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Optimal analysis and design of large-scale domes with frequency constraints

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Abstract. Structural optimization involves a large number of structural analyses. When optimizing large structures, these analyses require a considerable amount of computational time and effort. However, there are specific types of structure for which the results of the analysis can be achieved in a much simpler and quicker way thanks to their special repetitive patterns. In this paper, frequency constraint optimization of cyclically repeated space trusses is considered. An efficient technique is used to decompose the large initial eigenproblem into several smaller ones and thus to decrease the required computational time significantly. Some examples are presented in order to illustrate the efficiency of the presented method.

Keywords: optimal analysis; optimal design; frequency constraint; truss; cyclic symmetry

1. Introduction

In low frequency vibration problems, the response of the structure primarily depends on its fundamental frequencies and mode shapes (Grandhi 1993). Therefore, the dynamic behavior of a structure can be controlled by constraining its fundamental frequencies. Mass minimization of a structure for which some natural frequencies should be upper and/or lower-bounded is known as a structural optimization problem with frequency constraints.

History of structural optimization with frequency constraints dates back to 1960s and since then has always received considerable attention by optimization experts utilizing a wide variety of algorithms (Taylor 1967, Armand 1971, Cardou and Warner 1974, Elwany and Barr 1978, Lin *et al*. 1982, Konzelman 1986, Grandhi and Venkayya 1988, Sedaghati *et al*. 2002, Lingyun *et al*. (2005), Gomes 2011, Kaveh and Zolghadr 2012, 2014a,b).

In a frequency constraint structural optimization problem large generalized eigenproblems should be solved in order to find the natural frequencies of the structure. The size of the structure affects the dimensions of the matrices involved and thus the required computational time and effort. On the other hand, as the number of optimization variables increases, more and more structural analyses are needed to be performed in order to obtain a near-optimal solution. There are numerous algebraic methods for eigensolution of large structural systems some of them utilizing

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such properties as sparsity and symmetry of the associated matrices. For general structures, utilization of general time consuming algebraic methods seems to be inevitable. However, fast and efficient techniques could be used for several types of structures, which enjoy specific characteristics such as symmetry. These methods utilize the characteristics of special categories of matrices whose eigenvalues and eigenvectors can be more easily obtained by using block diagonalization techniques. Several applications of these techniques could be found in the literature. Kaveh and Rahami (2006, 2007) utilized block diagonalization techniques for different types of canonical forms for applications in structural mechanics. Kaveh and Koohestani (2009) employed special canonical forms for the efficient eigensolutions of Laplacian and adjacency matrices of special graphs and free vibration and buckling load analysis of cyclically repeated space truss structures (Koohestani and Kaveh 2010).

Many different types of complex structural systems can be considered as the cyclic repetition of a simple substructure around a revolution axis. These structures, which are usually called cyclically symmetric, exhibit some special patterns in their structural matrices. Structures like domes and cooling towers fall into this category. These special patterns and the benefits they bring about in the analysis of such structures have been studied in the works of Courant (1943), Hussey (1967), Leung (1980), Williams (1986), Vakakis (1992), Karpov *et al*. (2002), and Liu and Yang (2007), El-Raheb (2011), Zingoni (2012,2014), Shi *et al*. (2013), Tran (2014), Rahami *et al*. (2014), and Shojai *et al*. (2015) among many others.

The aim of this paper is to incorporate previously existing efficient methods of analysis for cyclically repeated truss structures into the well-known frequency constraint optimization problem in order to achieve considerable computational savings. An efficient method for free vibration analysis of these structures, introduced by Koohestani and Kaveh (2010), is utilized to decompose the initial generalized eigenproblem to several smaller ones and to reduce the required computational time consequently. Other swift and efficient methods for analysis of different types of symmetric, regular and near regular structures could be found in (Kaveh 2013).

The remainder of this article is organized as follows. In section 2, the mathematical statement of the minimum weight optimization problem for a truss structure subject to frequency constraints is summarized. In section 3, basic formulation of free vibration analysis of a truss structure and the corresponding stiffness matrix are presented concisely. The efficient eigensolution of cyclically repeated dome trusses is then discussed in section 4 followed by three numerical examples, examined in section 5, in order to show the efficiency of the proposed method. Finally, some concluding remarks are presented in section 6.

2. Formulation of the optimization problem

Size optimization of a truss structure subject to frequency constraints where the objective is to minimize the weight of the structure can be mathematically stated as follows:

$$\begin{gathered}
\text{Find } X=[x_1,x_2,x_3,\ldots,x_n] \\
\text{to minimize } P(X) = f(X) \times f_{penalty}(X) \\
\text{subject to} \\
\omega_j \le \omega_j^* \quad \text{for some natural frequencies } j \\
\omega_k \ge \omega_k^* \quad \text{for some natural frequencies } k \\
x_{imin} \le x_i \le x_{imax}
\end{gathered} \tag{1}$$

where X is the vector of the design variables i.e., cross-sectional areas; n is the number of optimization variables which depends on the element grouping scheme; $f(X)$ is the cost function, which is taken as the weight of the structure in a weight optimization problem; $f_{penalty}(X)$ is the penalty function, which is used to make the problem unconstrained. When some constraints are violated in a particular solution, the penalty function magnifies the weight of the solution by taking values bigger than one; $P(X)$ is the penalized cost function or the objective function to be minimized. ω_j is the jth natural frequency of the structure with the corresponding upper bound ω_j^*, while ω_k is the kth natural frequency of the structure with the corresponding lower bound ω_k^*. x_{imin} and x_{imax} are the lower and upper bounds for the design variable x_i, respectively.

The cost function can be expressed as

$$f(X) = \sum_{i=1}^{nm} \rho_i L_i A_i \tag{2}$$

where nm is the number of structural members; ρ_i , L_i, and A_i are the material density, length, and cross-sectional area of the ith element.

The penalty function is defined as

$$f_{penalty}(X) = \left(1 + \varepsilon_1 . v\right)^{\varepsilon_2}, \quad v = \sum_{i=1}^{q} v_i \tag{3}$$

where q is the number of frequency constraints. The values for v_i can be considered as

$$v_i = \begin{cases} 0 & \textit{if the ith constraint is satisfied} \\ \left|1 - \dfrac{\omega_i}{\omega^*_i}\right| & \textit{else} \end{cases} \tag{4}$$

The parameters ε_1 and ε_2 determine the degree to which a violated solution should be penalized. In this study ε_1 is taken as unity, and ε_2 starts from 1.5 and then linearly increases to 6 for all test problems. Such a scheme penalizes the unfeasible solutions more severely as the optimization process proceeds. As a result, in the early stages the agents are free to explore the search space, but at the end they tend to choose solutions without violation.

3. Free vibration analysis of structures

3.1 Basic formulation

Abovementioned frequency constraint structural optimization involves a large number of free vibration analyses of the structural system under consideration. The mathematical formulation of the free vibration of a structure leads to a generalized eigenproblem of the following form

$$K\boldsymbol{\phi}_{\mathrm{i}} = \boldsymbol{\gamma}_{\mathrm{i}} M \boldsymbol{\phi}_{\mathrm{I}} \tag{5}$$

In which K is the elastic stiffness matrix and M is the mass matrix of the structure; $\boldsymbol{\phi}_{\mathrm{i}}$ is the ith eigenvector (mode shape) corresponding to the *i*th eigenvalue $\boldsymbol{\gamma}_i$; the *i*th period (T_i) and circular frequency ($\boldsymbol{\omega}_i$) are related to the *i*th eigenvalue by

$$\boldsymbol{\gamma}_i = \boldsymbol{\omega}_i^2 = (2\boldsymbol{\pi}/T_i)^2 \tag{6}$$

General methods to solve the generalized eigenproblem of Eq. (5) require manipulation of large matrices resulting in high computational costs. This is particularly the case when performing structural optimization, where the analysis part should be carried out thousands of times. Specifically, when the number of degrees of freedom of the structure is relatively large the required computational time becomes significant. In the next subsection a formulation is presented based on the works of Kaveh and Koohestani (Kaveh and Koohestani 2009, Koohestani and Kaveh 2010), which helps to obtain special patterns in the matrices involved in Eq. (5). Such a formulation, allows the initial eigenproblem to be decomposed into several smaller ones and results in a much faster solution to the problem at hand.

3.2 Elastic stiffness matrix of a 3D truss element

Fig. 1 Shows a three dimensional (3D) truss element in global Cartesian coordinate system together with the corresponding components of displacement. The elastic stiffness matrix of such an element is as follows

$$K_{ij}^{xyz} = \frac{EA_{ij}}{L_{ij}} \begin{bmatrix} d_{ii} & -d_{ii} \\ -d_{ii} & d_{ii} \end{bmatrix}, \qquad d_{ii} = \begin{bmatrix} l_{ij}^2 & l_{ij}m_{ij} & l_{ij}n_{ij} \\ m_{ij}l_{ij} & m_{ij}^2 & m_{ij}n_{ij} \\ n_{ij}l_{ij} & n_{ij}m_{ij} & n_{ij}^2 \end{bmatrix} \tag{7}$$

where E is the modulus of elasticity and A_{ij} and L_{ij} are the cross-sectional area and the length of the element, respectively. In the submatrix d_{ij}, l_{ij}, m_{ij}, and n_{ij} are the direction cosines of the element with respect to x, y, and z axes, respectively

$$l_{ij} = \frac{x_i - x_i}{L_{ij}}, \qquad m_{ij} = \frac{y_i - y_i}{L_{ij}}, \qquad n_{ij} = \frac{z_i - z_i}{L_{ij}} \tag{8}$$

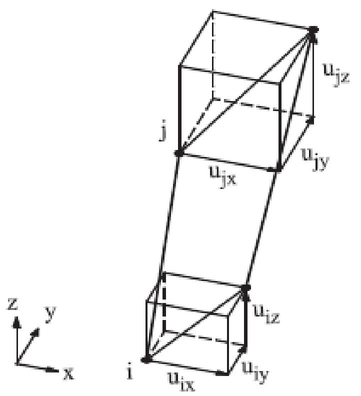

Fig. 1 A 3D truss element in the global Cartesian coordinate system

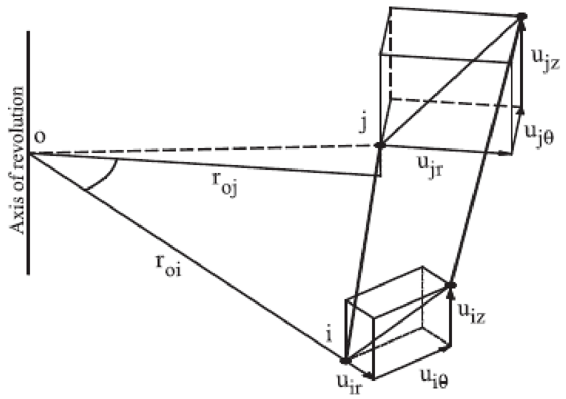

Fig. 2 Schematic of the 3D truss element in the global cylindrical coordinate system

It is apparent from Eq. (7) that the element stiffness matrix in Cartesian coordinates is not invariant against rotation about any axis. Therefore the global stiffness matrix of a cyclically repetitive structure does not generally exhibit any favorable pattern in Cartesian coordinates.

In order to use the desirable patterns of the global stiffness matrices of cyclically symmetric structures, the element global stiffness matrix should be developed in a cylindrical coordinate system. In such a coordinate system the element stiffness matrix is invariant against rotation about an axis of revolution. So the global stiffness matrix of a cyclically repeated structure exhibits a special pattern which is highly desired for efficient eigensolutions. A 3D truss element together with its displacement components in cylindrical coordinate system is shown in Fig. 2. It should be noted that for the group-theoretic formulation of stiffness matrices for symmetric finite elements, a similar procedure of choosing the coordinate system and numbering the nodes is also recommended by Zingoni (2005), making the matrices invariant with respect to both rotations and reflections.

The element stiffness matrix in Cartesian coordinate system can be transformed in to the cylindrical coordinate system by the following transformation

$$K_{ij}^{rz\theta} = R^t K_{ij}^{xyz} R \tag{9}$$

where R is a transformation matrix

$$R = \begin{bmatrix} R_{oi} & 0 \\ 0 & R_{oj} \end{bmatrix} \tag{10}$$

In which the submatrices R_{oi} and R_{oj} could be defined as

$$R_{oi} = \begin{bmatrix} l_{oi} & -m_{oi} & 0 \\ m_{oi} & l_{oi} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R_{oi} = \begin{bmatrix} l_{oj} & -m_{oj} & 0 \\ m_{oj} & l_{oj} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

In which we have

$$l_{oi}=\frac{x_i-x_0}{r_{oi}}, \qquad m_{oi}=\frac{y_i-y_0}{r_{oi}}, \qquad l_{oj}=\frac{x_j-x_0}{r_{oj}}, \qquad m_{oj}=\frac{y_j-y_0}{r_{oj}} \tag{12}$$

where

$$r_{oi}=\sqrt{x_i^2+y_i^2}, \qquad r_{oj}=\sqrt{x_j^2+y_j^2} \tag{13}$$

The expanded form of the element global stiffness matrix in cylindrical coordinates can then be derived as

$$K_{ij}^{rz\theta}=\frac{EA_{ij}}{L_{ij}}\begin{bmatrix} s_1^2 & -s_1s_2 & s_1n_{ij} & -s_1s_3 & s_1s_4 & -s_1n_{ij} \\ & s_2^2 & -s_2n_{ij} & s_2s_3 & -s_2s_4 & s_2n_{ij} \\ & & n_{ij}^2 & -s_3n_{ij} & s_4n_{ij} & -n_{ij}^2 \\ & & & s_3^2 & -s_3s_4 & s_3n_{ij} \\ & sym & & & s_4^2 & -s_4n_{ij} \\ & & & & & n_{ij}^2 \end{bmatrix} \tag{14}$$

where

$$\begin{aligned} s_1 &= l_{ij}l_{oi}+m_{ij}m_{oi} \\ s_2 &= l_{ij}m_{oi}+m_{ij}l_{oi} \\ s_3 &= l_{ij}l_{oj}+m_{ij}m_{oj} \\ s_4 &= l_{ij}m_{oj}+m_{ij}l_{oj} \end{aligned} \tag{15}$$

As it can be seen, this form of element stiffness matrix is invariant against rotation about the axis of revolution. Therefore, all similar substructures have the same stiffness matrix regardless of their rotational positions. Hence, the global stiffness matrix of the structure embodies some interesting patterns, which could be used for efficient eigensolution of the structure.

In relation with mass matrix, it should be noted that, both lumped and consistent mass matrices are invariant against rotation and therefore no transformation is needed. Since additional lumped masses are added to the free nodes, the difference between consistent and lumped mass matrices is negligible. A lumped mass matrix, which lumps the masses of the elements in their end nodes is utilized in this study. Therefore the mass matrix is a diagonal one.

4. Efficient eigensolution

Matrices related to a 3D truss element in cylindrical coordinate system are invariant against rotation about axis of revolution. Therefore, if the nodes of all similar substructures are labeled in a similar manner, the matrices corresponding to these substructures would be the same and the global mass and stiffness matrices of a cyclically repeated structure exhibit the canonical form shown in Eq. (16). This canonical form is called Block Tri-digonal Matrix with Corner Blocks (BTMCB).

$$\begin{bmatrix} A & B & & & & B^t \\ B^t & A & B & & & \\ & \cdot & \cdot & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot \\ & & & B^t & A & B \\ B & & & & B^t & A \end{bmatrix} \tag{16}$$

For a 3D truss structure which is formed of n cyclically repeated substructures each having m nodes, both mass and stiffness matrices are 3nm×3nm. Submatrices *A*, *B*, and B^t are square matrices with dimension 3 m. Although applying the support condition will change these dimensions, the canonical form of Eq. (16) will be preserved if the boundary conditions are also cyclically symmetric. Hence, the structural matrices could be decomposed using Kronecker products as

$$K_{(3nm\times 3nm)} = I_{n\times n} \otimes A_{K(3m\times 3m)} + H_{(n\times n)} \otimes B_{K(3m\times 3m)} + H^t_{(n\times n)} \otimes B^t_{K(3m\times 3m)} \tag{17}$$

$$M_{(3nm\times 3nm)} = I_{n\times n} \otimes A_{M(3m\times 3m)} + H_{(n\times n)} \otimes B_{M(3m\times 3m)} + H^t_{(n\times n)} \otimes B^t_{M(3m\times 3m)} \tag{18}$$

where subscripts *K*, and *M*, for *A*, *B* and B^t refer to stiffness, and mass matrices, respectively; *I* is an $n\times n$ identity matrix and *H* is an $n\times n$ unsymmetric permutation matrix as

$$H = \begin{bmatrix} 0 & 1 & & & & & 0 \\ 0 & 0 & 1 & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & 0 & 0 & 1 \\ 1 & & & & & 0 & 0 \end{bmatrix} \tag{19}$$

Kronecker product of two matrices $A_{m\times n}$ and $B_{p\times q}$, denoted by $A\otimes B$ is an $mp\times nq$ block matrix and could be defined as

$$A\otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix} \tag{20}$$

Block diagonalization of a BTMCB matrix is studied in (Kaveh and Koohestani 2009, Koohestani and Kaveh 2010) and could be summarized as follows. Eq. (5) has a non-trivial solution if and only if

$$det(\Omega_i) = det(K - \gamma_i M) = 0 \tag{21}$$

where "*det*" stands for determinant. Here the goal is to block diagonalize Ω_i and hence to decompose the main problem into some simpler subproblems. Let us consider the following definitions

$$\begin{aligned} A &= A_K - \gamma_i A_M \\ B &= B_K - \gamma_i B_M \\ B^t &= B_K^t - \gamma_i B_M^t \end{aligned} \tag{22}$$

Combining Eqs. (17) and (18) with the above equations Ω_i could be written as

$$\Omega_i = I \otimes A + H \otimes B + H^t \otimes B^t \tag{23}$$

This form of Ω_i could now be block diagonalized and its *j*th block is as follows

$$\Omega_i^j = A + \lambda_j B + \overline{\lambda_j} B^t \tag{24}$$

where λ_j is the *j*th eigenvalue of matrix *H* and the bar sign means conjugation of a general complex number. Thus the following equation holds

$$det(\Omega_i) = \prod_{j=1}^{n} det(\Omega_i^j) \tag{25}$$

The determinant of *j*th block of Ω_i is in turn a new generalized eigenproblem. Therefore, the original eigenproblem is decomposed into n highly smaller and simpler subproblems as

$$K_j x_i = \gamma_i M_j x_i, \quad j = 1,2,3,...,n \tag{26}$$

In which

$$\begin{aligned} K_j &= A_K + \lambda_j B_K + \overline{\lambda_j} B_K^t \\ M_j &= A_M + \lambda_j B_M + \overline{\lambda_j} B_M^t \end{aligned} \tag{27}$$

where x_i could be converted to the required eigenvector corresponding to $\boldsymbol{\gamma}_i$ (Kaveh and Koohestani 2009).

5. Numerical examples

In this section three numerical examples are studied in order to examine the viability and efficiency of the proposed method. Democratic Particle Swarm Optimization (DPSO) as introduced by Kaveh and Zolghadr (2014b) is utilized as the optimization algorithm. However, any other metaheuristic algorithm could be used. The algorithm as well as the finite element analysis were implemented by MATLAB R2009a on a laptop computer with an Intel (R) Core(TM)2 Duo 2.50 GHz processor, 4.00 GB RAM under the Microsoft Windows Vista™ Home Basic operating system. Matlab internal eigenvalue function was used equally for the initial eigenproblem and the decomposed ones. The overall computational times required for different optimization runs utilizing the standard method and the proposed one are compared. The results show that the proposed efficient method is significantly faster.

5.1 A 600-bar single layer dome

The first test problem is the 600-bar single layer dome structure shown in Fig. 3. The entire structure is composed of 216 nodes and 600 elements generated by cyclic repetition of a substructure having 9 nodes and 25 elements. The angle of cyclic symmetry between similar substructures is 15 degrees. A non-structural mass of 100 kg is attached to all free nodes. Table 1 summarizes the material properties, variable bounds, and frequency constraints for this example. Fig. 4 shows a substructure in more detail for nodal numbering and coordinates. Each of the elements of this substructure is considered as a design variable. Thus, this is a size optimization problem with 25 variables.

Table 1 Material properties, variable bounds and frequency constraints for the 600-bar single layer dome

Property/unit	**Value**
E (Modulus of elasticity)/ N/m^2	2×10^{11}
ρ (Material density)/ kg/m^3	7850
Added mass/kg	100
Design variable lower bound/m^2	1×10^{-4}
Design variable upper bound/m^2	100×10^{-4}
Constraints on first three frequencies/Hz	$\omega_1 \geq 5$, $\omega_3 \geq 7$

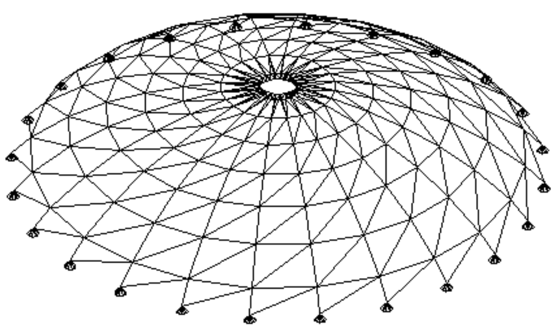

Fig. 3 Schematic of the 600-bar single layer dome

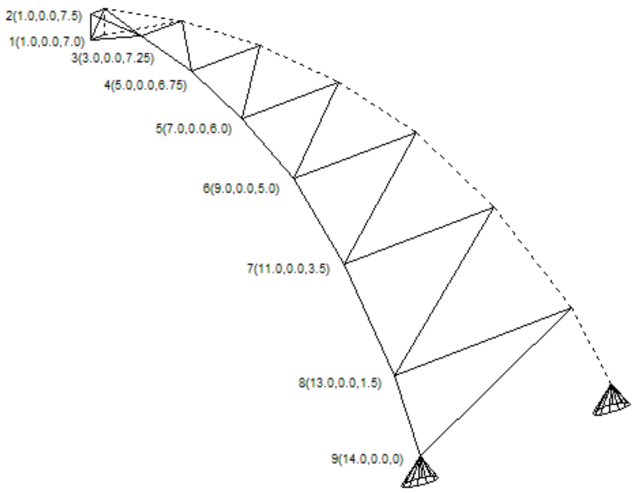

Fig. 4 Details of a substructure of the 600-bar single layer dome

Table 2 Optimized design for the 600-bar dome truss problem (added masses are not included)

Element no. (nodes)	**Cross-sectional area (cm^2)**	**Element no. (nodes)**	**Cross-sectional area (cm^2)**
1 (1-2)	1.365	14 (5-13)	5.529
2 (1-3)	1.391	15 (5-14)	7.007
3 (1-10)	5.686	16 (6-7)	5.462
4 (1-11)	1.511	17 (6-14)	3.853
5(2-3)	17.711	18 (6-15)	7.432
6 (2-11)	36.266	19 (7-8)	4.261
7(3-4)	13.263	20 (7-15)	2.253
8 (3-11)	16.919	21 (7-16)	4.337
9 (3-12)	13.333	22 (8-9)	4.028
10 (4-5)	9.534	23 (8-16)	1.954
11 (4-12)	9.884	24 (8-17)	4.709
12 (4-13)	9.547	25 (9-17)	1.410
13 (5-6)	7.866	Weight (kg)	6344.55

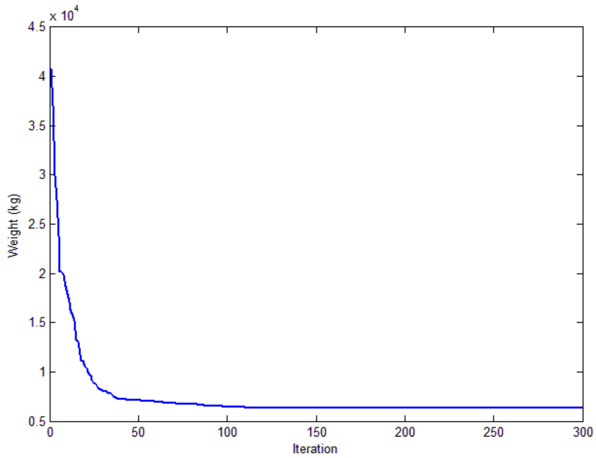

Fig. 5 Convergence curve of the best result for the 600-bar dome truss using the efficient method

Using the classical method it takes 2.6150 sec to perform a typical analysis for this structure while the efficient method needs 0.0198 sec i.e. the efficient method is about 132 times faster on a single analysis. Two different optimization cases are performed on this example as well as the other two. In case 1, the initial eigenproblem is solved directly using Matlab internal eigenvalue function; this is called the classical method. In case 2, the abovementioned efficient method is used for the analysis part i.e., the initial eigenproblem is decomposed into several smaller ones and then each of the subproblems are solved using the same Matlab function. In this example, 30 particles and 300 iterations (9000 analyses) are used for both cases. The required computational time to complete a single optimization run for cases 1 and 2 is 27326.25 sec and 190.77 sec, respectively. This means that the optimization procedure could be performed about 143 times faster using the efficient analysis method under the same circumstances. This example was solved 10 times using the efficient analysis method and the best result is presented in Table 2.

The total computational time to perform 10 optimization runs using the efficient method is 1906.68 sec (less than an hour), while it would have taken approximately 273112.84 sec (more than 3 days) to perform the same runs using the classical method. Table 3 presents the first 5 natural frequencies of the optimized structure. It can be seen that the constraints are fully satisfied. These frequencies are in full agreement with the results of the classical analysis method up to 10 significant digits. The mean weight of the structures found in 10 runs is 6674.71 kg with a standard deviation of 473.21 kg. Fig. 5 shows the convergence curve of the best result for the 600-bar dome truss using the efficient method.

5.2 A 1180-bar dome truss

The second test problem solved in this study was the weight minimization of the 1180-bar dome truss structure shown in Fig. 6. The entire structure is composed of 400 nodes and 1180 elements generated by cyclic repetition of a substructure with 20 nodes and 59 elements. The angle of cyclic symmetry between similar substructures is 18 degrees. A non-structural mass of 100 kg is attached to all free nodes. Table 4 summarizes the material properties, variable bounds, and frequency constraints for this example. Fig. 7 shows a substructure in more detail for nodal numbering. Table 5 summarizes the coordinates of the nodes in Cartesian coordinate system. Each of the elements of this substructure is considered as a design variable. Thus, this is a size optimization problem with 59 variables.

Table 3 Natural frequencies (Hz) evaluated at the optimized design for the 600-bar dome truss problem

Frequency number	Frequency value
1	5.000
2	5.000
3	7.000
4	7.000
5	7.000

Table 4 Material properties, variable bounds and frequency constraints for the 1180-bar dome truss

Property/unit	Value
E (Modulus of elasticity)/ N/m^2	2×10^{11}
ρ (Material density)/ kg/m^3	7850
Added mass/kg	100
Design variable lower bound/m^2	1×10^{-4}
Design variable upper bound/m^2	100×10^{-4}
Constraints on first three frequencies/Hz	$\omega_1 \geq 7$, $\omega_3 \geq 9$

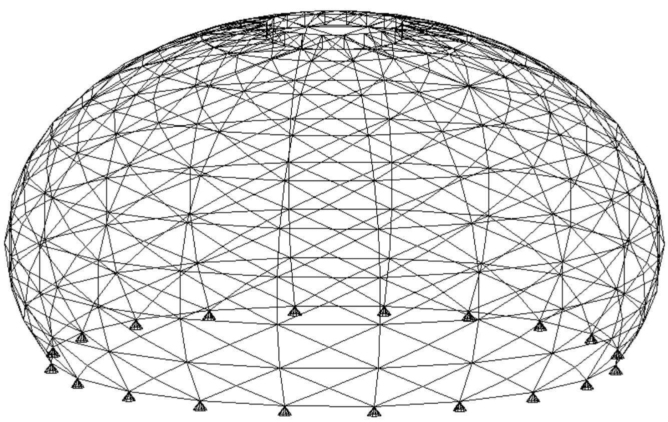

Fig. 6 Schematic of the 1180-bar dome truss

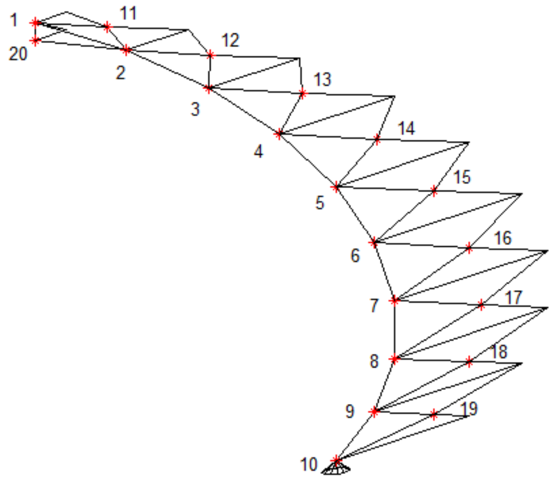

Fig. 7 Details of a substructure of the 1180-bar dome truss

Table 5 Coordinates of the nodes of the main structure (the 1180-bar dome truss)

Node no.	Coordinates (x, y, z)	Node no.	Coordinates (x, y, z)
1	(3.1181, 0.0, 14.6723)	11	(4.5788, 0.7252, 14.2657)
2	(6.1013, 0.0, 13.7031)	12	(7.4077, 1.1733, 12.9904)
3	(8.8166, 0.0, 12.1354)	13	(9.9130, 1.5701, 11.1476)
4	(11.1476, 0.0, 10.0365)	14	(11.9860, 1.8984, 8.8165)
5	(12.9904, 0.0, 7.5000)	15	(13.5344, 2.1436, 6.1013)
6	(14.2657, 0.0, 4.6358)	16	(14.4917, 2.2953, 3.1180)
7	(14.9179, 0.0, 1.5676)	17	(14.8153, 2.3465, 0.0)
8	(14.9179, 0.0, -1.5677)	18	(14.4917, 2.2953, -3.1181)
9	(14.2656, 0.0, -4.6359)	19	(13.5343, 2.1436, -6.1014)
10	(12.9903, 0.0, -7.5001)	20	(3.1181, 0.0, 13.7031)

A single analysis takes up to 11.3575 sec of computational time using the classical method. The required computational time for a similar analysis using the efficient method is only 0.0720 sec. This means that the efficient method is about 157 times faster for a single analysis. 100 particles and 500 iterations (50000 analyses) are used for optimization of this test problem. The required computational time to complete a single run for case 2 is 7095.56 sec. Fig. 8 shows the variation of the computational time with number of analyses for case 1. According to the figure, it is estimated that it would take 800160 sec to perform the same optimization run for case 1 (50000 analyses).

Table 6 Optimized design for the 1180-bar dome truss problem (added masses are not included)

Element no. (nodes)	Cross-sectional area (cm^2)	Element no. (nodes)	Cross-sectional area (cm^2)
1 (1-2)	7.926	31 (8-9)	34.642
2 (1-11)	10.426	32 (8-17)	19.860
3 (1-20)	2.115	33 (8-18)	25.079
4 (1-21)	14.287	34 (8-28)	18.965
5 (1-40)	3.846	35 (9-10)	47.514
6 (2-3)	5.921	36 (9-18)	28.133
7 (2-11)	7.955	37 (9-19)	33.023
8 (2-12)	6.697	38 (9-29)	32.263
9 (2-20)	1.889	39 (10-19)	33.401
10 (2-22)	11.881	40 (10-30)	1.344
11 (3-4)	7.121	41 (11-21)	9.327
12 (3-12)	6.080	42 (11-22)	7.202
13 (3-13)	6.599	43 (12-22)	6.792
14 (3-23)	7.772	44 (12-23)	6.228
15 (4-5)	9.358	45 (13-23)	6.601
16 (4-13)	6.213	46 (13-24)	6.584
17 (4-14)	8.200	47 (14-24)	8.320
18 (4-24)	7.799	48 (14-25)	8.844
19 (5-6)	11.752	49 (15-25)	11.254
20 (5-14)	7.494	50 (15-26)	12.162
21 (5-15)	9.696	51 (16-26)	13.854
22 (5-25)	9.177	52 (16-27)	13.844
23 (6-7)	17.326	53 (17-27)	17.536
24 (6-15)	11.797	54 (17-28)	20.551
25 (6-16)	14.002	55 (18-28)	24.072
26 (6-26)	11.562	56 (18-29)	27.287
27 (7-8)	23.981	57 (19-29)	32.965
28 (7-16)	12.996	58 (19-30)	36.940
29 (7-17)	16.591	59 (20-40)	3.837
30 (7-27)	15.910	Weight (kg)	37779.81

Table 7 Natural frequencies (Hz) evaluated at the optimized design for the 1180-bar dome truss problem

Frequency number	**Frequency value**
1	7.000
2	7.000
3	9.000
4	9.000
5	9.005

Therefore, the optimization procedure could be performed about 113 times faster under the same circumstances using the efficient analysis method. Again, this example was solved 10 times using the efficient analysis method and the best result is presented in Table 6.

It takes 68933.06 sec to perform 10 optimization runs using the efficient method for this example, while it would have taken approximately 7773580 sec (about 90 days) to perform the same runs using the classical method. Table 7 presents the first 5 natural frequencies of the optimized structure for this example. The mean weight of the structures found in 10 runs is 38294.45 kg with a standard deviation of 550.5 kg. Fig. 9 shows the convergence curve of the best result for the 1180-bar dome truss using the efficient method.

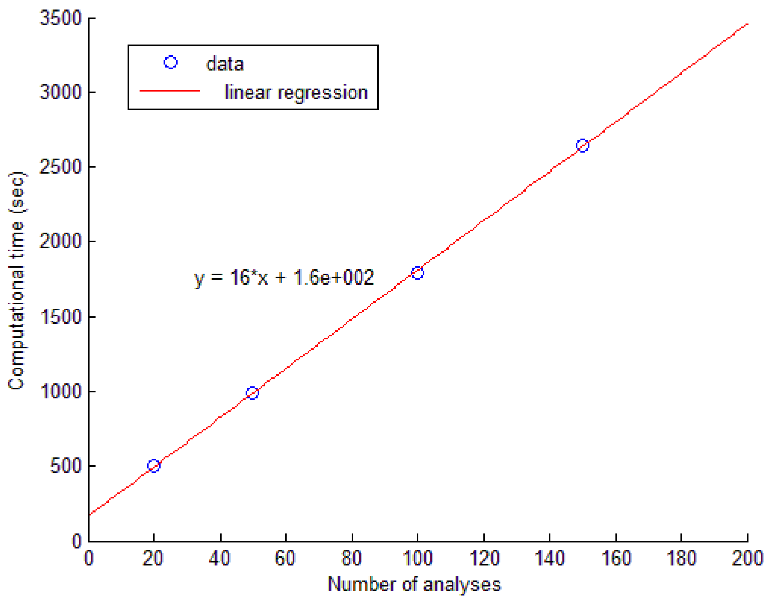

Fig. 8 Variation of the computational time with number of analyses for case 1 (1180-bar dome truss)

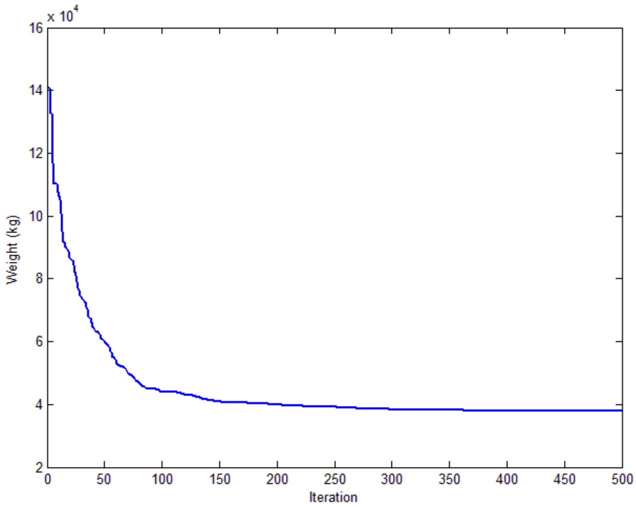

Fig. 9 Convergence curve of the best result for the 1180-bar dome truss using the efficient method

5.3 A 1410-bar double layer dome truss

The third test problem solved in this study was the weight minimization of the 1410- bar double layer dome truss as shown in Fig. 10. The entire structure is composed of 390 nodes and 1410 elements generated by cyclic repetition of a substructure with 13 nodes and 47 elements. The angle of cyclic symmetry between similar substructures is 12 degrees. A non-structural mass of 100 kg is attached to all free nodes. Table 8 summarizes the material properties, variable bounds, and frequency constraints for this example. Fig. 11 shows a substructure in more detail for nodal numbering. Table 9 presents the coordinates of the nodes in Cartesian coordinate system. Each of the elements of this substructure is considered as a design variable. Thus, this is a size optimization problem with 47 variables.

Required computational times for classical and efficient methods are 11.7101 and 0.0140 sec, respectively. Like the previous example, 100 particles and 500 iterations (50000 analyses) are used for optimization of this test problem. The required computational time to complete a single run for case 2 is 3871.62 sec. Fig. 12 shows the variation of the computational time with number of analysis for case 1. According to the figure, it is estimated that it would take 950240 sec to perform the same optimization run for case 1 (50000 analyses). Therefore, the optimization procedure could be performed about 245 times faster under the same circumstances using the efficient analysis method. This example was solved 10 times using the efficient analysis method and the best result is presented in Table 10.

It takes 38310.43 sec to perform 10 optimization runs using the efficient method for this example, while it would have taken approximately 9386055 sec (about 108 days) to perform the same runs using the classical method. Table 11 presents the first 5 natural frequencies of the optimized structure for this example. The mean weight of the structures found in 10 runs is 38294.45 kg with a standard deviation of 550.5 kg. Fig. 13 shows the convergence curve of the best result for the 1410-bar dome truss using the efficient method.

Table 8 Material properties, variable bounds and frequency constraints for the 1410-bar dome truss

Property/unit	**Value**
E (Modulus of elasticity)/ N/m^2	2×10^{11}
ρ (Material density)/ kg/m^3	7850
Added mass/kg	100
Design variable lower bound/m^2	1×10^{-4}
Design variable upper bound/m^2	100×10^{-4}
Constraints on first three frequencies/Hz	$\omega_1 \geq 7$, $\omega_3 \geq 9$

Table 9 Coordinates of the nodes of the main substructure (the 1410-bar dome truss)

Node no.	**Coordinates (x, y, z)**	**Node no.**	**Coordinates (x, y, z)**
1	(1.0, 0.0, 4.0)	8	(1.989, 0.209, 3.0)
2	(3.0, 0.0, 3.75)	9	(3.978, 0.418, 2.75)
3	(5.0, 0.0, 3.25)	10	(5.967, 0.627, 2.25)
4	(7.0, 0.0, 2.75)	11	(7.956, 0.836, 1.75)
5	(9.0, 0.0, 2.0)	12	(9.945, 1.0453, 1.0)
6	(11.0, 0.0, 1.25)	13	(11.934, 1.2543, -0.5)
7	(13.0, 0.0, 0.0)		

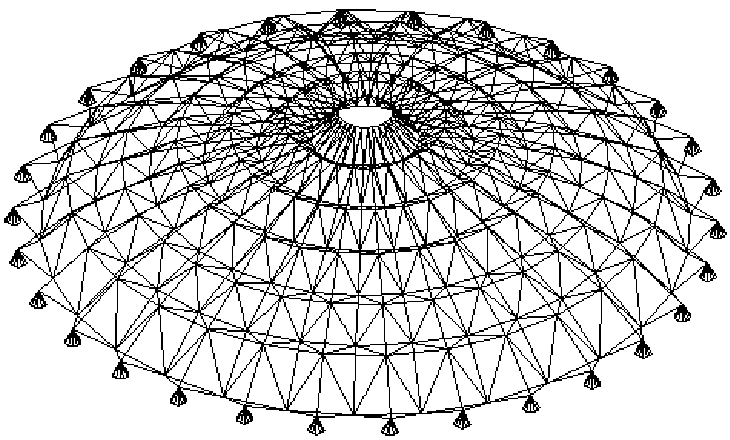

Fig. 10 Schematic of the 1410-bar dome truss

Table 10 Optimized design for the 1410-bar dome truss problem (added masses are not included)

Element no. (nodes)	Cross-sectional area (cm^2)	Element no. (nodes)	Cross-sectional area (cm^2)
1 (1-2)	7.209	25 (8-9)	2.115
2 (1-8)	5.006	26 (8-14)	4.923
3 (1-14)	38.446	27 (8-15)	4.047
4 (2-3)	9.438	28 (8-21)	5.906
5 (2-8)	4.313	29 (9-10)	3.392
6 (2-9)	1.494	30 (9-15)	1.902
7 (2-15)	8.455	31 (9-16)	4.381
8 (3-4)	9.488	32 (9-22)	8.442
9 (3-9)	3.480	33 (10-11)	5.011
10 (3-10)	3.495	34 (10-16)	3.577
11 (3-16)	16.037	35 (10-17)	2.805
12 (4-5)	9.796	36 (10-23)	2.024
13 (4-10)	2.413	37 (11-12)	6.709
14 (4-11)	5.681	38 (11-17)	5.054
15 (4-17)	15.806	39 (11-18)	3.259
16 (5-6)	8.078	40 (11-24)	1.063
17 (5-11)	3.931	41 (12-13)	5.934
18 (5-12)	6.099	42 (12-18)	7.057
19 (5-18)	10.771	43 (12-19)	5.745
20 (6-7)	13.775	44 (12-25)	1.185
21 (6-12)	4.231	45 (13-19)	7.274
22 (6-13)	6.995	46 (13-20)	4.798
23 (6-19)	1.837	47 (13-26)	1.515
24 (7-13)	4.397	Weight (kg)	10453.84

Table 11 Natural frequencies (Hz) evaluated at the optimized design for the 1410-bar dome truss problem

Frequency number	**Frequency value**
1	7.001
2	7.001
3	9.003
4	9.005
5	9.005

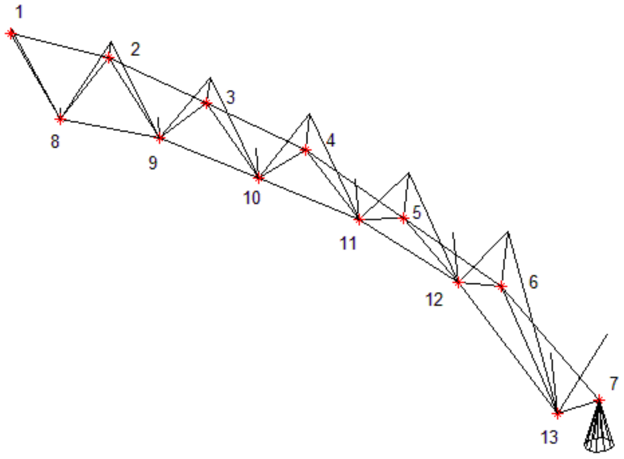

Fig. 11 Details of a substructure of the 1410-bar dome truss

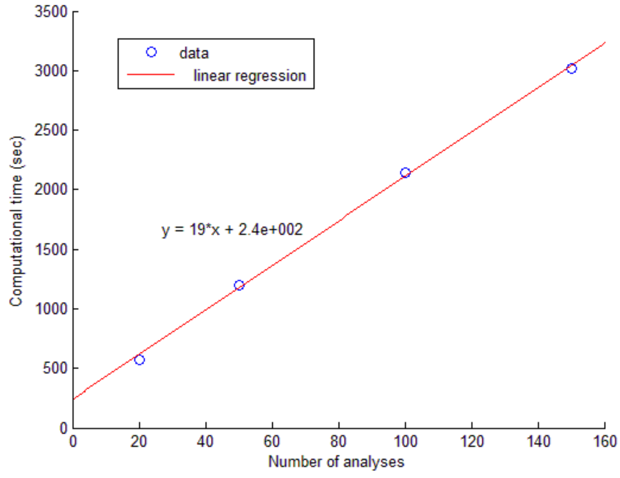

Fig. 12 Variation of the computational time with number of analysis for case 1 (1410-bar dome truss)

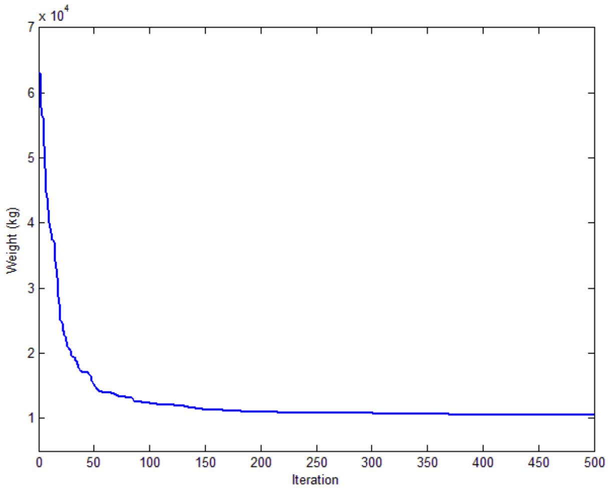

Fig. 13 Convergence curve of the best result for the 1410-bar dome truss using the efficient method

6. Conclusions

Structural optimization using meta-heuristic algorithms involves a large number of structural analyses, which requires a great amount of computational time, especially when optimizing large structural systems. In this paper simultaneous optimal analysis and design of cyclically repetitive dome trusses with frequency constraints is considered. These types of structures exhibit some favorable patterns in their structural matrices, which makes it possible to utilize some efficient analysis methods. These methods decompose the original eigenproblem into several smaller ones, which are simpler to solve and require less computational time. Democratic Particle Swarm Optimization (DPSO) introduced by Kaveh and Zolghadr (2014b) is utilized as the optimization algorithm.

Three different dome trusses are considered as numerical examples to show the efficiency of the proposed method. It can be seen that using the efficient method for analysis, the optimization procedure can be performed significantly faster. While all the runs are taken in less than 2 days using the efficient methods, it would have taken more than 200 days to do the same thing using classical methods. Such a substantial saving in computational time is due to the regular nature of the structures under consideration. Other types of efficient methods could also be used in order to deal with near-regular structures (Kaveh 2013).

The presented concepts can be generalized to optimization of other types of symmetric or regular structures as well as structural optimization with static constraints.

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