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Vibration characteristic analysis of sandwich cylindrical shells with MR elastomer

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Abstract. The vibration characteristic analysis of sandwich cylindrical shells subjected with magnetorheological (MR) elastomer and constraining layer are considered in this study. And, the discrete finite element method is adopted to calculate the vibration and damping characteristics of the sandwich cylindrical shell system. The effects of thickness of the MR elastomer, constraining layer, applied magnetic fields on the vibration characteristics of the sandwich shell system are also studied in this paper. Additionally, the rheological properties of the MR elastomer can be changed by applying various magnetic fields and the properties of the MR elastomer are described by complex quantities. The natural frequencies and modal loss factor of the sandwich cylindrical shells are calculated for many designed parameters. The core layer of MR elastomer is found to have significant effects on the damping behavior of the sandwich cylindrical shells.

Keywords: cylindrical shells; damping; magnetorheological; discrete layer finite element

1. Introduction

The vibration control is an important aspect in the design of mechanical structures and one of the methods to suppress excessive vibration is damping treatment. The passive damping methods are usually used to reduce vibration levels in those mechanical structures. The added viscoelastic materials which exhibit great material loss factors provide the major damping effects due to the deformations.

The relative studies on the vibration and damping effects of the structures with constrained layer damping treatments were presented by Ross *et al.* (1959), Mead and Markus (1969). And, in their work, the loss factor of the system was defined in terms of strain energy. Then, Pan (1969) investigated the axisymmetrical vibration of the finite length cylindrical shells with the viscoelastic core layer. After that, Markus (1976) investigated the damping properties of layered cylindrical shells in axially symmetric modes. EI-Raheb and Wagner (1986) adopted the transfer matrix method to study the problem of cylinder-absorber system with thin axial strips and a thin viscoelastic layer. Then, Ramesh and Ganesan (1994) presented the vibration and damping analysis of isotropic and orthotropic cylindrical shells with the constrained damping layer. The

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Rayleigh-Ritz method was used by Ip *et al.* (1996) to obtain the vibration analysis of orthotropic thin cylindrical shells with free ends. Then, Chen and Huang (1999) obtained the mathematical model for the cylindrical shells with partially constrained damping layer and the thin shell theory with Donnell-Mushtari-Vlasov assumption was adopted in their model. The modal strain energy method was used to calculate the energy dissipation of the cylindrical shells with viscoelastic layer by Saravanan *et al.* (2000). Then, Ganapathi *et al.* (2002) utilized the higher order theory to perform dynamic analysis of laminated cross-ply composite thick cylindrical shells.

In the past few years, the magnetorheological (MR) materials have great potential in applications for smart materials and structures. The effects of the MR materials were first discovered by Rabinow (1951) and the MR material had rapid change in the damping and stiffness properties with the applications of the magnetic fields (Weiss *et al.* 1994). Then, the study of the magnetoviscoelastic behaviors for composite gels was presented by Shiga *et al.* (1995). The magnetoviscoelastic response of the elastomer composites consisting of ferrous particles embedded in a polymer matrix was investigated by Jolly *et al.* (1996). Hereafter, Dyke *et al.* (1998) focused on designs of MR dampers and evaluations of their potential benefits in vibration suppression in structures and systems. The applications for the construction of smart components had been previously discussed and presented by Yalcintas and Dai (1999). Bellan and Bossis (2002) obtained the filed-dependence of viscoelastic properties of magnetorheological elastomers. Sun *et al.* (2003) utilized Hamilton's principle to develop the analytical model for the case of sandwich beam structure with MR core layer based and calculated the numerical results with simply-supported boundary condition. The investigation for the adjustable rigidity of magnetorheological-elastomer-based sandwich beams can be obtained by Zhou and Wang (2006). Then, Yeh and Shih (2006) calculated the Dynamic characteristics and dynamic instability of magnetorheological based adaptive beam structures. Ying and Ni (2009) calculated the micro-vibration response of a stochastically excited sandwich beam with a magnetorheological elastomer core. In recent, the dynamic analysis of magnetorheological elastomer-based sandwich beam with conductive skins for various boundary conditions was discussed by Nayak *et al.* (2011). Then, Rajamohan and Ramamoorthy (2012) studied the dynamic characterization of non-homogeneous magnetorheological fluids based multi-layer beam. Then, Aguib *et al.* (2014) presented the dynamic behavior analysis of a magnetorheological elastomer sandwich plate.

In this paper, the vibration and damping behaviors of the sandwich cylindrical shells with MR elastomer are calculated by the discrete layer finite element method. There is no work that has been done to study the sandwich cylindrical shells with MR elastomer to author's knowledge. The complex solutions can be obtained by adopting complex modulus representation of MR elastomer to substitute the material properties of the sandwich system. The variations of the natural frequencies and modal loss factors are discussed in this work. Besides, the effects of some designed parameters of the sandwich cylindrical shells, such as, applied magnetic field, thickness of the MR elastomer and thickness of the constraining layer are also calculated and discussed.

2. Problem formulation

2.1 Mathematical formulation

The sandwich cylindrical shells subjected with MR elastomer and constraining layer are considered in Fig. 1. The cylindrical shell layer is designated as layer 3 and assumed to be homogeneous, elastic and isotropic. Layer 2 is the MR elastomer and its material properties can be changed by applying various magnetic field magnitudes. And, layer 1 is the elastic, homogeneous, and isotropic constraining layer. Additionally, the thicknesses of the three layers of the sandwich cylindrical shells system are h_1, h_2, and h_3, respectively. Besides, the following assumptions must be mentioned before the derivation proceeds:

1. There is no slipping between the interfaces.
2. And, layer 1 and layer 3 are assumed to be undamped.
3. The deflections of the system are small.

In this study, the displacement relation of the elastic layer can be expressed in terms of the in-plane displacements of the adjacent layer interfaces and the transverse displacement as the following equation

$$\mathrm{u}_i = \begin{Bmatrix} u_i(x,\theta,z,t) \\ v_i(x,\theta,z,t) \\ w_i(x,\theta,z,t) \end{Bmatrix} = H_{1,i}(z) \begin{Bmatrix} U_i(x,\theta,t) \\ V_i(x,\theta,t) \\ U_{i+1}(x,\theta,t) \\ V_{i+1}(x,\theta,t) \\ W(x,\theta,t) \end{Bmatrix} \tag{1}$$

where u_i is the displacement field of layer i and the transverse thickness interpolation matrix for ith layer $H_{1,i}(z) = \begin{bmatrix} (\frac{1}{2}-\frac{z-R_i}{h_i}) & 0 & (\frac{1}{2}+\frac{z-R_i}{h_i}) & 0 & 0 \\ 0 & (\frac{1}{2}-\frac{z-R_i}{h_i}) & 0 & (\frac{1}{2}+\frac{z-R_i}{h_i}) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. In which, h_i is the thickness of layer i and R_i is the radius of the middle surface, respectively.

Then, the strain-displacement relation of the cylindrical shell can be expressed as follows

$$\varepsilon_i = D\mathrm{u}_i \tag{2}$$

where $\varepsilon_i = \{\varepsilon_{xx,i} \quad \varepsilon_{\theta\theta,i} \quad \gamma_{x\theta,i} \quad \gamma_{xz,i} \quad \gamma_{\theta z,i}\}^{\mathrm{T}}$ and the differential operator matrix

$$D = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{1}{z}\frac{\partial}{\partial \theta} & \frac{1}{z} \\ \frac{1}{z}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & \frac{\partial}{\partial z}-\frac{1}{z} & \frac{1}{z}\frac{\partial}{\partial \theta} \end{bmatrix}.$$

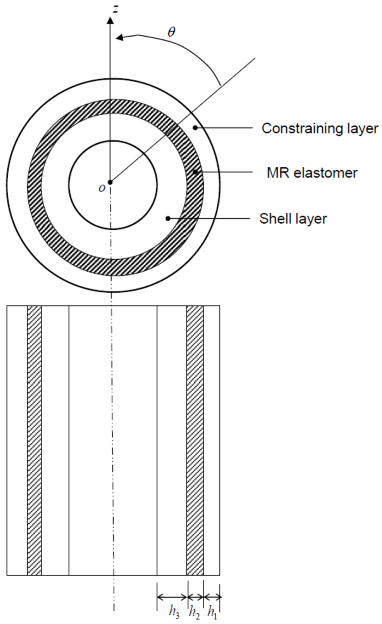


Fig. 1 The cylindrical shell with ER core layer and constraining layer

The stress-strain relations for the *i*th layer can be written as the following form

$$\sigma_i = C_i \varepsilon_i \tag{3}$$

where $\sigma_i = \{\sigma_{xx,i} \quad \sigma_{\theta\theta,i} \quad \tau_{x\theta,i} \quad \tau_{xz,i} \quad \tau_{\theta z,i}\}^T$ and the elasticity matrix $C_i = \begin{bmatrix} C_{11,i} & C_{12,i} & 0 & 0 & 0 \\ C_{21,i} & C_{22,i} & 0 & 0 & 0 \\ 0 & 0 & C_{66,i} & 0 & 0 \\ 0 & 0 & 0 & C_{44,i} & 0 \\ 0 & 0 & 0 & 0 & C_{55,i} \end{bmatrix}$. In which, $C_{11,i} = C_{22,i} = \dfrac{E_i}{1-\upsilon_i^2}$, $C_{12,i} = C_{21,i} = \dfrac{\upsilon_i E_i}{1-\upsilon_i^2}$, $C_{44,i} = C_{55,i} = \kappa^2 \dfrac{E_i}{2(1+\upsilon_i)}$ and $C_{66,i} = \dfrac{E_i}{2(1+\upsilon_i)}$, respectively. In the above equations, E_i is the Young's modulus, υ_i is the Poisson ration, and κ^2 is the shear correction factor ($\kappa^2 = \pi^2/12$ for layer 1,2 and $\kappa^2 = 1$ for layer 2).

2.2 Finite element formulation

As shown in Fig. 2, the discrete layer finite element is adopted to construct the sandwich cylindrical shell structure. The displacement of the interfaces can be expressed in terms of the nodal degrees of freedom by using interpolation in the *x*-direction and for the circumferential wave number m as follows

$$\begin{Bmatrix} U_i(x,\theta,t) \\ V_i(x,\theta,t) \\ U_{i+1}(x,\theta,t) \\ V_{i+1}(x,\theta,t) \\ W(x,\theta,t) \end{Bmatrix} = H_2(x,\theta) q_i^e(t) \tag{4}$$

where $q_i^e = \{U_i^A \quad V_i^A \quad U_{i+1}^A \quad V_{i+1}^A \quad W^A \quad \Theta^A \quad U_i^B \quad V_i^B \quad U_{i+1}^B \quad V_{i+1}^B \quad W^B \quad \Theta^B\}^T$ is the vector of the nodal displacements of the element, and the interpolation matrix

$$H_2(x,\theta) = \begin{bmatrix} \phi_u^A & 0 & 0 & 0 & 0 & 0 & \phi_u^B & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_v^A & 0 & 0 & 0 & 0 & 0 & \phi_v^B & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_u^A & 0 & 0 & 0 & 0 & 0 & \phi_u^B & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_v^A & 0 & 0 & 0 & 0 & 0 & \phi_v^B & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_w^A & \phi_\Theta^A & 0 & 0 & 0 & 0 & \phi_w^B & \phi_\Theta^B \end{bmatrix}.$$ In which,

$\phi_u^A = (1-\xi)\cos m\theta$, $\phi_u^B = \xi \cos m\theta$, $\phi_v^A = (1-\xi)\sin m\theta$, $\phi_v^B = \xi \sin m\theta$, $\phi_w^A = (1-3\xi^2 + 2\xi^3)\cos m\theta$, $\phi_w^B = (3\xi^2 - 2\xi^3)\cos m\theta$, $\phi_\Theta^A = (\xi - 2\xi^2 + \xi^3)\cos m\theta$, $\phi_\Theta^B = (-\xi^2 + \xi^3)\cos m\theta$, $\xi = \dfrac{x}{L_e}$, and L_e is the elemental length in the x-direction, respectively.

Then, the strain and kinetic energies of the element for the ith layer can be expressed as the following equations

$$V_i^e = \frac{1}{2}\int_V \rho_i \sigma_i^{\mathrm{T}} \, \varepsilon_{\mathrm{i}} \, dV \tag{5}$$

$$T_i^e = \frac{1}{2}\int_V \rho_i \dot{\mathrm{u}}_{\mathrm{i}}^{\mathrm{T}} \dot{\mathrm{u}}_{\mathrm{i}} dV \tag{6}$$

where ρ_i is the mass density of the ith layer. The strain and kinetic energies of the element can be rewritten as the following forms by substituting Eqs. (1)-(4) into Eqs. (5) and (6):

$$V_i^e = \frac{1}{2} \mathrm{U}_i^{e\mathrm{T}} \mathrm{K}_i^e \mathrm{U}_i^e \tag{7}$$

$$T_i^e = \frac{1}{2} \dot{\mathrm{U}}_i^{e\mathrm{T}} \mathrm{M}_i^e \dot{\mathrm{U}}_i^e \tag{8}$$

where K_i^e and M_i^e are the element stiffness and mass matrices, respectively and can be expressed as follows

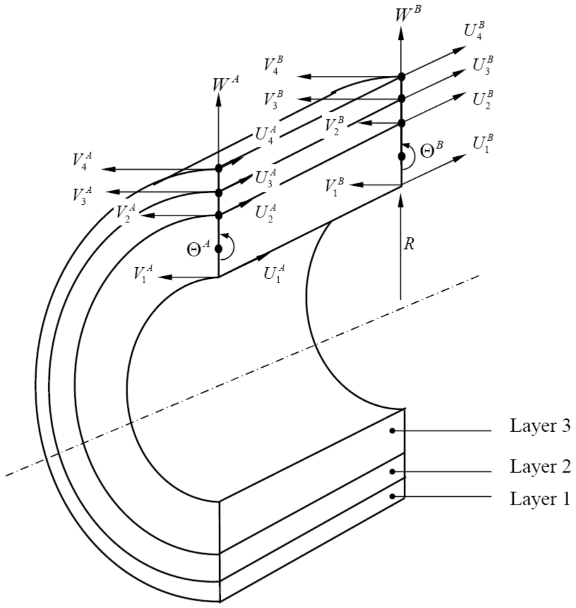


Fig. 2 Discrete layer finite element for three-layer element

$$\mathrm{K}_i^e = \int_V \rho_i (H_{1,i} H_2)^{\mathrm{T}} (H_{1,i} H_2) dV \tag{9}$$

$$\mathrm{M}_i^e = \int_V (DH_{1,i} H_2)^{\mathrm{T}} C_i^{\mathrm{T}} (DH_{1,i} H_2) dV \tag{10}$$

The following relations must be done first by assembling the elemental matrices into the global stiffness and mass matrices

$$\mathrm{U}_i^e = \mathrm{Tr}_i^e \mathrm{U} \tag{11}$$

in which, U and Tr_i^e are the global nodal co-ordinate vector and transformation matrix, respectively.

Then, the equation of motion of the sandwich system can be express as the following equations by assembling the contributions of all elements

$$\mathrm{M}\ddot{\mathrm{U}} + \mathrm{K}\mathrm{U} = 0 \tag{12}$$

where K and M are the global stiffness and mass matrices and written as follows

$$\mathrm{K} = \sum_{i=1}^{3} \left(\sum_{e=1}^{N_i} \mathrm{Tr}_i^{e\,\mathrm{T}} \mathrm{K}_i^e \mathrm{Tr}_i^e \right) \tag{13}$$

$$\mathrm{M} = \sum_{i=1}^{3}\left(\sum_{e=1}^{N_i} \mathrm{Tr}_i^{e\,\mathrm{T}} \mathrm{M}_i^e \mathrm{Tr}_i^e\right) \tag{14}$$

in which N_i is the element number of the *i*th layer.

The complex eigenvalues $\tilde{\lambda}$ of the above complex eigenvalue problems can be calculated and obtained numerically. And, the natural frequencies ω and modal loss factor η_v are extracted in the following equations

$$\omega = \sqrt{\mathrm{Re}(\tilde{\lambda})} \tag{15}$$

$$\eta_v = \frac{\mathrm{Im}(\tilde{\lambda})}{\mathrm{Re}(\tilde{\lambda})} \tag{16}$$

3. Results and discussions

The sandwich cylindrical shells with MR elastomer and constraining layer are presented in this study. The comparisons between the present results and the results of the references are made in order to validate the present method and calculations. The good agreements can be seen in Table 1 (Ip *et al.* 1996) and Fig. 3 (Bert *et al.* 1969), respectively. In this study, only the electric field dependence of the ER material in the pre-yield regime needs to be considered based on the existing results of MR elastomer. The complex shear modulus of the MR elastomer used in this study was estimated by performing a free oscillation experimental on the fully treatment MR sandwich system (Rajamohan *et al.* 2010). The complex shear modulus of the MR elastomer can be expressed as follows with respect to the intensity of magnetic field

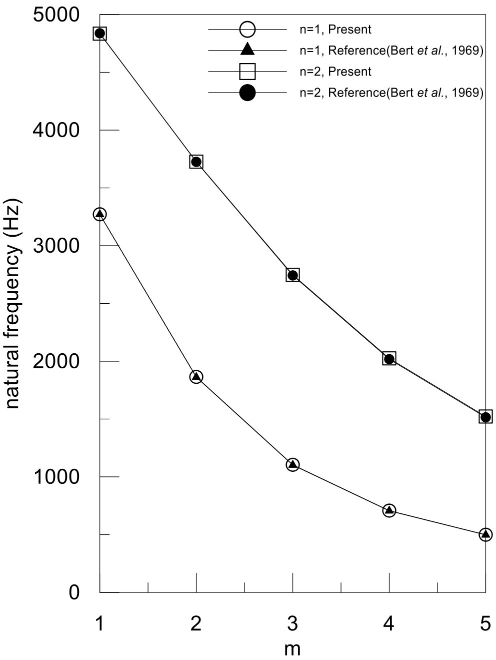
$$G^*(B) = G'(B) + jG''(B) \tag{17}$$

in which, the storage modulus $G^{'}(B) = -3.3691\times B^2 + 4997.5\times B + 873000$, the loss modulus $G^{''}(B) = -0.9\times B^2 + 812.4\times B + 185500$, B is the intensity of magnetic field in Gauss, and the mass density of the MR elastomer is $3500 kg/m^3$. Besides, the calculation results of the mode (*n*,*m*), where *n* is the axial mode number and *m* is circumferential mode number, are presented and the boundary condition is the simply-supported at two ends. Then, the following parameters are introduced

$$L = 0.3m,\ E_1 = E_3 = 70GPa,\ \upsilon_1 = \upsilon_3 = 0.3,\ \upsilon_2 = 0.499$$

$$\rho_1 = \rho_3 = 2700kg/m^3,\ \rho_2 = 3500kg/m^3,\ h_3 = 0.5mm$$

where L is the length of the cylindrical shell and *R* is the displacement from the central axis to the layer 3 (the base cylindrical shell).



Mode (n,m)

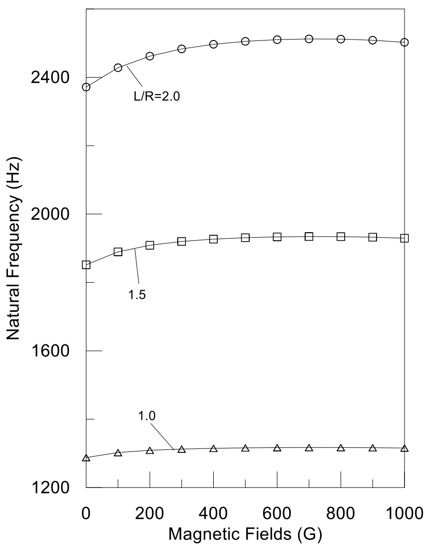
Fig. 3 Comparison between references and present method for simply-supported boundary condition

Table 1 Comparison between references and present method for free-free boundary condition

Mode	Natural frequency (Hz)					
	Case 1			Case 2		
(n,m)	Present	Rayleigh-Ritz (Ip *et al.* 1996)	Experimental result (Ip *et al.*1996)	Present	Rayleigh-Ritz (Ip *et al.* 1996)	Experimental result (Ip *et al.* 1996)
(1,2)	81.65	81.93	79.57	56.39	56.43	55.99
(2,2)	88.59	88.91	88.63	58.51	58.58	59.11
(1,3)	231.2	231.71	225.64	159.47	159.61	158.21
(2,3)	242.68	243.57	241.78	162.89	163.12	164.20
(1,4)	442.94	444.23	433.37	305.65	306.02	301.92
(2,4)	456.34	458.43	455.03	309.64	310.18	310.69

The effects of magnetic fields on natural frequencies and modal loss factors of the sandwich cylindrical shells with various parameters L/R are presented in Fig. 4. In the present paper, L is fixed as 0.3 m and the total $R_i=L*(R/L)+h_1/2+h_2/2+h_3/2$. According to the numerical results, it can be seen that the natural frequencies increase while the applied magnetic field magnitudes increase in lower magnitude of magnetic fields (0-400G) and will decrease under larger magnitude of magnetic fields (over 800G). Besides, the modal loss factors decrease when the magnetic field strength increases. Based on the numerical results, the tendency of the natural frequency and modal loss factor is similar for L/R=1.0, 1.5 and 2.0. Thus, the geometrical parameters of the sandwich system had significant effects on the natural frequency and modal loss factor of the sandwich system according to the above results. Fig. 5 shows the variations of the effects of magnetic fields on the natural frequency and modal loss factor of the sandwich system with various thickness of MR elastomer layer. It can be observed that the natural frequency decrease when the thickness of the MR elastomer layer of the sandwich system increase. As to the modal loss factor, it will increase as the thickness of the MR elastomer layer increases. The effects of magnetic fields on the natural frequency and modal loss factor of the sandwich system with various thickness of constraining layer are presented in Fig. 6. The natural frequency will increase and the modal loss factor will decrease as the thickness of constraining layer increases. The variation tendency of the system for parameter $h_1/h_3 = 0.1$, 0.2 and 0.3 are similar.

Fig. 7 shows the effects of MR elastomer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various parameters *L*/*R*. The natural frequency of the sandwich system will be smaller and the modal loss factor of the sandwich system will be getting larger as the thickness of MR elastomer increases. With various parameters *L*/*R*, the variations of the sandwich system on the natural frequency and modal loss factor are similar. According to the results, the larger parameter *L*/*R* is, the natural frequency increases and modal loss factor decreases as the parameter *L*/*R* increases. Then, the effects of the MR elastomer thickness on the natural frequencies and the modal loss factor of the sandwich cylindrical shell with various magnetic fields are presented in Fig. 8. The natural frequency decreases as the MR elastomer thickness increases and the tendency is similar for different applied magnetic fields according to the numerical results. On the other hand, it also can be observed that the modal loss factor increases while the MR thickness increases and the variations are similar with various magnetic fields. It is because that the MR elastomer thickness and the applied magnetic fields on the system can change the stiffness of the sandwich system. In Fig. 9, the numerical results for the effects of the MR elastomer thickness on the natural frequencies and the modal loss factor of the sandwich cylindrical shell with various thickness of constraining layer are plotted. It can be seen that the natural frequency increases and modal loss factor decreases as the thickness of the constraining layer increases from the results.



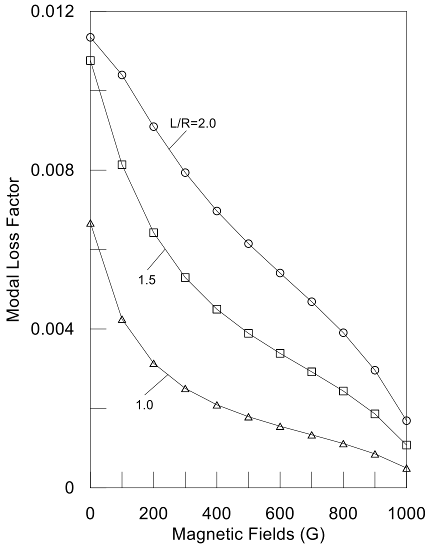


Fig. 4 Effects of magnetic fields on the natural frequency and modal loss factor of the sandwich cylindrical shells with various parameters L/R

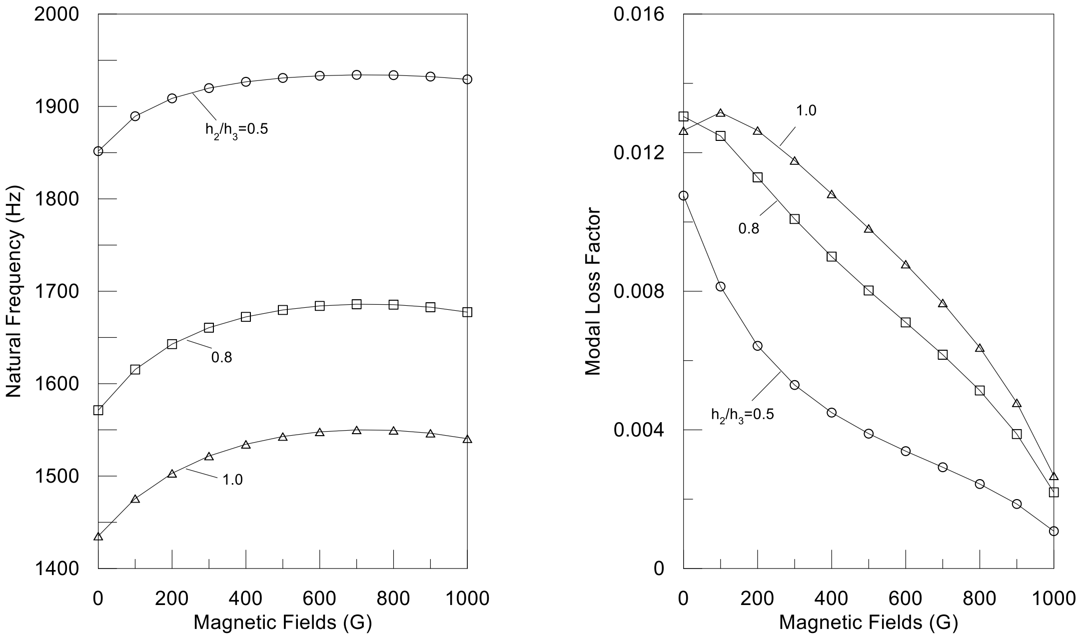


Fig. 5 Effects of magnetic fields on the natural frequency and modal loss factor of the sandwich cylindrical shells with various thickness of MR elastomer layer

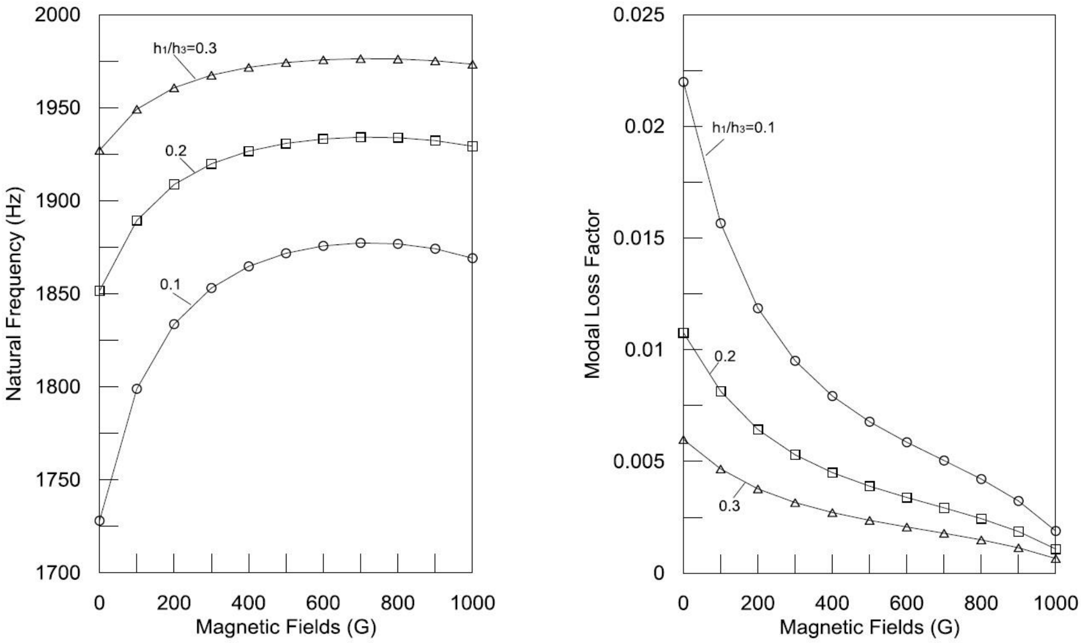


Fig. 6 Effects of magnetic fields on the natural frequency and modal loss factor of the sandwich cylindrical shells with various thickness of constraining layer

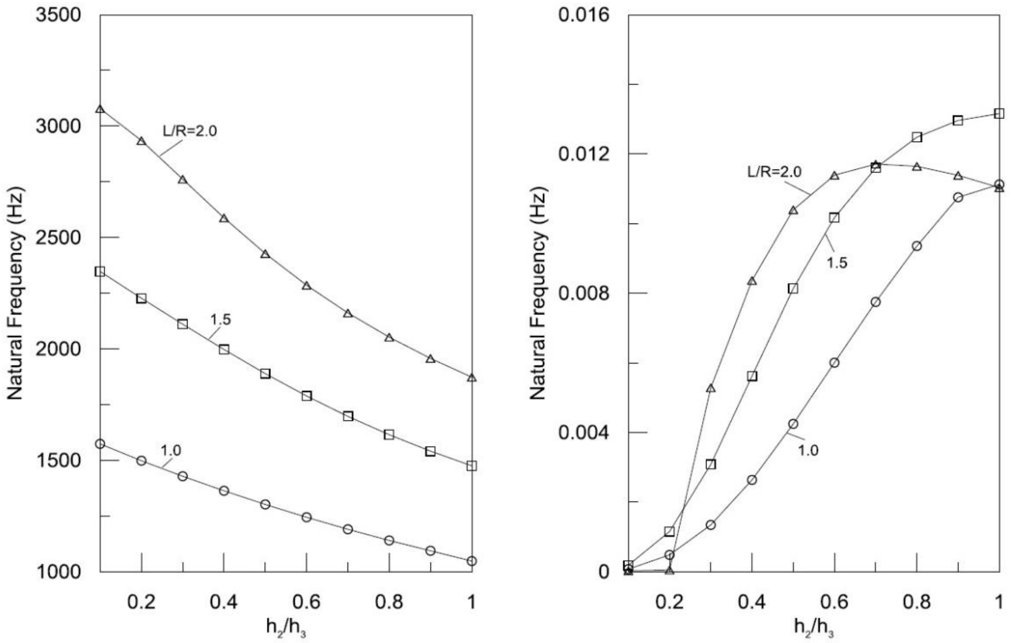


Fig. 7 Effects of MR elastomer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various parameters L/R

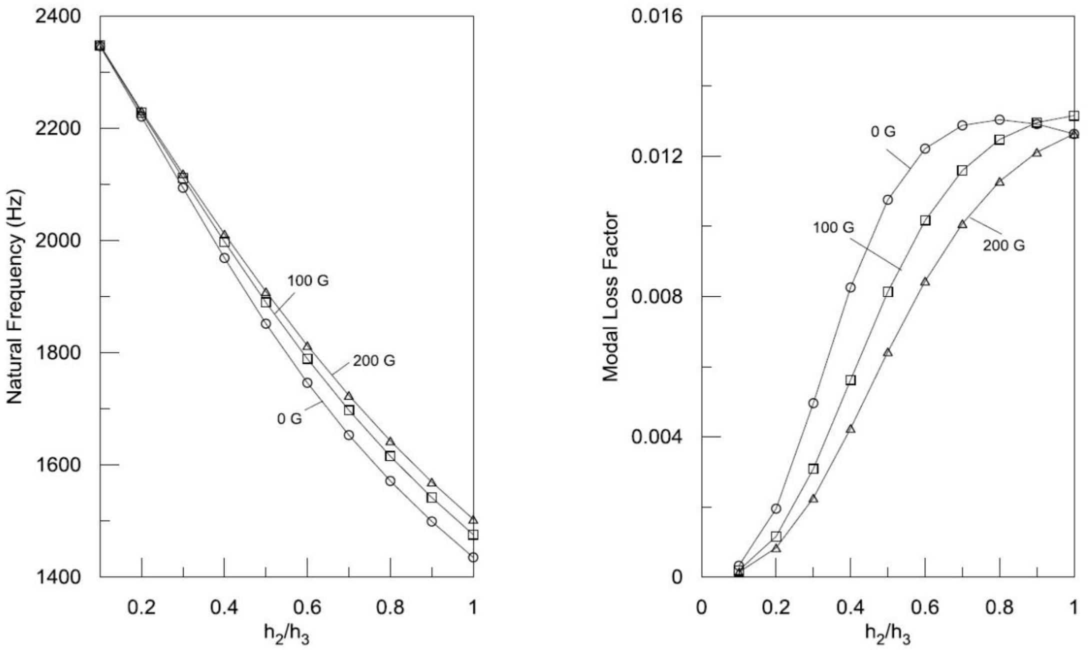


Fig. 8 Effects of MR elastomer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various magnetic fields

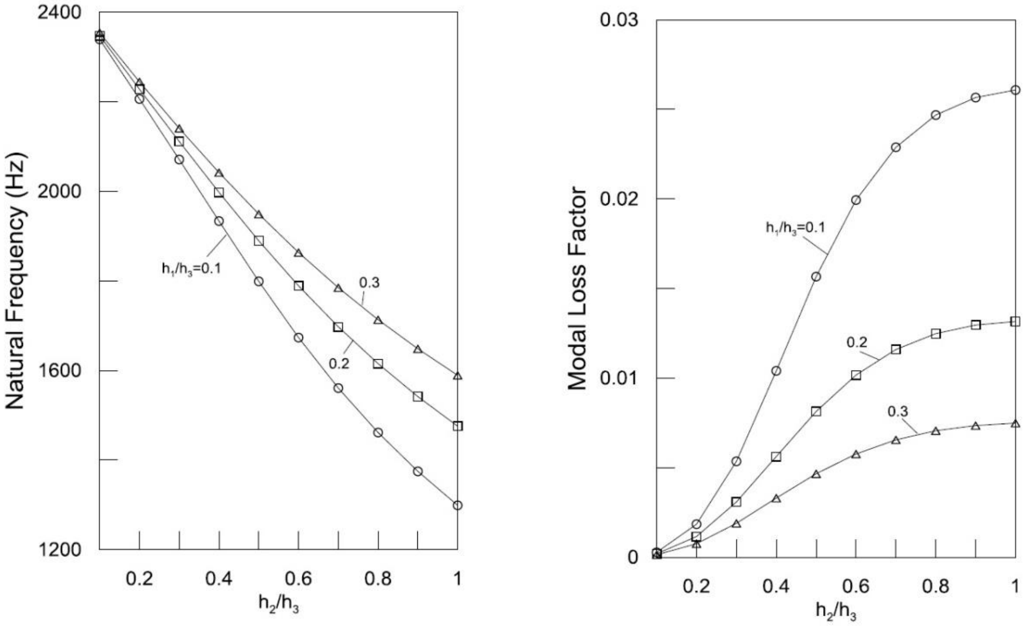


Fig. 9 Effects of MR elastomer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various thickness of constraining layer

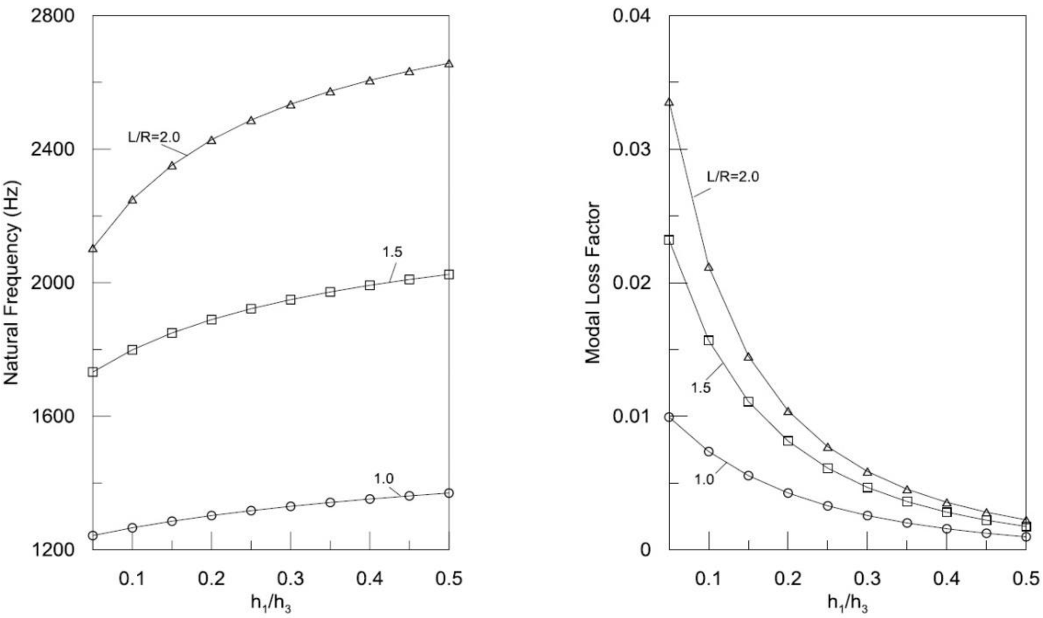


Fig. 10 Effects of constraining layer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various parameters L/R

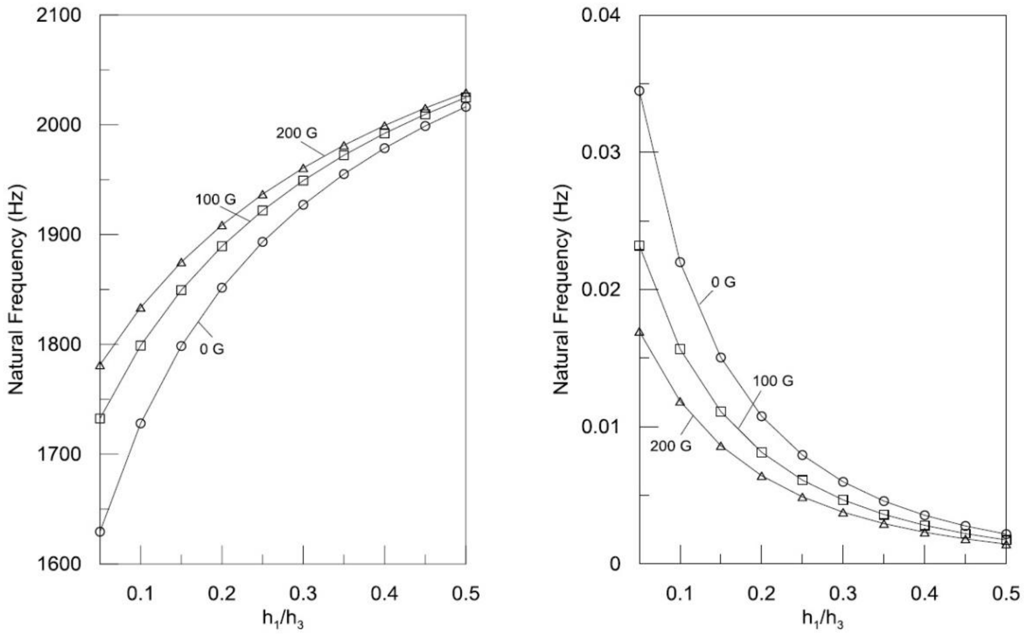


Fig. 11 Effects of constraining layer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various magnetic fields

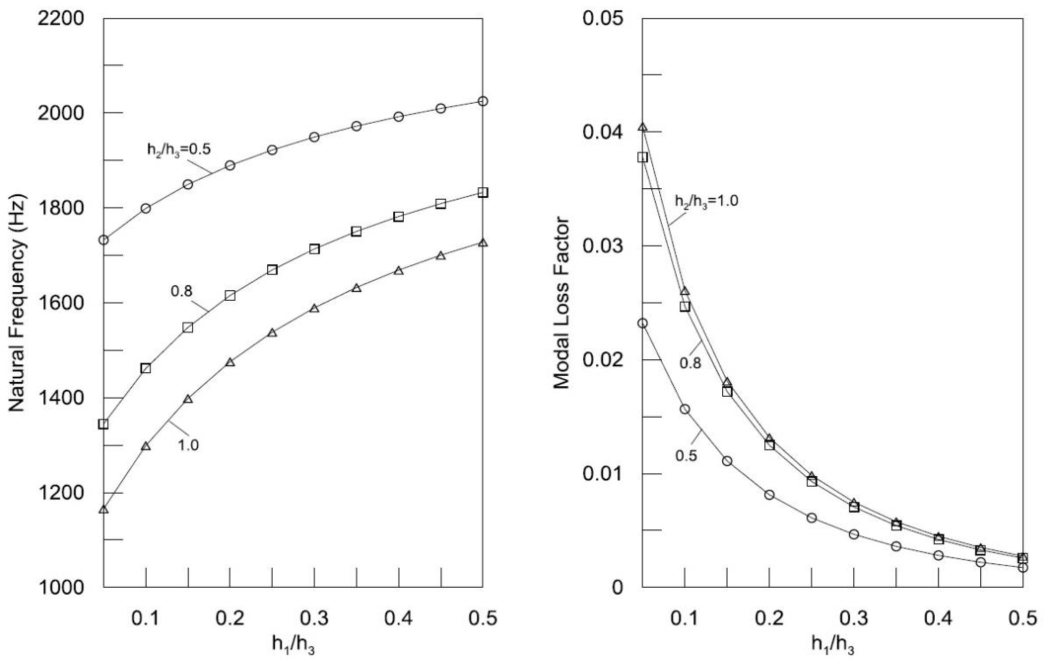


Fig. 12 Effects of constraining layer thickness on the natural frequency and modal loss factor of the sandwich cylindrical shells with various thickness of MR elastomer layer

The effects of the thickness of constraining layer on the natural frequency and modal loss factor of the sandwich cylindrical shell are plotted in Fig. 10. The natural frequency increases and modal loss factor decreases as the thickness of constraining layer increases based on the numerical results. Besides, the tendency of the sandwich system is the same for various parameters *L/R*. In Fig. 11, the effects of the thickness of constraining layer on the natural frequency and modal loss factor of the sandwich cylindrical shell with various applied magnetic fields are presented. From the figure, the natural frequency increases and the modal loss factor decreases as the applied magnetic fields increase.

Finally, the effects of the thickness of constraining layer on the natural frequency and modal loss factor of the sandwich cylindrical shell with various MR elastomer thicknesses can be obtained in Fig 12. The natural frequency decreases and the modal loss factor increases as the MR elastomer thickness increases according to the numerical results. Thus, the vibration characteristics and damping effects of the sandwich system can be controlled and changed with various parameters according to the above numerical results.

4. Conclusions

The vibration and damping characteristics of the sandwich cylindrical shells with MR elastomer and constraining layer are investigated in this study. The natural frequency and modal loss factor of the sandwich cylindrical shell system are calculated and analyzed by the discrete layer finite element method. Based on the numerical results, the following conclusions can be drawn:

- It can be observed that the applied magnetic fields will change the characteristics of the MR elastomer According to the above results. As the applied magnetic fields increase, the natural frequencies of the sandwich cylindrical shell system increase and the modal loss factor will decrease.
- Besides, it also can be observed that the larger MR elastomer thickness, the smaller natural frequency of the sandwich cylindrical shell system. As to the modal loss factor, the larger MR elastomer thickness will affect the values of the modal loss factor.
- Additionally, the thickness of the constraining layer also will change the characteristics of the sandwich system from the above results.

The material property of the MR elastomer is a function of the applied magnetic fields, and the change of the MR elastomer thickness also has significant effects on the natural frequency and modal loss factor. Thus, the thickness of MR elastomer, thickness of the constraining layer, and applied magnetic fields can be considered to obtain the high damping effects. So, we can use the characteristics to design some active, tunable and controllable mechanical devices.

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