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Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory

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Abstract. The hygro-thermo-mechanical bending behavior of sigmoid functionally graded material (S-FGM) plate resting on variable two-parameter elastic foundations is discussed using a four-variable refined plate theory. The material characteristics are distributed within the thickness direction according to the two power law variation in terms of volume fractions of the constituents of the material. By employing a four variable refined plate model, both a trigonometric distribution of the transverse shear strains within the thickness and the zero traction boundary conditions on the top and bottom surfaces of the plate are respected without utilizing shear correction factors. The number of independent variables of the current formulation is four, as against five in other shear deformation models. The governing equations are deduced based on the four-variable refined plate theory incorporating the external load and hygro-thermal influences. The results of this work are compared with those of other shear deformation models. Various numerical examples introducing the influence of power-law index, plate aspect ratio, temperature difference, elastic foundation parameters, and side-to-thickness ratio on the static behavior of S-FGM plates are investigated.

Keywords: refined plate theory; moisture concentration; thermal effect; functionally graded plate; variable elastic foundation

1. Introduction

Recently, a novel type of plates made up of functionally graded materials (FGM), in which the material characteristics continuously change across the thickness, has become popular in various engineering applications such as building constructions, automotive, aerospace, nuclear, ship and underwater (Hadji *et al*. 2014, Arefi 2015, Bennai *et al*. 2015, Al-Basyouni *et al*. 2015, Ebrahimi and Dashti 2015, Darılmaz 2015, Pradhan and Chakraverty 2015, Tagrara *et al*. 2015, Sallai *et al*. 2015, Akbaş 2015, Hadji *et al*. 2015a,b, Hadji *et al*. 2016, Ait Atmane *et al*. 2015, 2016, Bellifa *et al*. 2016, Abdelbari *et al*. 2016, Ebrahimi and Habibi 2016, Moradi-Dastjerdi 2016). The concept

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of FGM was introduced in 1984 by the material scientists in the Sendai area of Japan (Koizumi 1997), although it had been suggested much earlier by American scientists as a type of novel spatially-graded composite materials aimed at providing a structural tailoring tool to aerospace designers (Bever and Duwez 1972). FGMs are considered as new composite materials which are now widely employed in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical, and shipbuilding industries. Therefore, proposing theoretical and numerical investigation of FGM structures (such as plates or beams) has attracted considerable attention from researchers (Mantari and Guedes Soares 2012, Bourada *et al*. 2012, Ould Larbi *et al*. 2013, Neves *et al*. 2013, Jung and Han 2014, Belabed *et al*. 2014, Jung *et al*. 2014, Ait Yahia *et al*. 2014, Bourada *et al*. 2015, Mahi *et al*. 2015, Hebali *et al*. 2014, Bousahla *et al*. 2014, Larbi Chaht *et al*. 2015, Nguyen *et al*. 2015, Kar and Panda 2015a, Meradjah *et al*. 2015, Belkorissat *et al*. 2015, Bounouara *et al*. 2016, Bennoun *et al*. 2016).

Many studies on FGM structures have been investigated in literature. For example, Reddy (2000) discussed the bending response of FG rectangular plates using a third-order shear deformation plate theory. The deflection of a FGM plate was examined by Ferreira *et al*. (2005) using a third-order shear deformation theory. Malekzadeh (2009) studied the free vibration behavior of thick FG plates resting on two-parameter elastic foundation based on the three-dimensional elasticity theory. Matsunaga (2008) studied natural frequencies and buckling stresses of plates made of FGM by considering the influences of transverse shear and normal deformations and rotatory inertia. Lu *et al*. (2009) examined the free vibration behavior of FG thick plates on elastic foundation based on three-dimensional elasticity. Akavci (2014) investigated the efficiency of an improved version of a hyperbolic shear deformation theory proposed by Akavci (2007) for free vibration analysis of FG plates. Vibration and buckling response of exponentially graded sandwich plate resting on elastic foundations under various boundary conditions have investigated by Ait Amar Meziane *et al.* (2014) using an efficient and simple refined theory. Laoufi *et al*. (2016) studied the mechanical and hygrothermal response of FG plates resting on elastic foundation using hyperbolic shear deformation theory.

In general, the structures are subjected to mechanical load and temperature changes both internally and externally. Then, it is important to investigate the response of structural elements under a mechanical or thermal loads or a combination of both. In this sense, many researchers analyzed the thermo-mechanical response of FG plates by employing higher order shear deformation theories (HSDTs). Zhang *et al*. (1994) proposed an analytical method for FGM cylinder with axial symmetry based on thermal elasticity theory. Reddy and Chen (2001) proposed a 3D model for a FG plate under mechanical and thermal loads. Liew *et al*. (2003) analyzed the thermo-mechanical response of hollow circular cylinders made of FGM. Vel and Batra (2003) proposed a 3D solution for transient thermal stresses in FG rectangular plates. The stability investigation of shear deformable FG rectangular plates under thermo-mechanical loads was studied by Shukla *et al.* (2007) using the first-order shear deformation plate theory (FSDT). Shahrjerdi *et al.* (2011) presented a free vibration analysis of solar FG plates with temperature-dependent material properties using second order shear deformation theory. The thermal stability analysis of rectangular composite laminated plates is discussed by Moradi and Mansouri (2012) using the Differential Quadrature method. Using a novel shear deformation theory, Saidi *et al.* (2013) analyzed the thermo-mechanical bending response with stretching effect of FG sandwich plates. Houari *et al*. (2013) presented a thermoelastic bending analysis of FG sandwich plates using a new higher order shear and normal deformation theory. Zidi *et al*. (2014) studied the bending response of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. Khalfi *et al*. (2014) developed a refined and simple shear deformation theory for thermal buckling of solar FG plates on elastic foundation. Thermal buckling of the FG orthotropic plates has been examined by Mansouri and Shariyat (2014). Chakraverty and Pradhan (2014) studied the free vibration of exponential functionally graded rectangular plates in thermal environment with general boundary conditions. Kar and Panda (2015b) analyzed the free vibration responses of temperature dependent FG curved panels under thermal environment. Rad (2015) investigated the thermo-elastic analysis of FG circular plates resting on a gradient hybrid foundation. Hamidi *et al.* (2015) proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of FG sandwich plates. Tebboune *et al.* (2015) studied the thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Kar *et al*. (2015) presented a nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading. Mansouri and Shariyat (2015) investigated for the first time the biaxial buckling behavior of FG orthotropic plates under hygrothermal effects. Using various four variable refined plate theories, Attia *et al*. (2015) analyzed the free vibration response of FG plates with temperature-dependent properties. Bakora and Tounsi (2015) studied the thermo-mechanical post-buckling behavior of thick FG plates resting on elastic foundations. Bouchafa *et al.* (2015) discussed the thermal stresses and deflections of FG sandwich plates using a new refined hyperbolic shear deformation theory. Bouguenina *et al*. (2015) proposed a numerical analysis of FG plates with variable thickness subjected to thermal buckling. Mehar *et al*. (2016) studied the vibration response of FG carbon nanotube reinforced composite plate in thermal environment. Kar and Panda (2016) analyzed the nonlinear thermo-mechanical deformation behavior of P-FGM shallow spherical shell panel. Sobhy (2016) proposed an accurate shear deformation theory for vibration and buckling of FGM sandwich plates in hygrothermal environment. Bouderba *et al*. (2016) studied the thermal buckling of FG sandwich plates using a simple shear deformation theory.

Power-law function (Bao and Wang 1995, Jin and Paulino 2001), and exponential function (Delale and Erdogan, 1983; Erdogan and Chen, 1998) are commonly employed to describe the variations of material characteristics of FGM. However, in both power-law and exponential functions, the stress concentrations appear in one of the interfaces in which the material is continuously but rapidly changing. Therefore, Chung and Chi (2001) proposed a sigmoid FGM (S-FGM), which was composed of two power-law functions to define a novel volume fraction. Chi and Chung (2002) demonstrated that the use of an S-FGM can significantly diminish the stress intensity factors of a cracked body. Han *et al.* (2008) investigated vibration and stability responses of S-FGM plates and shells by employing finite element method. The non-linear analysis of anisotropic S-FGM structures was discussed by Han *et al*. (2009). Duc and Cong (2013) investigated the nonlinear post-buckling of symmetric S-FGM plates resting on elastic foundations using higher order shear deformation plate theory in thermal environments. Jung and Han (2013) studied an S-FGM nanoscale plates using the nonlocal elasticity theory. Jung *et al.* (2014) investigated the bending and vibration behavior of S-FGM microplates embedded in Pasternak elastic medium using the modified couple stress theory. Han *et al.* (2015) proposed a four-variable refined plate theory for dynamic stability analysis of S-FGM plates based on physical neutral surface. Quan *et al*. (2015) discussed the nonlinear dynamic and vibration of shear deformable eccentrically stiffened S-FGM cylindrical panels with metal–ceramic–metal layers resting on elastic foundations. Lee *et al.* (2015) used a refined higher order shear and normal deformation theory for E-, P-, and S-FGM plates on Pasternak elastic foundation. Chikh *et al*. (2016) presented an analytical formulation based on both hyperbolic shear deformation theory and stress function, to study the nonlinear post-buckling response of symmetric S-FG plates supported by elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. Thanga *et al.* (2016) presented an analytical formulation for nonlinear analysis of imperfect S-FGM plates with variable thickness resting on elastic medium.

It can be noticed in the above studies, that the hygro-thermo-mechanical bending behavior of FGM plates is studied in different works (Hamidi *et al*. 2015, Zidi *et al*. 2014, Tounsi *et al*. 2013, Bouderba *et al*. 2013, Houari *et al*. 2013), However, the hygro-thermo-mechanical bending behavior of S-FGM plates resting on variable elastic foundation is not treated. Thus, the aim of this work is to investigate the hygro-thermo-mechanical bending response of S-FGM plates resting on variable two-parameter elastic foundations using a four-variable trigonometric plate theory. The material properties are graded in the thickness direction according to the two power-law distribution in terms of volume fractions of the constituents of the material. The effective material properties are estimated by employing a simple power law based on rule of mixture. The equilibrium equations are obtained using the four-variable trigonometric plate theory containing the thermal effect and the interaction between the plate and the elastic foundations. The results obtained by the present four-variable trigonometric plate theory are compared with those obtained by the first-order shear deformation theory (FSDT) and the higher-order one. Some numerical examples are presented to demonstrate the influences of various parameters on the hygro-thermo-mechanical bending behavior of the S-FGM plates.

2. Fundamental formulations

In the present work, a functionally graded rectangular plate with an uniform thickness h, the length a, and the width b is examined. The geometry of the plate and coordinate system are indicated in Fig. 1. The volume fraction employing two power-law functions which ensure smooth variations of stresses is expressed by

$$\{V_1(z),\ V_2(z)\} = \left\{1 - \frac{1}{2}\left(\frac{h/2 - z}{h/2}\right)^k,\ \frac{1}{2}\left(\frac{h/2 + z}{h/2}\right)^k\right\} \tag{1}$$

By employing the rule of mixture, the material properties P like as Young's modulus E, Poisson's ratio ν, and thermal α and moisture expansion β coefficients of the S-FGM can be computed by

$$P(z) = P_1 V_1(z) + P_2\left(1 - V_1(z)\right) \quad \text{for} \quad 0 \le z \le h/2 \tag{2a}$$

$$P(z) = P_1 V_2(z) + P_2\left(1 - V_2(z)\right) \quad \text{for} \quad -h/2 \le z \le 0 \tag{2b}$$

where P_1 and P_2 denotes the property of the top and the bottom faces of the plate, respectively, and k is the volume fraction exponent. The Poisson's ratio ν is considered to be constant.
Based on the refined trigonometric four-variable plate theory (Tounsi *et al*. 2013), the displacement field can be expressed as

$$u(x,y,z) = u_0(x,y) \quad z\frac{\partial w_b}{\partial x} \quad f(z)\frac{\partial w_s}{\partial x} \tag{3a}$$

$$v(x,y,z)=v_0(x,y) \quad z\frac{\partial w_b}{\partial y} \quad f(z)\frac{\partial w_s}{\partial y} \tag{3b}$$

$$w(x,y,z)=w_b(x,y)+w_s(x,y) \tag{3c}$$

where u_0 and v_0 are the mid-plane displacements of the plate in the x and y direction, respectively; w_b and w_s are the bending and shear components of transverse displacement, respectively.

Also the shape function $f(z)$ is chosen according to Bouderba *et al*. (2013), Fekrar *et al*. (2014) and Draiche *et al*. (2014) as

$$f(z)=z-\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{4}$$

By employing the displacement field in Eq. (3) within the application of the linear, small-strain elasticity theory, normal and shear strains are determined as

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0+z\,k_x^b+f(z)\,k_x^s\\ \varepsilon_y^0+z\,k_y^b+f(z)\,k_y^s\\ \gamma_{xy}^0+z\,k_{xy}^b+f(z)\,k_{xy}^s\\ g(z)\gamma_{yz}^s\\ g(z)\gamma_{xz}^s\end{Bmatrix} \tag{5}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial w_s}{\partial y}\\ \dfrac{\partial w_s}{\partial x}\end{Bmatrix} \tag{6a}$$

and

$$g(z)=1-\frac{df(z)}{dz} \tag{6b}$$

For the FG plates, the stress–strain relationships for plane-stress state can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x-\alpha(z)\,\Delta T-\beta(z)\,\Delta C\\ \varepsilon_y-\alpha(z)\,\Delta T-\beta(z)\,\Delta C\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{7}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The elastic constants C_{ij} are defined as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \nu C_{11}, \quad C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{8}$$

and $\Delta T = T - T_0$ and $\Delta C = C - C_0$ in which T_0 is the reference temperature and C_0 is the reference moisture concentration.

We consider the plate under a thermal field and moisture concentration varying linearly within the thickness, i.e., $T(x,y,z) = T_1(x,y) + \frac{z}{h} T_2(x,y)$ and $C(x,y,z) = C_1(x,y) + \frac{z}{h} C_2(x,y)$. However, the temperature variation and the moisture concentration may be distributed nonlinearly across the thickness of the plate as Zidi *et al.* (2014)

$$T(x,y,z) = T_1(x,y) + \frac{z}{h} T_2(x,y) + \frac{1}{h} \Psi(z) T_3(x,y) \tag{9a}$$

$$C(x,y,z) = C_1(x,y) + \frac{z}{h} C_2(x,y) + \frac{1}{h} \Psi(z) C_3(x,y) \tag{9b}$$

where $\Psi(z) = \frac{h}{\pi} \sin(\pi z / h)$.

3. Governing equations

The governing equations of equilibrium can be obtained by employing the principle of virtual displacements. The principle of virtual work in the present case yields

$$\int_{-h/2}^{h/2} \int_{\Omega} \left[\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} + \tau_{yz} \delta \gamma_{yz} \right] d\Omega \, dz - \int_{\Omega} (q - f_e) \delta w \, d\Omega = 0 \tag{10}$$

where Ω is the top surface, and f_e is the density of reaction force of foundation. For the Pasternak foundation model

$$f_e = K_w(x) w(x,y) - K_G \nabla^2 w(x,y) \tag{11}$$

where K_w is Winkler parameter depended on x only. It is considered to be linear, parabolic or sinusoidal as (Zhou 1993, Pradhan and Murmu 2009, Sobhy 2015)

$$K_w(x) = \frac{J_1 h^3}{a^4} \begin{cases} 1 + \xi \left(\frac{x}{a} \right) & \text{Linear,} \\ 1 + \xi \left(\frac{x}{a} \right)^2 & \text{Parabolic,} \\ 1 + \xi \sin\left(\pi \frac{x}{a} \right) & \text{Sinusoidal,} \end{cases} \tag{12}$$

in which J_1 is a constant and ξ is a varied parameter. K_G is the shear layer foundation stiffness, ∇^2 is the Laplace operator in *x* and *y*.

Note that, if $\xi = 0$, the elastic foundation becomes Pasternak foundation and if the shear layer foundation stiffness is neglected, the Pasternak foundation becomes the Winkler foundation. Substituting Eqs. (5) and (7) into Eq. (10) and integrating across the thickness of the plate, Eq. (10) can be expressed as

$$\int_\Omega \Big[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\varepsilon_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \gamma_{yz}^s + S_{xz}^s \gamma_{xz}^s \Big] d\Omega - \int_\Omega (q - f_e)\,\delta w \, d\Omega = 0 \tag{13}$$

N, M and S are stress resultants and can be expressed as follows

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{14a}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) g(z) dz \tag{14b}$$

Substituting Eq. (7) into Eq. (14) and integrating across the thickness of the plate, the stress resultants are written as

$$\begin{Bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} & \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} & \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} & \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix} & \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \end{Bmatrix} - \begin{Bmatrix} \begin{Bmatrix} N_x^T \\ N_y^T \\ 0 \end{Bmatrix} \\ \begin{Bmatrix} M_x^{bT} \\ M_y^{bT} \\ 0 \end{Bmatrix} \\ \begin{Bmatrix} M_x^{sT} \\ M_y^{sT} \\ 0 \end{Bmatrix} \end{Bmatrix} - \begin{Bmatrix} \begin{Bmatrix} N_x^C \\ N_y^C \\ 0 \end{Bmatrix} \\ \begin{Bmatrix} M_x^{bC} \\ M_y^{bC} \\ 0 \end{Bmatrix} \\ \begin{Bmatrix} M_x^{sC} \\ M_y^{sC} \\ 0 \end{Bmatrix} \end{Bmatrix} \tag{15a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{15b}$$

and stiffness components are given as

$$(A_{ij}, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s) = \int_{-h/2}^{h/2} C_{ij}\,(1, z, f(z), z^2, z\, f(z), f(z)^2) dz, \; (i, j = 1,2,6), \tag{16a}$$

$$A_{ij}^{s} = \int_{-h/2}^{h/2} C_{ij} \left[g(z) \right]^{2} dz, \ \ (i, j = 4{,}5) \tag{16b}$$

The hygrothermal stress and moment resultants $N_x^{\theta} = N_y^{\theta}$, $M_x^{b\theta} = M_y^{b\theta}$ and $M_x^{s\theta} = M_y^{s\theta}$ are defined by

$$\left[N_x^{\theta}, M_x^{b\theta}, M_x^{s\theta} \right] = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu} \left[1, z, f(z) \right] \eta \, \theta \, dz \tag{17}$$

where

$$\theta = \begin{cases} T & \text{if } \eta = \alpha, \\ C & \text{if } \eta = \beta, \end{cases} \tag{18}$$

The governing equations of equilibrium can be obtained from Eq. (13) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0, δw_b and δw_s zero separately. Thus one can obtain the equilibrium equations associated with the present theory

$$\begin{aligned}
\delta u_0 &: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
\delta v_0 &: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
\delta w_b &: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - f_e + q = 0 \\
\delta w_s &: \ \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - f_e + q = 0
\end{aligned} \tag{19}$$

Substituting Eq. (15) into Eq. (19), the governing equations can be expressed in terms of displacements (u_0, v_0, w_b, w_s) as follows

$$\begin{aligned}
& A_{11} d_{11} u_0 + A_{66} d_{22} u_0 + \left(A_{12} + A_{66} \right) d_{12} v_0 - B_{11} d_{111} w_b - \left(B_{12} + 2B_{66} \right) d_{122} w_b \\
& \quad - \left(B_{12}^s + 2B_{66}^s \right) d_{122} w_s - B_{11}^s d_{111} w_s = p_1,
\end{aligned} \tag{20a}$$

$$\begin{aligned}
& A_{22} d_{22} v_0 + A_{66} d_{11} v_0 + \left(A_{12} + A_{66} \right) d_{12} u_0 - B_{22} d_{222} w_b - \left(B_{12} + 2B_{66} \right) d_{112} w_b \\
& \quad - \left(B_{12}^s + 2B_{66}^s \right) d_{112} w_s - B_{22}^s d_{222} w_s = p_2,
\end{aligned} \tag{20b}$$

$$\begin{aligned}
& B_{11} d_{111} u_0 + \left(B_{12} + 2B_{66} \right) d_{122} u_0 + \left(B_{12} + 2B_{66} \right) d_{112} v_0 + B_{22} d_{222} v_0 - D_{11} d_{1111} w_b - 2\left(D_{12} + 2D_{66} \right) d_{1122} w_b \\
& \quad - D_{22} d_{2222} w_b - D_{11}^s d_{1111} w_s - 2\left(D_{12}^s + 2D_{66}^s \right) d_{1122} w_s - D_{22}^s d_{2222} w_s = p_3
\end{aligned} \tag{20c}$$

$$\begin{aligned}
& B_{11}^s d_{111} u_0 + \left(B_{12}^s + 2B_{66}^s \right) d_{122} u_0 + \left(B_{12}^s + 2B_{66}^s \right) d_{112} v_0 + B_{22}^s d_{222} v_0 - D_{11}^s d_{1111} w_b - 2\left(D_{12}^s + 2D_{66}^s \right) d_{1122} w_b \\
& \quad - D_{22}^s d_{2222} w_b - H_{11}^s d_{1111} w_s - 2\left(H_{12}^s + 2H_{66}^s \right) d_{1122} w_s - H_{22}^s d_{2222} w_s + A_{55}^s d_{11} w_s + A_{44}^s d_{22} w_s = p_4
\end{aligned} \tag{20d}$$

where $\{p\} = \{p_1, p_2, p_3, p_4\}^t$ is a generalized force vector, d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1,2). \tag{21a}$$

The components of the generalized force vector $\{p\}$ are given by

$$p_1 = \frac{\partial N_x^T}{\partial x} + \frac{\partial N_x^C}{\partial x}, \quad p_2 = \frac{\partial N_y^T}{\partial y} + \frac{\partial N_y^C}{\partial y}, \quad p_3 = f_e - q + \frac{\partial^2 (M_x^{bT} + M_x^{bC})}{\partial x^2} + \frac{\partial^2 (M_y^{bT} + M_y^{bC})}{\partial y^2} \tag{21b}$$

$$p_4 = f_e - q + \frac{\partial^2 (M_x^{sT} + M_x^{sC})}{\partial x^2} + \frac{\partial^2 (M_y^{sT} + M_y^{sC})}{\partial y^2} \tag{21c}$$

4. Analytical solution for simply-supported S-FG plates

In this work, the exact solution of equations (20) is desired for a simply supported S-FG plate. The boundary conditions for simply supported plate according to the present formulation can be found in Refs (Benachour *et al.* 2011, Thai and Vo 2013). Navier solution is considered for the mechanical, temperature and moisture loads, q, T_i and C_i in the form of a double Fourier series as

$$\begin{Bmatrix} q \\ T_i \\ C_i \end{Bmatrix} = \begin{Bmatrix} q_0 \\ t_i \\ c_i \end{Bmatrix} \sin(\lambda x) \sin(\mu y), \quad (i = 1, 2, 3) \tag{22}$$

where $\lambda = \pi / a$, $\mu = \pi / b$, q_0, t_i and c_i are constants and T_i and C_i are defined in Eq. (9).

Following the Navier method, we suppose the following solution form for u_0, v_0, w_b and w_s that respects the boundary conditions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \begin{Bmatrix} U \cos(\lambda x) \sin(\mu y) \\ V \sin(\lambda x) \cos(\mu y) \\ W_b \sin(\lambda x) \sin(\mu y) \\ W_s \sin(\lambda x) \sin(\mu y) \end{Bmatrix}, \tag{23}$$

where U, V, W_b, and W_s are arbitrary coefficients to be determined. One obtains the following equation

$$[K]\{\Delta\} = \{P\}, \tag{24}$$

where $\{\Delta\} = \{U, V, W_b, W_s\}^t$ and $[K]$ is the symmetric matrix defined by

$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{12} & k_{22} & k_{23} & k_{24} \\ k_{13} & k_{23} & k_{33} & k_{34} \\ k_{14} & k_{24} & k_{34} & k_{44} \end{bmatrix} \tag{25}$$

in which

$$\begin{aligned}
k_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \\
k_{12} &= -\lambda\,\mu\left(A_{12} + A_{66}\right) \\
k_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2] \\
k_{14} &= \lambda\,[B^s_{11}\lambda^2 + (B^s_{12} + 2B^s_{66})\,\mu^2] \\
k_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right) \\
k_{23} &= \mu\,[(B_{12} + 2B_{66})\,\lambda^2 + B_{22}\mu^2] \\
k_{24} &= \mu\,[(B^s_{12} + 2B^s_{66})\,\lambda^2 + B^s_{22}\mu^2] \\
k_{33} &= -\left(D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 + \overline{K}_W + K_G(\lambda^2 + \mu^2)\right) \\
k_{34} &= -\left(D^s_{11}\lambda^4 + 2(D^s_{12} + 2D^s_{66})\lambda^2\,\mu^2 + D^s_{22}\,\mu^4 + \overline{K}_W + K_G(\lambda^2 + \mu^2)\right) \\
k_{44} &= -\left(H^s_{11}\lambda^4 + 2(H^s_{11} + 2H^s_{66})\lambda^2\mu^2 + H^s_{22}\mu^4 + A^s_{55}\lambda^2 + A^s_{44}\mu^2 + \overline{K}_W + K_G(\lambda^2 + \mu^2)\right)
\end{aligned} \tag{26}$$

The components of the generalized force vector $\{P\}=\{P_1, P_2, P_3, P_4\}^t$ are given by

$$\begin{aligned}
P_1 &= \lambda\left[\left(A^T t_1 + B^T t_2 + {}^aB^T t_3\right) + \left(A^C c_1 + B^C c_2 + {}^aB^C c_3\right)\right], \\
P_2 &= \mu\left[\left(A^T t_1 + B^T t_2 + {}^aB^T t_3\right) + \left(A^C c_1 + B^C c_2 + {}^aB^C c_3\right)\right], \\
P_3 &= -q_0 - h\left(\lambda^2 + \mu^2\right)\left[\left(B^T t_1 + D^T t_2 + {}^aD^T t_3\right) + \left(B^C c_1 + D^C c_2 + {}^aD^C c_3\right)\right], \\
P_4 &= -q_0 - h\left(\lambda^2 + \mu^2\right)\left[\left({}^sB^T t_1 + {}^sD^T t_2 + {}^sF^T t_3\right) + \left({}^sB^C c_1 + {}^sD^C c_2 + {}^sF^C c_3\right)\right]
\end{aligned} \tag{27}$$

$$\left\{A^T, B^T, D^T\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\alpha(z)\left\{1, \bar{z}, \bar{z}^2\right\}dz, \quad \left\{A^C, B^C, D^C\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\beta(z)\left\{1, \bar{z}, \bar{z}^2\right\}dz, \tag{28a}$$

$$\left\{{}^aB^T, {}^aD^T\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\alpha(z)\overline{\Psi}(z)\left\{1, \bar{z}\right\}dz, \quad \left\{{}^aB^C, {}^aD^C\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\beta(z)\overline{\Psi}(z)\left\{1, \bar{z}\right\}dz \tag{28b}$$

$$\left\{{}^sB^T, {}^sD^T, {}^sF^T\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\alpha(z)\bar{f}(z)\left\{1, \bar{z}, \overline{\Psi}(z)\right\}dz \tag{28c}$$

$$\left\{{}^sB^C, {}^sD^C, {}^sF^C\right\} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu}\beta(z)\bar{f}(z)\left\{1, \bar{z}, \overline{\Psi}(z)\right\}dz \tag{28d}$$

in which $\bar{z} = z/h$, $\bar{f}(z) = f(z)/h$, $\bar{K}_W = \int_0^a K_W \, dx/a$ and $\bar{\Psi}(z) = \frac{1}{\pi}\sin\left(\frac{\pi z}{h}\right)$.

5. Results and discussion

In this section, various numerical examples are proposed and discussed for studying the hygro-thermo-mechanical bending responses of S-FGM plates resting on variable two-parameter elastic foundations. Comparisons are carried out with the proposed model. The S-FGM plate is considered to be made of Titanium and Zirconia with the following material properties (Zidi *et al*. 2014):

- Ceramic (Zirconia, ZrO_2): $E_1 = 117.0$ GPa, $\nu = 1/3$, $\alpha_1 = 7.11 \times (10^{-6}/°C)$, $\beta_1 = 0$.
- Metal (Titanium, Ti-6Al-4V): $E_2 = 66.2$ GPa, $\nu = 1/3$, $\alpha_2 = 10.3 \times (10^{-6}/°C)$, $\beta_2 = 0.33$.

The reference temperature and moisture concentration are taken by $T_0 = 25\,°\mathrm{C}$ (room temperature) and $C_0 = 0\%$. It is assumed, unless otherwise stated, that $q_0 = 100$ GPa, $a/h = 10$, $t_1 = 0$, $c_1 = 0$. We also take the shear correction factor $\boldsymbol{K}$ = 5⁄6 in FSDT. Numerical results are provided in terms of dimensionless stresses and deflection. The various dimensionless parameters employed are

- center deflection $\bar{w} = \frac{10^2 h}{a^2 q_0} w\left(\frac{a}{2}, \frac{b}{2}\right)$,
- axial stress $\bar{\sigma}_x = \frac{-10h^2}{a^2 q_0} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right)$,
- longitudinal shear stress $\bar{\tau}_{xy} = \frac{10^2 h^2 b}{a^3 q_0} \tau_{xy}\left(0, 0, \frac{-h}{2}\right)$
- transversal shear stress $\bar{\tau}_{xz} = \frac{10hb}{a^2 q_0} \tau_{xz}\left(0, \frac{b}{2}, 0\right)$,
- Pasternak foundation parameter $J_2 = \frac{K_G a^2}{h^3}$

Comparisons are made with various plate models available in the scientific literature. The description of various displacement models is given in Table 1.

The correlation between the proposed model and different shear deformation theories is presented in Tables 2-5. These tables demonstrate also the influences of the volume fraction exponent k and elastic foundation parameters on the dimensionless transverse displacement and stresses of S-FGM plate.

Table 2 shows the effects of the elastic foundation parameters on the dimensionless deflections of S-FGM square plate subjected to a mechanical load. The present four-variable refined plate model provides almost identical results. It can be observed that the deflection is decreasing with the existence of the elastic foundations. The inclusion of the Winkler foundation parameter yields higher magnitude results than those with the inclusion of Pasternak foundation parameters.

Tables 3 and 4 illustrate similar results as those given in Table 2 including the influence of the temperature and moisture fields. An excellent agreement is demonstrated between the present four-variable refined plate model and the other plate models for all values of thickness ratios a/h and with or without the presence of the elastic foundation. Comparatively to the results given by Table 2, it can be observed that the incorporation of the hygrothermal loads affect clearly the deflections of S-FGM plates.

Table 5 presents the effects of the material distribution parameter k and elastic foundation parameters on the dimensionless deflections and stresses of S-FGM plate subjected to a hygro-thermo-mechanical load. It can be seen that both the deflections and stresses are sensitive to the variation of k. It can be shown that the deflection and stresses are decreasing with the existence of the elastic foundations. The inclusion of the Winkler foundation parameter yields higher magnitude results than those with the inclusion of Pasternak foundation parameters.

Table 1Displacement models

Model	Theory	Unknowns
FSDT	First-order shear deformation theory (Whitney 1969)	5
TSDT	Third-order shear deformation theory (Reddy 2000)	5
SSDT	Sinusoidal shear deformation theory (Touratier 1991)	5
Present	Four-variable refined plate theory	4

Table 2 The deflection $\bar{w}$ of S-FGM square plates with/without elastic foundations ($q_0 = 100$, $T = C = 0$, $k = 2$, $\xi = 0$)

J_1	J_2	Theory	a/h						
			5	10	15	20	25	30	50
0	0	FSDT	0.92581	3.26081	7.15246	12.60077	19.60574	28.16738	77.98053
		TSDT	0.95457	3.28993	7.18165	12.62999	19.63498	28.19662	78.00978
		SSDT	0.95417	3.28962	7.18136	12.62970	19.63469	28.19633	78.00949
		Present	0.95417	3.28962	7.18136	12.62970	19.63469	28.19633	78.00949
10^3	0	FSDT	0.67562	2.45898	5.42722	9.58219	14.92416	21.45316	59.43991
		TSDT	0.69080	2.47551	5.44401	9.59908	14.94109	21.47012	59.45690
		SSDT	0.69059	2.47533	5.44384	9.59891	14.94092	21.46995	59.45673
		Present	0.69059	2.47533	5.44384	9.59891	14.94092	21.46995	59.45673
10^3	10^3	FSDT	0.10666	0.42006	0.94201	1.67269	2.61211	3.76028	10.44053
		TSDT	0.10703	0.42054	0.94252	1.67320	2.61262	3.76080	10.44106
		SSDT	0.10702	0.42054	0.94251	1.67320	2.61262	3.76079	10.44105
		Present	0.10702	0.42054	0.94251	1.67320	2.61262	3.76079	10.44105

Figs. 2-6 demonstrate the effect of the type of elastic foundations (parabolic, linear, sinusoidal) on the deflection and the stresses when the S-FGM plates ($k = 2$) is subjected to thermo-mechanical or hygro-mechanical loads. It can be noted that the results depends on the type of elastic foundations. The thermo-mechanical load always amplifies the deflection and stresses magnitudes whereas the hygro-mechanical load always reduces these quantities.

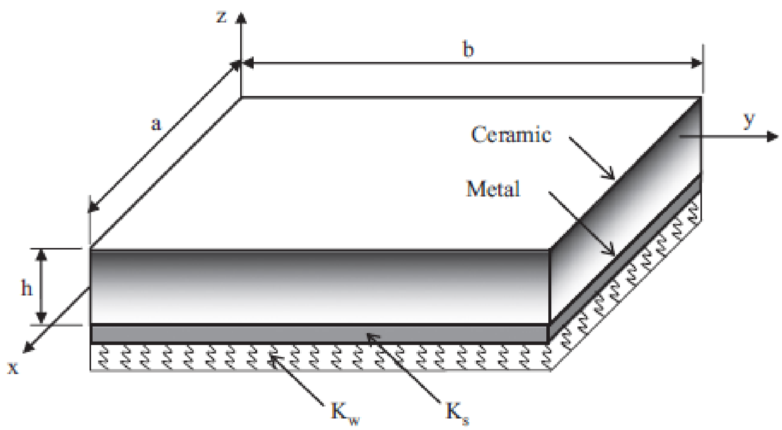


Fig. 1 Schematic representation of a rectangular FG plate resting on elastic foundation

Table 3 The deflection $\bar{w}$ of S-FGM square plates with/without elastic foundations ($q_0 = 100$, $t_1 = t_3 = 0$, $t_2 = 10$, $c_1 = c_3 = 0$, $c_2 = 100$, $k = 2$, $\xi = 0$)

J_1	J_2	Theory	a/h						
			5	10	15	20	25	30	50
0	0	FSDT	7.72551	10.06050	13.95215	19.40046	26.40544	34.96707	84.78022
		TSDT	7.75003	10.08856	13.98087	19.42942	26.43450	34.99619	84.80942
		SSDT	7.74913	10.08812	13.98052	19.42909	26.43419	34.99589	84.80913
		Present	7.74913	10.08812	13.98052	19.42909	26.43419	34.99589	84.80913
10^3	0	FSDT	5.63772	7.58664	10.58676	14.75299	20.10017	26.63202	64.62291
		TSDT	5.60854	7.59113	10.59812	14.76680	20.11514	26.64761	64.63940
		SSDT	5.60853	7.59098	10.59796	14.76664	20.11497	26.64744	64.63924
		Present	5.60853	7.59098	10.59796	14.76664	20.11497	26.64744	64.63924
10^3	10^3	FSDT	0.89001	1.29601	1.83757	2.57531	3.51804	4.66802	11.35092
		TSDT	0.86895	1.28959	1.83485	2.57398	3.51737	4.66771	11.35114
		SSDT	0.86917	1.28964	1.83487	2.57399	3.51737	4.66771	11.35114
		Present	0.86917	1.28964	1.83487	2.57399	3.51737	4.66771	11.35114

Figs. 2 and 3 display the variation of the center deflection $\bar{w}$ with the side-to-thickness a/h and aspect b/a ratios, respectively. The deflection is maximum for the plate resting on parabolic elastic foundation and minimum for the plate supported on sinusoidal elastic foundation irrespective of the values of temperature and moisture.

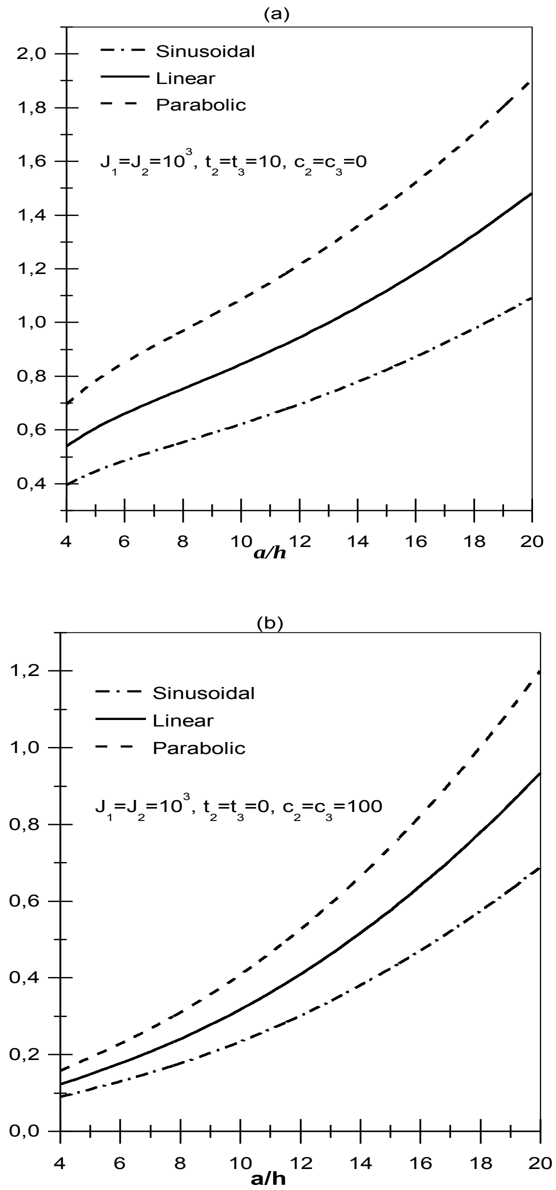


Fig. 2 The deflection $\bar{w}$ of the S-FGM plate ($k = 2$) versus the side-to-thickness ratio a/h under thermo-mechanical and hygro-mechanical loads for various types of Winkler parameter

Fig. 4 depicts the through-the-thickness distributions of the non-dimensional axial stress $\bar{\sigma}_x$ in the S-FGM rectangular plates. It can be seen that the effect of the type of elastic foundations (parabolic, linear, sinusoidal) on $\bar{\sigma}_x$ is not important particularly for the case of thermo-mechanical loads.

The effect of the type of elastic foundations (parabolic, linear, sinusoidal) on the shear stress $\bar{\tau}_{xz}$ through the thickness of S-FGM plate is demonstrated in Fig. 5 for thermo-mechanical or hygro-mechanical loads. It can be seen that the maximum value occurs at a point above the mid-plane of the S-FGM plate and it depends on the type of elastic foundations.

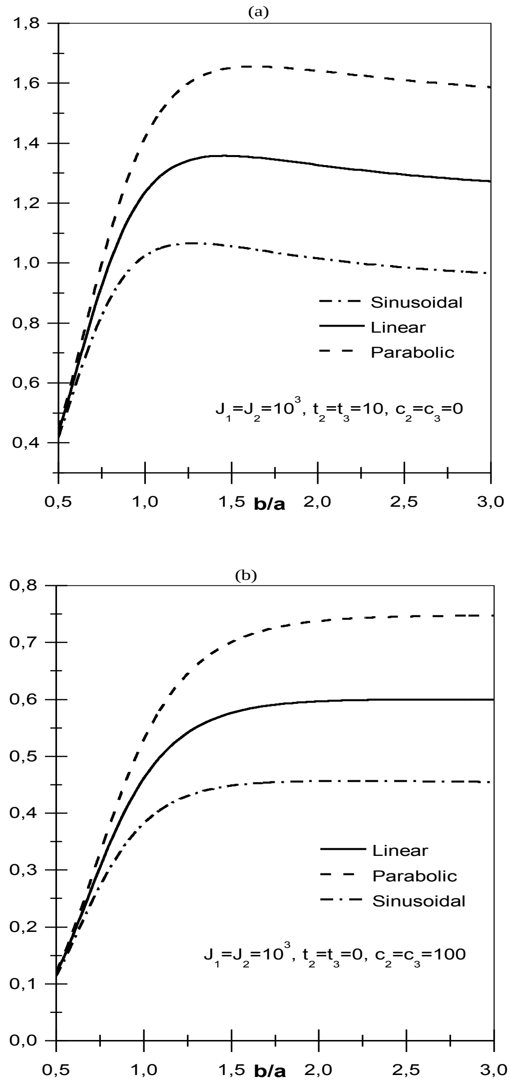


Fig. 3 The deflection $\bar{w}$ of the S-FGM plate ($k=2$) versus the aspect ratio b/a under thermo-mechanical and hygro-mechanical loads for various types of Winkler parameter

Table 4 The deflection $\bar{w}$ of S-FGM square plates with/without elastic foundations ($q_0 = 100$, $t_1 = 0$, $t_2 = t_3 = 10$, $c_1 = 0$, $c_2 = c_3 = 100$, $k = 2$, $\xi = 0$)

J_1	J_2	Theory	a/h 5	10	15	20	25	30	50
0	0	FSDT	13.00085	15.33585	19.22750	24.67581	31.68078	40.24242	90.05557
		TSDT	13.00621	15.35911	19.25409	24.70356	31.70908	40.27101	90.08458
		SSDT	13.00318	15.35813	19.25350	24.70311	31.70868	40.27064	90.08427
		Present	13.00318	15.35813	19.2535	24.70311	31.70868	40.27064	90.08427
10^3	0	FSDT	9.48742	11.56479	14.58964	18.76460	24.11584	30.64990	68.64399
		TSDT	9.41233	11.55695	14.59545	18.77528	24.12879	30.66408	68.65998
		SSDT	9.41122	11.55649	14.59514	18.77503	24.12857	30.66388	68.65980
		Present	9.41122	11.55649	14.59514	18.77503	24.12857	30.66388	68.65980
10^3	10^3	FSDT	1.49775	1.97559	2.53236	3.27559	4.22088	5.37227	12.05722
		TSDT	1.45829	1.96331	2.52690	3.27269	4.21920	5.37125	12.05718
		SSDT	1.45849	1.96335	2.52692	3.27269	4.21920	5.37125	12.05718
		Present	1.45849	1.96335	2.52692	3.27269	4.21920	5.37125	12.05718

Fig. 6 shows the variation of the in-plane shear stress $\bar{\tau}_{xy}$ through the thickness of S-FGM plate when is subjected to thermo-mechanical or hygro-mechanical loads. Different type of elastic foundations (parabolic, linear, sinusoidal) are considered in this example. It can be observed that this effect is more pronounced near the top and bottom surfaces of the plate.

The variation of the transverse displacement $\bar{w}$ of the S-FGM plate ($k = 2$) resting on parabolic elastic foundation and under thermo-mechanical load vs. the side-to-thickness ratio a/h is exhibited in Fig 7 for ζ = 10, 30, 50, 80. Also, in such case, the variations of the stresses through-the-thickness of S-FGM plate are presented in Figs. 8 to 10 with the case of hygro-thermo-mechanical loads. As is evident, the decrease of the parabolic parameter ζ leads to a significant increment in the variation of the transverse displacement and shear stresses ($\bar{\tau}_{xz}$ and $\bar{\tau}_{xy}$). However, this effect in the case of the normal stress $\bar{\sigma}_x$ is felt slightly at the top and bottom surfaces of the plate.

Table 5 Effect of the volume fraction exponent and elastic foundation parameters on the dimensionless and stresses of an S-FGM square plate ($a/h=10$, $q_0=100$, $t_1=0$, $t_2=t_3=10$, $c_1=0$, $c_2=c_3=100$, $\xi=0$)

k	J_1	J_2	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{\tau}_{xy}$
0	0	0	FSDT	15.54469	1.20274	1.59155	52.19976
			TSDT	15.56983	1.18323	2.32855	52.29730
			SSDT	15.56909	1.18205	2.39624	52.30321
			Present	15.56909	1.18205	2.39624	52.30321
	10^3	0	FSDT	11.83355	3.60072	-0.29182	40.20988
			TSDT	11.82624	3.60337	-0.49277	40.19664
			SSDT	11.82593	3.60362	-0.51494	40.19536
			Present	11.82593	3.60362	-0.51494	40.19536
	10^3	10^3	FSDT	2.07150	9.90852	-5.24595	8.67088
			TSDT	2.05814	9.91818	-7.85439	8.62256
			SSDT	2.05821	9.92268	-8.11165	8.60008
			Present	2.05821	9.92268	-8.11165	8.60008
0.5	10^3	10^3	FSDT	2.03173	8.27104	-5.11466	6.79059
			TSDT	2.01864	8.28625	-7.67656	6.77087
			SSDT	2.01868	8.29173	-7.92987	6.75503
			Present	2.01868	8.29173	-7.92987	6.75503
1	10^3	10^3	FSDT	2.00121	8.40197	-5.01394	6.96572
			TSDT	1.98848	8.41890	-7.53181	6.95451
			SSDT	1.98851	8.42428	-7.78118	6.93921
			Present	1.98851	8.42428	-7.78118	6.93921
2	10^3	10^3	FSDT	1.97559	8.51057	-4.92939	7.10902
			TSDT	1.96331	8.52966	-7.40214	7.10529
			SSDT	1.96335	8.53504	-7.64710	7.09053
			Present	1.96335	8.53504	-7.64710	7.09053
5	10^3	10^3	FSDT	1.96114	8.58428	-4.88169	7.22509
			TSDT	1.94928	8.60561	-7.32235	7.22721
			SSDT	1.94933	8.61111	-7.56368	7.21292
			Present	1.94933	8.61111	-7.56368	7.21292
Metal	10^3	10^3	FSDT	2.06374	9.71180	-5.22032	6.69643
			TSDT	2.04996	9.71936	-7.81103	6.65862
			SSDT	2.05003	9.72385	-8.06660	6.63616
			Present	2.05003	9.72385	-8.06660	6.63616

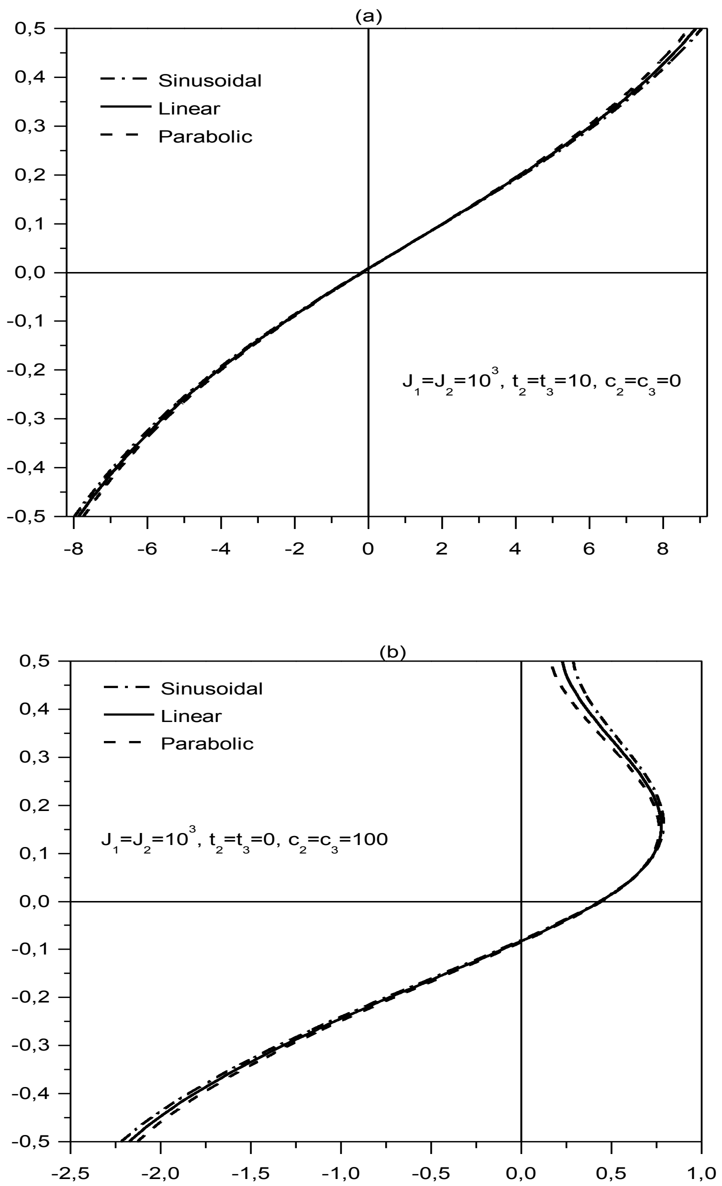
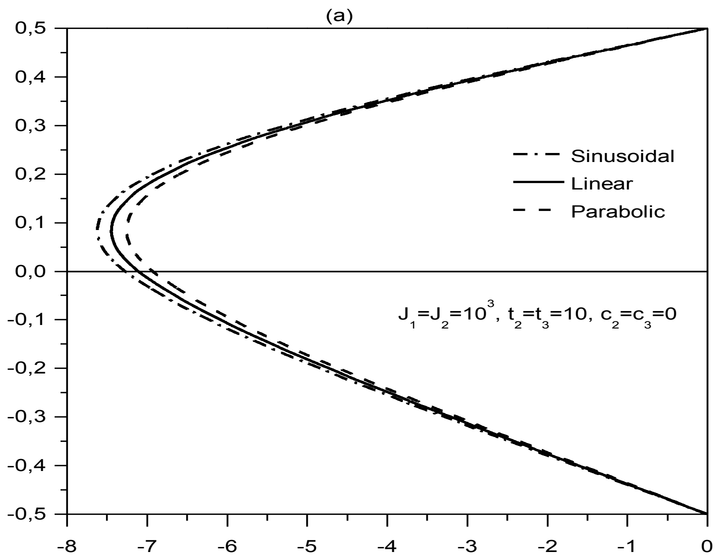


Fig. 4 Dimensionless axial stress $\bar{\sigma}_x$ through-the-thickness of S-FGM plate ($k=2$) under thermo-mechanical and hygro-mechanical loads for various types of Winkler parameter



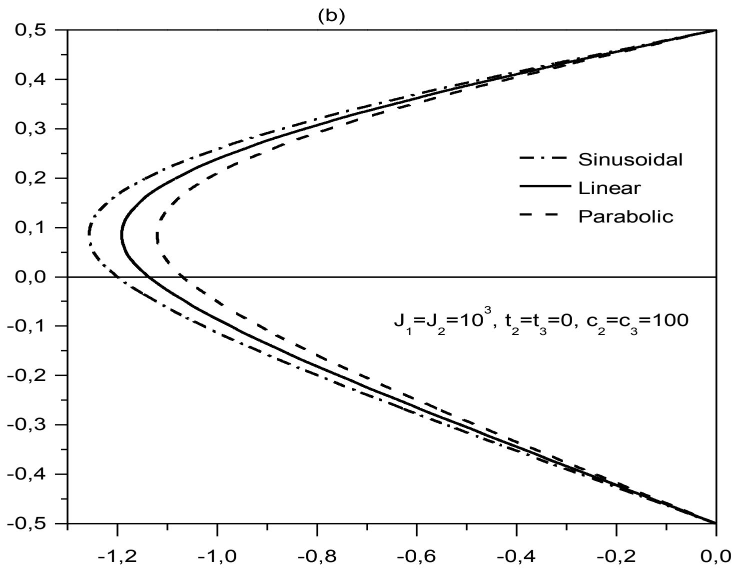


Fig. 5 Dimensionless shear stress $\bar{\tau}_{xz}$ through-the-thickness of S-FGM plate ($k=2$) under thermo-mechanical and hygro-mechanical loads for various types of Winkler parameter

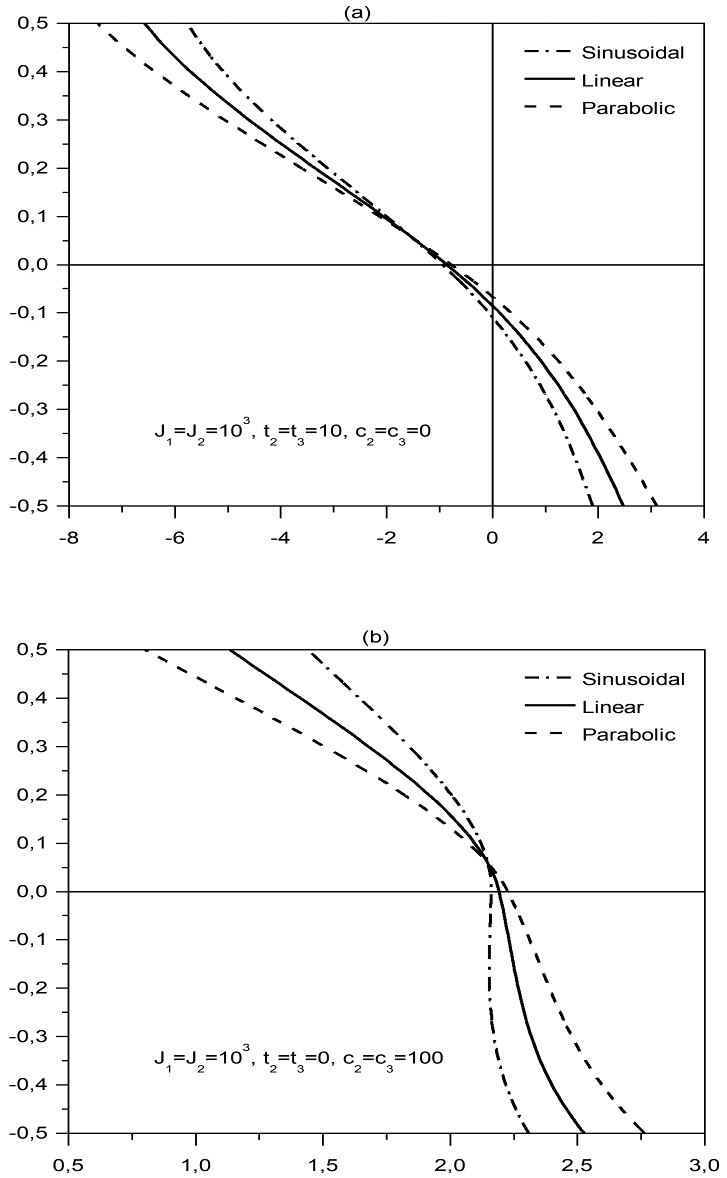


Fig. 6 Dimensionless in-plane shear stress $\bar{\tau}_{xy}$ through-the-thickness of S-FGM plate ($k = 2$) under thermo-mechanical and hygro-mechanical loads for various types of Winkler parameter

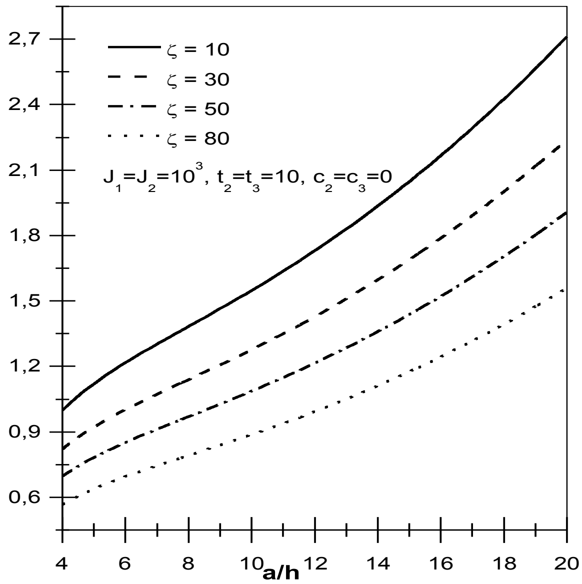


Fig. 7 The deflection $\bar{w}$ of the S-FGM plate ($k=2$) against the side-to-thickness ratio a/h under thermo-mechanical load for different values of the parabolic parameter ξ

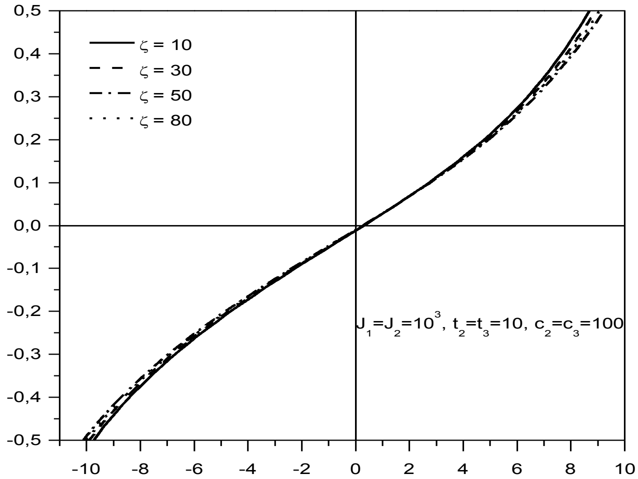


Fig. 8 Variation of the stress $\bar{\sigma}_x$ through-the-thickness of the S-FGM plate ($k=2$) under hygro-thermo-mechanical load for different values of the parabolic parameter ξ

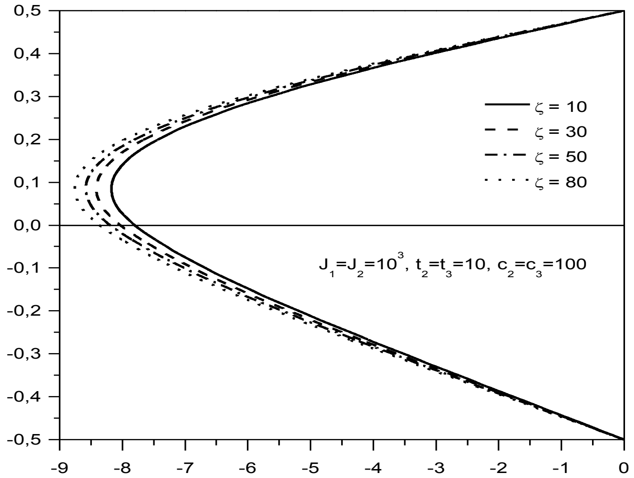


Fig. 9 Variation of the transverse shear stress $\bar{\tau}_{xz}$ through-the-thickness of the S-FGM plate $(k = 2)$ under hygro-thermo-mechanical load for different values of the parabolic parameter ξ

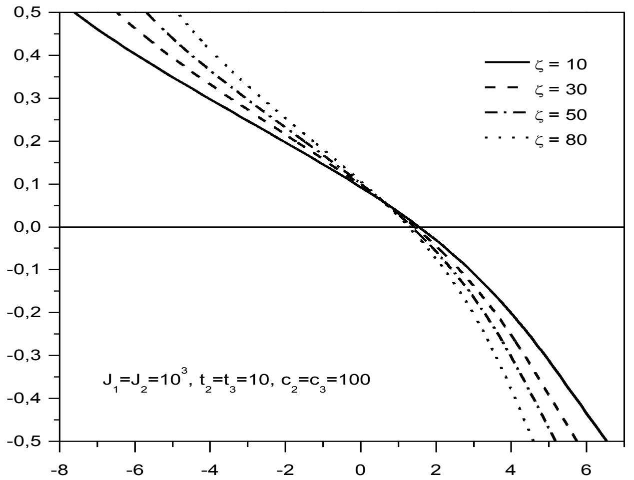


Fig. 10 Variation of the on-plane shear stress $\bar{\tau}_{xy}$ through-the-thickness of the S-FGM plate $(k = 2)$ under hygro-thermo-mechanical load for different values of the parabolic parameter ξ

Figs. 11 and 12 show the influence of the thermal and moisture loads, respectively, on the center deflection of S-FGM plate resting on parabolic elastic foundations (ζ = 50). It can be observed that the transverse displacement $\bar{w}$ increases as the temperature and the moisture parameters increase.

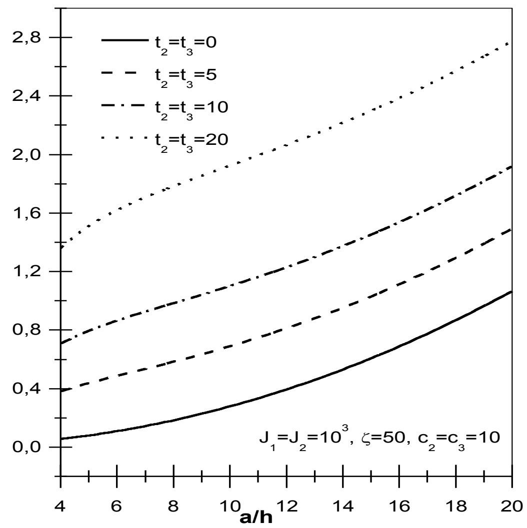


Fig. 11 Dimensionless center deflection $\bar{w}$ of S-FGM plate ($k=2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of temperatures

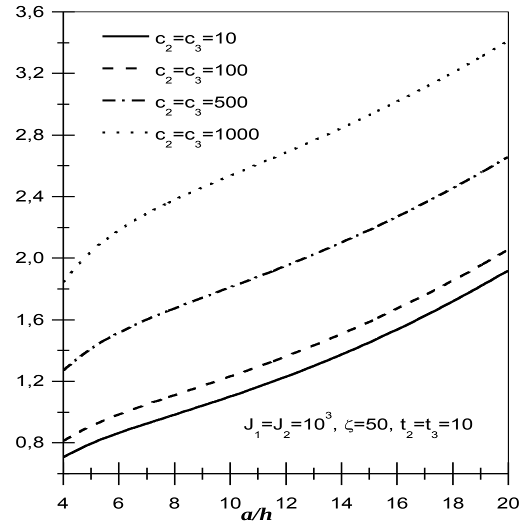


Fig. 12 Dimensionless center deflection $\bar{w}$ of S-FGM plate ($k=2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of moistures

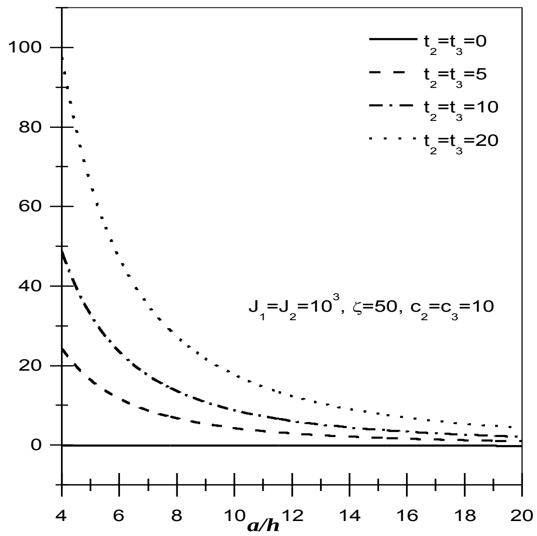


Fig. 13 Dimensionless normal stress $\bar{\sigma}_x$ of S-FGM plate ($k=2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of temperatures

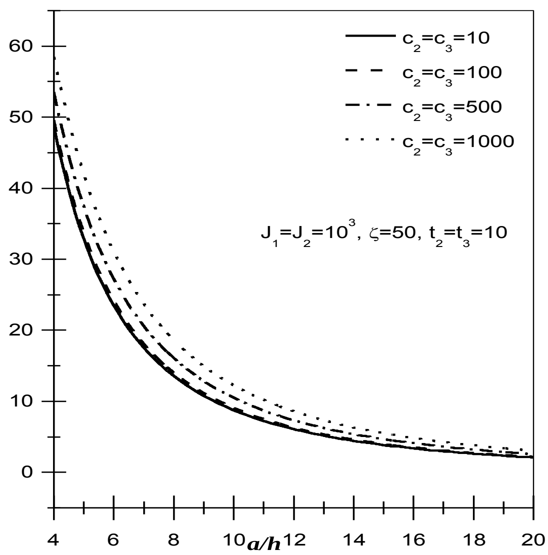


Fig. 14 Dimensionless normal stress $\bar{\sigma}_x$ of S-FGM plate ($k=2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of moistures

The effect of thermal and moisture loads on in-plane normal stress $\bar{\sigma}_x$ of S-FGM plate resting on parabolic elastic foundations ($\zeta = 50$) is demonstrated in Figs. 13 and 14, respectively. It can be seen that the increase of the temperature and the moisture parameters leads to an increase of the normal stress $\bar{\sigma}_x$.

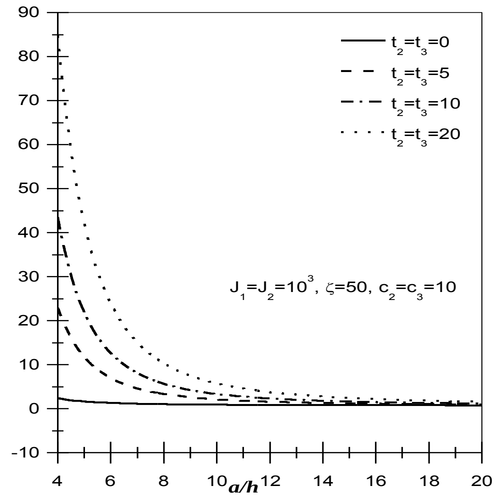


Fig. 15 Dimensionless in-plane shear stress $\bar{\tau}_{xy}$ of S-FGM plate ($k = 2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of temperatures

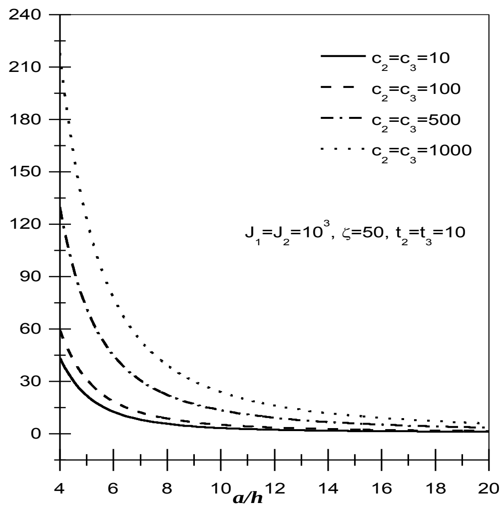


Fig. 16 Dimensionless in-plane shear stress $\bar{\tau}_{xy}$ of S-FGM plate ($k = 2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of moistures

The same observation are demonstrated in Figs. 15 and 16 for the case of in-plane shear stress $\bar{\tau}_{xy}$ and in Figs. 17 and 18. It can be also noticed from these figures that the increasing of the side-to-thickness ratio leads to a decrement in the variation of the stress $\bar{\sigma}_x$ and an increment in the variation of the shear stresses ($\bar{\tau}_{xy}$ and $\bar{\tau}_{xz}$).

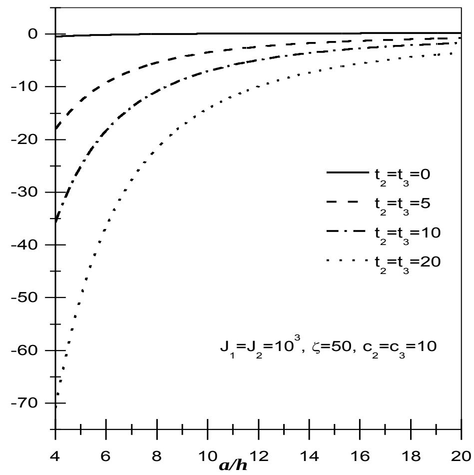


Fig. 17 Dimensionless transverse shear stress $\bar{\tau}_{xz}$ of S-FGM plate ($k=2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of temperatures

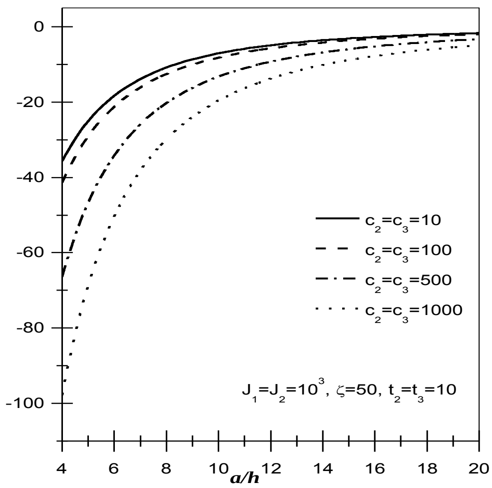


Fig. 18 Dimensionless transverse shear stress $\bar{\tau}_{xz}$ of S-FGM plate ($k=2$) on parabolic elastic foundations versus side-to-thickness ratio a/h for different values of moistures

6. Conclusions

The four-variable refined plate theory for hygro–thermo-mechanical response of S-FGM plates resting on two-parameter elastic foundations is investigated. The Winkler parameter is varying in the direction of x-axis as a linear, parabolic or sinusoidal functions of x. The second parameter represents the shear layer modulus that takes constant values. All comparison studies demonstrate that the deflection and stresses obtained by the proposed theory with four unknowns are almost identical with those predicted by other shear deformation theories containing five unknowns. The effect of moisture concentration as well as other parameters is shown to be significant. The formulation and techniques derived herein should be useful in further studies and should provide engineers with the capability for the design of functionally graded plates for special technical applications including rocket launch pad foundation structures.

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