Smart Structures and Systems, *Vol. 15, No. 6 (2015) 1569-1582*
DOI: http://dx.doi.org/10.12989/sss.2015.15.6.1569

A comparative study for bending of cross-ply laminated plates resting on elastic foundations

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(Received March 29, 2014, Revised June 22, 2014, Accepted June 29, 2014)

Abstract. Two hyperbolic displacement models are used for the bending response of simply-supported orthotropic laminated composite plates resting on two-parameter elastic foundations under mechanical loading. The models contain hyperbolic expressions to account for the parabolic distributions of transverse shear stresses and to satisfy the zero shear-stress conditions at the top and bottom surfaces of the plates. The present theory takes into account not only the transverse shear strains, but also their parabolic variation across the plate thickness and requires no shear correction coefficients in computing the shear stresses. The governing equations are derived and their closed-form solutions are obtained. The accuracy of the models presented is demonstrated by comparing the results obtained with solutions of other theories models given in the literature. It is found that the theories proposed can predict the bending analysis of cross-ply laminated composite plates resting on elastic foundations rather accurately. The effects of Winkler and Pasternak foundation parameters, transverse shear deformations, plate aspect ratio, and side-to-thickness ratio on deflections and stresses are investigated.

Keywords: hyperbolic expressions; cross-ply laminated plate; elastic foundations

1. Introduction

Laminated composite plates are increasingly used as structural components in engineering applications, and many approximate analytical and numerical methods have been developed for calculating their mechanical behavior. Composite laminated structures are widely used, particularly in aerospace engineering due to their superior mechanical properties. By virtue of their high strength to weight ratios and because of their mechanical properties in various directions, they can be tailored as per requirements. Further, they combine a number of unique properties, including corrosion resistance and high damping. These unique properties have resulted in the expanded use of the advanced composite materials in structures subjected to mechanical loading. Examples are provided by structures used in high-speed aircraft, spacecraft, etc. A laminated plate consists of several laminae each with a fiber oriented at a specified angle. This is generally made by stacking several thin layers of fibers at the desired locations and angles in a matrix and consolidating them to give the required thickness. The fiber orientation in each thin layer can be

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

arranged in a specific manner so as to achieve the required properties of the structural member (Rao 1999).

There is a number of plate theories that are used to represent the kinematics of deformation. The most widely used is the classical plate theory (CPT), in which straight lines or planes normal to the neutral plate axis remain straight and normal after deformation. This theory thus neglects the effect of transverse shear deformations, a condition that holds only in the case of slender plates. It is well-known that for the case of foundation plates with small side-to-thickness ratio this contribution cannot be neglected. In such cases, it becomes necessary to take into account shear deformation effects. To confront this problem, various improved plate theories such as first-order shear deformation plate theory (FPT) (see Akavci *et al*. 2007, Cheng and Batra 2000a and Lanhe 2004), in which the normality assumption is removed and the effect of transverse shear deformation is considered, can be used. In fact, FPT needs a shear correction factor, which depends not only on the material and geometric parameters, but also on the loading and boundary conditions. Higher-order shear deformation plate theory (HPT) was developed for plates with rectangular cross-sections that account for the strain distribution through the depth to satisfy the boundary conditions on the upper and lower surfaces without the need for a correction coefficient (see Cheng and Batra 2000a, b, Reddy 2000). Also, the sinusoidal shear deformation plate theory (SPT) was developed by Zenkour (2004a, b, 2006). In this theory, trigonometric terms are used for the displacements in addition to the initial terms of a power series through the thickness. The form of the assumed displacements of this theory is simplified by enforcing traction-free boundary conditions at the plate faces. No transverse shear correction factors are needed because a correct representation of the transverse shear strain is given. Carrera (2002) has presented an overview of available theories and finite elements for multilayered, anisotropic, composite structures. In Carrera and Ciuffreda (2005), a unified formulation has been used to compare about 40 theories for multilayered, composites and sandwich plates subjected to transverse pressure with various in-plane distributions.

A plate on an elastic foundation belongs to the problem of mutual action between two media. Plates supported by elastic foundations are commonly encountered technical problems in many engineering applications. The studies of plates resting on elastic foundations have attracted the attentions of many researchers (Jaiswal and Iyengar 1993, Chudinovich and Constanda 2000, Dumir 2003, Tsiatas 2010, Akgoz and Civalek 2011 and Al Khateeb and Zenkour 2014). Early research adopted single-parameter Winkler model to simulate the foundation. It considered that the displacement on a foundation surface is limited only on the loaded domain, which conflicts with the practical response situation. In some analyses of plates on elastic foundations, a single parameter is used to describe the foundation behavior (Akavci *et al.* 2007). In this model it is assumed that there is a proportional interaction between the external forces and the deflection of the applied point in the foundation. Liew *et al.* (1996) studied the differential quadrature method for Mindlin's plates on Winkler foundation. Eratll and Akoz (1997) used a new function to examine Mindlin's plate on Winkler foundation.

The response of structural elements resting on the one- and two-parameter foundation is usually analyzed by assuming that the foundation supports compressive as well as tensile stresses, which simplifies the analysis considerably. Two-parameter elastic foundation model can reflect the practical deformation of a foundation, so it is widely accepted by investigators. Many researchers have modeled the foundations with two parameters. One of these models is the Pasternak model. This two-parameter model takes into account the effect of shear interaction among the points in the foundation (Chien and Chen 2006), and the well-known Winkler model is one of its special cases.

Han and Liew (1997) investigated numerical differential quadrature method for Reissner-Mindlin's plates on two-parameter elastic foundations. Omurtag and Kadioglu (1998) investigated the vibration of Kirchhoff's plates on Winkler and Pasternak foundations. Singh *et al.* (2007) investigated the post buckling response of laminated composite plate on elastic foundation with random system properties. Zenkour (2009) discussed the refined sinusoidal theory for functionally graded plates on elastic foundations. Based on a refined sinusoidal plate theory, Zenkour *et al.* (2010) presented the bending response of functionally graded viscoelastic beams resting on elastic foundations. Also, Zenkour *et al.* (2011) investigated the bending of a fiber-reinforced viscoelastic composite plate resting on elastic foundations. Shen and Zhu (2012) investigated postbuckling of sandwich plates with nanotube-reinforced composite face sheets resting on elastic foundations. Recently, Zenkour *et al.* (2013) studied the bending of cross-ply laminated plates resting on elastic foundations under thermo-mechanical loading. Also, Zenkour *et al.* (2014) investigated the effects of hygrothermal conditions on cross-ply laminated plates resting on elastic foundations.

In this study, additional two hyperbolic displacement models for laminated composite plates resting on elastic foundations are proposed. Analytical solutions for the bending response of symmetric and anti-symmetric cross-ply laminated plates resting on elastic foundations are obtained. Based on the principle of virtual displacements, the governing equations are deduced. The interaction between the plate and the elastic foundations is considered and included in the equilibrium equations. Pasternak model is used here to describe the two-parameter elastic foundation, and getting a special case of Winkler foundation model. Numerical results for deflections and stresses are presented. It is found that both the models are able to provide accurate solutions.

2. Governing equations

Consider a rectangular laminated plate of length a, width b and uniform thickness h (see Fig. 1). The plate is composed of n orthotropic layers oriented at angles $\theta_1, \theta_2, \ldots, \theta_n$. The material of each layer is assumed to possess one plane of elastic symmetry parallel to the *x*-*y* plane. Perfect bending between the orthotropic layers and mechanical properties is assumed. Let the plate be subjected to a transverse load $q(x, y)$.

2.1 Winkler-Pasternak foundations

Pasternak model is the most natural extension of the Winkler one. It considers a shear interaction between the spring elements by connecting the ends of the springs to a plate of an incompressible shear layer. The reaction-deflection relation of the shear layer and spring elements is given by

$$E_f = K_{\mathrm{W}} w - K_{\mathrm{P}} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w \tag{1}$$

where E_f, w, K_{W} and K_{P} are the density of reaction of foundation, transverse displacement, normal (Winkler) and shear (Pasternak) foundation stiffnesses, respectively, If the foundation is modeled as the linear Winkler foundation, the coefficient K_{P} in Eq. (1) is zero.

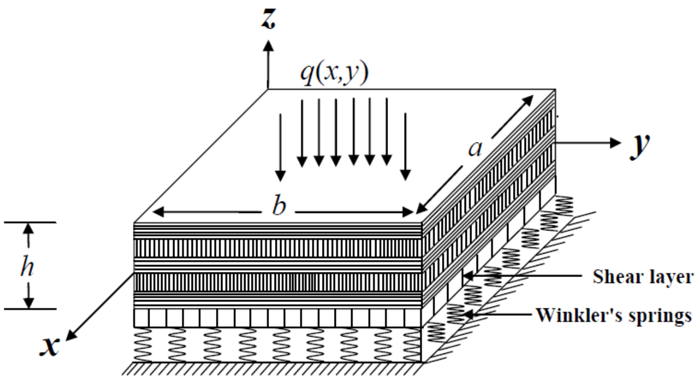


Fig. 1 Schematic diagram for the laminated plate resting on elastic foundations

2.2 Hyperbolic shear-deformation plate theory

The displacement field models are chosen such that to ensure zero transverse shear stresses on top and bottom surfaces of the plate. The displacement field can be written in the unified form as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + \Phi(z)u_1(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + \Phi(z)v_1(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

where u, v, and w are the displacements in the x, y, and z directions, u_0, v_0, and w_0 are the mid-plane displacements; u_1 and v_1 are the rotations of normals to the mid-plane about the x- and y-axis, respectively, $\Phi(z)$ is a hyperbolic shape function. The shape function $\Phi(z)$ satisfies the conditions

$$\left.\frac{\mathrm{d}\Phi}{\mathrm{d}z}\right|_{z=\pm h/2} = 0, \qquad \int_{-h/2}^{h/2}\Phi(z)\mathrm{d}z = 0 \tag{3}$$

By substituting the displacement relations given in Eq. (2) into the strain-displacement equations of elasticity, the normal and shear strain components are obtained as

$$\begin{aligned} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} &= \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial v_0}{\partial x} + \dfrac{\partial u_0}{\partial y} \end{Bmatrix} - z\begin{Bmatrix} \dfrac{\partial^2 w_0}{\partial x^2} \\ \dfrac{\partial^2 w_0}{\partial y^2} \\ 2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} + \Phi(z)\begin{Bmatrix} \dfrac{\partial u_1}{\partial x} \\ \dfrac{\partial v_1}{\partial y} \\ \dfrac{\partial v_1}{\partial x} + \dfrac{\partial u_1}{\partial y} \end{Bmatrix} \\ \{\gamma_{xz}, \gamma_{yz}\} &= \frac{\mathrm{d}\Phi}{\mathrm{d}z}\{u_1, u_2\}, \quad \varepsilon_{zz} = 0 \end{aligned} \tag{4}$$

The stress-strain relationships, accounting for transverse shear deformation and thermal effects, in the plate coordinates for the rth layer can be expressed as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{16}\\ c_{12} & c_{22} & c_{26}\\ c_{15} & c_{26} & c_{66}\end{bmatrix}^{r}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix},\quad \begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}c_{44} & c_{45}\\ c_{45} & c_{55}\end{bmatrix}^{r}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{5}$$

where c_{ij}^{r} are the transformed elastic coefficients.

2.3 Equations of equilibrium

The governing equilibrium equations can be derived by using the principle of virtual displacements. They given by the following forms as associated with the present unified hyperbolic shear deformation theory

$$\begin{gathered}\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0,\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=0\\ \frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+q-E_f=0\\ \frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-Q_{xz}=0,\quad \frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y}-Q_{yz}=0\end{gathered} \tag{6}$$

where N_{ij} and $M_{ij}(i,j=x,y)$ are the basic components of stress resultants and stress couples, S_{ij} are additional stress couples associated with the transverse shear effects, and Q_{iz} are the transverse shear stress resultants of a laminated composite plate that made up of n layers of orthotropic lamina. They can be obtained by integrating Eq. (5) over the thickness of the plate.

4. Bending of cross-ply laminated plates

The determination of transverse deflections and stresses are of fundamental importance in the design of many structural components. The present boundary-value problem associated with the equilibrium of laminated plates involves solving the differential Eq. (6), subjected to a given set of boundary conditions. An exact closed-form solution to Eq. (6) can be constructed when the plate is of a rectangular geometry (Fig. 1) with the following edge conditions, loading, and plate construction.

4.1 Boundary conditions

The following simply-supported boundary conditions along the edges of the plate are considered

$$\begin{gathered}v_0=w_0=v_1=N_{xx}=M_{xx}=S_{xx}=0\quad \text{at}\quad x=0,a\\ u_0=w_0=u_1=N_{yy}=M_{yy}=S_{yy}=0\quad \text{at}\quad y=0,b\end{gathered} \tag{7}$$

4.2 Distributed loading

We assume that the applied transverse load q can be expanded in the double-Fourier series as

$$q(x,y)=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}q_{ij}\sin(\lambda x)\sin(\mu y) \tag{8}$$

where $\lambda = i\pi/a$, $\mu = j\pi/b$; i and j are mode numbers. For the case of uniformly distributed load, $q_{ij} = 16q_0/(ij\pi^2)$ for odd i and j and $q_{ij} = 0$ otherwise. However, in the case of sinusoidally distributed load, $i = j = 1$ and $q_{11} = q_0$ in which q_0 represents the intensity of the load at the plate centre.

4.3 Plate construction

The plane stress-reduced material stiffness of the lamina are given by

$$c_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad c_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}, \quad c_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$
$$c_{44} = G_{23}, \quad c_{55} = G_{13}, \quad c_{66} = G_{12} \tag{9}$$

where E_i, G_{ij} and ν_{ij} stand for Young's moduli, shear moduli and Poisson's ratios, respectively. The plate construction may be cross-ply, i.e., θ_r should be either 0° or 90°. This requires that the index 1 in Eq. (9) in the first layer with $\theta_1 = 0^\circ$ should be changed to 2 in the second layer with $\theta_2 = 90^\circ$ and vice versa.

Under the above specific conditions, the appropriate solution $(u_0, v_0, w_0, u_1, v_1)$ to Eq. (6) is given by

$$\begin{Bmatrix} (u_0, u_1) \\ w_0 \\ (v_0, v_1) \end{Bmatrix} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \begin{Bmatrix} (U_{ij}, X_{ij})\cos(\lambda x)\sin(\mu y) \\ W_{ij}\sin(\lambda x)\sin(\mu y) \\ (V_{ij}, Y_{ij})\sin(\lambda x)\cos(\mu y) \end{Bmatrix} \tag{10}$$

where U_{ij}, V_{ij}, W_{ij}, X_{ij} and Y_{ij} are arbitrary parameters.

5. Numerical results and discussions

The static behaviours of the simply-supported, orthotropic rectangular plates resting on elastic foundations and subjected to mechanical loading are considered. The famous form of the shape function $\Phi(z)$ is introduced in many articles of the author and other investigators in the form

$$\Phi(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{11}$$

which satisfies the conditions given in Eq. (3). The above sinusoidal form proves itself to present accurate results when compared with the 3-D solution or any higher-order shear deformation theory solutions. However, two additional forms of the shape function $\Phi(z)$ will be examined here. They are given for the higher-order shear deformation theories HSDT1 and HSDT2 in the forms

$$\text{HSDT1:}\, \Phi(z) = \frac{z\cosh\left(\frac{\pi}{2}\right) - \frac{h}{\pi}\sinh\left(\frac{\pi z}{h}\right)}{\cosh\left(\frac{\pi}{2}\right) - 1} \tag{12}$$

and

$$\text{HSDT2:}\, \Phi(z) = h\sinh\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{2}\right) \tag{13}$$

It is to be noted that the above two forms also satisfy the two conditions given in Eq. (3).

All of the lamina are assumed to be of the same thickness and made of the same orthotropic material. The lamina properties are given for two materials.

Material I (Brischetto 2012 and Reddy and Hsu 1980)

$$E_1 = 25E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.2E_2, \quad \nu_{12} = 0.25 \tag{14}$$

Material II (Şahin 2005)

$$E_1 = 2.5E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.25E_2, \quad \nu_{12} = 0.3 \tag{15}$$

The results for cross-ply orthotropic rectangular plates resting on elastic foundations using both HSDT1 and HSDT2 are reported in Tables 1-3. However, Figs. 2-8 have displayed by using HSDT1 only. The improvement in the prediction of displacements and stresses by the present theory will be discussed. All figures and Tables 2 and 3 are given by using material II. The deflection and stresses of the bending response are presented. Here, the 3-D solution of Pagano (1970) and the solution of Brischetto (2012) for simply-supported rectangular plates under sinusoidal mechanical loading are used to assess the improvement.

The following dimensionless forms are used

$$\begin{gathered} \bar{w} = \frac{10^2 h^3}{a^4 q_0} w, \quad \kappa_W = h K_W, \quad \kappa_P = \frac{1}{h} K_P, \quad \sigma_1 = \frac{h}{a q_0} \sigma_{xx} \\ \sigma_2 = \frac{h}{a q_0} \sigma_{yy}, \ \sigma_4 = \frac{10h}{a q_0} \tau_{yz}, \ \sigma_5 = \frac{10^2 h}{a q_0} \tau_{xz}, \ \sigma_6 = \frac{10^2 h^2}{a^2 q_0} \tau_{xy} \end{gathered} \tag{16}$$

Table 1 Mechanical load applied to a three-layered $(0^\circ/90^\circ/0^\circ)$ laminated plate $(b = 3a)$ using material I

a/h		Pagano (1970)	Brischetto (2012)				Present	
		3-D	FSDT	ED2	ED4	LD4	HSDT1	HSDT2
4	$\bar{w}$	2.82	2.05	2.03	2.62	2.82	2.61568	2.63848
	$\bar{\sigma}_1\left(\pm\frac{h}{2}\right)$	1.14	0.614	0.637	1.11	1.14	1.00961	1.03301
		-1.10	-0.614	-0.591	-1.06	-1.10	-1.00961	-1.03301
	$\bar{\sigma}_2\left(\pm\frac{h}{6}\right)$	0.109	0.0833	0.0791	0.100	0.109	0.10208	0.10271
		-0.119	-0.0833	-0.0901	-0.111	-0.119	-0.10208	-0.10271
	$\bar{\sigma}_6\left(\pm\frac{h}{2}\right)$	-0.0269	-0.0187	-0.0177	-0.0254	-0.0269	-0.02588	-0.02627
		0.0281	-0.0187	-0.0189	0.0266	0.0281	0.02588	0.02627
	$\bar{\sigma}_4(0)$	0.0334	0.0234	0.0246	0.0346	0.0334	0.03418	0.03474
100	$\bar{w}$	0.508	0.506	0.506	0.507	0.508	0.50693	0.50699
	$\bar{\sigma}_1\left(\pm\frac{h}{2}\right)$	0.624	0.623	0.623	0.624	0.624	0.62392	0.62396
		-0.624	-0.623	-0.623	-0.624	-0.624	-0.62392	-0.62396
	$\bar{\sigma}_2\left(\pm\frac{h}{6}\right)$	0.0253	0.0252	0.0251	0.0252	0.0253	0.02528	0.02529
		-0.0253	-0.0252	-0.0251	-0.0252	-0.0253	-0.02528	-0.02529
	$\bar{\sigma}_6\left(\pm\frac{h}{2}\right)$	-0.0083	-0.0083	-0.0083	-0.0083	-0.0083	-0.00831	-0.00831
		0.0083	0.0083	0.0083	0.0083	0.0083	0.00831	0.00831
	$\bar{\sigma}_4(0)$	0.0108	0.0106	0.0108	0.0121	0.0108	0.01280	0.01292

Table 1 presents the deflection and stresses of a three-layered $(0^{\circ}/90^{\circ}/0^{\circ})$ composite plate under mechanical load. Brischetto (2012) used some models as layerwise (LW) and the equivalent single layer (ESL). ESL models are indicated with acronyms from ED1 to ED4 where E means ESL approach, D indicates the use of the principle of virtual displacements (PVD). LW models are indicated with acronyms from LD1 to LD4 where L means LW approach. The present results agree extremely well with those obtained by Pagano (1970) and Brischetto (2012) where $(\bar{\sigma}_1, \bar{\sigma}_2) = \frac{h}{a}(\sigma_1, \sigma_2)$ and $(\bar{\sigma}_4, \bar{\sigma}_6) = (0.1\sigma_4, 0.01\sigma_6)$. It is to be noted that the deflection and stresses are both decreasing with the increase of the side-to-thickness ratio a/h. Table 2 displays the effects of the number of layers on the defection $\bar{w}$ of rectangular plates resting on elastic foundations using the two models HSDT1 and HSDT2 $(a = 2b)$. A good agreement between HSDT1 and HSDT2 for deflections is occurred. In addition, it is observed that the deflection $\bar{w}$ decreases as Winkler parameter κ_W and Pasternak parameter κ_P increase. Table 3 shows the effects of the aspect ratio a/b and the foundation parameters κ_W and κ_P on the stresses of the four-layer $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ laminated plates resting on elastic foundations $(a = 4h)$. It is clear that the stresses σ_1, σ_2, σ_4, σ_5, and σ_6 decrease as the elastic foundation parameters decrease. Also, they decrease by increasing the aspect ratio a/b. Once again, a good agreement between the two models HSDT1 and HSDT2 has been occurred.

In all figures, except otherwise stated, $a = 10h$. Fig. 2 shows the dimensionless central deflection $\bar{w}$ versus the aspect ratio a/b (Fig. 2(a)) and the side-to-thickness ratio a/h (Fig. 2(b)). The deflection of the three-layer, cross-ply $(0^{\circ}/90^{\circ}/0^{\circ})$ rectangular plates resting on elastic foundations is illustrated for different elastic foundations. The deflection $\bar{w}$ decreases as the aspect ratio a/b and side-to-thickness ratio a/h increase regardless of the type of the elastic foundation. It observed from Fig. 2 that a decrement occurs for the central deflection $\bar{w}$ when the plate is resting on Pasternak foundations.

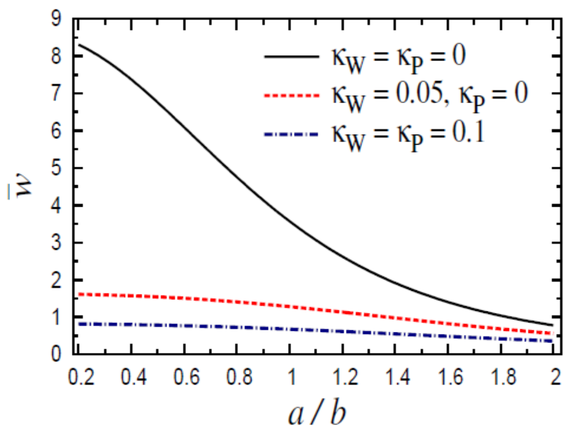
Table 2 Effects of the thickness and number of layers on the deflection $\bar{w}$ of rectangular plates resting on elastic foundations $(a = 2b)$

a/h	κ_W	κ_P	0°		$(0^{\circ}/90^{\circ})_1$		$(0^{\circ}/90^{\circ})_2$	
			HSDT1	HSDT2	HSDT1	HSDT2	HSDT1	HSDT2
4	0.0	0.0	1.53733	1.53627	1.22034	1.21875	4.57624	4.55647
	0.1	0.0	1.47912	1.47813	1.18337	1.18187	4.09634	4.08049
	0.1	0.1	1.32444	1.32365	1.08225	1.08099	3.09524	3.08618
10	0.0	0.0	0.82917	0.82925	0.66674	0.66669	3.74181	3.73896
	0.1	0.0	0.45330	0.45333	0.40002	0.40001	0.78911	0.78898
	0.1	0.1	0.37044	0.37045	0.33408	0.33406	0.56795	0.56788
12.5	0.0	0.0	0.77981	0.77987	0.62831	0.62829	3.68235	3.68054
	0.1	0.0	0.26855	0.26855	0.24796	0.24795	0.36859	0.36858
	0.1	0.1	0.22248	0.22248	0.20816	0.20816	0.28702	0.28701
15	0.0	0.0	0.75295	0.75299	0.60741	0.60739	3.64992	3.64867
	0.1	0.0	0.15648	0.15648	0.14906	0.14906	0.18739	0.18739
	0.1	0.1	0.13332	0.13332	0.12789	0.12789	0.15512	0.15511

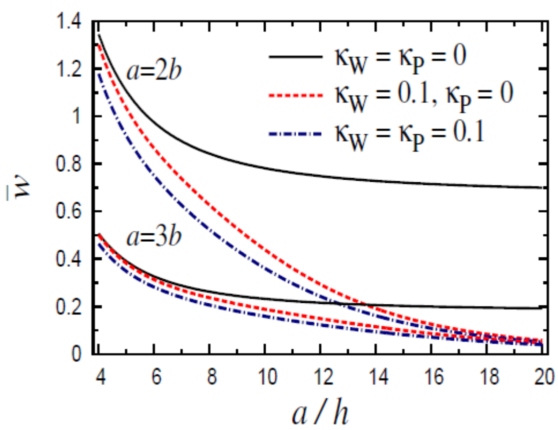
Table 3 Effects of the foundation parameters and aspect ratio on the stresses in the $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ laminated plates $(a = 4h)$

Stresses	κ_W	κ_P	$a = b$		$a = 2b$		$a = 3b$	
			HSDT1	HSDT2	HSDT1	HSDT2	HSDT1	HSDT2
σ_1	0.0	0.0	7.93925	7.93956	1.10946	1.11106	0.35069	0.35223
	0.1	0.0	5.58833	5.59126	1.04628	1.04795	0.34425	0.34579
	0.1	0.1	4.09307	4.09648	0.88996	0.89173	0.30921	0.31074
σ_2	0.0	0.0	5.77299	5.77895	2.50855	2.51915	1.37746	1.38877
	0.1	0.0	4.06353	4.06969	2.36569	2.37606	1.35216	1.36338
	0.1	0.1	2.97625	2.98169	2.01227	2.02185	1.21456	1.22519
σ_4	0.0	0.0	4.12793	4.16455	2.71779	2.74364	1.93934	1.95589
	0.1	0.0	2.90559	2.93279	2.56302	2.58780	1.90372	1.92013
	0.1	0.1	2.12815	2.14873	2.18011	2.20203	1.70999	1.72551
σ_5	0.0	0.0	17.83374	18.08135	7.32581	7.47089	3.63521	3.70648
	0.1	0.0	12.55293	12.73338	6.90863	7.04654	3.56845	3.63871
	0.1	0.1	9.19416	9.32920	5.87649	5.99609	3.20531	3.26991
σ_6	0.0	0.0	27.84769	27.88724	6.54852	6.58248	2.48975	2.51325
	0.1	0.0	19.60161	19.63895	6.17560	6.20859	2.44402	2.46730
	0.1	0.1	14.35684	14.38862	5.25298	5.28305	2.19531	2.21723

Figs. 3 and 4 display the distributions of the transverse shear stresses σ_4 and σ_5 through-the-thickness of four-layer, cross-ply, symmetric $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ and anti-symmetric $(0^{\circ}/90^{\circ}/0^{\circ}/90^{\circ})$ square (Fig. 3) and rectangular, $a = 2b$ (Fig. 4) plates resting on elastic foundations. It is to be noted that both σ_4 and σ_5 decrease as the foundation parameters κ_W and κ_P increase.

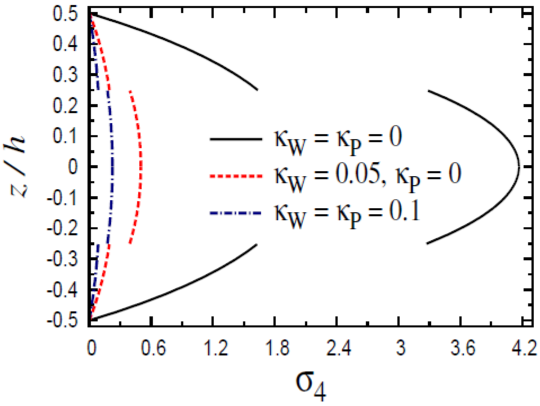


(a) $\bar{w}$ versus a/b

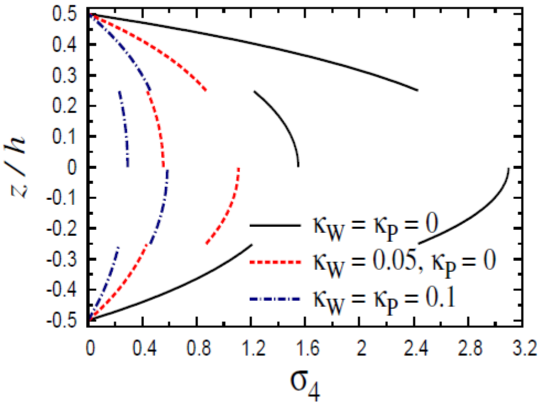


(b) $\bar{w}$ versus a/h

Fig. 2 Dimensionless deflection $\bar{w}$ of a $(0^{\circ}/90^{\circ}/0^{\circ})$ rectangular plate resting on elastic foundations

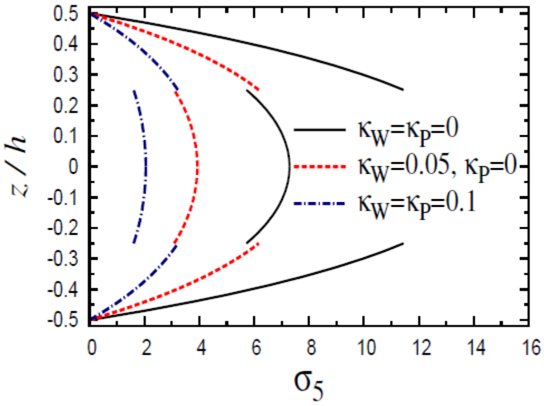


(a) A $(0^\circ/90^\circ/90^\circ/0^\circ)$ plate

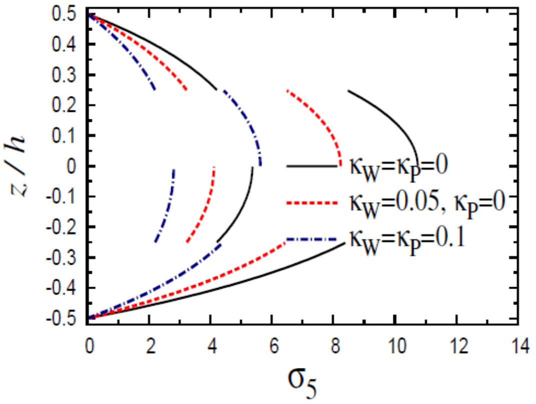


(b) A $(0^\circ/90^\circ/0^\circ/90^\circ)$ plate

Fig. 3 Distribution of σ_4 through-the-thickness of square plates for different elastic foundations

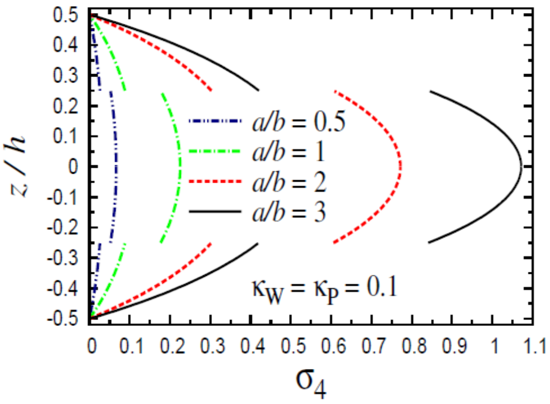


(a) A $(0^\circ/90^\circ/90^\circ/0^\circ)$ plate

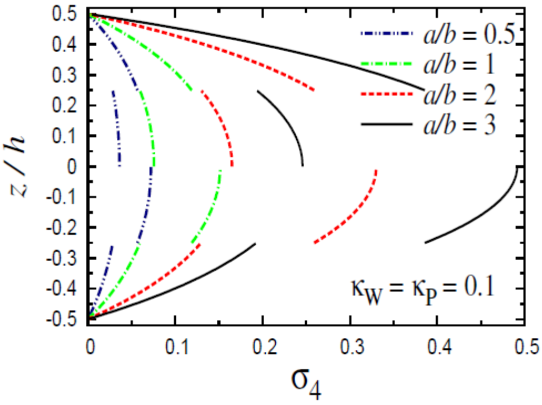


(b) A $(0^\circ/90^\circ/0^\circ/90^\circ)$ plate

Fig. 4 Distribution of σ_5 through-the-thickness of rectangular plates for different elastic foundations

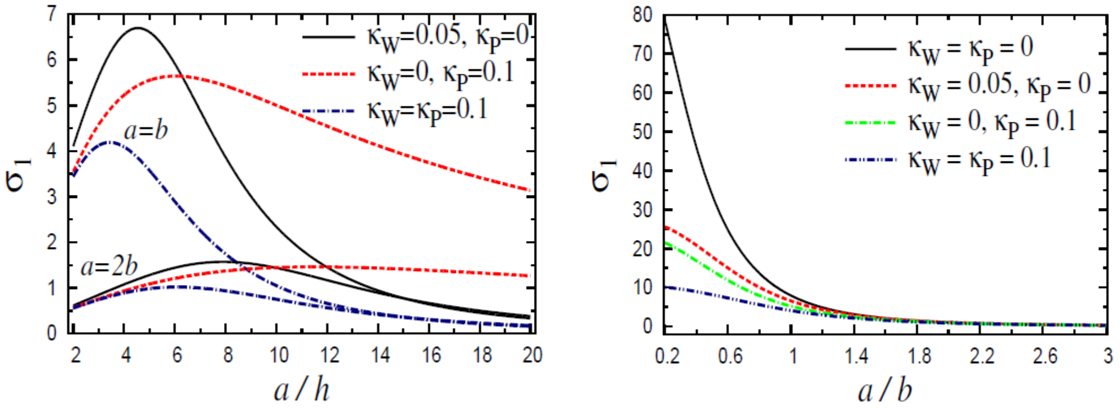


(a) A $(0^\circ/90^\circ/90^\circ/0^\circ)$ plate



(b) A $(0^\circ/90^\circ/0^\circ/90^\circ)$ plate

Fig. 5 Distribution of σ_4 through the thickness of cross-ply square plates for different aspect ratios



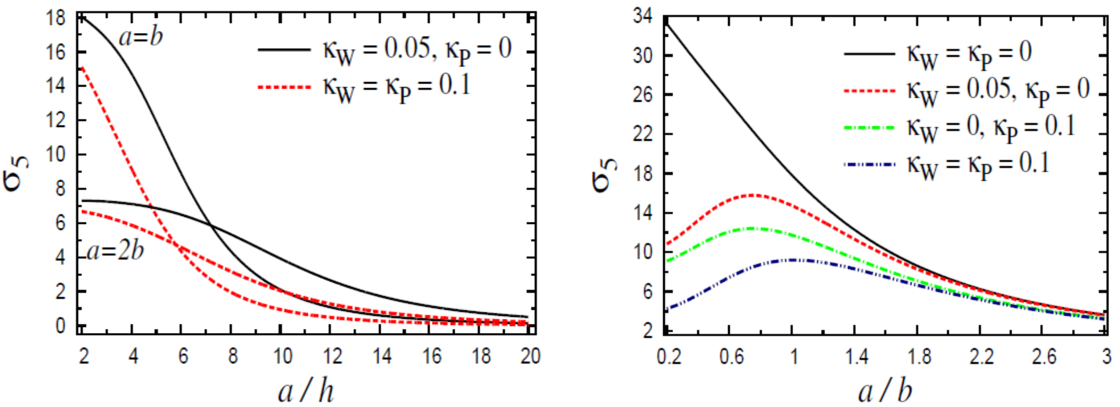
(a) σ_1 versus a/h (b) σ_1 versus a/b $(a = 4h)$

Fig. 6 Variation of $\sigma_1(0.5)$ in a $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ plate for different elastic foundations

Fig. 5 displays the distribution of the transverse shear stress σ_4 through-the-thickness of four-layer, cross-ply, symmetri $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ and anti-symmetric $(0^{\circ}/90^{\circ}/0^{\circ}/90^{\circ})$ rectangular plates resting on Winkler/Pasternak elastic foundations. The shear stress σ_4 increases as the aspect ratio increases.

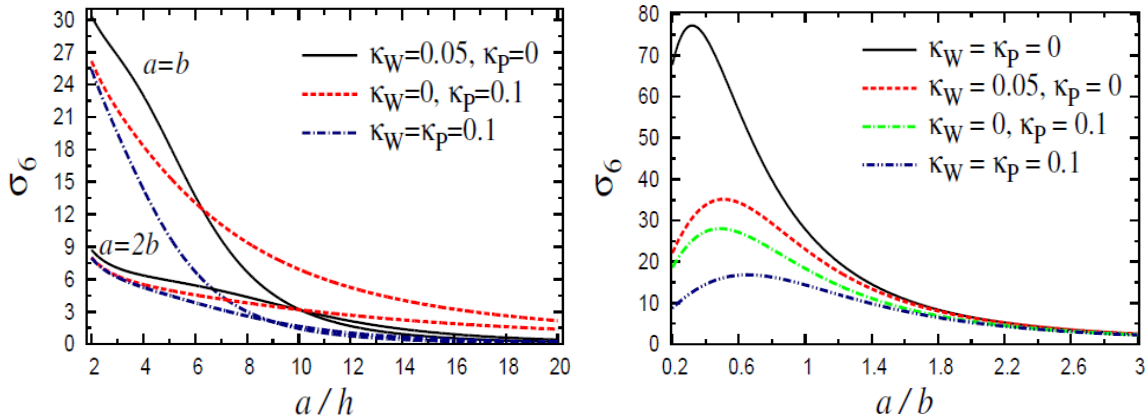
Fig. 6 shows the variation of the in-plane longitudinal stress $\sigma_1(0.5)$ versus the side-to-thickness ratio a/h and the aspect ratio a/b in four-layer, cross-ply, symmetric $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ plates resting on elastic foundations. The in-plane longitudinal stress σ_1 is decreasing with the increase of the ratios a/h and a/b. As it is well known, the stress σ_1 is decreasing with the increase of the foundation parameters κ_W and κ_P.

Fig. 7 displays the variation of the transverse shear stress $\sigma_5(0)$ versus the side-to-thickness ratio a/h and the aspect ratio a/b in four-layer, cross-ply, symmetric $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ rectangular plates resting on elastic foundations. Once again, the shear stress σ_5 is decreasing with the increase of the ratios a/h and a/b and with the increase of the foundation parameters.



(a) σ_5 versus a/h (b) σ_5 versus a/b

Fig. 7 Variation of $\sigma_5(0)$ in a $(0^{\circ}/90^{\circ}/90^{\circ}/0^{\circ})$ plate for different elastic foundations



(a) σ_6 versus a/h (b) σ_6 versus a/b

Fig. 8 Variation of $\sigma_6(-0.5)$ in a $(0^\circ/90^\circ/90^\circ/0^\circ)$ plate for different elastic foundations

Finally, Fig. 8 shows the variation of the in-plane tangential stress $\sigma_6(-0.5)$ versus the side-to-thickness ratio a/h andthe aspect ratio a/b in four-layer, cross-ply, symmetric $(0^\circ/90^\circ/90^\circ/0^\circ)$ rectangular plates resting on elastic foundations $(a = 4h)$. It is to be noted that the stress σ_6 decreases by increasing the ratio a/h and the foundation parameters κ_W and κ_P. Also, σ_6 is no longer increasing with the increase of the aspect ratio a/b and has its maximum for different aspect ratios depending on the foundation parameters κ_W and κ_P.

6. Conclusions

The laminated composite plates resting on elastic foundations are described and discussed using two models (HSDT1 and HSDT2). Closed-form solutions for bending analysis of cross-ply orthotropic laminated plates are developed on the assumption that the transverse shear displacement varies as a hyperbolic function across the thickness of the plates. For symmetric and anti-symmetric cross-ply laminated rectangular plates, the equilibrium equations and associated boundary conditions are obtained by the principle of virtual displacements. Navier's method is used to obtain analytical solutions for the laminated plate subjected to simply-supported boundary conditions. In order to verify the accuracy of the proposed models, numerical results are compared with other theories models given in the literature and a good agreement is found to exist between them. The interaction between the plate and the foundations is included in the formulations. The results of our calculations for different parameters a/h, a/b, κ_W, and κ_P are investigated. The subject of this paper is important in many fields and has received wide applications in modern industries. Such plate structures can be found in various kinds of industrial applications like raft foundations, storage tanks, swimming pools and in most civil engineering constructions. Also, advanced composites are used extensively in aerospace and other structural applications because of their low density, high strength and high stiffness.

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