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Data fusion based improved Kalman filter with unknown inputs and without collocated acceleration measurements

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Abstract. The classical Kalman filter (KF) can provide effective state estimation for structural identification and vibration control, but it is applicable only when external inputs are measured. So far, some studies of Kalman filter with unknown inputs (KF-UI) have been proposed. However, previous KF-UI approaches based solely on acceleration measurements are inherently unstable which leads to poor tracking and fictitious drifts in the identified structural displacements and unknown inputs in the presence of measurement noises. Moreover, it is necessary to have the measurements of acceleration responses at the locations where unknown inputs applied, i.e., with collocated acceleration measurements in these approaches. In this paper, it aims to extend the classical KF approach to circumvent the above limitations for general real time estimation of structural state and unknown inputs without using collocated acceleration measurements. Based on the scheme of the classical KF, an improved Kalman filter with unknown excitations (KF-UI) and without collocated acceleration measurements is derived. Then, data fusion of acceleration and displacement or strain measurements is used to prevent the drifts in the identified structural state and unknown inputs in real time. Such algorithm is not available in the literature. Some numerical examples are used to demonstrate the effectiveness of the proposed approach.

Keywords: Kalman filter; unknown inputs; data fusion; structural identification; least-square estimation

1. Introduction

The identification of structural dynamic systems using the measurements of structural vibration data is essential for structural health monitoring and vibration control (Wang *et al*. 2009; Sirca and Adeli 2012, Li and Chen 2013, Yuen and Mu 2015). As it is impractical to measure all structural responses, structural identification using only partial measurements of structural responses have received great attentions (Papadimitriou *et al*. 2011, Yi *et al*. 2013, Xu and Jia 2012, Xu *et al.* 2015, Lei *et al*. 2014, 2015). In this regard, the Kalman filter (KF), which was proposed by R.E. Kalman in the early sixties (Kalman 1960), provides a particularly practical and efficient state estimation algorithm with partial measurements of structural responses. Also, KF can inherently take the uncertainty in the model into account (Hung *et al*. 2010, Yuen *et al*. 2013, Azam *et al.* 2015, Naets *et al.* 2015). However, in the classical KF approach, it is requested that all external inputs are known (measured).

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To circumvent the limitation of the classical KF approach, many improved approaches have been proposed for Kalman filter based identification of joint structural state and external inputs, e.g., an iterative identification procedure consisting of the least-squares identification technique and a modification process between each iterative step (Chen and Li 2004); the unbiased minimum-variance input and state estimation with direct feed through (Gillijns and Moor 2007); a Kalman filter with unknown inputs approach derived by the weighted least-squares estimation method (Pan *et al.* 2010); a two-stage Kalman estimator in which the classical Kalman filter is first adopted to establish a regression model between the residual innovation and then a recursive least-squares estimator is proposed to identify the input excitation forces (Liu *et al.* 2000, Ma *et al.* 2003, Wu *et al.* 2009); an augmented Kalman filter (AFK) for force identification in structural dynamics, in which the unknown forces are included in the state vector and estimated in conjunction with the states (Lourens *et al.* 2012); an average acceleration discrete algorithm with regularization (Ding *et al.* 2013) and implicit Newmark-β algorithm with regularization (Liu *et al.* 2014); a weighted adaptive iterative least-squares estimation with incomplete measured excitations (Xu *et al.* 2015); Kalman estimator with unknown inputs (Lei *et al.* 2012, 2014) and a two-stage and two-step algorithm (Lei *et al.* 2015). However, it has been demonstrated that previous Kalman filter with unknown input (KF-UI) using limited number of acceleration measurements are inherently unstable which leads poor tracking and so-called drifts in the estimated unknown external inputs and structural displacements (Azam *et al.* 2015, Naets *et al.* 2015). Although regularization approaches (Ding *et al.* 2013, Liu *et al.* 2015) or post-signal processing schemes (Lei *et al.*2012, 2014, 2015) can be used to treat the drift in the identified results, these treatments prohibits the on-line and real-time identification of coupled structural state and unknown inputs.

Recently, the authors have proposed an improved Kalman filter with unknown inputs based on data fusion of partial acceleration and displacement measurements for real time estimation of joint structural states and the unknown inputs (Liu *et al*. 2016). However, like other previous Kalman filter with unknown input (KF-UI), it is necessary to have the measurements of acceleration responses at the locations where unknown inputs applied, i.e., collocated acceleration measurements at the locations of unknown inputs are requested.

In this paper, it aims to extend the classical KF approach and overcome the drawbacks of existing KF-UI approaches for real time estimation of structural states and unknown inputs without using collocated acceleration measurements. Compared with the recent KF-UI (Liu *et al*. 2016), an improved Kalman filter with unknown excitations (KF-UI) and without collocated acceleration measurements is derived. Since accelerations and displacements contains high and low frequencies vibration characteristics, respectively (Smith *et al.* 2007, Ay and Wang 2014, Kim and Sohn 2014), data fusion of acceleration and displacement or strain measurements is used to prevent the low-frequency drifts in the identified structural state vector and unknown external inputs in real time. Numerical examples of the identification of joint structural state and unknown inputs of a multi-story shear type building and a plane truss are used to demonstrate the effectiveness and versatilities of the proposed algorithm.

2. Recent KF-UI approach

To circumvent the limitation of the classical KF approach, many improved approaches have been proposed for Kalman filter based identification of joint structural state and external inputs. Recently, the authors have presented an improved Kalman filter with unknown inputs (KF-UI) using data fusion of partially measured acceleration and displacement responses (Liu *et al.* 2016). For comparison, the main scheme of the recent KF-UI is briefly introduced as follows,

The equation of motion of a linear structural system under unknown external inputs can be described by

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(\mathrm{t})+\boldsymbol{C}\dot{\boldsymbol{x}}(\mathrm{t})+\boldsymbol{K}\boldsymbol{x}(\mathrm{t})=\boldsymbol{E}\boldsymbol{f}^{u}(\mathrm{t}) \tag{1}$$

in which $\boldsymbol{x}(\mathrm{t})$, $\dot{\boldsymbol{x}}(\mathrm{t})$ and $\ddot{\boldsymbol{x}}(\mathrm{t})$ are vectors of displacements, velocity and acceleration responses, respectively; $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping, and stiffness matrices, $\boldsymbol{f}^{u}(\mathrm{t})$ is the unmeasured external input vector with influence matrix $\boldsymbol{E}$.

Based on the zero-order holder (ZOH) discretization of the above equation and the consideration of uncertainty in modeling, the state equation of the system in the discrete form can be expressed as

$$\boldsymbol{X}_{k+1}=\boldsymbol{A}_k\boldsymbol{X}_k+\boldsymbol{B}_k\boldsymbol{f}_k^{u}+\boldsymbol{w}_k \tag{2}$$

where $\boldsymbol{X}_k$ is the state vector at time $t=k\Delta t$ with Δt being the sampling time step. $\boldsymbol{A}_k$ is the state transformation matrix, $\boldsymbol{B}_k$ is the influence matrix of unknown input vector $\boldsymbol{f}_k^{u}$, and $\boldsymbol{w}_k$ is the model uncertainty which is assumed a noise with zero mean and a covariance matrix $\boldsymbol{Q}_k$.

In practice, only partial structural responses can be measured. The discrete form of the observation equation can be expressed as

$$\boldsymbol{Y}_{k+1}=\boldsymbol{C}_{k+1}\boldsymbol{X}_{k+1}+\boldsymbol{D}_{k+1}\boldsymbol{f}_{k+1}^{u}+\boldsymbol{v}_{k+1} \tag{3}$$

where $\boldsymbol{Y}_{k+1}$ is the measured response vector at time $t=(k+1)\Delta t$, $\boldsymbol{C}_{k+1}$ and $\boldsymbol{D}_{k+1}$ are two known measurement matrices associated with structural state and external force vectors, respectively, and $\boldsymbol{v}_{k+1}$ is the measurement noise vector, which is assumed a Gaussian white noise vector with zero mean and a covariance matrix $\boldsymbol{R}_{k+1}$.

Analogous to the classical KF scheme, the proposed KF-UI also contains two procedures. First, $\tilde{\boldsymbol{X}}_{k+1|k}$ is predicted as

$$\tilde{\boldsymbol{X}}_{k+1|k}=\boldsymbol{A}_k\hat{\boldsymbol{X}}_{k|k}+\boldsymbol{B}_k\hat{\boldsymbol{f}}_{k|k}^{u} \tag{4}$$

where $\tilde{\boldsymbol{X}}_{k+1|k}$, $\hat{\boldsymbol{X}}_{k|k}$ and $\hat{\boldsymbol{f}}_{k|k}^{u}$ denote the predicted $\boldsymbol{X}_{k+1}$, estimated $\boldsymbol{X}_k$ and the estimated $\boldsymbol{f}^{u}$ at time at time $t=k\Delta t$, respectively.

Then, the estimated $\boldsymbol{X}_{k+1}$ in the measurement update (correction) procedure is derived as

$$\hat{\boldsymbol{X}}_{k+1|k+1}=\tilde{\boldsymbol{X}}_{k+1|k}+\boldsymbol{K}_{k+1}(\boldsymbol{Y}_{k+1}-\boldsymbol{C}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k}-\boldsymbol{D}_{k+1}\hat{\boldsymbol{f}}_{k+1|k+1}^{u}) \tag{5}$$

where $\hat{\boldsymbol{X}}_{k+1|k+1}$ and $\hat{\boldsymbol{f}}_{k+1|k+1}^{u}$ are the estimated $\boldsymbol{X}_{k+1}$ and $\boldsymbol{f}_{k+1}^{u}$ given the observations ($\boldsymbol{Y}_1, \boldsymbol{Y}_{2,\ldots,}$ $\boldsymbol{Y}_{k+1}$) , respectively, $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix which can be derived as

$$\boldsymbol{K}_{k+1}=\tilde{\mathbf{P}}_{k+1|k+1}\boldsymbol{C}_{k+1}^{\mathrm{T}}(\boldsymbol{C}_{k+1}\tilde{\mathbf{P}}_{k+1|k}\boldsymbol{C}_{k+1}^{\mathrm{T}}+\boldsymbol{R}_{k+1})^{-1} \tag{6}$$

in which $\tilde{\mathbf{P}}_{k+1|k}$ is the error covariance of the predicted $\tilde{\boldsymbol{X}}_{k+1|k}$.

Under the condition that the number of response measurements (sensors) is no less than the number of the unknown inputs, $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ can be estimated based on the least-squares estimation as

$$\hat{\boldsymbol{f}}^u_{k+1|k+1} = \boldsymbol{S}_{k+1}\boldsymbol{D}^{\mathrm{T}}_{k+1}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\left(\boldsymbol{Y}_{k+1}-\boldsymbol{C}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k}\right) \tag{7}$$

where $\boldsymbol{I}$ denotes a unit matrix, and

$$\boldsymbol{S}_{k+1} = \left[\boldsymbol{D}^{\mathrm{T}}_{k+1}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\boldsymbol{D}_{k+1}\right]^{-1} \tag{8}$$

The covariance matrix for error $\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1}$ can be derived as (Liu *et al.* 2016).

$$\hat{\boldsymbol{P}}^{\boldsymbol{f}}_{k+1|k+1} = \boldsymbol{S}_{k+1}\boldsymbol{D}^{\mathrm{T}}_{k+1}\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)^T\boldsymbol{R}^{-T}_{k+1}\boldsymbol{D}_{k+1}\boldsymbol{S}^{\mathrm{T}}_{k+1} = \boldsymbol{S}_{k+1} \tag{9}$$

Also, other error covariance matrices can be derived as (Liu *et al.* 2016)

$$\hat{\boldsymbol{P}}^{\boldsymbol{X}}_{k+1|k+1} = \left(\boldsymbol{I}+\boldsymbol{K}_{k+1}\boldsymbol{D}_{k+1}\boldsymbol{S}_{k+1}\boldsymbol{D}^{\mathrm{T}}_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{C}_{k+1}\right)\left(\boldsymbol{I}-\boldsymbol{K}_{k+1}\boldsymbol{C}_{k+1}\right)\tilde{\boldsymbol{P}}^{\boldsymbol{X}}_{k+1|k} \tag{10}$$

$$\hat{\boldsymbol{P}}^{\boldsymbol{Xf}}_{k+1|k+1} = \left(\hat{\boldsymbol{P}}^{\boldsymbol{Xf}}_{k+1|k+1}\right)^T = E\left[\hat{\boldsymbol{e}}^{\boldsymbol{X}}_{k+1|k+1}\left(\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1}\right)^T\right] = -\boldsymbol{K}_{k+1}\boldsymbol{D}_{k+1}\boldsymbol{S}_{k+1} \tag{11}$$

However, it is noted that matrix $\boldsymbol{D}$ in Eq. (7) should exist for the recursive estimation of unknown input by Eq. (7). This requires the measurements of acceleration responses at the locations where unknown inputs applied, i.e., collocated acceleration measurements at the locations of unknown inputs are requested. This limitation is also requested in other previous KF-UI methodologies.

3. The improved KF-UI without collocated acceleration measurements

To further circumvent the limitation of the existing KF-UI approaches, it is proposed to extend the recent KF-UI for the identification of joint structural states and unknown inputs without using collocated acceleration measurements.

Instead of using the conventional zero-order holder (ZOH) discretization (constant interpolated), in which the unknown input force vector in a sampling period is assumed to be constant in Eq. (2), the first-order holder (FOH) discretization is used herein (Ding *et al.* 2013). This discretizationit within a sampling period is treated differently in the FOH discrete method, namely the triangle hold linear discrete (linear interpolated) method, in which the discrete data is interpolated as

$$\boldsymbol{f}^u(\mathrm{t}) = \boldsymbol{f}^u_k + \frac{\boldsymbol{f}^u_{k+1}-\boldsymbol{f}^u_k}{\Delta t}(t-k\Delta t) \quad ; \quad \mathrm{k}\Delta t \le \mathrm{t} \le (\mathrm{k}+1)\Delta t \tag{12}$$

Then, Eq. (1) can be converted into the following state equation as follows

$$\boldsymbol{X}_{k+1} = \boldsymbol{A}_k\boldsymbol{X}_k + \boldsymbol{B}_k\boldsymbol{f}^u_k + \boldsymbol{G}_{k+1}\boldsymbol{f}^u_{k+1} + \boldsymbol{w}_k \tag{13}$$

where matrices $\boldsymbol{A}_k$,$\boldsymbol{B}_k$ and $\boldsymbol{G}_{\mathrm{k}+1}$ have been explained after Eq. (2).

In this paper, it is considered that the acceleration responses at the location of external input are not measured. Therefore, the discrete observation equation is described by

$$\boldsymbol{Y}_{k+1} = \boldsymbol{C}_{k+1}\boldsymbol{X}_{k+1} + \boldsymbol{v}_{k+1} \tag{14}$$

where $\boldsymbol{Y}_{k+1}$ is also the measured response vector, $\boldsymbol{C}_{k+1}$ is the measurement matrix, and $\boldsymbol{v}_{k+1}$ is the measurement noise vector with zero mean and a covariance matrix $\boldsymbol{R}_{k+1}$.

Analogously, $\tilde{\boldsymbol{X}}_{k+1|k}$ is first predicted as

$$\tilde{\boldsymbol{X}}_{k+1|k} = \boldsymbol{A}_k\hat{\boldsymbol{X}}_{k|k} + \boldsymbol{B}_k\hat{\boldsymbol{f}}^u_{k|k} + \boldsymbol{G}_{k+1}\hat{\boldsymbol{f}}^u_{k+1|k+1} \tag{15}$$

Then, the measurement updated (corrected) $\hat{\boldsymbol{X}}_{k+1|k+1}$ can be obtained by

$$\hat{\boldsymbol{X}}_{k+1|k+1} = \tilde{\boldsymbol{X}}_{k+1|k} + \boldsymbol{K}_{k+1}(\boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k}) \tag{16}$$

where $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix.

Under the condition that the number of measurements (sensors) is no less than that of the unknown inputs, $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ can be estimated by minimizing the error vector $\boldsymbol{\Delta}_{k+1}$ defined by

$$\boldsymbol{\Delta}_{k+1} = \boldsymbol{y}_{k+1} - \boldsymbol{C}_{k+1}\hat{\boldsymbol{X}}_{k+1|k+1} \tag{17}$$

By inserting the expression of $\hat{\boldsymbol{X}}_{k+1|k+1}$ and $\tilde{\boldsymbol{X}}_{k+1|k}$ in Eqs. (16) and (15), respectively into the above error vector, $\boldsymbol{\Delta}_{k+1}$ can be expressed by

$$\boldsymbol{\Delta}_{k+1} = \left(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\left[\boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}\left(\boldsymbol{A}_k\hat{\boldsymbol{X}}_{k|k} + \boldsymbol{B}_k\hat{\boldsymbol{f}}^u_{k|k}\right)\right] - \left(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\boldsymbol{C}_{k+1}\boldsymbol{G}_{k+1}\hat{\boldsymbol{f}}^u_{k+1|k+1} \tag{18}$$

Then, $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ can be estimated from Eq.(18) based on least-squares estimation as

$$\hat{\boldsymbol{f}}^u_{k+1|k+1} = \boldsymbol{M}_{k+1}\left[\boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}(\boldsymbol{A}_k\hat{\boldsymbol{X}}_{k|k} + \boldsymbol{B}_k\hat{\boldsymbol{f}}^u_{k|k})\right] \tag{19}$$

where $\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\boldsymbol{G}_{k+1} = \boldsymbol{I}$

$$\boldsymbol{M}_{k+1} = (\boldsymbol{C}_{k+1}\boldsymbol{G}_{k+1})^T\left[(\boldsymbol{C}_{k+1}\boldsymbol{G}_{k+1})(\boldsymbol{C}_{k+1}\boldsymbol{G}_{k+1})^T\right]^{-1} \tag{20}$$

The error of state estimation defined as $\hat{\boldsymbol{e}}^X_{k+1|k+1} = \boldsymbol{X}_{k+1} - \hat{\boldsymbol{X}}_{k+1|k+1}$ can be derived from Eqs. (14)-(16) as

$$\hat{\boldsymbol{e}}^X_{k+1|k+1} = \left(\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{C}_{k+1}\right)\tilde{\boldsymbol{e}}^X_{k+1|k} - \boldsymbol{K}_{k+1}\boldsymbol{v}_{k+1} \tag{21}$$

where $\tilde{\boldsymbol{e}}^X_{k+1|k}$ is defined as $\tilde{\boldsymbol{e}}^X_{k+1|k} = \boldsymbol{X}_{k+1} - \tilde{\boldsymbol{X}}_{k+1|k}$. Form Eqs. (14) and (15), it is known that

$$\tilde{\boldsymbol{e}}^X_{k+1|k} = \boldsymbol{A}_k\hat{\boldsymbol{e}}^X_{k|k} + \boldsymbol{G}_{k+1}\hat{\boldsymbol{e}}^f_{k+1|k+1} + \boldsymbol{B}_k\hat{\boldsymbol{e}}^f_{k|k} + \boldsymbol{w}_k \tag{22}$$

where $\hat{\boldsymbol{e}}^{f}_{k+1|k+1}$ is error of estimated $\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$ defined as $\hat{\boldsymbol{e}}^{f}_{k+1|k+1}=\boldsymbol{f}^{u}_{k+1}-\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$. By inserting $\boldsymbol{Y}_{k+1}$ in Eq. (14) into Eq. (19), $\hat{\boldsymbol{e}}^{f}_{k+1|k+1}$ can be derived by

$$\hat{\boldsymbol{e}}^{f}_{k+1|k+1}=-\boldsymbol{M}_{k+1}\left[\boldsymbol{C}_{k+1}\left(\boldsymbol{A}_k\hat{\boldsymbol{e}}^{X}_{k|k}+\boldsymbol{B}_k\hat{\boldsymbol{e}}^{f}_{k|k}\right)+\boldsymbol{C}_{k+1}\boldsymbol{w}_k+\boldsymbol{v}_{k+1}\right] \tag{23}$$

From Eq. (21), the error covariance matrix $\hat{\boldsymbol{P}}^{X}_{k+1|k+1}$ is estimated as

$$\hat{\boldsymbol{P}}^{X}_{k+1|k+1}=\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\tilde{\boldsymbol{P}}^{X}_{k+1|k}\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)^{\mathrm{T}}+\boldsymbol{K}_{k+1}\boldsymbol{R}_{k+1}\boldsymbol{K}^{\mathrm{T}}_{k+1} \tag{24}$$

To minimize the error covariance matrix $\hat{\boldsymbol{P}}^{X}_{k+1|k+1}$, $\boldsymbol{K}_{k+1}$ should be selected as

$$\boldsymbol{K}_{k+1}=\tilde{\boldsymbol{P}}^{X}_{k+1|k}\boldsymbol{C}^{T}_{k+1}(\boldsymbol{C}_{k+1}\tilde{\boldsymbol{P}}^{X}_{k+1|k}\boldsymbol{C}^{T}_{k+1}+\boldsymbol{R}_{k+1})^{-1} \tag{25}$$

Then, $\hat{\boldsymbol{P}}^{X}_{k+1|k+1}$ in Eq. (24) can be simplified

$$\hat{\boldsymbol{P}}^{X}_{k+1|k+1}=\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\tilde{\boldsymbol{P}}^{X}_{k+1|k} \tag{26}$$

The error covariance matrix $\hat{\boldsymbol{P}}^{f}_{k+1|k+1}$ can be estimated from Eqs. (23) as

$$\begin{aligned}\hat{\boldsymbol{P}}^{f}_{k+1|k+1}&=\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\left[\boldsymbol{A}_k\ \ \boldsymbol{B}_k\right]\begin{bmatrix}\hat{\boldsymbol{P}}^{X}_{k|k} & \hat{\boldsymbol{P}}^{Xf}_{k|k}\\ \hat{\boldsymbol{P}}^{fX}_{k|k} & \hat{\boldsymbol{P}}^{f}_{k|k}\end{bmatrix}\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}_{k}\\ \boldsymbol{B}^{T}_{k}\end{bmatrix}\left(\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)^{\mathrm{T}}+\boldsymbol{M}_{k+1}\boldsymbol{R}_{k+1}\boldsymbol{M}^{\mathrm{T}}_{k+1}\\&+\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\boldsymbol{Q}_k\left(\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)^{\mathrm{T}}\end{aligned} \tag{27}$$

where $\hat{\boldsymbol{P}}^{Xf}_{k|k}$ and $\hat{\boldsymbol{P}}^{fX}_{k|k}$ are the two error covariance matrices defined as

$$\hat{\boldsymbol{P}}^{Xf}_{k|k}=\mathrm{E}\left(\boldsymbol{e}^{X}_{k|k}\boldsymbol{e}^{f\,\mathrm{T}}_{k|k}\right);\quad \hat{\boldsymbol{P}}^{fX}_{k|k}=\mathrm{E}\left(\boldsymbol{e}^{f}_{k|k}\boldsymbol{e}^{X\,\mathrm{T}}_{k|k}\right) \tag{28}$$

From Eqs. (19) and (20), the error of predicted state $\tilde{\boldsymbol{e}}^{X}_{k+1|k}$ can be rewritten as

$$\tilde{\boldsymbol{e}}^{X}_{k+1|k}=(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1})(\boldsymbol{A}_k\hat{\boldsymbol{e}}^{X}_{k|k}+\boldsymbol{B}_k\hat{\boldsymbol{e}}^{f}_{k|k}+\boldsymbol{w}_k)-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{v}_{k+1} \tag{29}$$

So the error covariance matrix $\tilde{\boldsymbol{P}}^{x}_{k+1|k}$ is expressed as

$$\begin{aligned}\tilde{\boldsymbol{P}}^{X}_{k+1|k}&=\left(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)\left[\boldsymbol{A}_k\ \ \boldsymbol{B}_k\right]\begin{bmatrix}\hat{\boldsymbol{P}}^{X}_{k|k} & \hat{\boldsymbol{P}}^{Xf}_{k|k}\\ \hat{\boldsymbol{P}}^{fX}_{k|k} & \hat{\boldsymbol{P}}^{f}_{k|k}\end{bmatrix}\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}_{k}\\ \boldsymbol{B}^{T}_{k}\end{bmatrix}\left(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)^{\mathrm{T}}\\&+\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{R}_{k+1}\boldsymbol{M}^{\mathrm{T}}_{k+1}\boldsymbol{G}^{T}_{k+1}+\left(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)\boldsymbol{Q}_{k+1}\left(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)^{\mathrm{T}}\end{aligned} \tag{30}$$

and the error covariance matrix $\hat{\boldsymbol{P}}^{Xf}_{k+1|k+1}$ can be derived from Eq. (29) as

$$\hat{\boldsymbol{P}}^{\boldsymbol{Xf}}_{k+1|k+1} = -\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\left(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)\left[\boldsymbol{A}_k \;\; \boldsymbol{B}_k\right]\begin{bmatrix}\hat{\boldsymbol{P}}^{X}_{k|k} & \hat{\boldsymbol{P}}^{Xf}_{k|k}\\ \hat{\boldsymbol{P}}^{fX}_{k|k} & \hat{\boldsymbol{P}}^{f}_{k|k}\end{bmatrix}\begin{bmatrix}\boldsymbol{A}^{\mathrm{T}}_k\\ \boldsymbol{B}^{T}_k\end{bmatrix}\left(\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)$$
$$+\left[\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\boldsymbol{B}_k\boldsymbol{M}_{k+1}+\boldsymbol{K}_{k+1}\right]\boldsymbol{R}_{k+1}\boldsymbol{M}^{\mathrm{T}}_{k+1}-\left(\boldsymbol{I}-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\right)\left(\boldsymbol{I}-\boldsymbol{G}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)\boldsymbol{Q}_{k+1}\left(\boldsymbol{M}_{k+1}\boldsymbol{C}_{k+1}\right)^{\mathrm{T}} \tag{31a}$$

and

$$\hat{\boldsymbol{P}}^{\boldsymbol{fX}}_{k+1|k+1} = \left(\hat{\boldsymbol{P}}^{\boldsymbol{Xf}}_{k+1|k+1}\right)^{\mathrm{T}} \tag{31b}$$

In practice, accelerometers are often used in structural dynamics applications. However, previous KF-UI approaches based only on sparse noisy acceleration measurements are inherently unstable which leads to the so-called spurious drifts in the estimated unknown inputs and structural displacements. The reason can be considered from a physical point of view. At any given time it is not clear whether acceleration is the effect of external forces or from the elastic restoring force. Although either regularization scheme or post-signal processing can be used to treat the drift problem. These approaches prohibit the on-line identification of coupled structural state and unknown inputs. In this paper, it is proposed to add partial measured displacements to the measured accelerations since accelerations and displacements contains high and low frequencies vibration characteristics, respectively. Data fusion of acceleration and displacement or strain measurements is used in the observation equation (Smith *et al.* 2007, Ay and Wang 2014, Kim and Sohn 2014).

In summary, the procedures of the proposed KF-UI without collocated acceleration measurements are shown in Fig. 1.

4. Numerical validations of the proposed KF-UI

Two numerical examples are used to validate the proposed KF-UI for the identification of joint structural states and unknown inputs. The theoretically computed displacement and acceleration responses are superimposed with corresponding white noises to consider the influence of measurement noises. These polluted responses are treated as “measured responses” for the identification problem.

4.1 Identification of multi-story shear building and unknown input

A twenty-story shear building is used as an example. Parameters of the building are assumed as: floor mass m_i=60kg, floor stiffness k_i=1.2x10^6 N/m, floor damping c_i=1000Ns/m (i=1,2,..,20), respectively. An input of wide-banded white noise is applied to the top floor of the building.

First, only three acceleration measurements at the 6th, 14th and 19th story floors are used in the proposed KF-UI. Measurement noises with 5% noise-to-signal ratio in root mean square (RMS) are considered. As shown in Fig. 2, significant drifts occur in the identified displacement and the identified unknown input.

To circumvent the spurious low-frequency drift problem, partial measured displacements are added in the improved KF-UI. For this relatively simple structural model, the displacement at the 2nd floor of the shear building is added in the measured signals. Data fusion of the measured displacement and the above three accelerations are used.

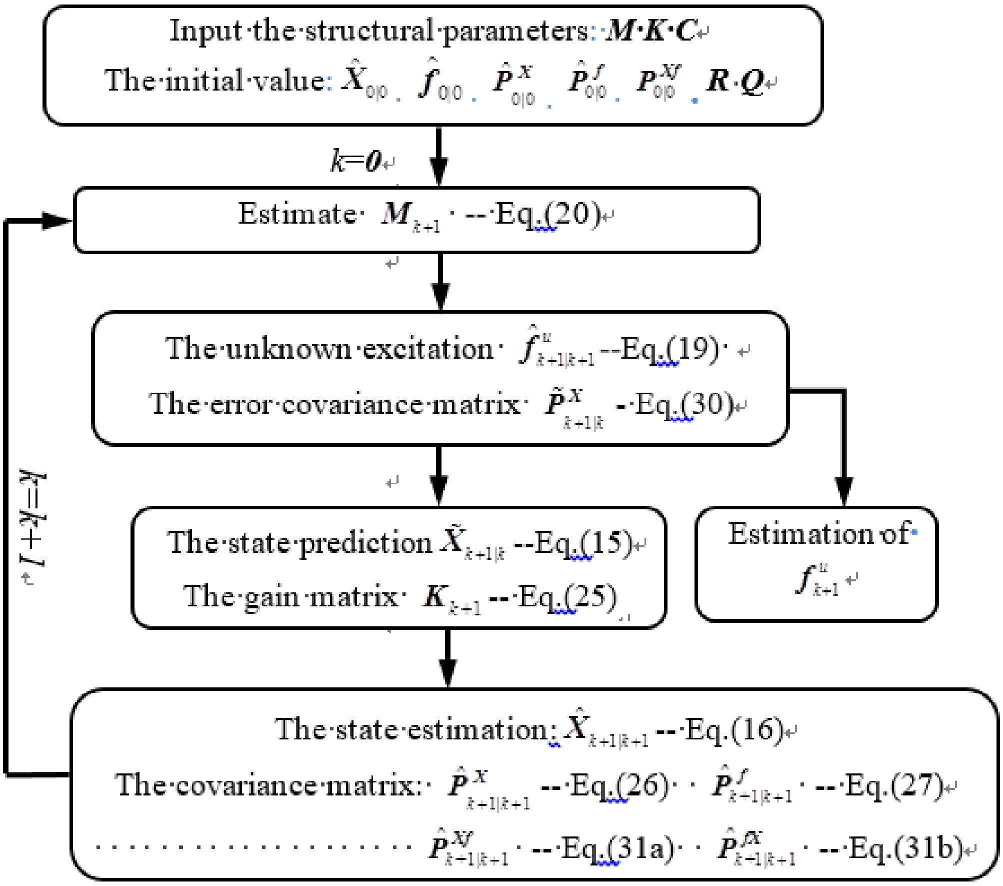
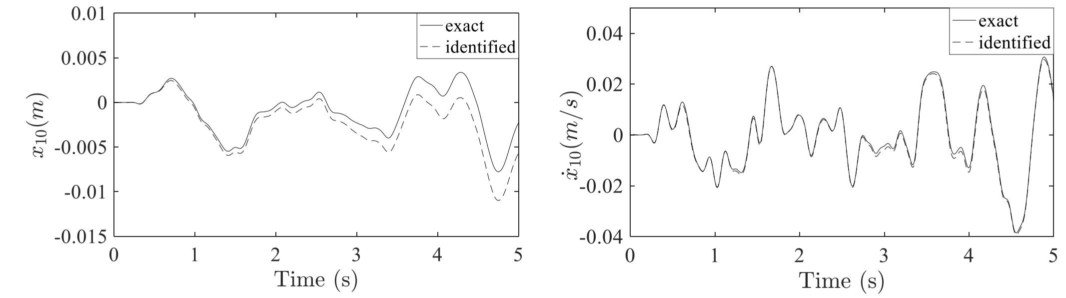


Fig. 1 Procedures of the proposed KF-UI without collocated acceleration measurements

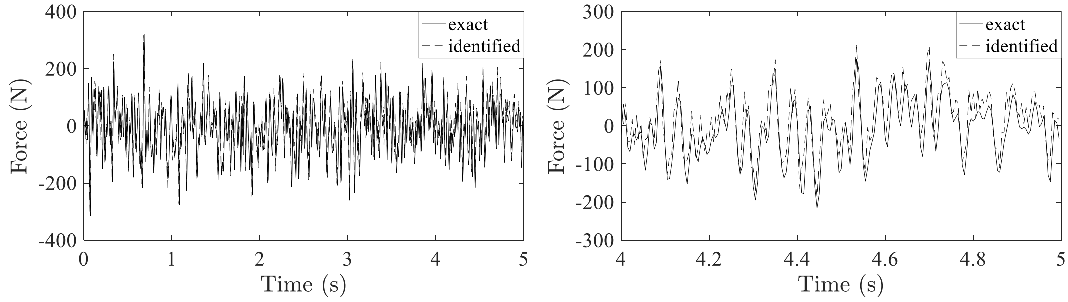
Fig. 3 shows the comparisons of identified structural state and input with their exact values. It is clearly demonstrated that the so-called drift can be avoided by the proposed KF-UI with data fusion of 5% noisy acceleration and displacement measurements.

From the above comparisons, it is noted that the recursive identification results of both structural state and unknown input by the proposal algorithm quickly coverage to the corresponding actual values and the identification accuracies are satisfactory. However, the identification accuracy decreases with the increase of measurement noise level.



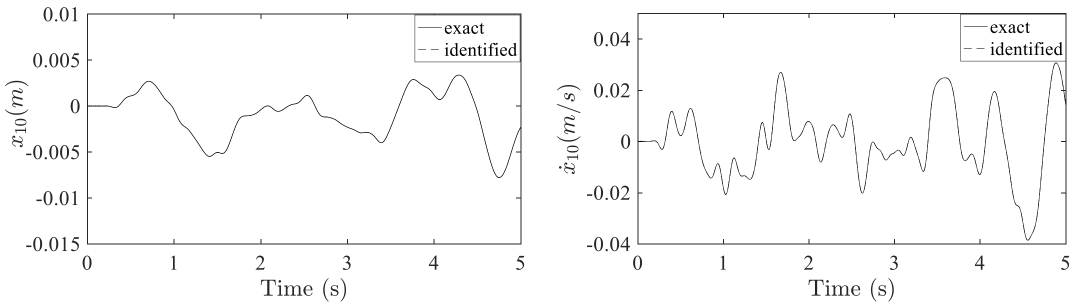
(a) Drift in the identified displacement

(b) Identified velocity



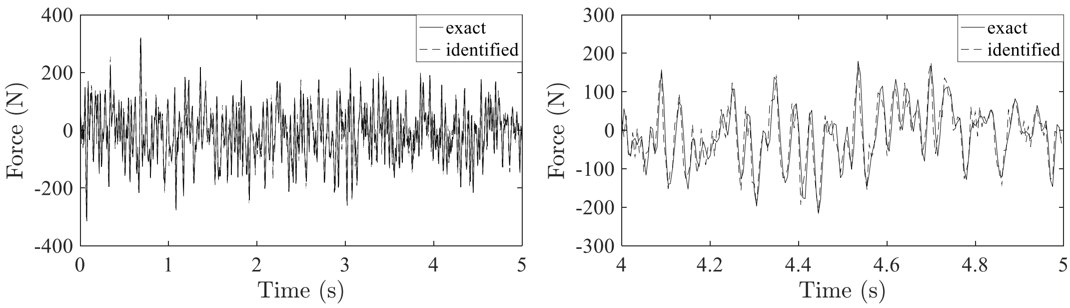
(c) Oscillation in the identified unknown input

Fig. 2 Comparisions of identified structural state and input with noisy acceleration measurements



(a) Comparison of identified displacement

(b) Comparison of identified velocity



(c) Comparison of identified unknown input

Fig. 3 Comparisions of identified results with data fusion of 5% noisy measuremnts

4.2 Identification of a truss-structure and unknown input

To validate the proposed KF-UI for the identification of other type structures with unknown inputs, the identification of a plane truss and unknown input is studied. As shown in Fig. 4, the truss consists of 11 uniform members. The length of each horizontal and inclined bar are $2m, \sqrt{2}$ m, respectively. Other parameters of the truss are: cross section area $A = 7.854\times10^{-5}$ m^2, Young's module $E = 2\times10^{11}$ pa, mass density of truss member $\rho = 7.8\times10^{3} kg/m^{3}$ and the mass is concentrated on each node. The truss is subjected to an unknown input in the vertical direction at node 4. In this example, Rayleigh damping $\boldsymbol{C}=\alpha\boldsymbol{M}+\beta\boldsymbol{K}$ is employed with α=0.6993 and β=0.0011.

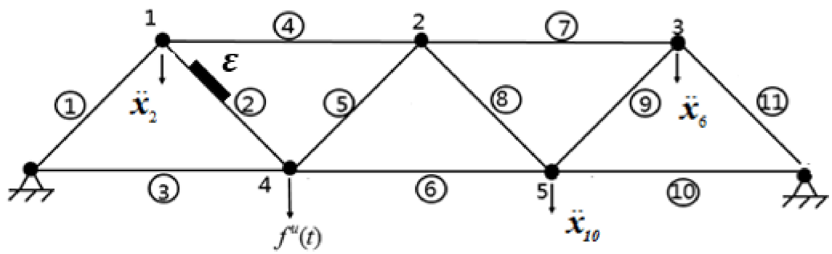
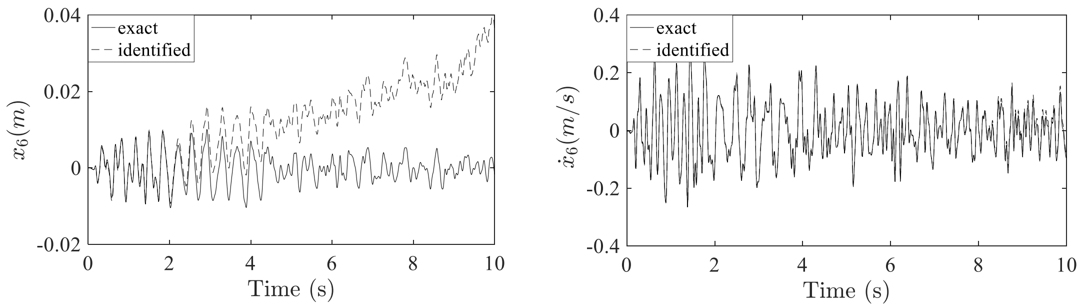
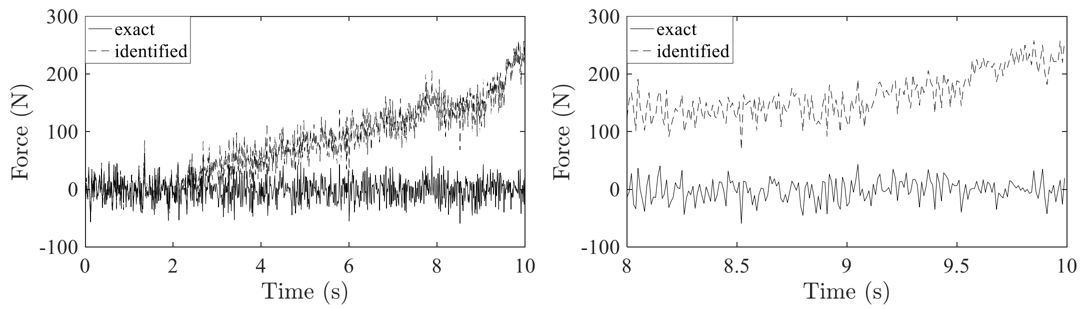


Fig. 4 A plane truss under unknown input

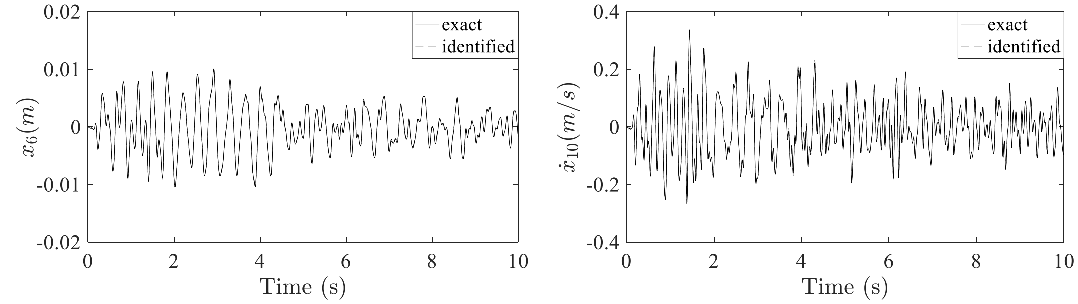


(a) Drift in the identified displacement (b) Identified velocity

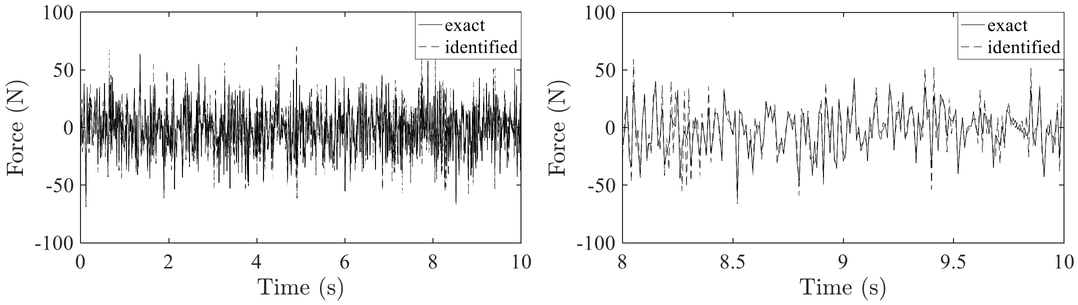


(c) Oscillation in the identified unknown input

Fig. 5 Comparisions of identified structural state and input with noisy acceleration measuremnts



(a) Comparison of identified displacement (b) Comparison of identified velocity



(c) Comparison of identified unknown input

Fig. 6 Comparisions of identified results with data fusion of 5% noisy measuremnts

As indicted in Fig.4, acceleration responses in the vertical directions of nodes 1, 3 and 5 are measured. If only these three measured noisy acceleration responses are used in the identification by the KF-UI, significant drifts in the identified displacement and unknown input are shown in Fig. 5 where measured accelerations contains noises with a 5% noise-to-signal ratio in RMS.

In practice, displacement measurements may be absent but strain measurements are easily available. Displacement measurements can be replaced by strain measurements in the KF-UI based on data fusion. Therefore, partially measured strains are added in combination with the partial acceleration measurements to prevent the above drifts in the identification problem. For this relatively small size structural model, the strain at the second bar in Fig. 4 is measured. Data fusion of this measured strain and the above three accelerations are used in the observation equation. As shown by the comparisons of identified structural state and input with their exact values in Fig. 6, it is demonstrated that the so-called drifts in estimated structural state and input are avoided by the proposed KF-UI.

5. Conclusions

Previous KF-UI approaches based solely on acceleration measurements are inherently unstable which leads the drifts in the estimated unknown inputs and structural displacements. Moreover, it is necessary to have the measurements of acceleration responses at the locations where unknown inputs applied for the recursive estimation of unknown inputs. In this paper, an algorithm is proposed to circumvent these limitations for the estimation of structural states and unknown inputs without using collocated acceleration measurements. Based on the scheme of the classical KF, an improved Kalman filter with unknown excitations (KF-UI) and without collocated acceleration measurements is derived. Then, data fusion of acceleration and displacement or strain measurements is used to prevent the drifts in the identified structural state vector and unknown external inputs in real time.

The proposed algorithm is not available previous literature and the advantages of the proposed algorithm are obvious since it provides an efficient algorithm of real time estimation of joint structural states and the unknown inputs. Some numerical examples have demonstrated the effectiveness and versatilities of the proposed approach. However, more numerical example demonstrations by complex structural configurations and experimental validations are needed. Also, measurement noise is assumed as white noise this paper; the effect of other different type of noise distribution on the identification results should be investigated.

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