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A developed hybrid method for crack identification of beams

Ali. R. Vosoughi*

Department of Civil and Environmental Engineering, School of Engineering Shiraz University, Shiraz, Iran

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Abstract. A developed hybrid method for crack identification of beams is presented. Based on the Euler-Bernouli beam theory and concepts of fracture mechanics, governing equation of the cracked beams is reformulated. Finite element (FE) method as a powerful numerical tool is used to discritize the equation in space domain. After transferring the equations from time domain to frequency domain, frequencies and mode shapes of the beam are obtained. Efficiency of the governed equation for free vibration analysis of the beams is shown by comparing the results with those available in literature and via ANSYS software. The used equation yields to move the influence of cracks from the stiffness matrix to the mass matrix. For crack identification measured data are produced by applying random error to the calculated frequencies and mode shapes. An objective function is prepared as root mean square error between measured and calculated data. To minimize the function, hybrid genetic algorithms (GAs) and particle swarm optimization (PSO) technique is introduced. Efficiency, Robustness, applicability and usefulness of the mixed optimization numerical tool in conjunction with the finite element method for identification of cracks locations and depths are shown via solving different examples.

Keywords: a hybrid inverse method; crack identification; reformulated governing equation; optimization; hybrid GAs- PSO

1. Introduction

Presentation of cracks and damages in structural elements such as beams may lead to reduction in stiffness and serviceability of structures. So, different researches are performed for detecting defects considering measured data. Karthikeyan *et al*. (2007) identified crack location and its depth from vibration measurements. They used Timoshenko beam theory and finite element method in conjunction with direct iterative regularisation technique for solving the problem. Vakil-Baghmisheh *et al.* (2008) employed analytical modal analysis of a cracked cantilever beam and genetic algorithms (GAs) to identify the crack location of beam and its depth. Khaji *et al*. (2009) used bending vibration measurement for crack detection of beams via Timoshenko beam theory and analytical approach. Lee (2009) introduced a method for identifying multi cracks in cantilever beams based on combined Euler–Bernoulli beam theory and the Newton–Raphson method. Sayyad and Kumar (2011) presented theoretical and experimental study for crack identification of cantilever beams using natural frequencies measurement. They employed a finite element method package for analysis of the cracked beams. Saeed *et al*. (2012) presented artificial

∗Corresponding author, Assistant Professor, E-mail: vosoughi@shirazu.ac.ir

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intelligence (AI) techniques for crack identification in curvilinear beams based on changes in vibration characteristics. They used finite element method to compute natural frequencies and frequency response functions. Moradi *et al*. (2011) employed separation of variable method and bee algorithm for crack identification of cantilever Euler beams. They used bee algorithm for minimizing the error function between measured and estimated frequencies. Khorram *et al*. (2013) introduced a method for cracks detection in a simply supported beam. They used continuous wavelet transform (CWT) in conjunction with factorial design method. They considered mid span deflection time history in their analysis for detecting cracks locations and depths. Gillich and Praisach (2014) used time frequency analysis and signal processing for crack detecting of beams via natural frequency changes. Khiem and Tran (2014) derived a new form for solving eigenvalue problem of cracked beams. They employed regularization method to obtain both location and size of multiple cracks from noisy measured data. Barad *et al*. (2013) used direct search method for obtaining crack location and depth of thin beams. Muñoz-Abella *et al*. (2012) presented a nondestructive method for crack identification of shafts. They used stress wave propagation in conjunction with genetic algorithms. Xiang and Liang (2012) presented two steps method for crack identification of beams. They found the locations of the cracks by applying the wavelet transform to the modal shape of beams. They estimated the depths of the cracks using the measured natural frequencies as inputs from a database established by wavelet finite element method. Jiang *et al*. (2012) presented a method for crack identification of beams. They used the slope of the mode shape of the beams to detect cracks. They introduced the angle coefficients of complex continuous wavelet transform for detecting the location of the nonpropagating transverse crack. Jafarkhani and Masri (2011) presented the performance of an evolutionary strategy in the finite element model updating approach for damage detection of beam-like structures. Tanaka *et al*. (2013) presented fracture mechanics analysis using the wavelet Galerkin method and extended finite element method. Singh and Tiwari (2013) presented an algorithm for crack identification of stepped shafts. Varghese and Shankar (2014) combined transient power flow balance and acceleration matching technique for damage detection of beams. Using a multiobjective optimization formulation they detected and quantified crack damage in beam structures at different locations. Wang and Chen (2013) proposed a moving-window least squares fitting method for rapid identification of cracks and flexural rigidities in beam structures.

To the best of author's knowledge and based on the efficiency of combined methods for solving engineering problems (Malekzadeh *et al*. 2014), as a first attempt, finite element (FE), genetic algorithms (GAs) and particle swarm optimization (PSO) techniques are combined for crack identification of thin beams. Based on Euler-Bernouli beam theory and fracture mechanic concepts the governing equation is reformulated. Then, finite element method as a powerful numerical tool is used to obtain frequencies and mode shapes of the beam. By applying random error to the arbitrary number of frequencies and mode shapes of cracked beams measured data are produced. An objective function is generated via calculating root mean square between the measured and the estimated data. A hybrid genetic algorithms and particle swarm optimization technique is used as a simple and efficient numerical optimization tool to obtain cracks depths and locations. More details are presented in the following sections.

2. Governing equation and solution procedure

Consider a cantilever beam with length L, width b, thickness h, crack location C_i and depth of crack a_i (see Fig. 1).

Using the classical thin beam theory, the transverse flexural equation of motion due to small deflection, at an arbitrary point on the mid-plane of elastic beam with no crack can be written as

$$-D\frac{\partial^4 w}{\partial x^4} = I_0 \frac{\partial^2 w}{\partial t^2} \tag{1}$$

where, D and I_o are the stiffness coefficient and moment inertia of the beam, respectively and are as

$$D = \int_{-\frac{h}{2}}^{\frac{h}{2}} bEz^2 dz\,,\quad I_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho b dz$$

Transferring Eq. (1) from temporal domain into frequency domain yields to

$$D\frac{d^4 w}{dx^4} - \omega^2 I_0 w = 0 \tag{2}$$

Considering concepts of fracture mechanics and using Heavyside function, rotation of the cracked section at any point of the beam, x_i, can be stated as (Broek 1986, Ranjbaran 2010)

$$\theta = \frac{dw_{crack}}{dx} = c_{bi}\frac{d^2 w}{dx^2}\mathrm{H}\left(x - x_i\right) \tag{3}$$

where, c_{bi} is equivalent flexibility of cross section at i^{th} crack due to bending, which can be obtained with respect to depth of cracks as (Lee 2009)

$$c_{bi} = \frac{h\xi_i(2-\xi_i)}{0.9(\xi_i - 1)^2} \tag{4}$$

where, $\xi_i = \dfrac{a_i}{h}$.

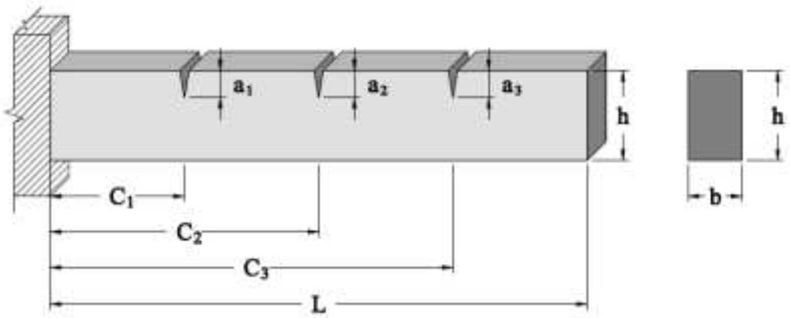

Fig. 1 A cantilever beam with three cracks

Considering Heavyside function properties the derivative of Eq. (3) would be expressed as

$$\frac{d\theta}{dx} = \frac{d^2 w_{crack}}{dx^2} = c_{bi} \frac{d^2 w}{dx^2} \delta(x - x_i) \tag{5}$$

where, δ is Dirac-delta function.

Two times integration of Eq. (2) would be as follows

$$\frac{d^2 w}{dx^2} - \lambda^2 \left(\int \left(\int w dx \right) dx + B_1 x + B_2 \right) = 0 \tag{6}$$

where, $\lambda^2 = \frac{\omega^2 I_o}{D}$ and B_1, B_2 are constant coefficients of the integral.

$$\int \left(\int w dx \right) dx + B_1 x + B_2 = V \Rightarrow \frac{d^2 V}{dx^2} = w \tag{7}$$

Substituting Eq. (7) in Eq. (6) gets

$$\frac{d^2 w}{dx^2} - \lambda^2 V = 0 \tag{8}$$

Using Eq. (8) in Eq. (5) can write

$$\frac{d^2 w_{crack}}{dx^2} = c_{bi} \lambda^2 V \delta(x - x_i) \tag{9}$$

In the cracked section of the beam with respect to intact and crack parts participation the following equation can be written

$$\frac{d^2 w_{\text{int}act}}{dx^2} = \frac{d^2 w}{dx^2} - \frac{d^2 w_{crack}}{dx^2} = \frac{d^2 w}{dx^2} - c_{bi} \lambda^2 V \delta(x - x_i) \tag{10}$$

Substituting Eqs. (7)-(10) in Eq. (8) gives different form of equation of motion with respect to considering the cracks influence at their locations as follows

$$\frac{d^4 V}{dx^4} - \lambda^2 \left[1 + c_{bi} \delta(x - x_i) \right] V = 0 \tag{11}$$

The boundary conditions of Eq. (11) should be determined. Considering Eq. (7) one can write

$$\frac{d^2 V}{dx^2} = w \quad \text{and} \quad \frac{d^3 V}{dx^3} = \frac{dw}{dx} \tag{12 a-b}$$

Which means clamped boundary condition ($w = 0$ and $\frac{dw}{dx} = 0$) in Eq. (1) is equal to zero shear and moment ($\frac{d^2V}{dx^2}=0$ and $\frac{d^3V}{dx^3}=0$) in Eq. (11). This means that clamped boundary condition changes to free boundary condition.

For solving Eq. (12), finite element method (FEM) is used. Using FEM rules (Reddy 2004) and by considering transverse and rotation as two degrees of freedom at each nodes of elements the stiffness and mass matrix of the problem can be stated as follows

$$k_{ij}^e=\int_0^{L^e}\frac{d^2\varphi_i}{dx^2}\frac{d^2\varphi_j}{dx^2}dx\,,\quad m_{ij}^e=\int_0^{L^e}\lambda^2\varphi_i\varphi_j dx\,,$$

$$m_{ij}^{ec}=c_{ib}\lambda^2\varphi_i(x_i)\varphi_j(x_i)\,,\ i,j=1,\ \ldots,\ 4 \tag{13a-c}$$

where, k_{ij}^e, m_{ij}^e and m_{ij}^{ec} are components of stiffness, mass and cracked mass matrices of the beam elements, respectively. Also, $\varphi_i\left(i=1,\ldots,4\right)$, are Hermitian interpolation shape functions.

Assembling of the elements matrices the eigen-value problem is obtained as follows

$$([K]-\lambda^2[NM])\{V\}=\{0\} \tag{14}$$

where, $\left[K\right]$ is the global stiffness matrix and $\left[NM\right]$ is the new global mass matrix which includes influences of cracks.

For identification of cracks depths and locations, measured data should be produced. By applying random error to the arbitrary number of calculated frequencies and mode shapes from the direct solution, measured data are generated.

The following functional is considered as an objective function for identification of the parameters.

$$J\left[\lambda\right]=\sum_{s=1}^{M}\left[\lambda_s-\lambda_s^*\right]^2 \tag{15}$$

where, M denotes the number of used eigen-pairs including frequencies and mode shapes; λ_s and λ_s^* are the estimated (computed) and measured eigen-pairs, respectively. For minimizing the functional optimization technique should be used. First, an introduction to GAs and PSO optimization techniques is presented separately. Then, mixing procedure of GAs-PSO would explain.

In GAs, a candidate solution for an optimization problem is a chromosome, and consists of a linear list of genes. Each chromosome shows a point in the optimization search space, which can be a possible solution of the problem. A finite number of chromosomes produce a population. Fitness value of each chromosome would be obtained. Based on the fitness value and using genetic operators, a new population is generated iteratively, while the best solution obtains.

The PSO solves optimization problem using a population same as chromosomes in GAs. A population of particles is randomly generated initially. Each particle can be a possible solution and has its position is shown by a position vector. A swarm of particles moves through the optimization search space, with a velocity vector. At each step, the fitness value of the problem will obtain. This procedure will be repeated until the best solution obtains.

Based on the brief review of GAs and PSO mentioned above, for minimizing Eq. (15) genetic algorithms and particle swarm optimization methods are mixed to create an efficient optimization tool. First GAs is used to generate first population and using the FEM the fitness values of the population would be calculated. Then, the top-percent of best-performing ones are marked. These individuals are regarded as elites. Instead of reproducing the elites directly to the next generation first the elites are enhanced by PSO. By using these enhanced elites as parents, the generated offsprings will usually achieve better performance than those bred by original elites. The group constituted by the elites is regarded as a swarm, and each elite corresponds to a particle in it. More details of the used mixed method for solving the problem are briefly stated as follows,

Step 1: first population is generated using GAs.
Step 2: fitness of the population is calculated.
Step 3: the selected percent of elites would be used in PSO method as an input data.
Step 4: new population is generated using PSO method as enhanced elites and by using GAs considering tournament selection and mutation in conjunction with the crossover operation as offspring.
Step 5: fitness of the new population is calculated.
Step 6: the bellow convergence criteria should be checked.

$$\left| J[\lambda]^{n+1} - J[\lambda]^{n} \right| \Big/ J[\lambda]^{n} \le \varepsilon_0 \tag{16}$$

where n denotes the number of iteration and ε_0 is a small value number and in the present analysis is taken to be 10^{-6}.

Step 7: if convergence is not achieved solution procedure from step 2 should be repeated.
Step 8: if convergence is achieved depth of cracks and their locations can be obtained.
The solution procedure for crack identification of beams is shown in Fig. 2.

3. Numerical results

In this section, first convergence and accuracy of the presented solution is investigated. Then, convergence, accuracy and robustness of the hybrid optimization technique for crack identification of beams are shown. For all solved problems a steel beam with the following properties is considered.

L=0.5m, b=0.01m, h=0.02m, E=210Gpa, ρ=7860kg/m^3, $\beta_i=C_i b/L$

As a first example, the convergence and accuracy of the FEM for solving the reformulated governing equation is investigated in Table 1. It can be seen that the presented results are in very excellent agreement with those of Lee (2009). Without any extra effort, hereafter for solving the problems N_e=20 is considered.

In order to show the applicability of the presented method for different crack ratios Fig. 3 is prepared. From this figure one can see that the obtained (ω_c/ω), frequencies of cracked beam to frequency of beam with no crack ratio, from the presented method are in good agreement with those obtained from ANSYS software. For solving the problem in ANSYS software 1000 two dimensional (2D) plane stress elements were used.

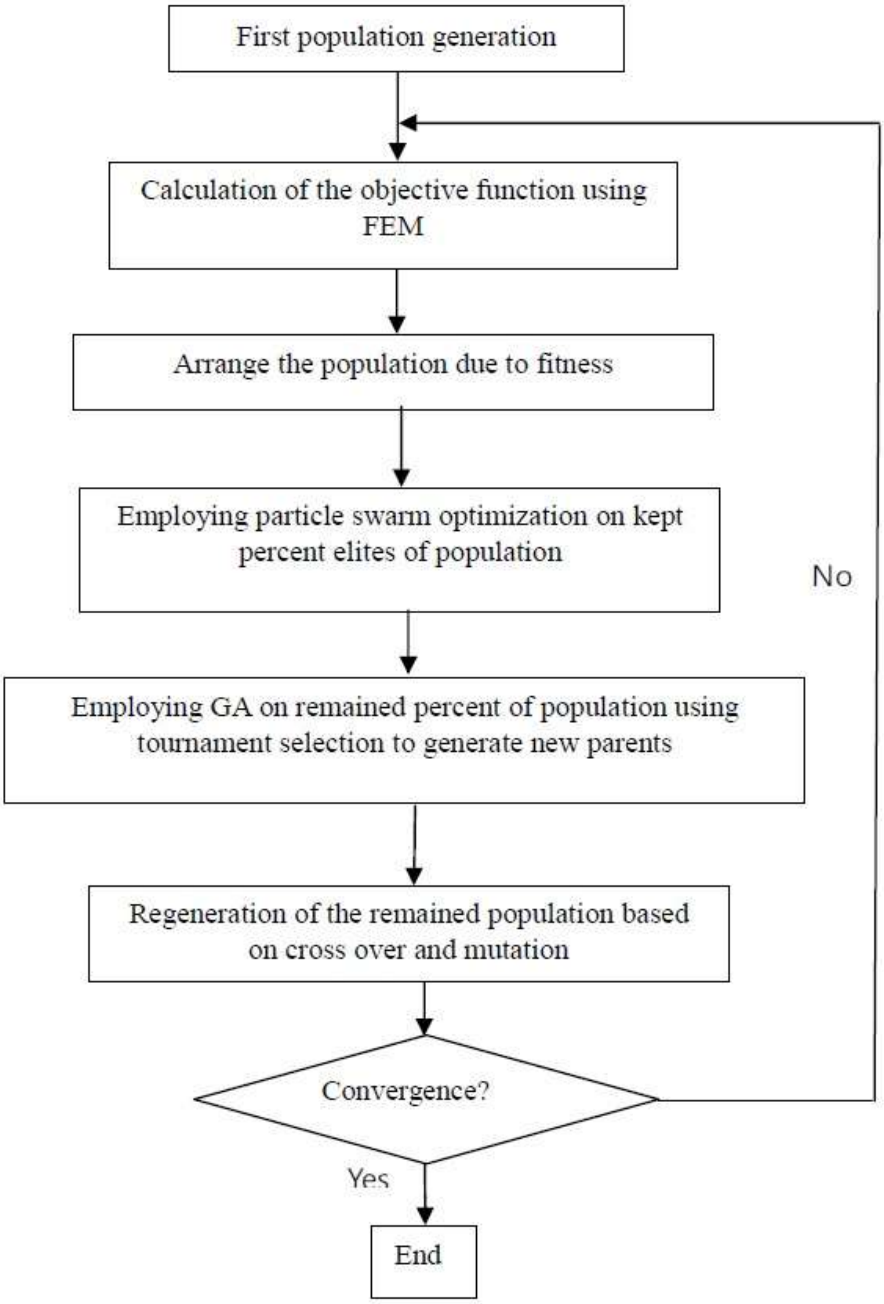

Fig. 2 Flowchart of the hybrid optimization solution

Table 1 Convergence and accuracy of the first four natural frequencies of cantilever isotropic steel beams ($\xi_1 = \xi_2 = \xi_3 = 0.1, \beta_3 = 1.5\beta_2 = 3\beta_1 = 0.6$)

		Mode sequence			
	N_e	1	2	3	4
Present	5	416.9212	2613.481	7349.902	14521.52
	10	416.9161	2612.296	7326.031	14370.67
	20	416.9157	2612.217	7324.325	14358.15
	40	416.9157	2612.212	7324.215	14357.33
Lee (2009)		416.8933	2612.065	7323.879	14356.68
% Relative error		0.005%	0.006%	0.005%	0.005%

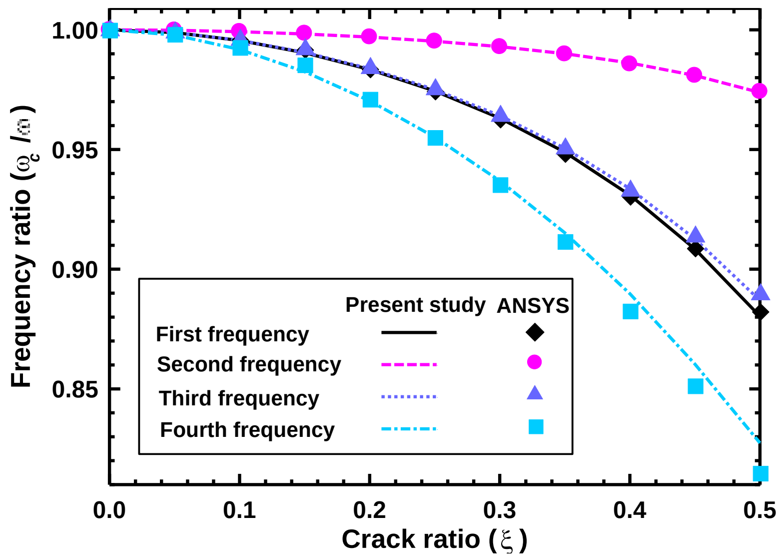

Fig. 3 Comparison of the hybrid method via ANSYS software ($\beta_3 = 1.5\beta_2 = 3\beta_1 = 0.6$)

To show how efficient the mixed optimization technique is different examples are solved. In the all solved examples (otherwise mentioned) eigen-pair(s) are calculated for a beam with $\xi_1 = \xi_2 = \xi_3 = 0.3, \beta_3 = 1.5\beta_2 = 3\beta_1 = 0.6$ using the FEM. Then random error is applied to the eigen-pair(s) with respect to the averages and standard deviations presented in Table 2 and the generated eigen-pairs are used as input for solving the crack identification problems.

Table 2 Average (AVE) and standard deviation (SD) of applied random errors used in the crack identification problems

Random error		Number of used eigen-pairs		
		1	2	3
+5%	AVE	3.35%	2.94%	2.98%
	SD	2.39%	1.85%	1.95%
+10%	AVE	6.42%	7.15%	6.32%
	SD	3.96%	4.86%	3.95%

In Figs. 4 and 5 influences of different population sizes and keeping percent on convergence of the hybrid optimization technique are studied, respectively. To verify efficiency of the hybrid method convergence of GAs for solving the problem are shown on the figures. From these figures, it is obvious that by increasing the applied random error, the obtained root mean square errors (RMSE) are increased.

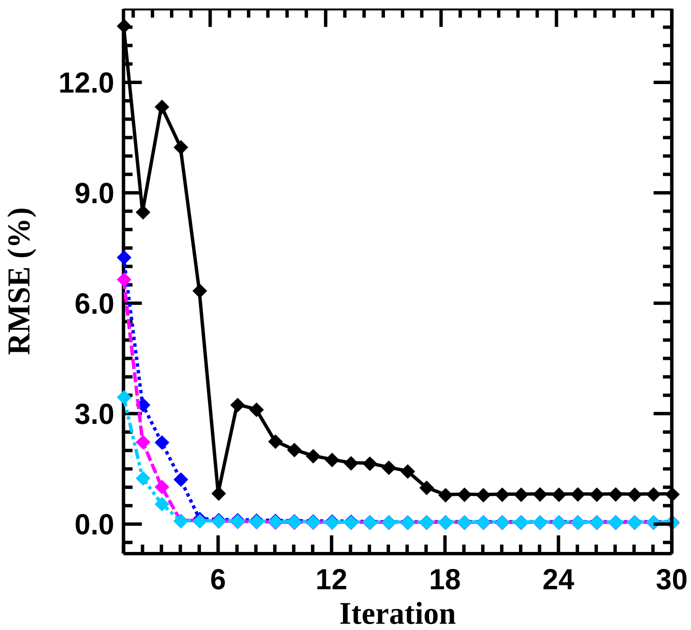

Fig. 4 Convergence and robustness of the presented hybrid approach for the crack identification of the beams in presence and absence of the PSO with +10% applied random error and keeping percent=70% (GAs with population size=30: , GAs-PSO with; population size=10: , population size=20: , population size=30:)

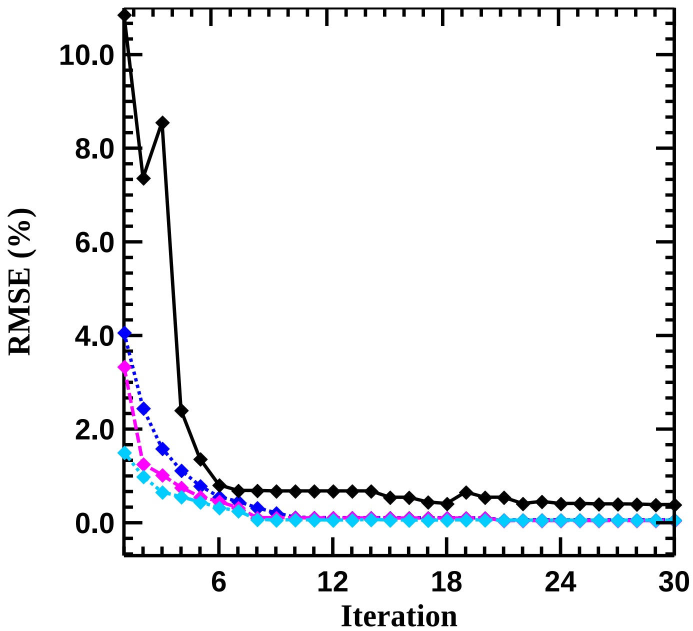

Fig. 5 Convergence and robustness of the presented hybrid approach for the crack identification of the beams in presence and absence of the PSO with +5% applied random error and with population size=30 (GAs: , GAs-PSO with; keeping percent =30%: , keeping percent=50%: and keeping percent=70%:)

Table 3 Accuracy of the presented method for crack identification of beams using different eigen-pairs with +5% applied random errors.

Eigen-pair(s)		Obtained parameters	Crack number i=1	i=2	i=3	RMSE (%)
1	GAs-PSO	β_i	0.2231	0.4170	0.6214	0.3596 (16) [a]
		ξ_i	0.3233	0.3194	0.2887	
	GAs	β_i	0.2468	0.4658	0.6789	0.9693 (24)
		ξ_i	0.3683	0.2765	0.3685	
2	GAs-PSO	β_i	0.2151	0.4112	0.6135	0.2866 (14)
		ξ_i	0.3149	0.3059	0.3111	
	GAs	β_i	0.2392	0.4368	0.6685	0.8105 (23)
		ξ_i	0.3475	0.3365	0.3498	
3	GAs-PSO	β_i	0.2090	0.4075	0.6012	0.2866 (14)
		ξ_i	0.3085	0.2965	0.3042	
	GAs	β_i	0.2291	0.4305	0.6598	0.8615 (25)
		ξ_i	0.3235	0.3102	0.3552	

[a] Numbers in the parentheses is the iteration number at which the convergence is achieved

Table 4 Accuracy of the presented method for crack identification of beams using different eigen-pairs and +10% applied random errors

Eigen-pair(s)		Obtained parameters	Crack number			RMSE (%)
			i=1	i=2	i=3	
1	GAs-PSO	β_i	0.2414	0.4280	0.6394	0.4712 (22) [a]
		ξ_i	0.3312	0.3298	0.3236	
	GAs	β_i	0.2825	0.4971	0.7021	0.6925(27)
		ξ_i	0.3854	0.3568	0.4001	
2	GAs-PSO	β_i	0.2198	0.4199	0.6249	0.3685 (19)
		ξ_i	0.3276	0.3118	0.3199	
	GAs	β_i	0.2652	0.4568	0.6872	0.5833 (24)
		ξ_i	0.3796	0.3433	0.3865	
3	GAs-PSO	β_i	0.2115	0.4126	0.6089	0.2973 (18)
		ξ_i	0.3125	0.3068	0.2963	
	GAs	β_i	0.2452	0.4461	0.6635	0.4985 (27)
		ξ_i	0.3698	0.3323	0.3589	

[a] See the footnote of Table 3

Table 5 Accuracy of the presented method for crack identification of beams using different eigen-pairs with +5% applied random errors

Eigen-pair(s)		Obtained parameters	Crack number			RMSE (%)
			i=1	i=2	i=3	
1	GAs-PSO	β_i	0.2213	0.4273	0.6351	0.4261 (18) [a]
		ξ_i	0.1121	0.1091	0.0912	
	GAs	β_i	----	0.5168	----	1.4251 (44)
		ξ_i	----	0.3325	----	
2	GAs-PSO	β_i	0.2194	0.4203	0.6217	0.3987 (19)
		ξ_i	0.0985	0.0973	0.1045	
	GAs	β_i	0.2131	----	0.7925	1.2115 (46)
		ξ_i	0.1734	----	0.2471	
3	GAs-PSO	β_i	0.2090	0.4075	0.6012	0.3325 (17)
		ξ_i	0.1084	0.0987	0.1074	
	GAs	β_i	0.2319	0.4376	0.5894	1.1845 (48)
		ξ_i	0.0875	0.1026	0.1402	

[a] See the footnote of Table 3

With respect to Figs. 4 and 5, fast rate of convergence and robustness of the new hybrid optimization method for crack identification of the beam is interesting than using the GAs. So, for generating the numerical results hereafter without any extra effort population size=20 and keeping percent=70% are used.

In Tables 3 and 4 for different number of measured eigen-pairs the cracks depth and their locations are identified and tabulated. The results are presented for the two cases of the GAs-PSO and GAs optimization techniques separately. From these tables, one can see that the more accurate identification procedure and faster rate of convergence are obtained using the hybrid presented GAs-PSO technique.

Table 5 is prepared to examine the accuracy of the hybrid method for crack identification of beam with $\xi_1 = \xi_2 = \xi_3 = 0.1, \beta_3 = 1.5\beta_2 = 3\beta_1 = 0.6$. From the obtained results one can see that by decreasing the crack depth to height ratio the presence of PSO is necessary for finding depth and location of cracks.

4. Conclusions

A developed hybrid method for crack identification of beams is presented. Using fracture mechanic concepts, the governing equation of thin cracked beam is reformulated. Finite element method as a powerful numerical tool is adopted to discretize the equation in space domain then the obtained equation is transferred from time domain to frequency domain. To show the applicability of the equation comparisons are made with those available in literature and via ANSYS software. By applying random error to arbitrary number of obtained frequencies and mode shapes (eigen-pairs) of cracked beams, measured data are produced. An objective function is generated by calculating root mean square error between the measured and the estimated data. To minimize the function and obtain the cracks depths and locations, genetic algorithms and particle swarm optimization techniques are combined. Convergence, efficiency, robustness and accuracy of the optimization method in conjunction with the finite element method are demonstrated via solving different examples.

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Appendix A. Nomenclature

a_i	depth of i^{th} crack
b	width of beam
C_i	location of i^{th} crack
c_{bi}	equivalent flexibility of cross section at i^{th} crack
D	bending stiffness
E	Young's modulus
h	thickness of beam
I_o	moment of inertia
K	global stiffness matrix of beam
k_{ij}^{e}	stiffness matrix of eleemnts
L	length of beam
M	number of used eigen-pairs
m_{ij}^{e}	mass matrix of elements
m_{ij}^{ec}	mass matrix of cracked elements
N_e	number of elements
NM	new global mass matrix
w	displacement component in the transverse direction of a point on natural axis of beam
$\{V\}$	vector of the transverse displacement degrees of freedom
β_i	crack location to beam length ratio
ε_0	tolerance of convergence
δ	Dirac-delta function
λ_s	estimated (computed) eigen-pairs
λ_s^*	measured eigen-pairs
$\varphi_i\left(i=1,\ldots,4\right)$	Hermitian interpolation shape functions
ρ	beam density
ω	frequency of beam with no crack
ω_c	frequency of cracked beam
ξ_i	crack depth to beam height ratio