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Optimal placement of piezoelectric actuators and sensors on a smart beam and a smart plate using multi-objective genetic algorithm

Tamara Nestorović*, Miroslav Trajkov[a] and Seyedmehdi Garmabi[b]

Ruhr-Universität Bochum, Mechanics of Adaptive Systems Universitätsstr, 150, D-44801 Bochum, Germany

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Abstract. In this paper a method of finding optimal positions for piezoelectric actuators and sensors on different structures is presented. The genetic algorithm and multi-objective genetic algorithm are selected for optimization and H_∞ norm is defined as a cost function for the optimization process. To optimize the placement concerning the selected modes simultaneously, the multi-objective genetic algorithm is used. The optimization is investigated for two different structures: a cantilever beam and a simply supported plate. Vibrating structures are controlled in a closed loop with feedback gains, which are obtained using optimal LQ control strategy. Finally, output of a structure with optimized placement is compared with the output of the structure with an arbitrary, non-optimal placement of piezoelectric patches.

Keywords: optimal location; smart structures; H_∞ norm; genetic algorithm; multi-objective genetic algorithm

1. Introduction

In recent years, many researchers in the field of vibration control have focused their investigation on implementation of active piezoelectric materials due to their numerous advantages. For instance, they response very fast to changes of circumstances and they can be flexibly used as a sensor or an actuator. They are also lightweight materials that can be embedded on different structures. Many researchers have done analytical and experimental studies concerning the actuation and vibration control of smart piezoelectric structures, as examples the contributions by Sadri *et al.* (1999), Nestorović *et al.* (2005, 2012), Kumar *et al.* (2006), Vasques *et al.* (2006), Gupta *et al.* (2010), Lee (2013), Schoeftner and Buchberger (2013), etc. can be named.

The overall procedure in active control of piezoelectric structures for vibration suppression involves several steps: model identification, controller design, simulation, experimental verification and implementation, Nestorović *et al.* (2012). In Azizi *et al.* (2009) the authors have investigated effects of different controllers on reduction of vibration. Various methods and techniques are suggested to control the system and to reduce the vibration such as: feedback

∗Corresponding author, Professor, E-mail: tamara.nestorovic@rub.de
[a] Dipl.-Ing., E-mail: miroslav.trajkov@rub.de
[b] M.Sc., E-mail: seyed.garmabi@rub.de

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control with a time delay, H_∞ controller, feedback control with augmented dynamics, filtered-X LMS algorithm, optimal controller, and Kalman estimator.

All above mentioned techniques are vital to minimize the vibration but to obtain the minimum vibration, the positions of piezoelectric patches on the structures should be optimized to have highest effect to the systems whether as actuator or sensor. Finding these positions has become a crucial issue in active vibration control. Lots of research has been done on this problem and lots of solutions have been recommended. Gupta *et al.* (2010) has made a technical review over the optimization criteria for optimal placement of piezoelectric sensors and actuators. He has overviewed the results of different criteria such as: maximizing modal forces/moments, maximizing deflection of the host structure, minimizing control effort, maximizing degree of controllability and observability, minimizing spill-over effects.

Barbol *et al.* (2000) based on the modal approach, have optimized the geometrical conditions of several cases of active beams with different boundary conditions. Bruant and Proslier (2005) have proposed a modified optimization criterion under consideration of the spill-over effects. Performance of an optimal LQR controller was used by Kumar and Narayanan (2008) as optimization criterion for finding optimal actuator/sensor positions for piezoelectric beams. Liu and Hu (2011) have introduced a modal correlation criterion for the eigenmodes. They have applied the controllability and observability Gramians as optimization criteria. Liu (2004) has used efficiency indices based on the mode shapes of a clamped piezoelectric beam.

Similar investigations have been performed also for plate structures. Finding optimal positions for piezoelectric patches on a plate structure is more cumbersome than finding positions on a beam, since all previous investigations should be executed in two directions of a coordinate system. For optimization process usually classical gradient based algorithms are used. The gradient based algorithms are able to find an optimum next to the selected starting point, which is normally not the global one if a lot of local optima exist. In this case the behavior of the objective function has to be investigated carefully and probably several starting points have to be calculated in order to find a global optimum. Unfortunately, there is no general simple proof available to prove that the optimum has been really found.

To overcome this problem genetic algorithm (GA) has been proposed in several papers. Bruant *et al.* (2010) have applied their modified optimization criteria using GA considering residual modes and controllability and observability of the structure. Similar criteria were used by (Han 1999) also in combination with GA and tested with a plate structure clamped along one edge. Kumar and Narayanan (2007) have applied their LQR controller based criteria to find optimal location of piezoelectric actuators/sensors for vibration control of plates using GA for solving a zero-one optimization problem. Peng *et al.* (2005) involved maximizing of the controllability Gramian as the optimization criterion for optimal placement on a clamped plate using GA. Similar approach with modal controllability and observability Gramians and GA was also used by Sadri *et al.* (1999).

The placement problem grows when not only optimization based on one mode, but also based on several simultaneously considered modes is demanded. Nestorović and Trajkov (2013) have used placement indices based on H_2 and H_∞ norms, as well as controllability and observability Gramians, to optimize the locations of piezoelectric actuators and sensors on a beam and a plate. They have revealed their results to optimize positions of piezoelectric patches regarding one eigenmode individually and also they have considered some eigenmodes simultaneously. They have observed that the best location of piezoelectric patches for one eigenmode can be improper or the worst place regarding the other eigenmodes. Hence, they found that the best way is to consider some eigenmodes simultaneously with giving acceptance tolerance and weighting to some of them.

In this paper, similarly like in Nestorović and Trajkov (2013), the H_∞ norm is selected as a criterion or fitness function for optimization process but instead of using placement indices, the GA and multi-objective GA has been used. The advantage of the GA over some other placement methods can be seen in the fact, that there is no need to specify in advance limited number of predefined places of piezoelectric patches and in multi-objective GA easily as many as needed objectives can be optimized simultaneously (in our case norms of different modes can be our objectives).

The structure of the paper is organized in the following way. In this study GA's commands of Global Optimization Toolbox of MATLAB are utilized. To clarify the used parameters in those commands, section 2 is devoted to a general description of genetic algorithm. In the next section, the multi-objective genetic algorithm is reviewed since the optimization will be performed not only for one mode. The multi-objective genetic algorithm makes it possible to find optima of a problem with multiple objectives.

In the following section, the controllability and observability Gramians are introduced as criteria for the comparison purpose i.e., to find out which system is more controllable and more observable. This objective function for optimal placement is introduced and approximately determined in terms of the H_∞ norm. Subsequent section contains the overall procedure of optimal placement in this paper. It includes the results of optimization of different host structures of piezoelectric patches. These host structures are: a cantilever beam and a simply supported plate. At last, the outputs of different systems are depicted to show how well piezo-patches are placed on the structure. A common trend of the compared results documents the reliability of the proposed approach.

2. Genetic algorithm

For very complicated cases, which cannot be optimized with other numeric optimization strategies, the evolution strategies are good choice of design optimization. The evolution strategies are slow and they have convergence problems. But they are well suited for complex situations: multi-modal problems that do not have only one single local optimum, problems where the objective function or the constraints or parts of constraints are not differentiable, problems with discontinuous solution spaces with local optima, etc. This strategy is a member of the down-hill-climbing methods since we are considering minimization as a goal of an optimization process (Preumont 2001).

One genetic algorithm consists of three parts:

1. **Chromosomes** (Individuals) are selected from the solution space. They can be optima or non-optima but generally they are result of a problem. In GA the chromosomes are built of genes, which encode the independent variables. These codes can be Boolean, integers, floating point, string variables or any combination of them. Traditionally, genes or codes are binary numbers as strings of 0s and 1s. One set of different chromosomes forms a generation.
2. **Cost or Fitness function** is a criterion to evaluate each of chromosomes.
3. **Operators** are used to create new chromosomes from old ones.

The goal of GA is to minimize fitness function. The sequences of this optimization are listed in the following steps.

2.1 Initialization of population

In this step as a first guess to find the best result, an amount of chromosomes is randomly created. The number of these chromosomes P_s, depends on the number of variables of the problem. Usually two methods for P_s are suggested (*n* is the number of variables)

$$P_s > n+1 \tag{1}$$

$$P_s = 10n \tag{2}$$

2.2 GA's Operators

With use of GA's operators, new individuals will be created. The GA's operators are: Mutation and Crossover.

2.2.1 Mutation

This is a random process where one allele of a gene is replaced by another to produce a new chromosome. This operator is mostly used to avoid the local optima but it can cause convergence problem and reduce the speed of convergence. Mutation is randomly applied with low probability in range of 0.001 and 0.01 (Deb 2000).

The mutation process is used when the feasible area of the problem is non-continuous. When the feasible area is continuous, one continuous mutation operator should be used.

2.2.2 Crossover

This operator uses an exploitation method to create new individuals, where with combining of two chromosomes of current population it creates new chromosomes. This combination is done by replacement of genes of parent chromosomes (Fig. 2). This operator is important to reach the optimum point more quickly, therefore it executed with high probability in range of 0.5 to 0.9 (Wehrens 1998).

2.3 Termination factor

Two methods for termination of optimization iteration are introduced (Deb 2001):

- After reaching the specified number of generations.
- When not a big improvement for last N generations is observed.

An overview of a genetic algorithm is represented by the flowchart in Fig.3.

Point Mutation: 11101001000 ⟶ 11101011000

Fig. 1 Sample of mutation in binary representation

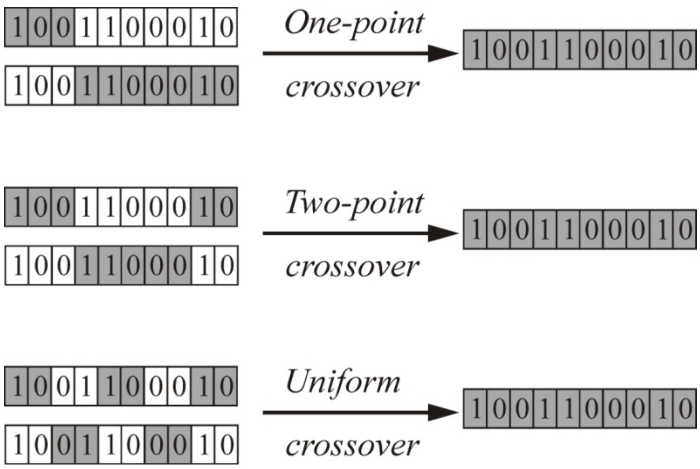

Fig. 2 Three kinds of crossover operator

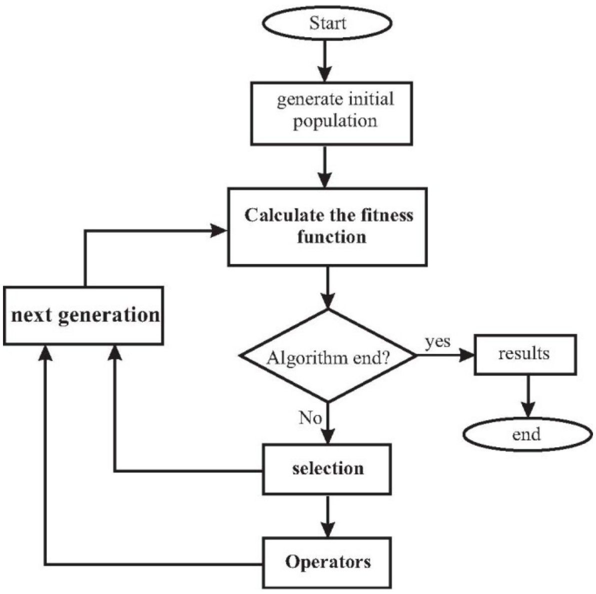

Fig. 3 General scheme of a genetic algorithm or any evolutionary algorithms

3. Multi-objective genetic algorithm

In many complex engineering problems simultaneous optimization of some objective functions is required, while optimizing one objective can cause an unaccepted result from other objectives. Traditionally GA was defined to solve one objective problem but lots of methods are developed to increase its capability. Generally there are two approaches for multi-objective optimization: Weighted sum method and Pareto optimal. In this paper, the Pareto optimal is used.

3.1 Pareto based approach

In many real-life problems, objectives are in conflict with each other. Hence, optimization with respect to one objective can cause unacceptable result for other objectives but a perfect multi-objective solution that simultaneously optimizes all objective functions is almost impossible. A reasonable solution to a multi-objective problem is to find a set of solutions, where each of them satisfies the objectives at an acceptable level without being dominated by any other solution.

To define a Pareto optimal, domination should be defined at first. A vector **v** dominates vector **u** if

$$\begin{aligned} &\forall i \in \{1,2,...,k\} : f_i(\mathbf{v}) \le f_i(\mathbf{u}); \\ &\exists\, j \in \{1,2,...,k\} : f_j(\mathbf{v}) < f_j(\mathbf{u}) \end{aligned} \tag{3}$$

A vector $\boldsymbol{x} \in S$ (S: Solution space) is a Pareto optimal solution, if and only if there would be no vector like $\boldsymbol{y} \in S$, where $f(\boldsymbol{y}) = (f_1(\boldsymbol{y}), ..., f_k(\boldsymbol{y}))$ dominates $f(\boldsymbol{x}) = (f_1(\boldsymbol{x}), ..., f_k(\boldsymbol{x}))$ (Popov 2005).

Finding the non-dominated set of solutions from a given set of solutions is similar in principle to finding the minimum of a set of a real numbers. A Pareto optimal cannot be improved without worsening at least one other objective. The set of all feasible non-dominated solutions in S is referred to as the Pareto optimal set, and for a given Pareto optimal set, the corresponding objective function values in the objective space are called the Pareto front (Konak 2005).

In Fig. 4 the Pareto optimal set for continuous curve with two objectives is depicted where the task is to minimize the both objectives. The solid curve marks the Pareto optimal solution set (Deb 2001).

4. Controllability and observability

Controllability and observability are structural properties that carry useful information for testing and control.

Definition 1: System states given by state equation are controllable if it is possible by admissible inputs to steer the states from any initial value to any final value within some time window. Observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs.

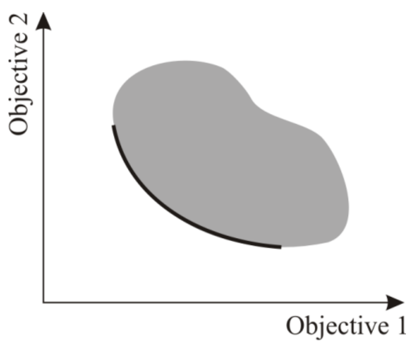

Fig. 4 Pareto optimal solution marked with continuous curve

Definition 2: A structure is controllable if the installed actuators excite all its structural modes. It is observable if the installed sensors detect the motions of all the modes (Gawronski 2004).

4.1 Continuous-time systems

A linear time invariant system (**A, B, C**) with *s* inputs is completely controllable when the controllability matrix

$$\mathbf{S}_{co} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \cdots & \mathbf{A}^{N-1}\mathbf{B} \end{bmatrix} \tag{4}$$

has rank N (N is number of states). And the system is completely observable if the observability matrix has rank N (Gawronski 2004).

$$\mathbf{S}_{ob} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{N-1} \end{bmatrix} \tag{5}$$

The above criteria are simple but they have two disadvantages: first that they answer the controllability and observability question in *yes* or *no* terms, and second they are not useful for a system of big dimensions, since it causes numeric problems and enlarge the calculation time. Gramians are the alternative approach for determining the system properties. Gramians are nonnegative matrices which can express the controllability and observability in qualitative form and they are free of the numerical difficulties. The controllability and observability Gramians are

$$\mathbf{W}_c(\mathrm{t}) = \int_0^{\mathrm{t}} \mathrm{e}^{\mathbf{A}t}\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathrm{e}^{\mathbf{A}^{\mathrm{T}}t}\mathrm{dt} \tag{6}$$

$$\mathbf{W}_o(\mathrm{t}) = \int_0^{\mathrm{t}} \mathrm{e}^{\mathbf{A}\mathrm{t}}\mathbf{C}^{\mathrm{T}}\mathbf{C}\,\mathrm{e}^{\mathbf{A}^{\mathrm{T}}\mathrm{t}}\mathrm{dt} \tag{7}$$

For a stable system the Gramians can be obtained by (6) and (7), which is called Lyapunov equations

$$\begin{gathered} \mathbf{AW}_c + \mathbf{W}_c\mathbf{A}^{\mathrm{T}} + \mathbf{BB}^{\mathrm{T}} = 0 \\ \mathbf{A}^{\mathrm{T}}\mathbf{W}_o + \mathbf{W}_o\mathbf{A} + \mathbf{C}^{\mathrm{T}}\mathbf{C} = 0 \end{gathered} \tag{8}$$

The eigenvalues of the product of Gramians are independent of coordinate transformation and can be denoted as

$$\gamma_i = \sqrt{\lambda_i(\mathbf{W}_c\mathbf{W}_o)} \tag{9}$$

They are referred to as the Hankel singular values of the system (Gawronski 2004).

4.2 Controllability and observability of a structural modal model

The modal state-space representation of flexible structures has specific controllability and observability properties, and its Gramians are of a specific form.

When the damping is small, the Gramians in modal coordinates are diagonally dominant and by using appropriate scaling they are approximately equal.

$$\begin{aligned} \mathbf{W}_c &\cong diag(w_{ci}\mathbf{I}_2) \\ \mathbf{W}_o &\cong diag(w_{oi}\mathbf{I}_2);\ \ i=1,...,n \end{aligned} \tag{10}$$

Therefore, the approximated Hankel singular values are obtained as a geometric mean of the modal controllability and observability factors.

$$\gamma_i \cong \sqrt{w_{cii}w_{oii}} \tag{11}$$

For flexible structures the Gramians of each mode can be expressed in a closed form. This allows for their speedy determination for structures with a large number of modes, and allows for the insight into the Gramian physical interpretation. In modal coordinates the diagonal entries of the controllability and observability Gramians are in form

$$w_{ci} = \frac{\|B_{mi}\|_2^2}{4\zeta_i\omega_i}, \quad w_{oi} = \frac{\|C_{mi}\|_2^2}{4\zeta_i\omega_i} \tag{12}$$

where ζ and ω are damping ratio and eigenfrequency of related mode *i*, respectively. Therefore, the approximated Hankel singular values will read as (Gawronski 2004)

$$\gamma_i = \frac{\|B_{mi}\|_2\|C_{mi}\|_2}{4\zeta_i\omega_i} \tag{13}$$

The form of (13) is similar to norm of a system. System norm serves as a measure of intensity of its response to standard excitations such as unit impulse or white noise of unit standard deviation. Typical system norms are: H_2, H_∞, and Hankel. In this work H_∞ is used, which is defined as

$$\mathrm{H}_\infty = \|G\|_\infty = \sup_{u(t)\neq 0}\frac{\|y(t)\|_2}{\|u(t)\|_2} \tag{14}$$

where $y(t)$ and $u(t)$ are output and input of system in domain of time. It can alternatively be written as

$$\|G\|_\infty = \max_\omega \sigma_{\max}(G(\omega)) \tag{15}$$

where $G(\omega) = C(j\omega\mathbf{I}-\mathbf{A})^{-1}$ is the transfer function of a system and $\sigma_{\max}(G(\omega))$ is the largest singular value of *G*. The peak of the transfer function magnitude is the H_∞ norm of a single-input-single-output (SISO) system.

The H_∞ norm in modal coordinate for each mode is expressed as following

$$\|G\|_\infty = \sigma_{\max}(G_i(\omega_i)) = \frac{\sigma_{\max}(C_{mi}B_{mi}))}{2\zeta_i\omega_i} = \frac{\|B_{mi}\|_2\|C_{mi}\|_2}{2\zeta_i\omega_i} \tag{16}$$

The acquired equation is similar to (13), hence it is a good criterion for measurement of controllability and observability of a system.

5. Application of optimal placement

In this section the results of placement optimization procedure for two different types of host structures with piezoelectric patches are presented. The host structures are cantilever beam and simply supported plate. For comparison purposes, the responses of the structures with an optimized and arbitrary patch positions are presented.

In Fig. 5 the general algorithm, which has been used in this work to find optimal places of piezoelectric patches, is depicted.

Firstly the input file of commercial FE software ANSYS is written to model the host structure and piezoelectric patches in a way that the locations of piezo-patches are introduced parametrically without determining specific value for them. The properties of the beam, plate and piezoelectric materials are inserted according to Table 1 and Table 2. In these FE analyses the free mesh and the mapped mesh generation are selected for the beam and the plate, respectively. The Block Lanczos algorithm is chosen to execute the modal analysis. Through the modal analysis the eigenfrequencies and eigenvectors of the first four modes of the structure are obtained, which are used in the reduced-order model.

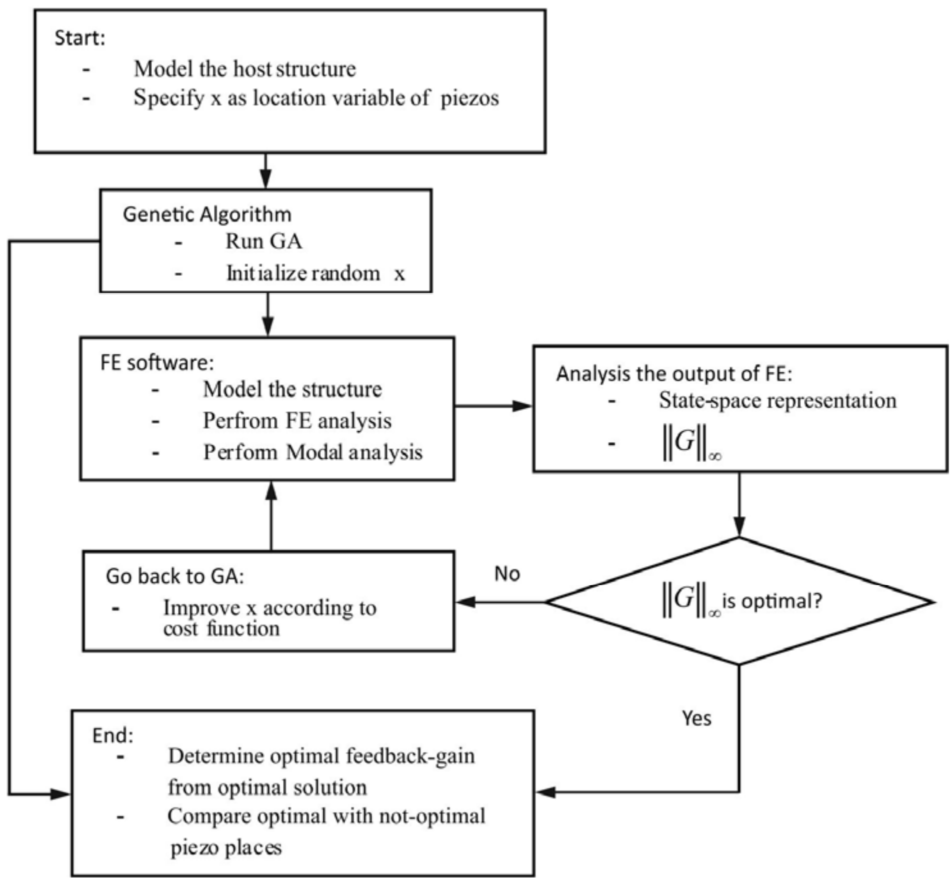

Fig. 5 Flow-chart of the optimization procedure applied throughout this work

Table 1 Material properties of the host structures

Element type in Ansys		Solid45
Elastic modulus	E	$0.7\times10^{11}\ N/m^2$
Poisson's ratio	υ	0.33
Density	ρ	$2800\quad kg/m^3$

Table 2 Material properties of the piezoelectric material

Element type in Ansys		Solid5
Stiffness matrix	C	$\begin{bmatrix} 12.29\times10^{10} & 7.66\times10^{10} & 7.02\times10^{10} & 0 & 0 & 0 \\ 7.66\times10^{10} & 12.29\times10^{10} & 7.02\times10^{10} & 0 & 0 & 0 \\ 7.02\times10^{10} & 7.02\times10^{10} & 9.71\times10^{10} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.32\times10^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.32\times10^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.32\times10^{10} \end{bmatrix}$
Piezoelectric constant matrix	d	$\begin{bmatrix} 0 & 0 & 7.07 \\ 0 & 0 & 7.07 \\ 0 & 0 & -13.82 \\ 0 & 0 & 0 \\ 0 & -11.91 & 0 \\ -11.91 & 0 & 0 \end{bmatrix}$
Permittivity matrix	ε	$\begin{bmatrix} 929 & 0 & 0 \\ 0 & 929 & 0 \\ 0 & 0 & 857 \end{bmatrix}$

In next step, which is starting point of the optimization loop, the written code in MATLAB opens the input-file, specifies randomly the value for locations of piezoelectric patches, feeds the completed input-file to ANSYS, and reads the related outputs. Based on outputs, the further calculations will be performed to build the state-space matrices and the cost function $\|G\|_\infty^{sys}$. The GA will compare this result with $\|G\|_\infty^{fully}$ of fully controllable and observable system. The fully controllable and observable system is a system in which the structure is completely covered with piezoelectric actuators and sensors on both sides. The $\|G\|_\infty^{fully}$ of different structures were initially calculated before running the whole optimization procedure. The GA will try to reduce and minimize $\|G\|_\infty^{diff}$, which is the difference between current system and fully controllable and observable system.

$$\|G\|_\infty^{diff} = \left| \|G\|_\infty^{sys} - \|G\|_\infty^{fully} \right| \tag{17}$$

This procedure will be continued till reaching one of the termination factors explained in section 2.3 of this paper. At initiation of the optimization procedure it is assumed that our optimization problem would converge before reaching the specified number of generations as one of the termination factors.

Finally to validate the results, extra MATLAB code is written to perform the optimal control analysis and to find the feedback gain matrix of optimized cases. The feedback gain, as it can be seen in Fig. 6, is used to compare the output (deflection) of the system in two cases. In optimized case, the optimal places of piezoelectric patches are obtained from proposed optimization procedure. In not-optimized case, the places of patches are selected randomly without any optimization.

5.1 Optimal locations of four piezo-patches on a cantilever beam

In this section the results of the placement optimization are presented for a cantilever beam with four piezoelectric patches. The optimization procedure is first performed based on the first mode of the system, which is the most important mode, and then for first four modes of the structure by implementation of multi objective genetic algorithm. The schematic of the cantilever beam is depicted in Fig. 7, where the left side of the beam is completely clamped to prevent movements in x, y, and z directions. Geometrical parameters of the investigated beam are given in Table 3.

5.1.1 Optimization based on first mode

We consider the total number of four patches, where it is assumed that two of them act as sensors and other two as actuators. The corresponding sensor and actuator patches have the same location regarding the coordinate in the length direction of the beam, but they are placed on opposite sides (collocated actuators and sensors). With regard to stability and spill-over issues, it has been shown in literature, e.g., Collet (2001), that the collocated actuators and sensors are advantageous over the non-collocated ones.

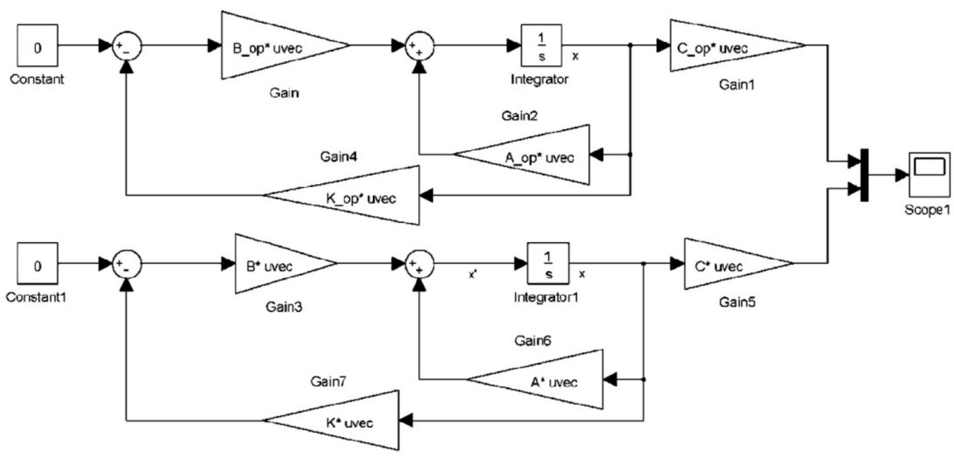

Fig. 6 A block diagram for comparison of optimized case with not-optimized case

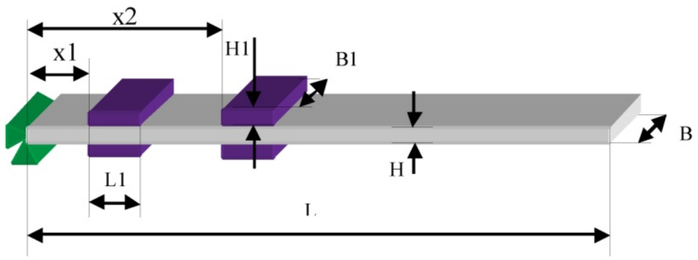

Fig. 7 Schematic of the cantilever beam with embedded piezoelectric patches on its both sides

Table 3 Geometrical parameters of the cantilever beam

Length of beam	L	0.44 m
Width of beam	B	0.04 m
Height of beam	H	0.0028 m
Length of patch	L1	0.05 m
Width of patch	B1	0.03 m
Height of patch	H1	0.0005 m

Casciati *et al.* (2006) have made extensive research on advantages and disadvantages of collocated and non-collocated systems. The most advantage of collocated systems relies on their robustness. In collocated systems the spill-over effects due to modal reduction can be avoided, since they are inherently stable. The other advantages of collocated systems are their performance, realization aspects, simplicity, and economical aspects.

In our study, we assume collocated actuator/sensor pairs. On one hand, with this assumption the complexity of the optimization procedure is reduced, since the coordinates x_1 and x_2 unambiguously define the location of the patches. On the other hand, the advantages of the collocated patches represent another motivation for this choice. We have also assumed that the precision of the placement can be achieved to a satisfactory extent on a real model as well. The justification of this assumption can be seen in the fact, that the piezoelectric patches cover a wider surface and are not strictly related to particular points, as related to the nodes with electric degrees of freedom in the FE simulation. Therefore small geometrical tolerances of the structure and the piezoelectric material can be compensated by the influence of the piezoelectric material covering relatively larger surface of the structure at prescribed locations. For the worst case scenarios of resonant states caused by periodic excitations with frequencies close to the eigenfrequencies of the structure, the control law has to incorporate information about the periodic excitations and compensates in turn for possible spill-over effects. Such cases are in the focus of interest in the active vibration reduction problems.

In this case the genetic algorithm is run with population size of 20 in 100 generations, where the MATLAB code is written in a way to have optimal locations with accuracy of 0.1 mm (maximum number of 4 decimals.) As the Fig. 8 shows, the optimization is converged after 11 generations. In this figure the best fitness is referring to the population with best fitness value, Eq. 17, in corresponding generation and mean fitness means, average value of fitness of all populations in related generation.

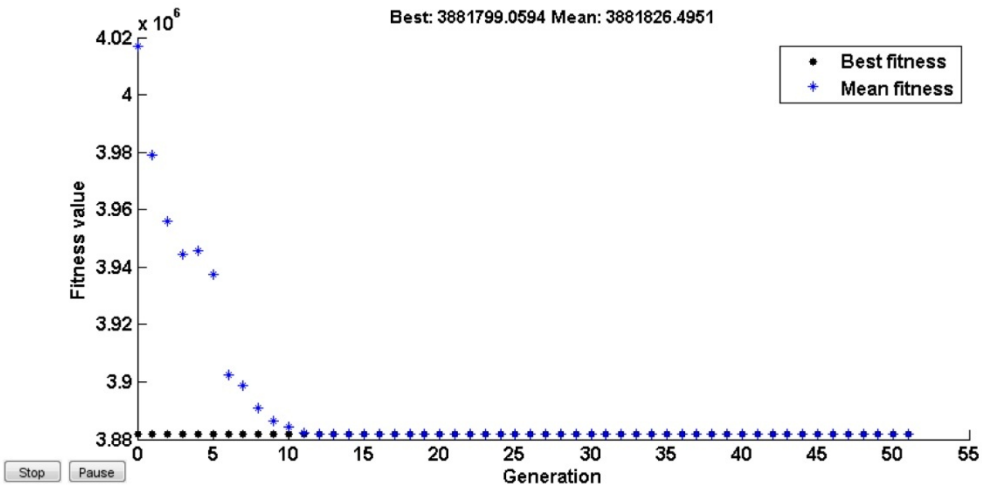

Fig. 8 The genetic algorithm process for a cantilever beam with four patches (Pop.=20, Gen.=100)

After 11 generations the genetic algorithm is converged resulting in best values for piezoelectric locations at x_1 = 0.01; x_2 = 0.07. These values are close to the root of the structure (clamped part) as it has also been reported in the literature, e.g., Crawley and de Luis (1986).

In order to demonstrate the performance of the optimization procedure, an optimal LQ feedback control system is designed and the system outputs are compared according to block-diagram in Fig. 6 for optimized and non-optimized case. In addition the performance of the controller is shown by comparison of the outputs for the cases with and without controller.

We have first considered the case which corresponds to free vibrations of the beam, without external excitation, actually vibrations due to initial condition. Initial condition for the states in the reduced state-space model of the beam can be related to initial displacement of the tip of the beam. The outputs of the sensor-patches are observed – voltages as function of time (in seconds) on the patch #1 (at distance x_1 from the clamped end of the beam) and on the patch #2 at distance x_2. Comparison of the results for optimal placement and an arbitrarily selected non-optimal placement is represented in Fig. 9. Fig.10 represents the comparison of the output voltages on the sensor patches #1 and #2 for uncontrolled system with optimal placement (dashed line), and for the closed loop system with optimal LQ feedback-gain controller, designed based on the model with optimal placement of collocated actuator-sensor patches (solid line).

Influence of the periodic excitation has also been investigated and demonstrated for optimized and non-optimized placement. Harmonic (sinusoidal) voltage excitation with amplitude 100V and frequency 10.38 Hz, corresponding to the first eigenfrequency of the beam for given placement, acts on both actuator patches. Fig. 11 represents controlled responses of corresponding output sensor patches during simulation time of 5 seconds for optimized and an arbitrary placement of patches. Similarly as in the previous case of the initial condition vibrations, the controlled and uncontrolled behavior for the system with optimal placement is compared in Fig. 12, but now in the presence of mentioned harmonic excitation.

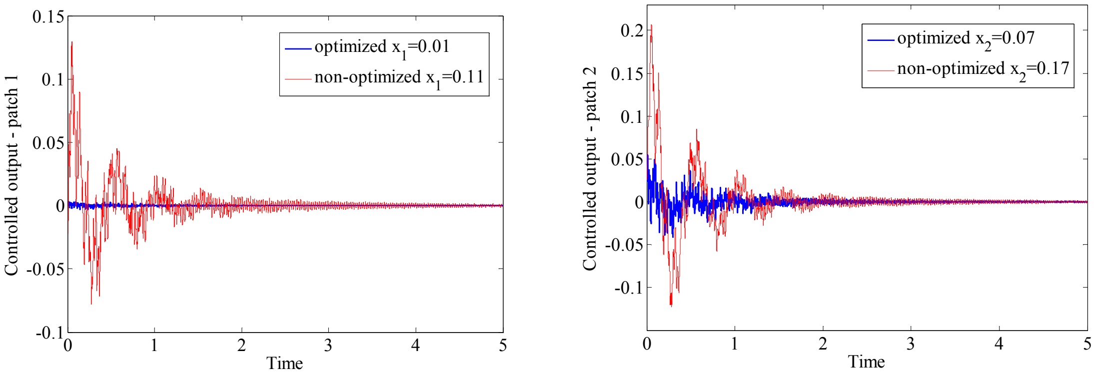

Fig. 9 Output voltages at sensor patches #1 and #2 for optimal and non-optimal placement (vibrations due to initial condition)

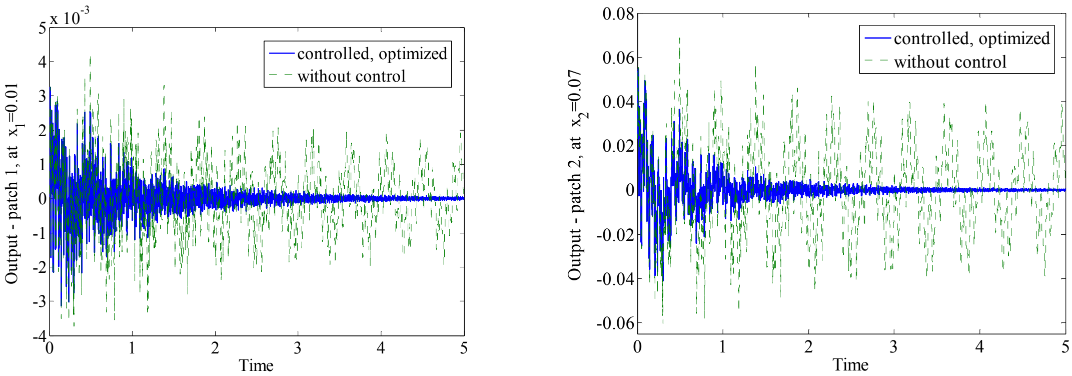

Fig. 10 Output voltages at sensor patches #1 and #2 for controlled and uncontrolled vibrations due to initial condition

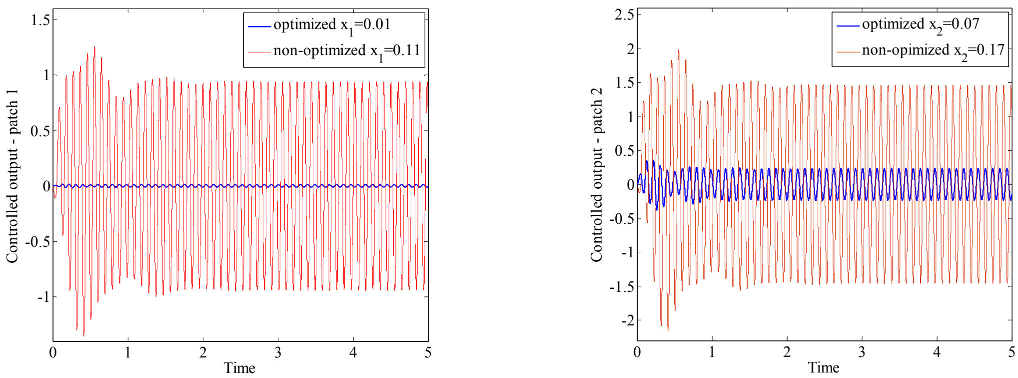

Fig. 11 Output voltages at sensor patches #1 and #2 for optimal and non-optimal placement (vibrations due to sinusoidal excitation with frequency corresponding to the first eigenfrequency of the beam)

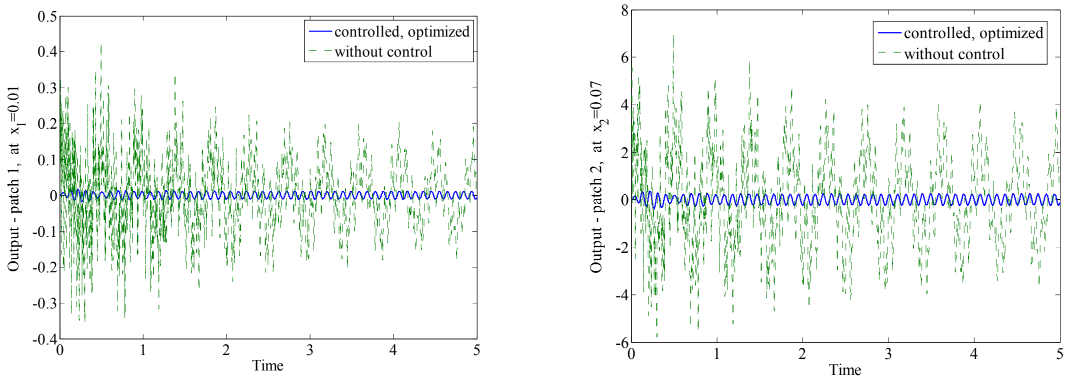

Fig. 12 Output voltages at sensor patches #1 and #2 for controlled and uncontrolled vibrations due to sinusoidal excitation with frequency corresponding to the first eigenfrequency of the beam

5.1.2 Optimization based on first four modes

The procedure of finding optimal places of four piezo-patches on a cantilever beam regarding the first four modes of the structure is almost like the previous procedure with some small changes. In this optimization process, the multi-objective genetic algorithm has been used. The multi-objective GA has run with 40 as population size in 200 generations.

The multi-objective GA uses Pareto front approach and gives the best Pareto front. Fig. 13 shows the best Pareto front of this optimization in 2 dimensions: optimization with respect to mode 1 and mode 2. The Pareto front is not easily recognizable in Fig. 13, due to impossibility of showing a diagram in 4 dimensions. Because of mentioned problem just generating points of the Pareto front can be plotted in 2D diagram.

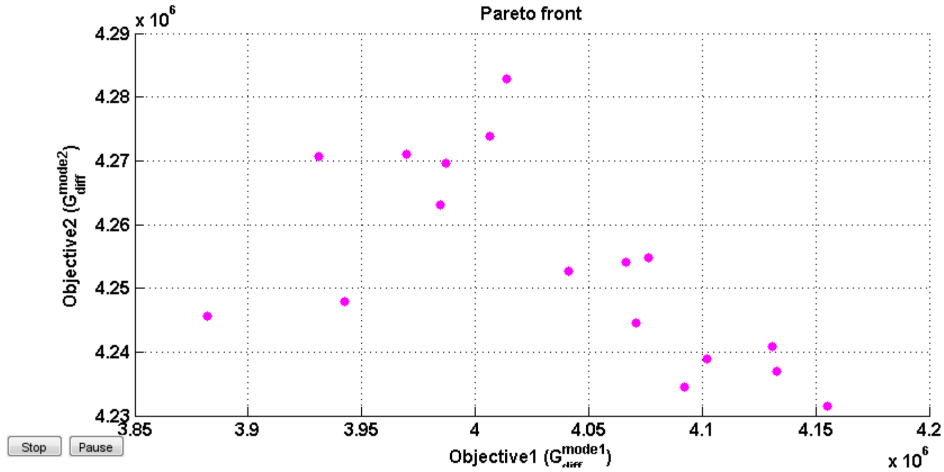

Fig. 13 The best Pareto front after 150 generations for optimization based on first four modes (Pop.=40, Gen.=200)

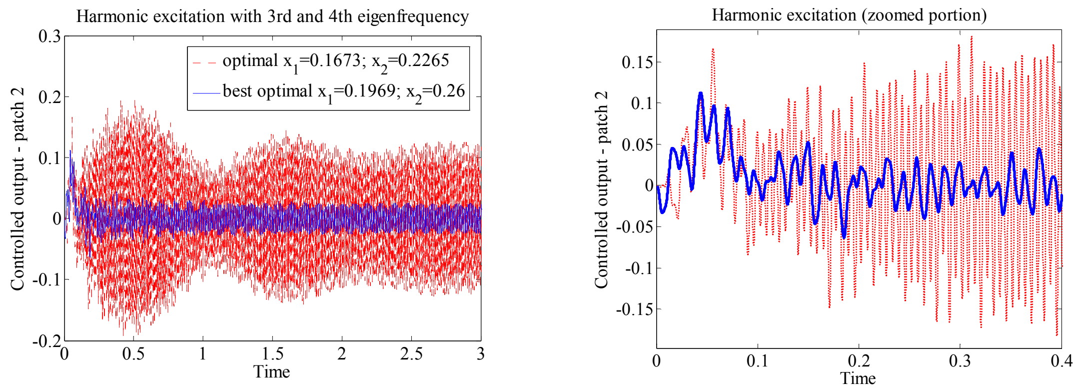

Fig. 14 Controlled response of the patch #2 due to harmonic excitation: comparison of the optimal candidate location and the best candidate location (left); zoomed portion (right)

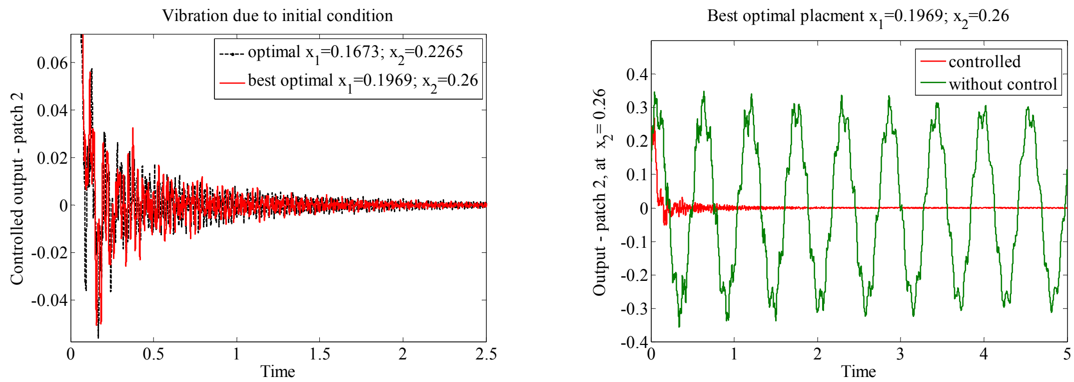

Fig. 15 Comparison of the controlled output at patch #2 for two optimal candidates (left); comparison of the controlled and uncontrolled output signal at sensor-patch #2 (right)

The optimization procedure has resulted in 14 good places – candidates for optimal locations of the two piezo-patches with respect to four modes regarding their infinity norms. Based on the free vibration response (due to initial condition) it is difficult to judge about only one optimal location, since the controlled outputs do not differ significantly (see Fig. 15).

Unlike the free vibration response, the response to harmonic excitation involving several modes of interest, is stronger influenced by the actuator/sensor placement. By comparing the responses of the controlled systems with placement corresponding to 14 optimal candidates, it has been concluded that the values x_1=0.1969; x_2=0.2600 represent best position for piezoelectric pairs. The controlled response of the sensor-patch #2 is represented in Fig. 14 (left) as comparison of best location and another optimal candidate location. Zoomed portion of the response is represented in Fig. 14 (right). Excitation is periodic, composed of the sum of sinusoidal signals with amplitude 100 V and frequencies corresponding to the 3rd (160.19 Hz) and the 4th (205.61Hz) eigenfrequency of the beam, for considered optimized placement.

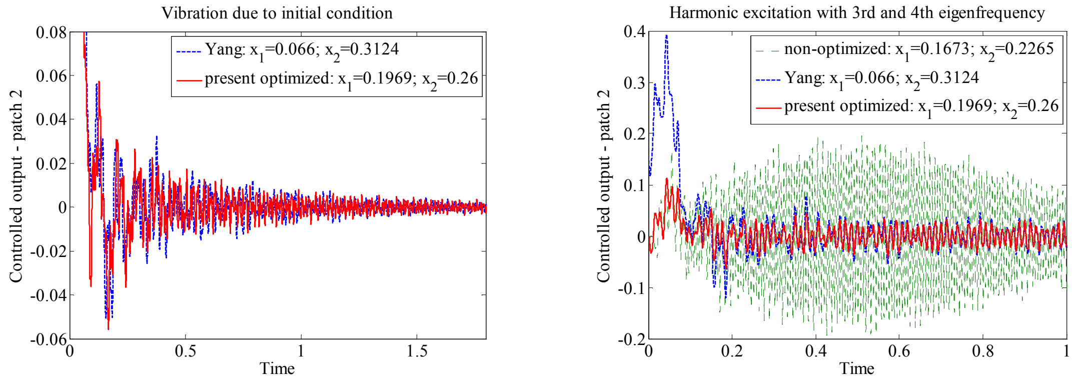

Fig. 16 Comparison of the calculated best locations with Yang's results for a collocated sensor-actuator on a cantilever beam regarding first four modes

Comparison of the controlled responses of the patch #2 for one optimal candidate location and the best location in case of free vibration ("initial condition" type) is represented in Fig. 15 (left). The results are obtained for the closed-loop optimal LQ feedback control system. Fig. 15 (right) represents comparison of the controlled and uncontrolled sensor signal at patch #2.

For the same scenario Yang *et al.* (2005) have performed the investigations and found that the first collocated sensor-actuator pair should be embedded at 0.15 times the beam length and the second one at 0.71 times the beam length, measured from the beam root. If we apply these values to our beam, we will obtain: x1_Yang=0.066; x2_Yang=0.3124. Fig. 16 represents the comparison of the controlled output signals at sensor patch #2 for two types of excitation: "initial condition" type (left) and harmonic composed of sinusoids with amplitude 100V and frequencies equal to the 3rd (160.19 Hz) and the 4th (205.61 Hz) eigenfrequency of the beam, for considered optimized placement (right).

5.2 Optimal placement of four piezoelectric patches on a plate

The structure under consideration is a plate, clamped along all four edges. In this approach, the same procedure is followed as for the cantilever beam but now there are four design variables x_1, x_2, x_3, and x_4, which are defined as in Fig. 17.

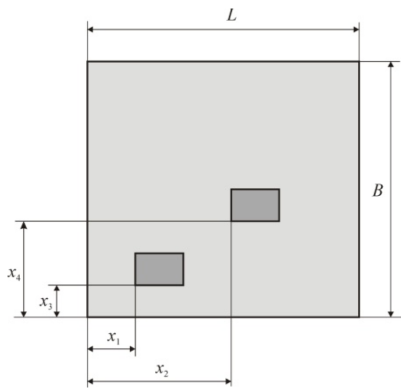

Fig. 17 A schematic of the plate L = B = 0.44 m

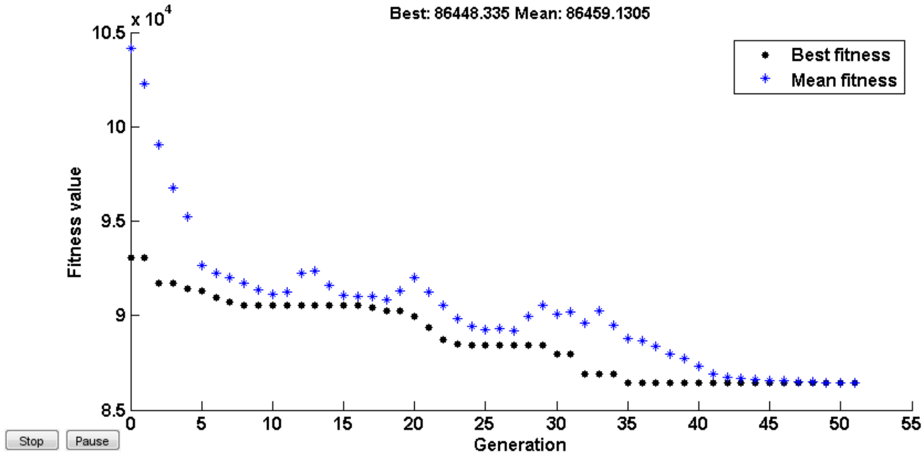

Fig. 18 The GA performance to find optimal places of piezos on a plate regarding its first mode (Pop.=40, Gen.=150)

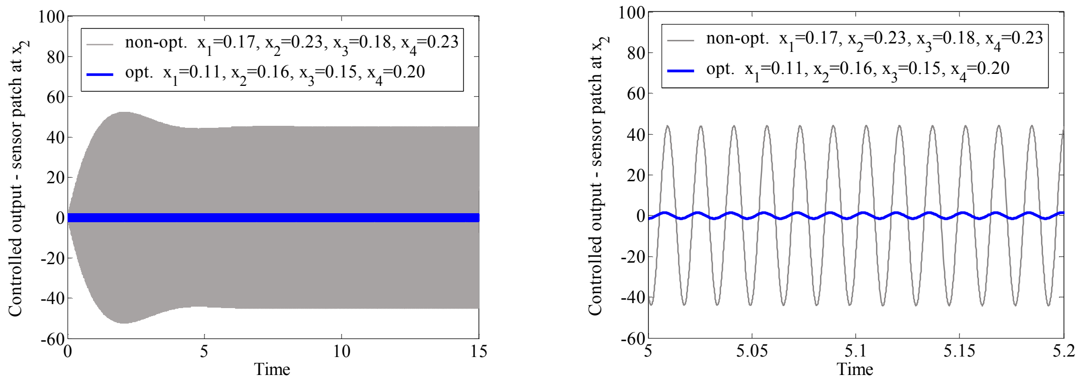

Fig. 19 Comparison of controlled outputs at the sensor patch placed at x2 for optimal and non-optimal positions (left) and zoomed portion of the diagram (right)

5.2.1 Optimal places of piezo-patches for plate regarding first mode of the structure

After defining the geometry and material properties of the structure (the same as represented in Tables 1 and 2), the optimization of the placement for piezoelectric actuators and sensors is performed. The GA is used again to find the optimal places. The GA was run for population size 40 in 150 generations and it has converged to best result after 52 generations (Fig. 18).

Similarly as in the case of placement optimization for the beam, the efficiency of the optimization can be best observed in the presence of periodic excitations. At the same time such excitation can be considered as the worst case due to possibility of resonance with very high vibration amplitudes. For the optimization performed based on the first eigenmode of the plate, the periodic sinusoidal excitation with the amplitude 100V and frequency corresponding to the first eigenfrequency of the plate (125.48 HZ) is applied to both top actuator patches. Comparison of the controlled outputs (using optimal LQ controller) at the second bottom patch (placed at coordinate x_2) is presented in Fig. 19 for the optimal and one arbitrarily selected non-optimal placement. Fig. 20 presents the comparison of the uncontrolled and controlled output at the sensor patch placed at x_2 for optimal placement shown in Fig. 19.

5.2.2 Optimal places of piezo-patches regarding first four modes of the structure

For parallel consideration of the four eigenmodes, the multi-objective GA was executed with the 40 population size in 150 generations. This optimization process results in final Pareto front which is depicted in Fig. 21. In a similar way as for the beam, for best placement the following positions of the patches are obtained: $x_1 = 0.19$, $x_2 = 0.25$, $x_3 = 0.19$, $x_4 = 0.23$.

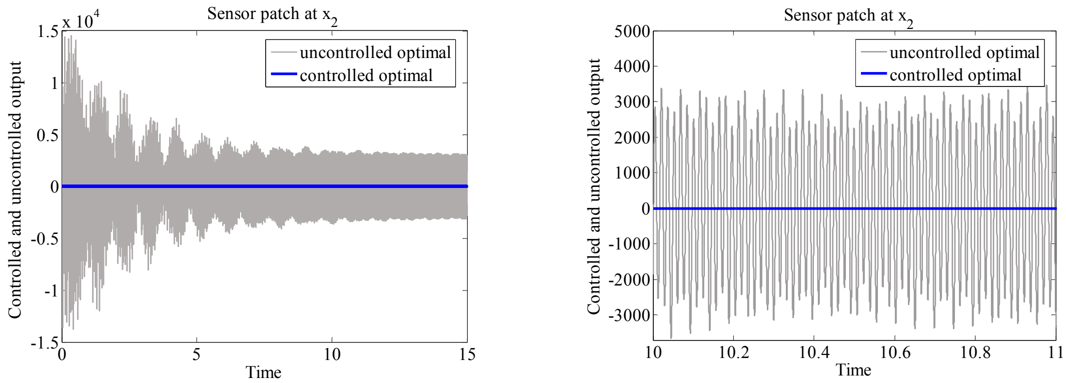

Fig. 20 Comparison of the uncontrolled and controlled output at the sensor patch placed at x2 for optimal placement (left) and zoomed portion of the diagram (right)

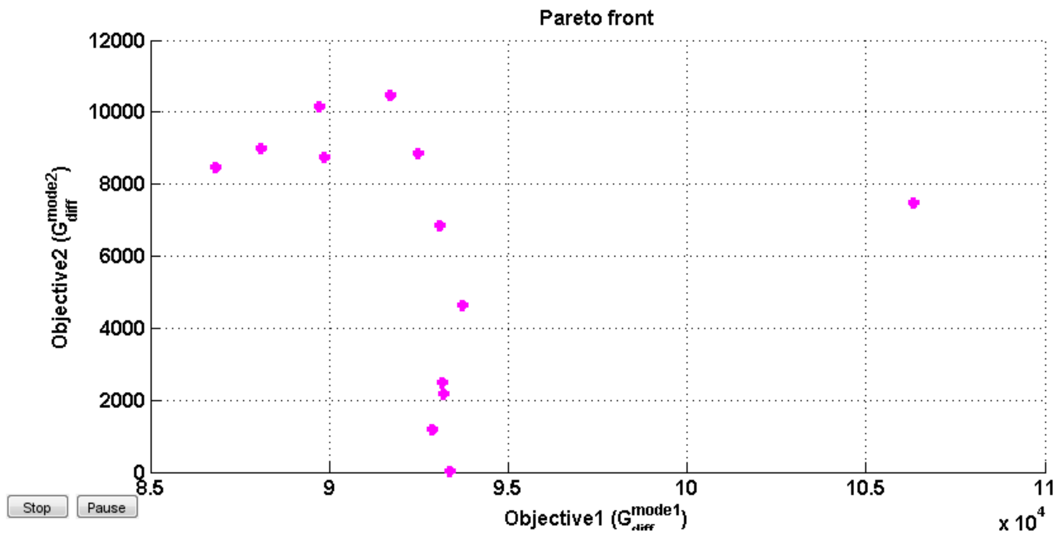

Fig. 21 The final Pareto front for consideration of four eigenmodes (Pop.=40, Gen.=150)

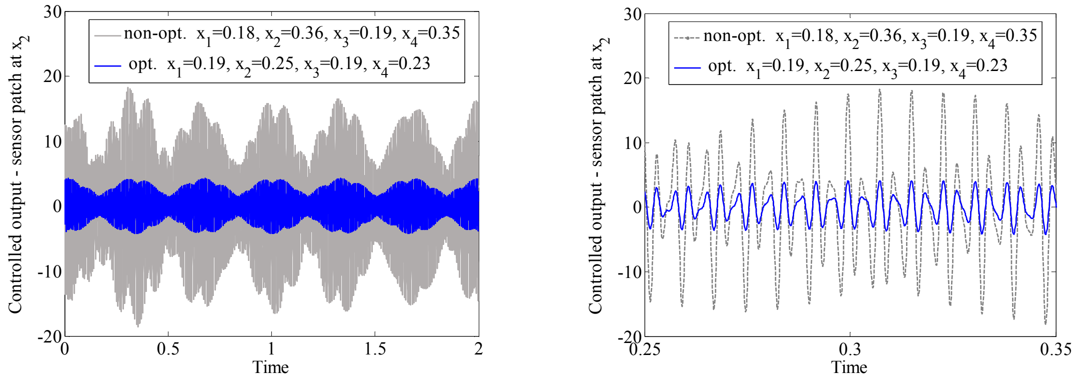

Fig. 22 Comparison of controlled outputs at the sensor patch placed at x2 for optimal and arbitrarily selected non-optimal positions (left) and zoomed portion of the diagram (right); excitation: superposition of sinusoids

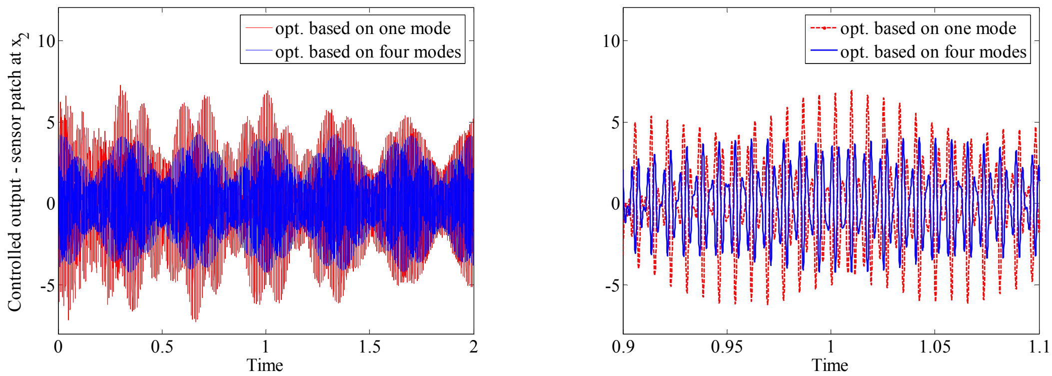

Fig. 23 Comparison of controlled outputs at the sensor patch placed at x2 for optimal placement based on one and on four modes (left) and zoomed portion of the diagram (right); excitation: superposition of sinusoids

For the optimal placement determined based on the first four eigenmodes, the placement is verified by comparison of the responses of the sensor patch placed at position x2 for optimal and arbitrary non-optimal placement. The result presented in Fig. 22 shows the response to periodic excitation obtained as superposition of sinusoids with frequencies corresponding to eigenfrequencies of the plate: second 258.22 Hz, third 261.20 Hz and fourth 384.96 Hz.

Importance of the parallel consideration of multiple eigenmodes in optimization procedure is shown in Fig. 23. Results are compared for the optimal placement based on the first four modes and optimal placement based only on one − first eigenmode. The excitation is the same as for Fig. 22. It can be observed, that the optimal placement based on four modes results in better suppression of the controlled amplitudes in comparison with optimal placement which has considered only one mode.

6. Conclusions

It is very important to find the places on the structure with the highest value of controllability and observability using limited number of piezoelectric patches. In this case, the GA has been used to find the optimal places. At first finite element analysis with use of ANSYS has been applied to obtain the eigenmodes and eigenfrequencies of the structure. Afterwards, the outputs of ANSYS have been fed into MATLAB for further analysis and building the state space realization of the system. Meanwhile a criterion, which in our case was H_∞ norm of system, has been calculated and introduced for the optimization process within GA or multi-objective GA. The use of GA or multi-objective GA depends on the optimization purpose that whether optimization should be based on one mode or more than one mode. From design point of view, it is very important beforehand to make a decision which mode shape or mode shapes are important to be controlled. The multi-objective GA gives different positions for piezo patches as optimal positions when optimization is based on more than one mode. All of them can be regarded as optima but in this work to find the best one, the outputs of the systems with optimal placements have been visually compared and which one had less amplitude of oscillation has been selected as the best.

The process has been applied to two different structures: a cantilever beam and a plate clamped along all four edges. The optimization process gives very convenient results as it can be seen from figures and in comparison with other similar works. Hence, the GA and multi-objective GA can be recommended for optimization of location of piezoelectric patches. The advantage of the GA over some other placement methods can be seen in the fact, that there is no need in advance to specify limited number of predefined places of piezoelectric patches and in multi-objective GA easily as many as needed objectives can be optimized simultaneously. In the GA, it is also possible to define some constraints regarding the position of piezo-patches since in some designs there are some predefined places where the patches cannot be embedded. It is also possible to apply this optimization process to other structures.

In this paper, the optimization is done with a limited number of piezo-patches but it is also possible to generalize the code to find optimal number of piezo-patches. Optimization of the number of pizo-patches as well as implementation of the procedure with complex geometries is a part of the ongoing research.

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