG	0.1629	183	698
	M-BFGS	0.1625	72	564
	Lev-Mar	0.1621	-	709
F	M-NCG	0.0885	224	702
	M-BFGS	0.0915	77	551
	Lev-Mar	0.0906	-	469

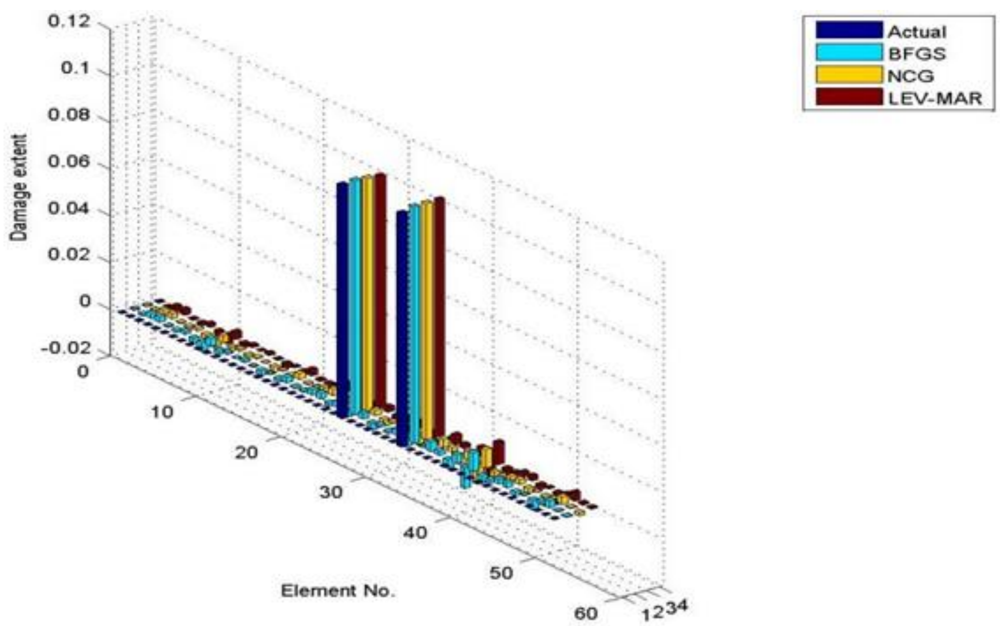

Fig. 8 Damage detection result for 52-element dome structure for Condition F

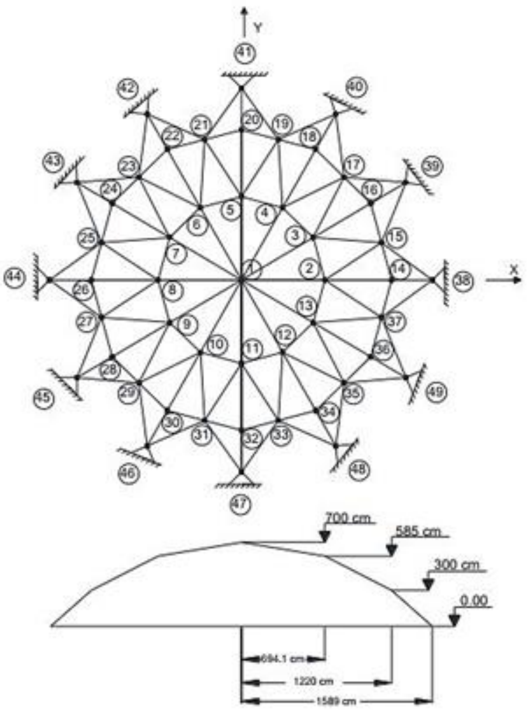

Fig. 9 120-element dome structure

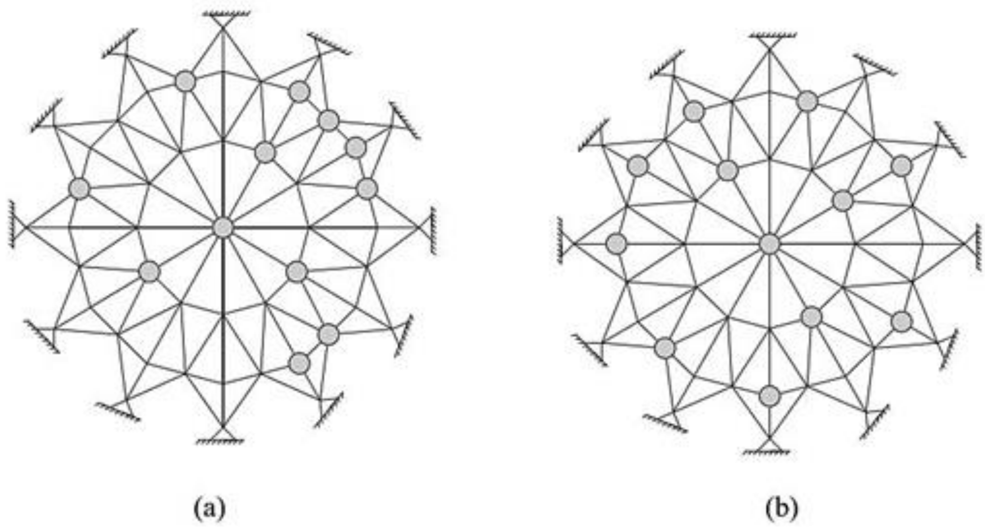

Fig. 10 Two sensor patterns in 120-element dome structure

Fig. 11 shows the results of damage detection of Condition A. The accuracy of proposed method is demonstrated in noise free data. In this condition, sensors location is asymmetric which shows this method is insensitive to sensors location.

Fig. 12, shows the results of damage detection of condition B in which sensors location is almost symmetric and data is free of noise. According to this figure and Table 6, it is noticeable when sensors condition is symmetric, result is accurate and norm of difference between actual values and calculate values is tend to zero.

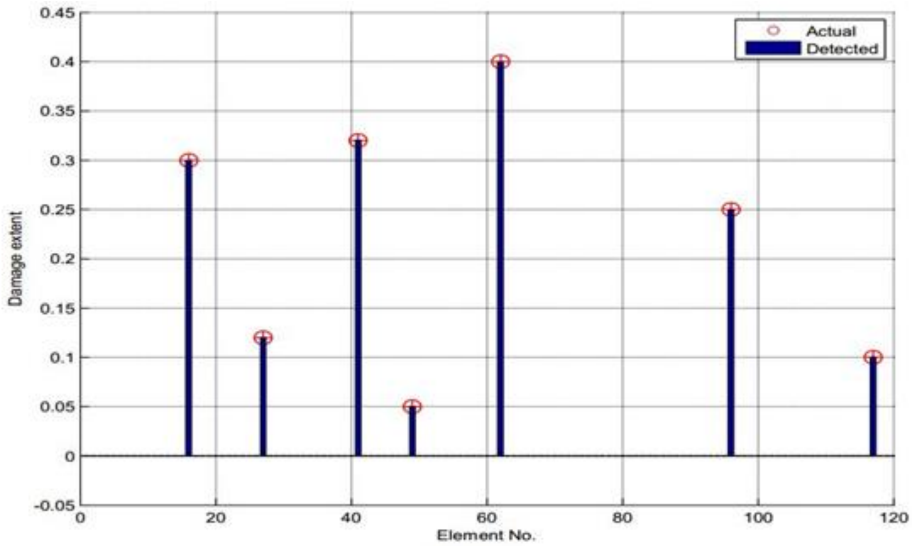

Fig. 11 Damage detection result for 120-element dome structure for Condition A using M–BFGS and M–NCG methods

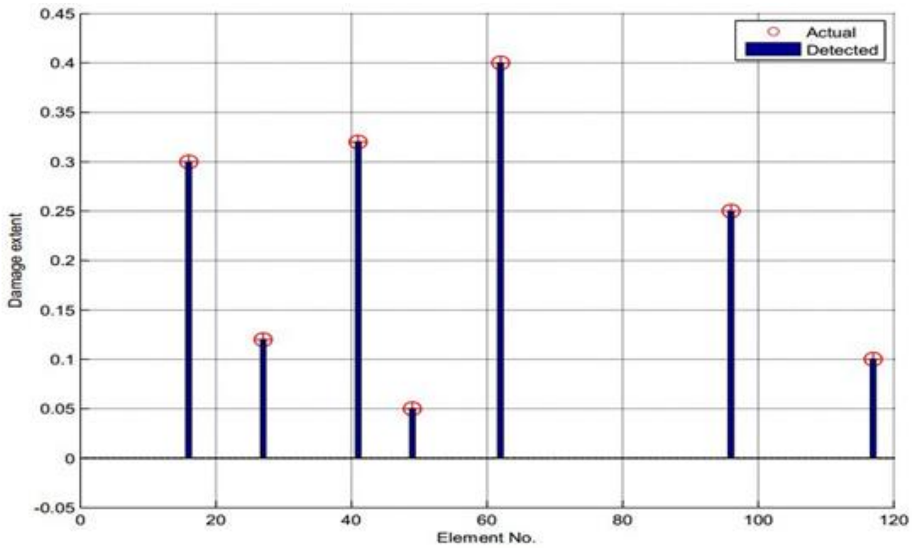

Fig. 12 Damage detection result for 120-element dome structure for Condition B employing M–BFGS and M–NCG methods

Results of damage detection for Condition C is shown in Fig. 13. In this condition sensors location is asymmetric and data is free of noise. This result show the proposed method is capable of assessing adjacent damages without error. Also insensitivity to sensors location is observed.

Results of damage detection in Condition D show accuracy of method to detect adjacent damages as before. It is demonstrated the accuracy of method is increased in the case of symmetric sensors location. In this condition sensors location is almost symmetric and data is free of noise (see Fig. 14).

Fig. 15 shows the results of damage detection in condition E. In this case, the location of sensors is almost symmetrical and 5% noise is applied to acceleration data. These show proposed method is capable to assessment location and magnitude of damage in large structure with noisy data.

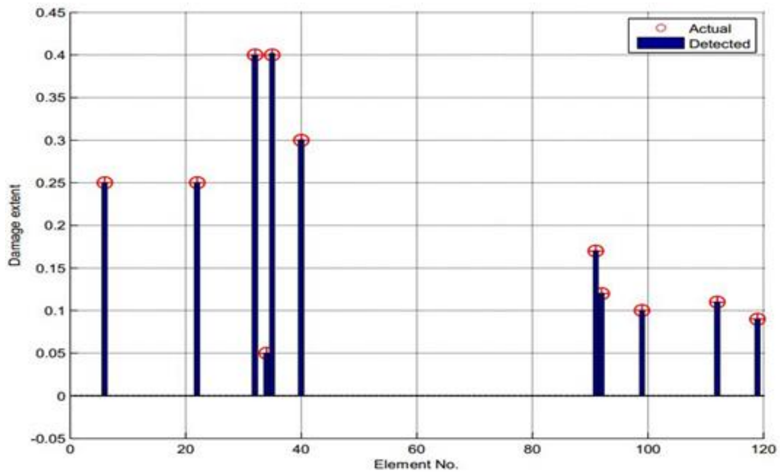

Fig. 13 Damage detection result for 120-element dome structure for Condition C using M–BFGS and M–NCG methods

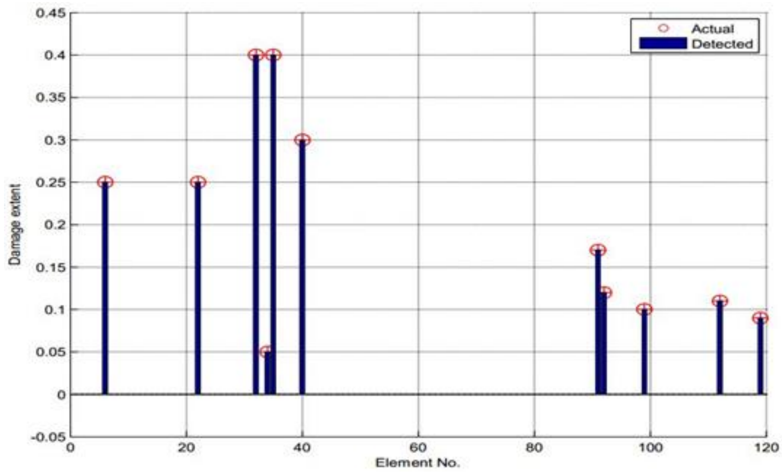

Fig. 14 Damage detection result for 120-element dome structure for Condition D, M–BFGS and M–NCG methods

According above figure which shows result of condition F, efficiency of method in noisy data is demonstrated and the ability of assessment adjacent damage is shown.

The comparison between convergence speed of Conditions A and B is shown in above figure. The good convergence speed of this method is proved.

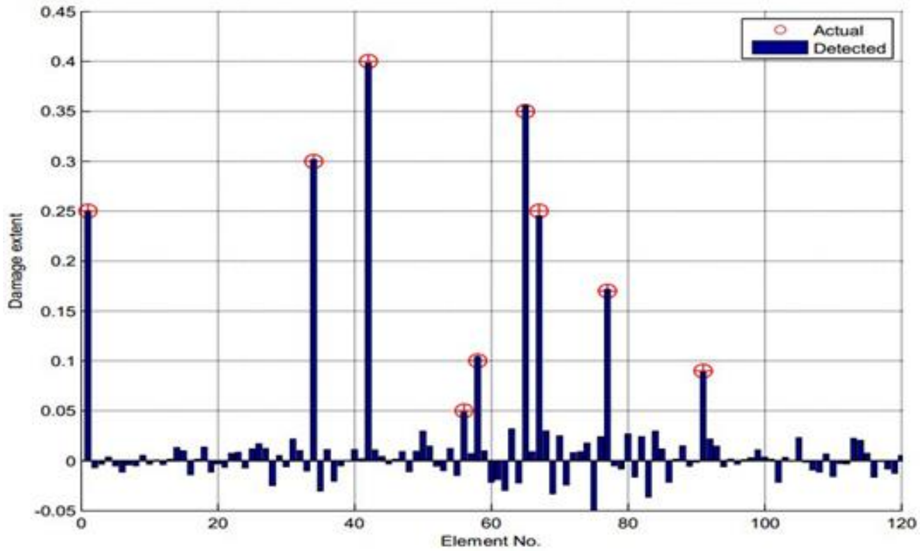

Fig. 15 Damage detection result for 120-element dome structure for Condition E, M–BFGS and M–NCG methods

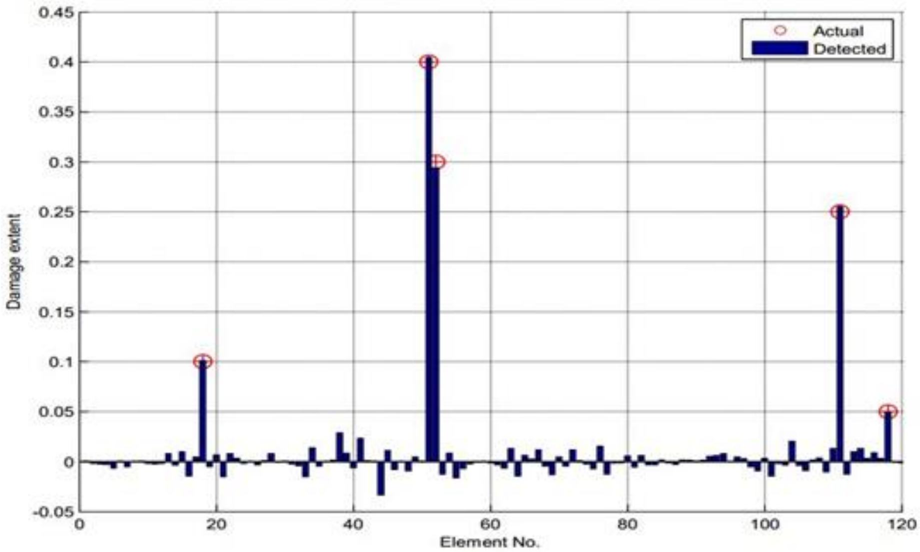

Fig. 16 Damage detection result for 120-element dome structure for Condition F, M–BFGS and M–NCG methods

Table 7 The result summary for 120-member dome in different conditions

Case study name	Number of nodes	Number of element	Noise %	Number of sensor	$\frac{\text{Sensor}}{\text{node}}$ %	Norm of residual (actual-detected)
120 member dome	49	120	0	4	8 %	0.0001
120 member dome	49	120	2	9	18 %	0.0911
120 member dome	49	120	5	11	22 %	0.1821

The comparison between convergence speed of Conditions C and D is shown in above figure. If sensors location is almost symmetrical, convergence speed is increased increased compared with asymmetric sensors location.

In Table 7, the minimum number of sensors required for acceptable identification process is given for 120 member structure. Here are three noise level is tested and for each level of the noise, the minimum number of sensors and the ratio of the number of sensor on number of nodes is calculated. As is noticeable, with an increase in the level of noise in the data, the number of sensors required for damage detection operations increases. It should be noted that the sensor position is almost symmetrical.

5.3 Double layer grid space structure with 800 members

For more investigation, the proposed algorithm is implanted on a full-member structure for damage detection. Physical model of the structure is composed of 221 nodes and 555 active degrees of freedom.

The label of points and members are shown in Figure 19. Patterns and condition of damage are respectively given in Tables 8 and 9. In this example, sensor pattern are randomly considered as shown in Fig. 20.

Table 8 Condition for 800-Element Double Layer Grid

Condition	Scenario	Sensor Pattern	Noise Level %
A	1	a	1

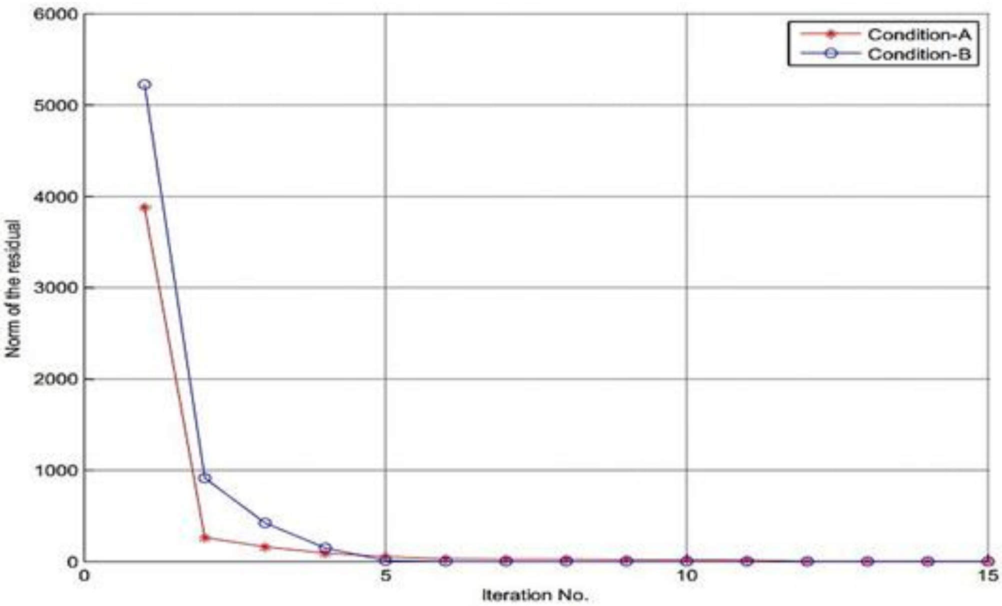

Fig. 17 Convergence diagram for 120-element dome structure for Condition A&B, M-BFGS and M–NCG

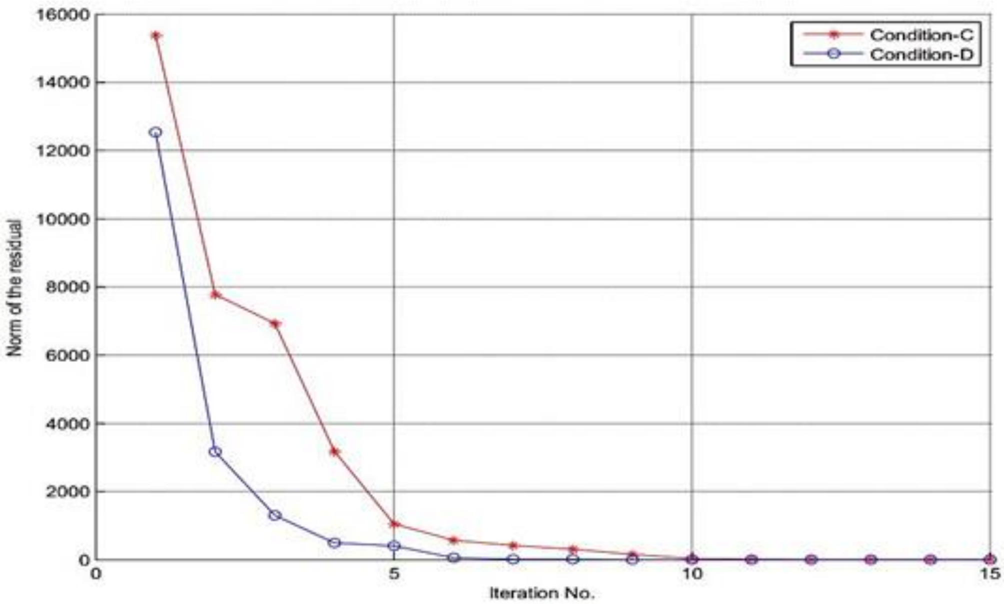

Fig. 18 Convergence diagram for 120-element dome structure for Condition C&D, M-BFGS and M–NCG

Table 9 Different damage ratio in 800-element double layer grid

Number of element	Damage ratio %
12	15
35	25
50	8
94	24
124	17
202	23
241	15
245	31
264	30
293	17
306	25
377	37
422	11
455	40
465	17
483	27
570	33
682	15
687	8
750	22

Table 10 Results summary for 800-element double layer grid

Scenario	Method	Norm of residual (actual-detected)	Max iteration	Time (s)
A	M-NCG	0.0816	203	N/C
	M-BFGS	0.0631	145	N/C

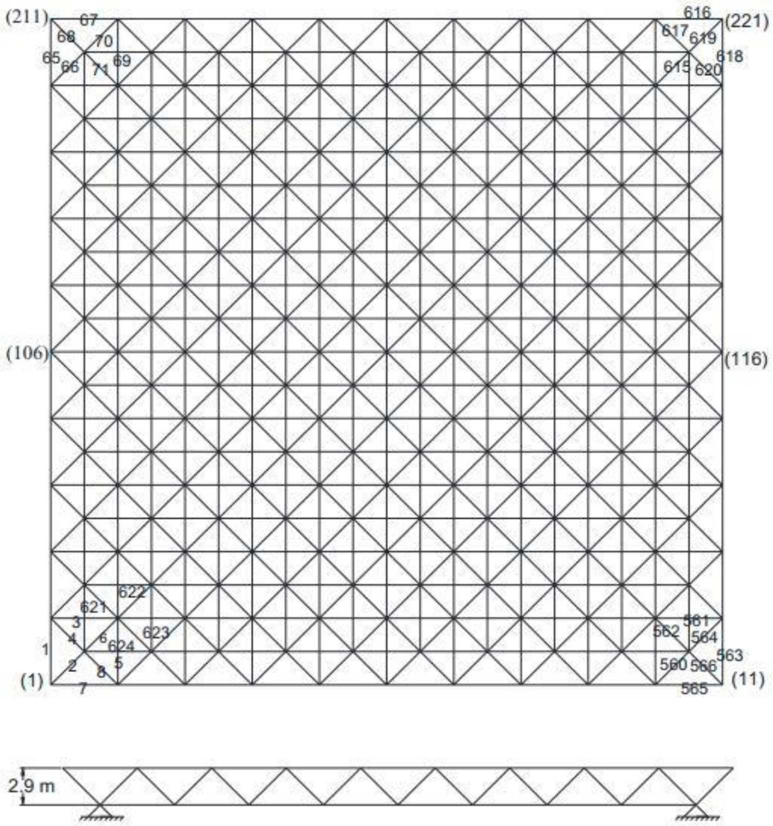

Fig. 19 800-element double layer grid

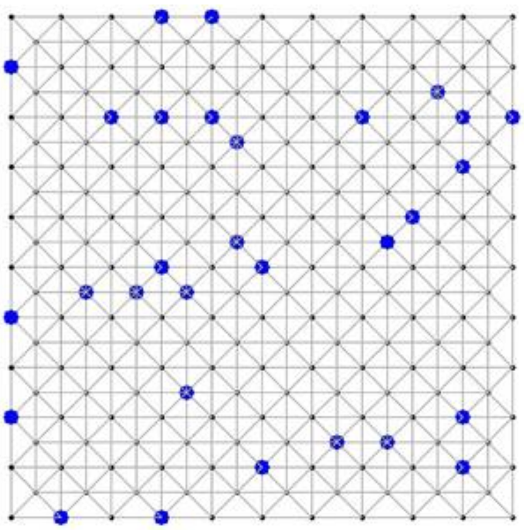

Fig. 20 Sensor pattern in 800-element double layer grid

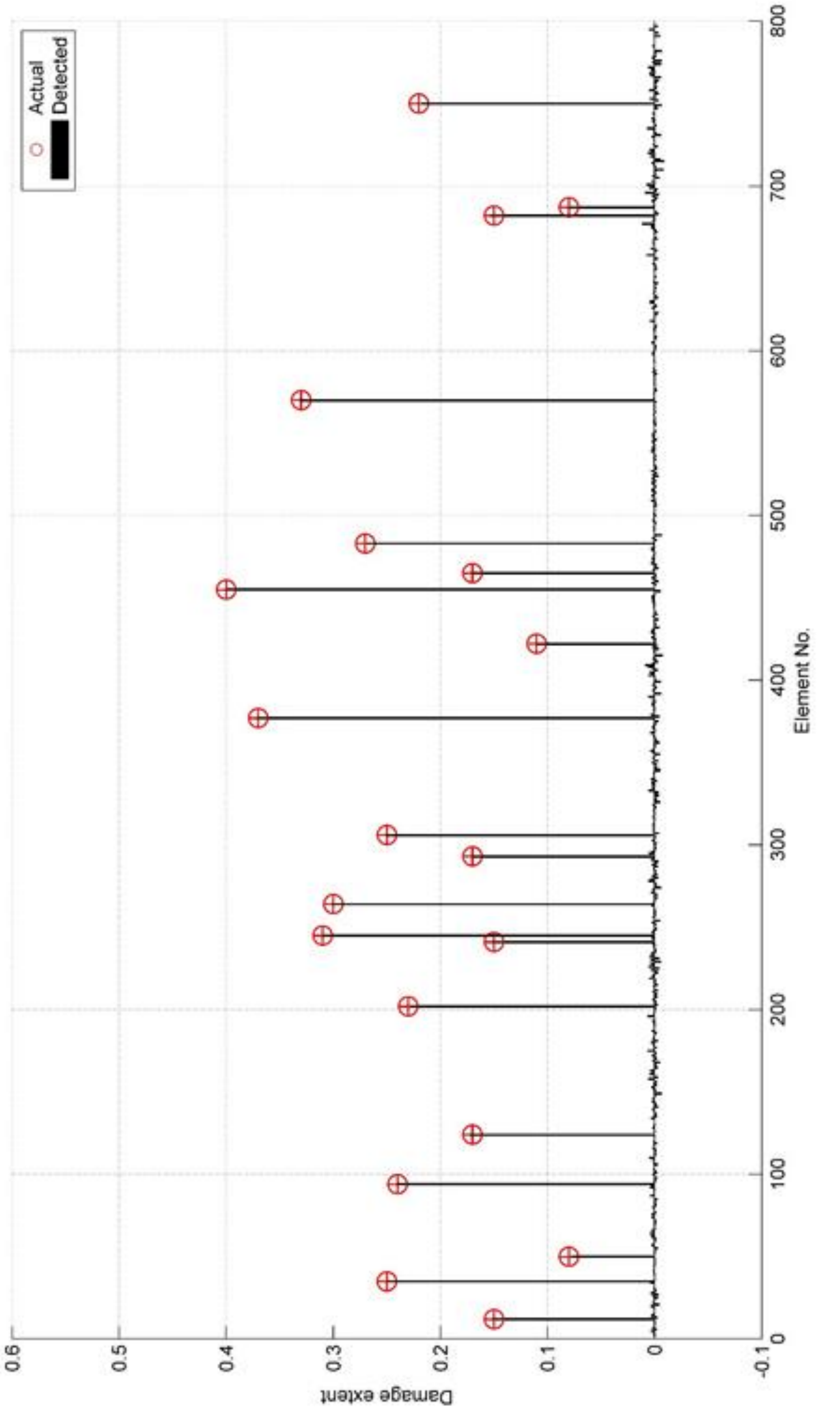

Fig. 21 Damage detection result for 800-element double layer grid for Condition A using M-BFGS and M-NCG methods

7. Conclusions

This work proposes damage identification of large structures using the M-NCG and M-BFGS algorithms based on sensitivity analysis. Due to the high cost of data-receiving systems (including installation of sensors and instruments for keeping them), it is demonstrated that the proposed method can effectively detect damage with the assumption of putting limited number of sensors in structure and therefore it has more advantages in this respect. Number of sensors are dependent on level of noise.

Since sensor placement is carried out randomly, the results show that the proposed method with different sensor placement patterns is able to detect the damage, but it should be noted the location of sensors is significant in terms of precision and convergence speed. From different patterns of sensors placement it is concluded that the quantity of damage with symmetrical sensors in structures is detected with more accuracy and speed. The results show that the sensitivity of the method is higher compared to the little acceleration change and even in big structures the gradual loading in one of the points is sufficient and is able to detect the damage with high precision and speed.

To investigate the performance of M-NCG and M-BFGS methods some examples are solved to detect damage in full-member discrete structures.

Also the results from damage detection of 800-element double layer grid show that the proposed method has high precision and efficiency for the damage detection in large scale structures and it can be used in practical issues.

References

Bakir, P.G., Reynders, E. and De Roeck, G. (2007), “Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm”, *J. Sound Vib*., **305**(1-2), 211-225.

Basseville, M. Mevel, L. and Goursat, M. (2004), “Statistical model-based damage detection and localization: subspace-based residuals and damage-to-noise sensitivity ratios”, *J. Sound Vib.*, **275**(3), 769-794.

Chou, J.H. and Ghaboussi, J. (2001), “Genetic algorithm in structural damage detection”, *Comput. Struct*., **79**(14), 1335-1353.

Esfandiari, A., Bakhtiari-Nejad, F., Rahai, A. and Sanayei, M. (2009), “Structural model updating using frequency response function and quasi-linear sensitivity equation”, *J. Sound Vib*., **326**(3), 557-573.

Fan, W. and Qiao, P. (2011), "Vibration-based damage identification methods: A review and comparative study”, *Struct. Health Monit*., **10**, 83-111.

Jaishi, B. and Ren, W.X. (2006), “Damage detection by finite element model updating using modal flexibility residual”, *J. Sound Vib*., **290**(1-2), 369-387.

Kaveh, A. and Maniat, M. (2014), “Damage detection in skeletal structures based on charged system search optimization using incomplete modal data”, *Int. J. Civil Eng*., **12**(2), 291-298.

Kaveh, A. and Talatahari, S. (2009), “Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures”, *Comput. Struct*., **87**(5), 267-283.

Lee, J. (2009), “Identification of multiple cracks in a beam using natural frequencies”, *J. Sound Vib*., **320**(3), 482-490.

Li, X. and Law, S. (2010), “Adaptive Tikhonov regularization for damage detection based on nonlinear model updating”, *Mech. Syst. Signal Pr*., **24**(6), 1646-1664.

Luenberger, D. and Ye, Y. (2007), “Linear and Nonlinear Programming”, International Series in Operations Research & Management Science.

Naseralavi, S.S., Salajegheh, E., Salajegheh, J. and Fadaee M.J. (2012), “Detection of damage in cyclic

structures using an eigenpair sensitivity matrix”, *Comput. Struct*., **110-111**, 43-59.
Naseralavi, S.S., Salajegheh, E., Salajegheh. J. and Ziaee, M. (2012), “A two-stage method for damage detection of large-scale structures”, *Int. J. Optim. Civil Eng*., **2**(4), 489-510.
Pandey, A.K., Biswas, M. and Samman, M.M. (1991), “Damage detection from changes in curvature mode shapes”, *J. Sound Vib*., **145**(2), 321 -332.
Rytter, A. (1993), “Vibrational based inspection of civil engineering structures”, Ph.D. Dissertation Department of Building Technology and Structural Engineering.
Salawu, O.S. (1997), “Detection of structural damage through changes in frequency: a review”, *Eng. Struct*., **19**(9), 718-723.
Sarvi, F., Shojaee, S. and Torkzadeh, P. (2014), “Damage identification of trusses by finite element model updating using an enhanced levenberg –marquardt algorithm”, *Int. J. Optim. Civil Eng*., **4**(2), 207-231.
Teughels, A. and De Roeck, G. (2005), “Damage detection and parameter identification by finite element model updating”, *Revue Européenne de Génie Civil*, **9**(1-2), 109-158.
Teughels, A., Maeck, J. and De Roeck, G. (2002), “Damage assessment by FE model updating using damage functions”, *Comput. Struct*., **80**(25), 1869-1879.
Torkzadeh, P., Goodarzi, Y. and Salajegheh, E. (2013), “A two-stage damage detection method for large-scale structures by kinetic and modal strain energies using heuristic particle swarm optimization”, *Int. J. Optim. Civil Eng*., **3**(3), 465-482.
Wang, Y., Liu, J., Shi, F. and Xiao, J. (2011), “A method to identify damage of roof truss under static”, *Artif. Intell. Comput. Intell.,* 9-15.
Yan, Y.J., Cheng, L., Wu, Z.Y. and Yam, L.H. (2007), “Development in vibration-based structural damage detection technique”, *Mech. Syst. Signal Pr.,* **21**(5), 2198-2211.

CC